A set of unconstrained optimization test problems

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Abstract. In this paper I present a collection of 80 unconstrained optimization test functions. The purpose of this collection is to put together some test problems for testing unconstrained optimization algorithms, as well as to perform comparisons among codes. For each function, its algebraic expression and a standard initial point are provided. A large part of the test problems in this collection is taken from CUTE by Bongartz, Conn, Gould and Toint [6], others are from Andrei [1, 2, 3, 4, 5], as well as from other publications.

1. Freudenstein & Roth FREUROTH (CUTE)

$$f(x)=\sum_{i=1}^{n/2}\left(-13+x_{2i-1}+((5-x_{2i})x_{2i}-2)x_{2i}\right)^2 + \left(-29+x_{2i-1}+((x_{2i}+1)x_{2i}-14)x_{2i}\right)^2,$$

$$x_0=[0.5,-2,0.5,-2,\ldots,0.5,-2].$$

2. Extended Trigonometric ET1

$$f(x)=\sum_{i=1}^{n}\left(\left(n-\sum_{j=1}^{n}\cos x_j\right)+i(1-\cos x_i)-\sin x_i\right)^2,$$

$$x_0=[0.2,0.2,\ldots,0.2].$$

3. Extended Rosenbrock SROSENBR (CUTE)

$$f(x)=\sum_{i=1}^{n/2}c\left(x_{2i}-x_{2i-1}^2\right)^2+\left(1-x_{2i-1}\right)^2,\quad x_0=[-1.2,1,\ldots,-1.2,1].\quad c=1000.$$

4. Extended White & Holst

$$f(x)=\sum_{i=1}^{n/2}c\left(x_{2i}-x_{2i-1}^3\right)^2+\left(1-x_{2i-1}\right)^2,\quad x_0=[-1.2,1,\ldots,-1.2,1],\quad c=1.$$

5. Extended Beale BEALE (CUTE)

$$f(x)=\sum_{i=1}^{n/2}\left(1.5-x_{2i-1}\left(1-x_{2i}\right)\right)^2+\left(2.25-x_{2i-1}\left(1-x_{2i}^2\right)\right)^2+\left(2.625-x_{2i-1}\left(1-x_{2i}^3\right)\right)^2,$$

$$x_0=[1,0.8,\ldots,1,0.8].$$

6. Extended Penalty

$$f(x)=\sum_{i=1}^{n-1}\left(x_i-1\right)^2+\left(\sum_{j=1}^{n}x_j^2-0.25\right)^2,\quad x_0=[1,2,\ldots,n].$$

7. Perturbed Quadratic

$$f(x)=\sum_{i=1}^{n}ix_i^2+\frac{1}{100}\left(\sum_{i=1}^{n}x_i\right)^2,\quad x_0=[0.5,0.5,\ldots,0.5]$$

8. Raydan 1

$$f(x)=\sum_{i=1}^{n}\frac{i}{10}\left(\exp(x_i)-x_i\right),\quad x_0=[1,1,\ldots,1].$$

9. Raydan 2

$$f(x)=\sum_{i=1}^{n}\left(\exp(x_i)-x_i\right),\quad x_0=[1,1,\ldots,1].$$

10. TR-SUMM of quadratics

$$f(x)=\sum_{i=1}^{n-1}x_i^2+c(x_{i+1}+x_i^2)^2\quad x_0=[1,1,\ldots,1].\quad c=100000.$$

11. Diagonal 1

$$f(x)=\sum_{i=1}^{n}\left(\exp(x_i)-\frac{x_i}{i}\right),\quad x_0=[1/n,1/n,\ldots,1/n].$$

12. Diagonal 2

$$f(x)=\sum_{i=1}^{n}\left(\exp(x_i)-i\sin(x_i)\right),\quad x_0=[1/1,1/2,\ldots,1/n].$$

13. Hager

$$f(x)=\sum_{i=1}^{n}\left(\exp(x_i)-\sqrt{i}x_i\right),\quad x_0=[1,1,\ldots,1].$$

14. Generalized Tridiagonal 1

$$f(x)=\sum_{i=1}^{n-1}\left(x_i+x_{i+1}-3\right)^2+\left(x_i-x_{i+1}+1\right)^4,\quad x_0=[2,2,\ldots,2].$$

15. Extended Tridiagonal 1

$$f(x)=\sum_{i=1}^{n/2}\left(x_{2i-1}+x_{2i}-3\right)^2+\left(x_{2i-1}-x_{2i}+1\right)^4,\quad x_0=[2,2,\ldots,2].$$

