Unconstrained Optimization Test Functions

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October 15, 2004
April 28, 2005

In this work we present the algebraic expression of a number of unconstrained optimization test functions in order to be considered in different packages for testing unconstrained optimization sotware. Some of these functions are from CUTE library. Some of them have been constructed for this purpose. All these functions are in generalized or extended form. At the same time we present the standard initial point.

Extended Freudenstein & Roth Function

$$f(x)=\sum_{i=1}^{n/2}\left(-13+x_{2i-1}+((5-x_{2i})x_{2i}-2)x_{2i}\right)^2 + \left(-29+x_{2i-1}+((x_{2i}+1)x_{2i}-14)x_{2i}\right)^2,$$

$$x_0=[0.5,-2,0.5,-2,\ldots,0.5,-2].$$

Extended Trigonometric Function

$$f(x)=\sum_{i=1}^{n}\left(\left(n-\sum_{j=1}^{n}\cos x_j\right)+i(1-\cos x_i)-\sin x_i\right)^2,$$

$$x_0=[0.2,0.2,\ldots,0.2].$$

Extended Rosenbrock Function

$$f(x)=\sum_{i=1}^{n/2}c\left(x_{2i}-x_{2i-1}^2\right)^2+\left(1-x_{2i-1}\right)^2,$$

$$x_0=[-1.2,1,\ldots,-1.2,1]. \qquad c=100$$

Extended White & Holst Function

$$f(x)=\sum_{i=1}^{n/2} c\left(x_{2i}-x_{2i-1}^{3}\right)^{2}+\left(1-x_{2i-1}\right)^{2},$$

$$x_0=[-1.2,1,\ldots,-1.2,1], \qquad \text{c=100.}$$

Extended Beale Function

$$f(x)=\sum_{i=1}^{n/2}\left(1.5-x_{2i-1}\left(1-x_{2i}\right)\right)^{2}+\left(2.25-x_{2i-1}\left(1-x_{2i}^{2}\right)\right)^{2}+\left(2.625-x_{2i-1}\left(1-x_{2i}^{3}\right)\right)^{2},$$

$$x_0=[1,0.8,\ldots,1,0.8].$$

Extended Penalty Function

$$f(x)=\sum_{i=1}^{n-1}\left(x_i-1\right)^{2}+\left(\sum_{j=1}^{n} x_j^{2}-0.25\right)^{2},$$

$$x_0=[1,2,\ldots,n].$$

Perturbed Quadratic Function

$$f(x)=\sum_{i=1}^{n} i x_i^{2}+\frac{1}{100}\left(\sum_{i=1}^{n} x_i\right)^{2},$$

$$x_0=[0.5,0.5,\ldots,0.5]$$

Raydan 1 Function

$$f(x)=\sum_{i=1}^{n}\frac{i}{10}\left(\exp(x_i)-x_i\right),$$

$$x_0=[1,1,\ldots,1].$$

Raydan 2 Function

$$f(x)=\sum_{i=1}^{n}\left(\exp(x_i)-x_i\right),$$

$$x_0=[1,1,\ldots,1].$$

Diagonal 1 Function

$$f(x)=\sum_{i=1}^{n}(\exp(x_i)-i x_i),$$

$$x_0=[1/n,1/n,\ldots,1/n].$$

Diagonal 2 Function

$$f(x) = \sum_{i=1}^{n} \left(\exp(x_i) - \frac{x_i}{i} \right),$$

$$x_0 = [1/1, 1/2, \ldots, 1/n].$$

Diagonal 3 Function

$$f(x) = \sum_{i=1}^{n} \left(\exp(x_i) - i \sin(x_i) \right),$$

$$x_0 = [1, 1, \ldots, 1].$$

Hager Function

$$f(x) = \sum_{i=1}^{n} \left(\exp(x_i) - \sqrt{i}\, x_i \right),$$

$$x_0 = [1, 1, \ldots, 1].$$

Generalized Tridiagonal-1 Function

$$f(x) = \sum_{i=1}^{n-1} \left(x_i + x_{i+1} - 3 \right)^2 + \left(x_i - x_{i+1} + 1 \right)^4,$$

