



Eduard Stiefel (1909-1978)

Eduard Stiefel's birthday centenary

The conjugate gradient algorithm which now is more than 55 years old comprise a class of very elegant unconstrained optimization algorithms characterized by strong local and global convergence properties and by low memory requirements. The history of these algorithms begins with the research of Eduard Stiefel at *Eidgenössische Technische Hochschule Zürich - Swiss Federal Institute of Technology*, and Cornelius Lanczos (1893-1974) and Magnus Hestenes (1906-1991) at the *Institute for Numerical Analysis (National Applied Mathematics Laboratories of the United States National Bureau of Standards in Los Angeles)*.

In 1951 Stiefel established the n-step method for solving systems of algebraic linear equations systems with symmetric and positive definite matrix, published as: *Über einige Methoden der Relaxationsrechnung*. Z. Angew. Math. Phys., 3 (1952), pp.1-33 (submitted in July 1951). In July 1951, invited by Olga Taussky-Todd (1906-1995), he visited the Institute for Numerical Analysis (INA) of the National Bureau of Standards - UCLA, Los Angeles, until February 1952. There he found out that Magnus Hestenes in collaboration with others of the INA (Lanczos, Forsythe, Motzkin, Rosser, Stein) had discovered the same method too. Hestenes, who called it the conjugate gradient method, published an internal report as: *Iterative methods for solving equations*. NAML Report 52-9, National Bureau of Standards, Los Angeles, CA, July 1951. During this visit Hestenes and Stiefel combined their efforts and published a 28-page two-columns conjugate gradient paper as: *Methods of Conjugate Gradients for Solving Linear Systems*. Journal of Research of the National Bureau of Standards, vol.49, No. 6, December 1952 (Research Paper 2379), being one of the most influential papers in numerical analysis. Some details and insights on this paper have been prepared by Dianne P. O'Leary [O'Leary, D.P., *Conjugate gradients and related KMP algorithms: the beginnings*. in *Linear and Nonlinear Conjugate Gradient – Related Methods*, L. Adams and J.L. Nazareth (Eds.) SIAM, 1996, pp.1-8]. Since than over 50 nonlinear conjugate gradient algorithms have been established, and practically an innumerable number of papers have been published on this subject insisting both on their theoretical and practical aspects. An excellent survey of development of different versions of nonlinear conjugate gradient methods, with special attention to global convergence properties, is presented by Hager and Zhang [Hager, W.W., Zhang, H., *A survey of nonlinear conjugate gradient methods*. Pacific Journal of Optimization, 2 (2006), pp.35-58.] Some open problems in nonlinear conjugate gradient algorithms for unconstrained optimization have been articulated by Andrei [Andrei, N., *Open Problems in Nonlinear Conjugate Gradient Algorithms for Unconstrained Optimization*. ICI Technical Report No. 28/08, July 30, 2008].

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