16. Extended Three Expo Terms

$$f(x)=\sum_{i=1}^{n/2}\Big(\exp(x_{2i-1}+3x_{2i}-0.1)+\exp(x_{2i-1}-3x_{2i}-0.1)+\exp(-x_{2i-1}-0.1)\Big),$$
$$x_0=[0.1,0.1,\ldots,0.1].$$

17. Generalized Tridiagonal 2 (Pentadiagonal)

$$f(x)=\Big((5-3x_1-x_1^2)x_1-3x_2+1\Big)^2+$$
$$\sum_{i=1}^{n-1}\Big((5-3x_i-x_i^2)x_i-x_{i-1}-3x_{i+1}+1\Big)^2+\Big((5-3x_n-x_n^2)x_n-x_{n-1}+1\Big)^2,$$
$$x_0=[-1,-1,\ldots,-1].$$

18. Diagonal 3

$$f(x)=\frac{1}{2}\sum_{i=1}^{n/2}\Big(x_{2i-1}^2+cx_{2i}^2\Big),\qquad x_0=[1,1,\ldots,1].\quad c=10000.$$

19. Diagonal Full Borded

$$f(x)=(x_1-1)^4+(x_n^2-x_1^2)^2+\sum_{i=1}^{n-2}(\sin(x_{i+1}-x_n)-x_1^2-x_{i+1}^2)^2,\quad x_0=[0.1,0.1,\ldots,0.1].$$

20. Extended Himmelblau HIMMELBC (CUTE)

$$f(x)=\sum_{i=1}^{n/2}\Big(x_{2i-1}^2+x_{2i}-11\Big)^2+\Big(x_{2i-1}+x_{2i}^2-7\Big)^2,\qquad x_0=[1,1,\ldots,1].$$

21. Extended Powell

$$f(x)=\sum_{i=1}^{n/4}\Big(x_{4i-3}+10x_{4i-2}\Big)^2+5\Big(x_{4i-1}-x_{4i}\Big)^2+$$
$$\Big(x_{4i-2}-2x_{4i-1}\Big)^4+10\Big(x_{4i-3}-x_{4i}\Big)^4,$$
$$x_0=[3,-1,0,1,\ldots,3,-1,0.1].$$

22. Tridiagonal Double Borded Arrow Up

$$f(x)=(x_1-1)^2+\sum_{i=1}^{n-1}(x_1-0.5x_i^2-05x_{i+1}^2)^2,\qquad x_0=[-1,-1,\ldots,-1].$$

23. Extended PSC1

$$f(x)=\sum_{i=1}^{n/2}\Big(x_{2i-1}^2+x_{2i}^2+x_{2i-1}x_{2i}\Big)^2+\sin^2(x_{2i-1})+\cos^2(x_{2i}),$$
$$x_0=[3,0.1,\ldots,3,0.1].$$

24. Extended Block-Diagonal BD1

$$f(x)=\sum_{i=1}^{n/2}\Big(x_{2i-1}^2+x_{2i}^2-2\Big)^2+\Big(\exp(x_{2i-1})-x_{2i}\Big)^2,\quad x_0=[0.1,0.1,\ldots,0.1].$$

25. Extended Maratos

$$f(x)=\sum_{i=1}^{n/2} x_{2i-1}+c\left(x_{2i-1}^2+x_{2i}^2-1\right)^2, \quad x_0=[1.1,0.1,\ldots,1.1,0.1], \qquad c=1.$$

26. Full Hessian FH1 (Sum of quadratics, quadratic inside)

$$f(x)=\sum_{i=1}^{m}\left(\sum_{j=1}^{n} ijx_j^2-1\right)^2, \quad m=50,$$
$$x_0=[0.01,0.01,\ldots,0.01].$$

27. Extended Cliff

$$f(x)=\sum_{i=1}^{n/2}\left(\frac{x_{2i-1}-3}{100}\right)^2-\left(x_{2i-1}-x_{2i}\right)+\exp\left(2(x_{2i-1}-x_{2i})\right),$$
$$x_0=[0,-0.1,\ldots,0,-0.1].$$

28. Quadratic Diagonal Perturbed

$$f(x)=\left(\sum_{i=1}^{n} x_i\right)^2+\sum_{i=1}^{n}\frac{i}{100}x_i^2, \qquad x_0=[0.5,0.5,\ldots,0.5].$$