$$x_0 = [2, 2, \ldots, 2].$$

Extended Tridiagonal-1 Function

$$f(x) = \sum_{i=1}^{n/2} \left(x_{2i-1} + x_{2i} - 3 \right)^2 + \left(x_{2i-1} - x_{2i} + 1 \right)^4,$$

$$x_0 = [2, 2, \ldots, 2].$$

Extended Three Exponential Terms Function

$$f(x) = \sum_{i=1}^{n/2} \left(\exp(x_{2i-1} + 3x_{2i} - 0.1) + \exp(x_{2i-1} - 3x_{2i} - 0.1) + \exp(-x_{2i-1} - 0.1) \right),$$

$$x_0 = [0.1, 0.1, \ldots, 0.1].$$

Generalized Tridiagonal-2 Function

$$f(x) = \left((5 - 3x_1 - x_1^2) x_1 - 3x_2 + 1 \right)^2 + \sum_{i=1}^{n-1} \left((5 - 3x_i - x_i^2) x_i - x_{i-1} - 3x_{i+1} + 1 \right)^2 + \left((5 - 3x_n - x_n^2) x_n - x_{n-1} + 1 \right)^2,$$

$$x_0 = [-1, -1, \ldots, -1].$$

Diagonal 4 Function

$$f(x) = \sum_{i=1}^{n/2} \frac{1}{2}\left(x_{2i-1}^2 + c x_{2i}^2\right),$$

$$x_0 = [1,1,\ldots,1], \qquad c = 100.$$

Diagonal 5 Function

$$f(x) = \sum_{i=1}^{n} \log\left(\exp(x_i) + \exp(-x_i)\right),$$

$$x_0 = [1.1,1.1,\ldots,1.1].$$

Extended Himmelblau Function

$$f(x) = \sum_{i=1}^{n/2} \left(x_{2i-1}^2 + x_{2i} - 11\right)^2 + \left(x_{2i-1} + x_{2i}^2 - 7\right)^2,$$

$$x_0 = [1,1,\ldots,1].$$

Generalized Rosenbrock Function (c=100)

$$f(x) = \sum_{i=1}^{n-1} c\left(x_{i+1} - x_i^2\right)^2 + \left(1 - x_i\right)^2,$$

$$x_0 = [-1.2,1.\ldots,-1.2,1], \qquad c = 100.$$

Generalized White & Holst Function (c=100)

$$f(x) = \sum_{i=1}^{n-1} c\left(x_{i+1} - x_i^3\right)^2 + \left(1 - x_i\right)^2,$$

$$x_0 = [-1.2,1.\ldots,-1.2,1], \qquad c = 100.$$

Generalized PSC1 Function

$$f(x) = \sum_{i=1}^{n-1} \left(x_i^2 + x_{i+1}^2 + x_i x_{i+1}\right)^2 + \sin^2(x_i) + \cos^2(x_i),$$

$$x_0 = [3,0.1,\ldots,3,0.1].$$

Extended PSC1 Function

$$f(x) = \sum_{i=1}^{n/2} \left(x_{2i-1}^2 + x_{2i}^2 + x_{2i-1} x_{2i}\right)^2 + \sin^2(x_{2i-1}) + \cos^2(x_{2i}),$$

$$x_0 = [3,0.1,\ldots,3,0.1].$$

Extended Powell Function

$$f(x)=\sum_{i=1}^{n/4}\left(x_{4i-3}+10x_{4i-2}\right)^2+5\left(x_{4i-1}-x_{4i}\right)^2+\left(x_{4i-2}-2x_{4i-1}\right)^4+10\left(x_{4i-3}-x_{4i}\right)^4,$$