29. Full Hessian FH2

$$f(x)=\left(\sum_{i=1}^{n} x_i\right)^2+\sum_{i=1}^{n} i(\sin(x_i)+\cos(x_i))/1000, \qquad x_0=[1,1,\ldots,1].$$

30. Full Hessian FH3

$$f(x)=\left(\sum_{i=1}^{n} x_i^2\right)^2+\sum_{i=1}^{n} i(\sin(x_i)+\cos(x_i))/1000, \qquad x_0=[1,1,\ldots,1].$$

31. Tridiagonal Double Borded – NONDQUAR

$$f(x)=(x_1-x_2)^2+\sum_{i=1}^{n-2}\left(x_i+x_{i+1}+x_n\right)^4+(x_{n-1}+x_n)^2, \quad x_0=[1.,-1.,\ldots,1.,-1.,].$$

32. Tridiagonal White & Holst (c=4)

$$f(x)=\sum_{i=1}^{n-1} c(x_{i+1}-x_i^3)^2+(1-x_i)^2, \quad x_0=[-1.2,1,\ldots,-1.2,1]. \qquad c=4.$$

33. Diagonal Double Borded Arrow Up

$$f(x)=\sum_{i=1}^{n} 4(x_i^2-x_1)^2+(x_i-1)^2, \qquad x_0=[4,0,\ldots,4,0].$$

34. TRIDIA (CUTE)

$$f(x)=\gamma\left(\delta x_1-1\right)^2+\sum_{i=2}^{n} i\left(\alpha x_i-\beta x_{i-1}\right)^2,$$
$$\alpha=2, \quad \beta=1, \quad \gamma=1, \quad \delta=1, \quad x_0=[1,1,\ldots,1].$$

35. ARWHEAD (CUTE)

$$f(x)=\sum_{i=1}^{n-1}\left(-4x_i+3\right)+\sum_{i=1}^{n-1}\left(x_i^2+x_n^2\right)^2, \qquad x_0=[1.,1.,\ldots,1.].$$

36. NONDIA (CUTE) (Shanno)

$$f(x)=(x_1-1)^2+c(x_1-x_1^2)^2+\sum_{i=2}^{n}c\left(x_1-x_i^2\right)^2, \qquad x_0=[-1.,-1.,\ldots,-1.], \quad c=100.$$

37. Extended WOODS (CUTE)

$$f(x)=\sum_{i=1}^{n/4}100\left(x_{4i-3}^2-x_{4i-2}\right)^2+\left(x_{4i-3}-1\right)^2+90\left(x_{4i-1}^2-x_{4i}\right)^2+$$
$$\left(1-x_{4i-1}\right)^2+10.1\left\{\left(x_{4i-2}-1\right)^2+\left(x_{4i}-1\right)^2\right\}+19.8\left(x_{4i-2}-1\right)\left(x_{4i}-1\right),$$
$$x_0=[-3,-1,-3,-1,\ldots,-3,-1,-3,-1].$$

38. Extended Hiebert

$$f(x)=\sum_{i=1}^{n/2}\left(x_{2i-1}-10\right)^2+\left(x_{2i-1}x_{2i}-500\right)^2, \qquad x_0=[0,0,\ldots,0].$$

39. BDQRTIC (CUTE)

$$f(x)=\sum_{i=1}^{n-4}\left(-4x_i+3\right)^2+\left(x_i^2+2x_{i+1}^2+3x_{i+2}^2+4x_{i+3}^2+5x_n^2\right)^2,$$
$$x_0=[1.,1.,\ldots,1.].$$

40. DQDRTIC (CUTE)

$$f(x)=\sum_{i=1}^{n-2}\left(x_i^2+cx_{i+1}^2+dx_{i+2}^2\right), \qquad c=1000, \qquad d=1000, \qquad x_0=[3.,3.,\ldots,3.].$$

41. EG2 (CUTE)

$$f(x)=\sum_{i=1}^{n-1}\sin(x_1+x_i^2-1)+\frac{1}{2}\sin(x_n^2), \qquad x_0=[1.,1.,\ldots,1.].$$

42. EDENSCH (CUTE)

$$f(x)=16+\sum_{i=1}^{n-1}\left[(x_i-2)^4+(x_ix_{i+1}-2x_{i+1})^2+(x_{i+1}+1)^2\right], \quad x_0=[0,0,\ldots,0].$$

43. Broyden Pentadiagonal (CUTE)

$$f(x)=(3x_1-2x_1^2)^2+\sum(3x_i-2x_i^2-x_{i-1}-2x_{i+1}+1)^2+(3x_n+2x_n^2-x_{n-1}+1)^2,$$
$$x_0=[-1,-1,\ldots,-1].$$

44. Almost Perturbed Quadratic

$$f(x)=(x_1+x_n)^2/100+\sum_{i=1}^{n}ix_i^2, \qquad x_0=[0.5,0.5,\ldots,0.5].$$

45. Almost Perturbed Quartic

$$f(x) = (x_1 + x_n)^2 / 100 + \sum_{i=1}^{n} i x_i^4, \qquad x_0 = [0.5, 0.5, \ldots, 0.5].$$

46. FLETCHCR (CUTE)

$$f(x) = \sum_{i=1}^{n-1} c\left(x_{i+1} - x_i + 1 - x_i^2\right)^2, \quad x_0 = [0.5, 0.5, \ldots, 0.5], \qquad c = 100.$$

47. ENGVAL1 (CUTE)

$$f(x) = \sum_{i=1}^{n-1} (x_i^2 + x_{i+1}^2)^2 + \sum_{i=1}^{n-1} (-4x_i + 3), \qquad x_0 = [2, 2, \ldots, 2].$$

48. DENSCHNA (CUTE)

$$f(x) = \sum_{i=1}^{n/2} x_{2i-1}^4 + (x_{2i-1} + x_{2i})^2 + (-1 + \exp(x_{2i}))^2, \qquad x_0 = [1., 1., \ldots, 1.].$$

49. DENSCHNB (CUTE)

$$f(x) = \sum_{i=1}^{n/2} \left(x_{2i-1} - 2\right)^2 + \left(x_{2i-1} - 2\right)^2 x_{2i}^2 + \left(x_{2i} + 1\right)^2, \quad x_0 = [1, 1, \ldots, 1].$$

50. DENSCHNC (CUTE)

$$f(x) = \sum_{i=1}^{n/2} (-2 + x_{2i-1}^2 + x_{2i}^2)^2 + (-2 + \exp(x_{2i-1} - 1) + x_{2i}^3)^2, \qquad x_0 = [0.1, 0.1, \ldots, 0.1].$$

51. DENSCHNF (CUTE)

$$f(x) = \sum_{i=1}^{n/2} \left(2(x_{2i-1} + x_{2i})^2 + (x_{2i-1} - x_{2i})^2 - 8\right)^2 + \left(5x_{2i-1}^2 + (x_{2i} - 3)^2 - 9\right)^2,$$
$$x_0 = [2., 0., 2., 0., \ldots, 2., 0.].$$

52. SINQUAD (CUTE)

$$f(x) = (x_1 - 1)^4 + (x_n^2 - x_1^2)^2 + \sum_{i=1}^{n-2} (\sin(x_{i+1} - x_n) - x_1^2 + x_{i+1}^2)^2, \quad x_0 = [0.1, 0.1, \ldots, 0.1].$$

53. HIMMELBG (CUTE)

$$f(x) = \sum_{i=1}^{n/2} \left(2x_{2i-1}^2 + 3x_{2i}^2\right) \exp\left(-x_{2i-1} - x_{2i}\right), \quad x_0 = [1.5, 1.5, \ldots, 1.5].$$

54. HIMMELBH (CUTE)

$$f(x) = \sum_{i=1}^{n/2} (-3x_{2i-1} - 2x_{2i} + 2 + x_{2i-1}^3 + x_{2i}^2), \quad x_0 = [1.5, 1.5, \ldots, 1.5].$$

55. DIXON3DQ (CUTE)

$$f(x) = (x_1 - 1)^2 + \sum_{i=1}^{n-1} (x_i - x_{i+1})^2 + (x_n - 1)^2, \qquad x_0 = [-1., -1., \ldots, -1].$$

56. BIGGSB1 (CUTE)

$$f(x) = (x_1 - 1)^2 + (1 - x_n)^2 + \sum_{i=2}^{n} (x_i - x_{i-1})^2, \quad x_0 = [0,0,\ldots,0].$$

57. Perturbed Quadratic

$$f(x) = \left(\sum_{i=1}^{n} i x_i \right)^2 + \sum_{i=1}^{n} i x_i^2, \qquad x_0 = [0.5,0.5,\ldots,0.5],$$