$$x_0=[3,-1,0,1,\ldots,3,-1,0.1].$$

Full Hessian FH1 Function

$$f(x)=(x_1-3)^2+\sum_{i=2}^{n}\left(x_1-3-2(x_1+x_2+\cdots+x_i)^2\right)^2,$$

$$x_0=[0.01,0.01,\ldots,0.01].$$

Full Hessian FH2 Function

$$f(x)=(x_1-5)^2+\sum_{i=2}^{n}\left(x_1+x_2+\cdots+x_i-1\right)^2,$$

$$x_0=[0.01,0.01,\ldots,0.01].$$

Extended Block Diagonal BD1 Function

$$f(x)=\sum_{i=1}^{n/2}\left(x_{2i-1}^2+x_{2i}^2-2\right)^2+\left(\exp(x_{2i-1}-1)-x_{2i}\right)^2,$$

$$x_0=[0.1,0.1,\ldots,0.1].$$

Extended Maratos Function (c=100)

$$f(x)=\sum_{i=1}^{n/2}x_{2i-1}+c\left(x_{2i-1}^2+x_{2i}^2-1\right)^2,$$

$$x_0=[1.1,0.1,\ldots,1.1,0.1],\qquad c=100.$$

Extended Cliff Function

$$f(x)=\sum_{i=1}^{n/2}\left(\frac{x_{2i-1}-3}{100}\right)^2-\left(x_{2i-1}-x_{2i}\right)+\exp\left(20(x_{2i-1}-x_{2i})\right),$$

$$x_0=[0,-1,\ldots,0,-1].$$

Quadratic Diagonal Perturbed Function

$$f(x)=\left(\sum_{i=1}^{n}x_i\right)^2+\sum_{i=1}^{n}\frac{i}{100}x_i^2,$$

$$x_0=[0.5,0.5,\ldots,0.5].$$

Extended Wood Function

$$f(x)=\sum_{i=1}^{n/4}100\left(x_{4i-3}^2-x_{4i-2}\right)^2+\left(x_{4i-3}-1\right)^2+90\left(x_{4i-1}^2-x_{4i}\right)^2+$$
$$\left(1-x_{4i-1}\right)^2+10.1\left\{\left(x_{4i-2}-1\right)^2+\left(x_{4i}-1\right)^2\right\}+19.8\left(x_{4i-2}-1\right)\left(x_{4i}-1\right),$$

$$x_0=[-3,-1,-3,-1,\ldots,-3,-1,-3,-1].$$

Extended Hiebert Function

$$f(x)=\sum_{i=1}^{n/2}\left(x_{2i-1}-10\right)^2+\left(x_{2i-1}x_{2i}-50000\right)^2,$$

$$x_0=[0,0,\ldots,0].$$

Quadratic QF1 Function

$$f(x)=\frac{1}{2}\sum_{i=1}^{n}ix_i^2-x_n,$$

$$x_0=[1,1,\ldots,1].$$

Extended Quadratic Penalty QP1 Function

$$f(x)=\sum_{i=1}^{n-1}\left(x_i^2-2\right)^2+\left(\sum_{i=1}^{n}x_i^2-0.5\right)^2,$$

$$x_0=[1,1,\ldots,1].$$

Extended Quadratic Penalty QP2 Function

$$f(x)=\sum_{i=1}^{n-1}\left(x_i^2-\sin x_i\right)^2+\left(\sum_{i=1}^{n}x_i^2-100\right)^2,$$

$$x_0=[1,1,\ldots,1].$$

Quadratic Function QF2

$$f(x)=\frac{1}{2}\sum_{i=1}^{n}i\left(x_i^2-1\right)^2-x_n,$$

$$x_0=[0.5,0.5,\ldots,0.5].$$

Extended EP1 Function

$$f(x)=\sum_{i=1}^{n/2}\left(\exp(x_{2i-1}-x_{2i})-5\right)^2+\left(x_{2i-1}-x_{2i}\right)^2\left(x_{2i-1}-x_{2i}-11\right)^2,$$

$$x_0=[1.5,1.5,\ldots,1.5].$$

Extended Tridiagonal-2 Function

$$f(x)=\sum_{i=1}^{n-1}\left(x_i x_{i+1}-1\right)^2+c\left(x_i+1\right)\left(x_{i+1}+1\right),$$