58. GENROSNB (CUTE)

$$f(x) = (x_1 - 1)^2 + \sum_{i=2}^{n} 100(x_i - x_{i-1}^2)^2, \qquad x_0 = [-1.2,1,\ldots,-1.2,1].$$

59. QP1 Extended Quadratic Penalty

$$f(x) = \left(\sum_{i=1}^{n} x_i^2 - 0.5 \right)^2 + \sum_{i=1}^{n-1} (x_i^2 - 2)^2, \qquad x_0 = [1.,1.,\ldots,1.].$$

60. QP2 Extended Quadratic Penalty

$$f(x) = \left(\sum_{i=1}^{n} x_i^2 - 100 \right)^2 + \sum_{i=1}^{n-1} (x_i^2 - \sin(x_i))^2, \qquad x_0 = [1.,1.,\ldots,1.].$$

61. Tridiagonal TS1 (Staircase S1)

$$f(x) = \sum_{i=1}^{n-1} (x_i + x_{i+1} - i)^2, \qquad x_0 = [1.,1.,\ldots,1.].$$

62. Tridiagonal TS2 (Staircase S2)

$$f(x) = \sum_{i=2}^{n} (x_{i-1} + x_i - i)^2, \qquad x_0 = [1.,1.,\ldots,1.].$$

63. Tridiagonal TS3 (Staircase S3)

$$f(x) = \sum_{i=2}^{n} (x_{i-1} + x_i + i)^2, \qquad x_0 = [2.,2.,\ldots,2.].$$

64. Extended Trigonometric ET2

$$f(x) = \sum_{i=1}^{n} \left(\left(n - \sum_{i=1}^{n} \sin(x_i) \right) + i(1 - \sin(x_i)) - \sin(x_i) \right)^2, \quad x_0 = [0.2,0.2,\ldots,0.2].$$

65. QP3 Extended Quadratic Penalty

$$f(x) = \left(\sum_{i=1}^{n} x_i^2 - 0.25 \right)^2 - \sum_{i=1}^{n-1} (x_i^2 - 1)^2, \qquad x_0 = [1.,1.,\ldots,1.].$$

66. EG1

$$f(x) = \frac{1}{2} \cos(x_n^2) + \sum_{i=1}^{n-1} \cos(x_1 + x_i^2 - 1), \qquad x_0 = [1.,1.,\ldots,1.].$$

67. GENROSEN-2

$$f(x) = (x_1 - 1)^2 + \sum_{i=2}^{n} c(x_{i-1}^2 - x_i)^2, \qquad x_0 = [-1.2, 1, \ldots, -1.2, 1]. \qquad c = 100.$$

68. PRODsin

$$f(x) = \left(\sum_{i=1}^{m} x_i^2\right)\left(\sum_{i=1}^{n} \sin(x_i)\right), \quad x_0 = [1., 1., \ldots, 1.]. \quad m = n - 1.$$

69. PROD1 (m=n)

$$f(x) = \left(\sum_{i=1}^{m} x_i\right)\left(\sum_{i=1}^{n} x_i\right), \quad x_0 = [1., 1., \ldots, 1.]. \quad m = n.$$

70. PRODcos

$$f(x) = \left(\sum_{i=1}^{m} x_i^2\right)\left(\sum_{i=1}^{n} \cos(x_i)\right), \quad x_0 = [1., 1., \ldots, 1.]. \quad m = n - 1.$$

71. PROD2 (m=1)

$$f(x) = \left(\sum_{i=1}^{m} x_i^4\right)\left(\sum_{i=1}^{n} i x_i\right), \qquad x_0 = [15, 15, \ldots, 15]. \quad m = 1.$$

72. ARGLINB (m=5)

$$f(x) = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} ij x_j - 1\right)^2, \quad x_0 = [0.01, 0.001, \ldots, 0.001]. \quad m = 5.$$

73. DIXMAANA (CUTE)

$$f(x) = 1 + \sum_{i=1}^{n} \alpha\, x_i^2 \left(\frac{i}{n}\right)^{k1} + \sum_{i=1}^{n-1} \beta\, x_i^2 \left(x_{i+1} + x_{i+1}^2\right)^2 \left(\frac{i}{n}\right)^{k2} +$$
$$\sum_{i=1}^{2m} \gamma\, x_i^2 x_{i+m}^4 \left(\frac{i}{n}\right)^{k3} + \sum_{i=1}^{m} \delta\, x_i x_{i+2m} \left(\frac{i}{n}\right)^{k4},$$
$$m = n/4,$$