$$x_0=[1.,1.,\ldots,1.].\qquad c=0.1$$

FLETCBV3 Function (CUTE)

$$f(x)=\frac{1}{2}p\left(x_1^2+x_n^2\right)+\sum_{i=1}^{n-1}\frac{p}{2}\left(x_i-x_{i+1}\right)^2-\sum_{i=1}^{n}\left(\frac{p(h^2+2)}{h^2}x_i+\frac{cp}{h^2}\cos(x_i)\right)$$

where:

$$p=1/10^8,\qquad h=1/(n+1),\qquad c=1,$$
$$x_0=[h,2h,\ldots,nh].$$

FLETCHCR Function (CUTE)

$$f(x)=\sum_{i=1}^{n-1}c\left(x_{i+1}-x_i+1-x_i^2\right)^2,$$

$$x_0=[0.,0.,\ldots,0.],\qquad c=100.$$

BDQRTIC Function (CUTE)

$$f(x)=\sum_{i=1}^{n-4}\left(-4x_i+3\right)^2+\left(x_i^2+2x_{i+1}^2+3x_{i+2}^2+4x_{i+3}^2+5x_n^2\right)^2,$$

$$x_0=[1.,1.,\ldots,1.].$$

TRIDIA Function (CUTE)

$$f(x)=\gamma\left(\delta x_1-1\right)^2+\sum_{i=2}^{n}i\left(\alpha x_i-\beta x_{i-1}\right)^2,$$

$$\alpha=2,\quad \beta=1,\quad \gamma=1,\quad \delta=1,$$

$$x_0=[1,1,\ldots,1].$$

ARGLINB (m=20) Function (CUTE)

$$f(x)=\sum_{i=1}^{m}\left(\sum_{j=1}^{n}ijx_j-1\right)^2,$$

$$x_0=[1,1,\ldots,1].$$

ARWHEAD Function (CUTE)

$$f(x)=\sum_{i=1}^{n-1}\left(-4x_i+3\right)+\sum_{i=1}^{n-1}\left(x_i^2+x_n^2\right)^2,$$

$$x_0=[1.,1.,\ldots,1.].$$

NONDIA (Shanno-78) Function (CUTE)

$$f(x)=(x_1-1)^2+\sum_{i=2}^{n}100\left(x_1-x_{i-1}^2\right)^2,$$

$$x_0=[-1.,-1.,\ldots,-1.].$$

NONDQUAR Function (CUTE)

$$f(x)=(x_1-x_2)^2+\sum_{i=1}^{n-2}\left(x_i+x_{i+1}+x_n\right)^4+(x_{n-1}+x_n)^2,$$

$$x_0=[1.,-1.,\ldots,1.,-1.,].$$

DQDRTIC Function (CUTE)

$$f(x)=\sum_{i=1}^{n-2}\left(x_i^2+cx_{i+1}^2+dx_{i+2}^2\right),$$

$$c=100., \quad d=100.,$$

$$x_0=[3.,3.,\ldots,3.].$$

EG2 Function (CUTE)

$$f(x)=\sum_{i=1}^{n-1}\sin(x_1+x_i^2-1)+\frac{1}{2}\sin(x_n^2),$$

$$x_0=[1.,1.,\ldots,1.].$$

CURLY20 Function (CUTE)

$$f(x)=\sum_{i=1}^{n}q_i^4-20q_i^2-0.1q_i,$$

where

$$q_i=\begin{cases}x_i+x_{i+1}+\cdots+x_{i+k}, & i\le n-k,\\ x_i+x_{i+1}+\cdots+x_n, & i>n-k,\end{cases} \quad k=20,$$

$$x_0=[0.001/(n+1),\ldots,0.001/(n+1)].$$

DIXMAANA - DIXMAANL Functions

$$f(x) = 1 + \sum_{i=1}^{n} \alpha\, x_i^2 \left(\frac{i}{n}\right)^{k1} + \sum_{i=1}^{n-1} \beta\, x_i^2 \left(x_{i+1} + x_{i+1}^2\right)^2 \left(\frac{i}{n}\right)^{k2} + \sum_{i=1}^{2m} \gamma\, x_i^2 x_{i+m}^4 \left(\frac{i}{n}\right)^{k3} + \sum_{i=1}^{m} \delta\, x_i x_{i+2m} \left(\frac{i}{n}\right)^{k4},$$
$$m = n/3,$$