	α	β	γ	δ	k1	k2	k3	k4
A	1	0	0.125	0.125	0	0	0	0

$$x_0 = [2., 2., \ldots, 2.].$$

74. DIXMAANB (CUTE)

$$f(x) = 1 + \sum_{i=1}^{n} \alpha\, x_i^2 \left(\frac{i}{n}\right)^{k1} + \sum_{i=1}^{n-1} \beta\, x_i^2 \left(x_{i+1} + x_{i+1}^2\right)^2 \left(\frac{i}{n}\right)^{k2} +$$
$$\sum_{i=1}^{2m} \gamma\, x_i^2 x_{i+m}^4 \left(\frac{i}{n}\right)^{k3} + \sum_{i=1}^{m} \delta\, x_i x_{i+2m} \left(\frac{i}{n}\right)^{k4},$$
$$m = n/4,$$

	α	β	γ	δ	k1	k2	k3	k4
B	1	0.0625	0.0625	0.0625	0	0	0	1

$$x_0 = [2., 2., \ldots, 2.].$$

75. DIXMAANC (CUTE)

$$f(x)=1+\sum_{i=1}^{n}\alpha\, x_i^2\left(\frac{i}{n}\right)^{k1}+\sum_{i=1}^{n-1}\beta\, x_i^2\left(x_{i+1}+x_{i+1}^2\right)^2\left(\frac{i}{n}\right)^{k2}+$$
$$\sum_{i=1}^{2m}\gamma\, x_i^2 x_{i+m}^4\left(\frac{i}{n}\right)^{k3}+\sum_{i=1}^{m}\delta\, x_i x_{i+2m}\left(\frac{i}{n}\right)^{k4},$$
$$m=n/4,$$

	α	β	γ	δ	k1	k2	k3	k4
C	1	0.125	0.125	0.125	0	0	0	0

$$x_0=[2.,2.,\ldots,2.].$$

76. DIXMAAND (CUTE)

$$f(x)=1+\sum_{i=1}^{n}\alpha\, x_i^2\left(\frac{i}{n}\right)^{k1}+\sum_{i=1}^{n-1}\beta\, x_i^2\left(x_{i+1}+x_{i+1}^2\right)^2\left(\frac{i}{n}\right)^{k2}+$$
$$\sum_{i=1}^{2m}\gamma\, x_i^2 x_{i+m}^4\left(\frac{i}{n}\right)^{k3}+\sum_{i=1}^{m}\delta\, x_i x_{i+2m}\left(\frac{i}{n}\right)^{k4},$$
$$m=n/4,$$

	α	β	γ	δ	k1	k2	k3	k4
D	1	0.26	0.26	0.26	0	0	0	0

$$x_0=[2.,2.,\ldots,2.].$$

77. DIXMAANL (CUTE)

$$f(x)=1+\sum_{i=1}^{n}\alpha\, x_i^2\left(\frac{i}{n}\right)^{k1}+\sum_{i=1}^{n-1}\beta\, x_i^2\left(x_{i+1}+x_{i+1}^2\right)^2\left(\frac{i}{n}\right)^{k2}+$$
$$\sum_{i=1}^{2m}\gamma\, x_i^2 x_{i+m}^4\left(\frac{i}{n}\right)^{k3}+\sum_{i=1}^{m}\delta\, x_i x_{i+2m}\left(\frac{i}{n}\right)^{k4},$$
$$m=n/4,$$

	α	β	γ	δ	k1	k2	k3	k4
L	1	0.26	0.26	0.26	2	0	0	2

$$x_0=[2.,2.,\ldots,2.].$$

78. VARDIM (CUTE)

$$f(x)=\sum_{i=1}^{n}(x_i-1)^2+\left(\sum_{i=1}^{n}ix_i-\frac{n(n+1)}{2}\right)^2+\left(\sum_{i=1}^{n}ix_i-\frac{n(n+1)}{2}\right)^4,$$
$$x_0=\left[1-\frac{1}{n},1-\frac{2}{n},\ldots,1-\frac{n}{n}\right].$$

79. DIAG-AUP1

$$f(x)=\sum_{i=1}^{n}4(x_i^2-x_1)^2+(x_i^2-1)^2,\quad x_0=[4,4,\ldots,4].$$

80. ENGVAL8

$$f(x) = \sum_{i=1}^{n-1} (x_i^2 + x_{i+1}^2)^2 - (7 - 8x_i), \qquad x_0 = [2, 2, \ldots, 2].$$

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