	α	β	γ	δ	k1	k2	k3	k4
A	1	0	0.125	0.125	0	0	0	0
B	1	0.0625	0.0625	0.0625	0	0	0	1
C	1	0.125	0.125	0.125	0	0	0	0
D	1	0.26	0.26	0.26	0	0	0	0
E	1	0	0.125	0.125	1	0	0	1
F	1	0.0625	0.0625	0.0625	1	0	0	1
G	1	0.125	0.125	0.125	1	0	0	1
H	1	0.26	0.26	0.26	1	0	0	1
I	1	0	0.125	0.125	2	0	0	2
J	1	0.0625	0.0625	0.0625	2	0	0	2
K	1	0.125	0.125	0.125	2	0	0	2
L	1	0.26	0.26	0.26	2	0	0	2

$$x_0 = [2.,2.,\ldots,2.].$$

Partial Perturbed Quadratic

$$f(x) = x_1^2 + \sum_{i=1}^{n} \left(i x_i^2 + \frac{1}{100}\left(x_1 + x_2 + \cdots + x_i\right)^2 \right)$$

$$x_0 = [0.5,0.5,\ldots,0.5].$$

Broyden Tridiagonal Function

$$f(x) = (3x_1 - 2x_1^2)^2 + \sum_{i=2}^{n-1} (3x_i - 2x_i^2 - x_{i-1} - 2x_{i+1} + 1)^2 + (3x_n - 2x_n^2 - x_{n-1} + 1)^2,$$

$$x_0 = [-1,-1,\ldots,-1].$$

Almost Perturbed Quadratic Function

$$f(x) = \sum_{i=1}^{n} i x_i^2 + \frac{1}{100}(x_1 + x_n)^2,$$

$$x_0 = [0.5,0.5,\ldots,0.5].$$

Tridiagonal Perturbed Quadratic Function

$$f(x) = x_1^2 + \sum_{i=2}^{n-1} i x_i^2 + (x_{i-1} + x_i + x_{i+1})^2,$$

$$x_0 = [0.5, 0.5, \ldots, 0.5].$$

Staircase 1 Function Function

$$f(x) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j \right)^2,$$

$$x_0 = [1, 1, \ldots, 1].$$

Staircase 2 Function

$$f(x) = \sum_{i=1}^{n} \left[\left(\sum_{j=1}^{i} x_j \right) - i \right]^2,$$

$$x_0 = [0, 0, \ldots, 0].$$

LIARWHD Function (CUTE)

$$f(x) = \sum_{i=1}^{n} 4(-x_1 + x_i^2)^2 + \sum_{i=1}^{n} (x_i - 1)^2,$$

$$x_0 = [4, 4, \ldots, 4].$$

POWER Function (CUTE)

$$f(x) = \sum_{i=1}^{n} (i x_i)^2,$$

$$x_0 = [1, 1, \ldots, 1].$$

ENGVAL1 Function (CUTE)

$$f(x) = \sum_{i=1}^{n-1} (x_i^2 + x_{i+1}^2)^2 + \sum_{i=1}^{n-1} (-4x_i + 3),$$

$$x_0 = [2, 2, \ldots, 2].$$

CRAGGLVY Function (CUTE)

$$f(x) = \sum_{i=1}^{m} \left(\exp(x_{2i-1}) - x_{2i} \right)^4 + 100(x_{2i} - x_{2i+1})^6 +$$
$$(\tan(x_{2i+1} - x_{2i+2}) + x_{2i+1} - x_{2i+2})^4 + x_{2i-1}^8 + (x_{2i+2} - 1)^2,$$

$$x_0 = [1,2,\ldots,2].$$

EDENSCH Function (CUTE)

$$f(x) = 16 + \sum_{i=1}^{n-1}\left[(x_i - 2)^4 + (x_i x_{i+1} - 2x_{i+1})^2 + (x_{i+1} + 1)^2\right],$$

$$x_0 = [0,0,\ldots,0].$$

INDEF Function (CUTE)

$$f(x) = \sum_{i=1}^{n} x_i + \sum_{i=2}^{n-1}\frac{1}{2}\cos(2x_i - x_n - x_1),$$

$$x_0 = \left[\frac{1}{n+1}, \frac{2}{n+1}, \ldots, \frac{n}{n+1}\right].$$

CUBE Function (CUTE)

$$f(x) = (x_1 - 1)^2 + \sum_{i=2}^{n} 100(x_i - x_{i-1}^3)^2,$$

$$x_0 = [-1.2,1,-1.2,1,\ldots,-1.2,1].$$

EXPLIN Function (CUTE)

$$f(x) = \exp(0.1 x_i x_{i+1}) - 10\sum_{i=1}^{n}(i x_i),$$

$$x_0 = [0,0,\ldots,0].$$

EXPLIN2 Function (CUTE)

$$f(x) = \sum_{i=1}^{m}\exp\left(\frac{i x_i x_{i+1}}{10m}\right) - 10\sum_{i=1}^{n}(i x_i),$$

$$x_0 = [0,0,\ldots,0].$$

ARGLINC Function (CUTE)

$$f(x) = 2 + \sum_{i=2}^{m-1}\left(\sum_{j=2}^{n-1} j x_j (i-1) - 1\right)^2,$$

$$x_0 = [1,1,\ldots,1].$$

BDEXP Function (CUTE)

$$f(x)=\sum_{i=1}^{n-2}\left(x_i+x_{i+1}\right)\exp\left(-x_{i+2}(x_i+x_{i+1})\right),$$
$$x_0=[1,1,\ldots,1].$$

HARKERP2 Function (CUTE)

$$f(x)=\left(\sum_{i=1}^{n}x_i\right)^2-\sum_{i=1}^{n}(x_i+\frac{1}{2}x_i^2)+2\sum_{j=2}^{n}\left(\sum_{i=j}^{n}x_i\right)^2,$$

$$x_0=[1,2,\ldots,n].$$

GENHUMPS Function (CUTE)

$$f(x)=\sum_{i=1}^{n-1}\sin(2x_i)^2\sin(2x_{i+1})^2+0.05(x_i^2+x_{i+1}^2),$$

$$x_0=[-506.,506.2,\ldots,506.2].$$

MCCORMCK Function (CUTE)

$$f(x)=\sum_{i=1}^{n-1}\left(-1.5x_i+2.5x_{i+1}+1+(x_i-x_{i+1})^2+\sin(x_i+x_{i+1})\right),$$

$$x_0=[1,1,\ldots,1].$$

NONSCOMP Function (CUTE)

$$f(x)=(x_1-1)^2+\sum_{i=2}^{n}4(x_i-x_{i-1}^2)^2,$$

$$x_0=[3,3,\ldots,3].$$

VARDIM Function (CUTE)

$$f(x)=\sum_{i=1}^{n}(x_i-1)^2+\left(\sum_{i=1}^{n}ix_i-\frac{n(n+1)}{2}\right)^2+\left(\sum_{i=1}^{n}ix_i-\frac{n(n+1)}{2}\right)^4,$$

$$x_0=\left[1-\frac{1}{n},1-\frac{2}{n},\ldots,1-\frac{n}{n}\right].$$

QUARTC Function (CUTE)

$$f(x)=\sum_{i=1}^{n}\left(x_i-1\right)^4,$$

$$x_0=[2.,2.,\ldots,2.].$$

Diagonal 6 Function

$$f(x) = \sum_{i=1}^{n} e^{x_i} - (1 - x_i),$$
$$x_0 = [1,1,\ldots,1].$$

SINQUAD Function (CUTE)

$$f(x) = (x_1 - 1)^4 + \sum_{i=2}^{n-1} \left(\sin(x_i - x_n) - x_1^2 + x_i^2\right)^2 + (x_n^2 - x_1^2)^2,$$

$$x_0 = [0.1,0.1,\ldots,0.1].$$

Extended DENSCHNB Function (CUTE)

$$f(x) = \sum_{i=1}^{n/2} \left(x_{2i-1} - 2\right)^2 + \left(x_{2i-1} - 2\right)^2 x_{2i}^2 + \left(x_{2i} + 1\right)^2,$$

$$x_0 = [1,1,\ldots,1].$$

Extended DENSCHNF Function (CUTE)

$$f(x) = \sum_{i=1}^{n/2} \left(2(x_{2i-1} + x_{2i})^2 + (x_{2i-1} - x_{2i})^2 - 8\right)^2 + \left(5x_{2i-1}^2 + (x_{2i} - 3)^2 - 9\right)^2,$$

$$x_0 = [2.,0.,2.,0.,\ldots,2.,0.].$$

LIARWHD Function (CUTE)

$$f(x) = \sum_{i=1}^{n} 4(x_i^2 - x_1)^2 + \sum_{i=1}^{n} (x_i - 1)^2,$$

$$x_0 = [4.,4.,\ldots,4.].$$

DIXON3DQ Function (CUTE)

$$f(x) = (x_1 - 1)^2 + \sum_{j=1}^{n-1} (x_j - x_{j+1})^2 + (x_n - 1)^2,$$

$$x_0 = [-1.,-1.,\ldots,-1].$$

COSINE Function (CUTE)

$$f(x) = \sum_{i=1}^{n-1} \cos\left(-0.5x_{i+1} + x_i^2\right),$$

$$x_0 = [1,1,\ldots,1].$$

SINE Function

$$f(x) = \sum_{i=1}^{n-1} \sin\left(-0.5x_{i+1} + x_i^2\right),$$

$$x_0 = [1,1,\ldots,1].$$

BIGGSB1 Function (CUTE)

$$f(x) = (x_1 - 1)^2 + \sum_{i=1}^{n-1} \left(x_{i+1} - x_i\right)^2 + (1 - x_n)^2,$$

$$x_0 = [0,0,\ldots,0].$$

Generalized quartic Function GQ1

$$f(x) = \sum_{i=1}^{n-1} x_i^2 + \left(x_{i+1} + x_i^2\right)^2,$$

$$x_0 = [1,1,\ldots,1].$$

Diagonal 7 Function

$$f(x) = \sum_{i=1}^{n} \exp(x_i) - 2x_i - x_i^2,$$

$$x_0 = [1,1,\ldots,1].$$

Diagonal 8 Function

$$f(x) = \sum_{i=1}^{n} x_i \exp(x_i) - 2x_i - x_i^2,$$

$$x_0 = [1,1,\ldots,1].$$

Full Hessian Function

$$f(x) = \left(\sum_{i=1}^{n} x_i\right)^2 + \sum_{i=1}^{n} (x_i \exp(x_i) - 2x_i - x_i^2),$$

$$x_0 = [1,1,\ldots,1].$$

SINCOS Function

$$f(x) = \sum_{i=1}^{n/2} \left(x_{2i-1}^2 + x_{2i}^2 + x_{2i-1}x_{2i}\right)^2 + \sin^2 x_{2i-1} + \cos^2 x_{2i},$$

$$x_0 = [3,0.1,3,0.1,\ldots,3,0.1].$$

Generalized quartic Function GQ2

$$f(x) = (x_1^2 - 1)^2 + \sum_{i=2}^{n} \left(x_i^2 - x_{i-1} - 2 \right)^2,$$

$$x_0 = [1,1,\ldots,1].$$

TEODOR Function

$$f(x) = \left(x_1^2 - 1 \right)^2 + \sum_{i=2}^{n} \left(\left(x_i^2 - x_{i-1} \right)^2 - i \right)^2,$$

$$x_0 = [3.2,3.2,\ldots,3.2].$$

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