The conjugate gradient method closest to the scaled memoryless BFGS preconditioned with standard, approximate and improved Wolfe line search.

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Abstract. Two line search procedures, namely the improved Wolfe line search of Dai and Kou (2013) versus the approximate Wolfe line search of Hager and Zhang (2005) are compared in the frame of the Dai and Kou conjugate gradient algorithm. Intensive numerical experiments showed that the improved Wolfe line search is an improvement of the performances of the Dai and Kou conjugate gradient algorithm versus the approximate Wolfe line search.

1. Introduction

Consider the unconstrained optimization problem

$$\min f(x), \tag{1}$$

where $f:\mathbb{R}^n \to \mathbb{R}$ is a continuous differentiable function and the gradient g is available. Suppose that:

(i) The function f is bounded below, namely $f(x) > -\infty$ for all $x \in \mathbb{R}^n$.

(ii) The function f is differentiable and its gradient g is Lipschitz continuous, namely, there exists a positive constant $L > 0$ such that

$$\|g(x) - g(y)\| \le L\|x - y\|,$$

for all $x, y \in \mathbb{R}^n$, where $\|.\|$ stands for the Euclidian norm.

One of the most important methods for solving (1) is the conjugate gradient method. For a given initial point x_0, a conjugate gradient method generates the sequence $\{x_k\}$ defined by:

$$x_{k+1} = x_k + \alpha_k d_k, \qquad k = 0,1,\ldots, \tag{2}$$

where the sequence of the search directions $\{d_k\}$ is defined by:

$$d_{k+1} = -g_{k+1} + \beta_k d_k, \quad k = 0,1,\ldots, \tag{3}$$

where $d_0 = -g_0 = -\nabla f(x_0)$. The scalar α_k in (2) is the stepsize, often computed by the standard Wolfe conditions:

$$f(x_k+\alpha_k d_k)\le f(x_k)+\rho\alpha_k d_k^T g_k, \tag{4}$$

$$\nabla f(x_k+\alpha_k d_k)^T d_k \ge \sigma d_k^T g_k, \tag{5}$$

where $0<\rho<\sigma<1$. (See Wolfe, 1969, 1971.) The conjugate gradient parameter β_k is a scalar which defines the conjugate gradient algorithm. There is a great variety of conjugate gradient methods. All of them are identified after the procedure for β_k computation. Some of them are known as *standard conjugate gradient* methods: HS (Hestenes & Stiefel, 1952), FR (Fletcher & Reeves, 1964), PRP (Polak & Ribiére, 1969; Polyak, 1969), CD (Fletcher, 1987), LS (Liu & Storey, 1991) and DY (Dai & Yuan, 1999). If function f is strongly convex quadratic and the line search is exact, i.e. $\alpha_k=\arg\min_{\alpha\ge 0} f(x_k+\alpha d_k)$, then in theory, all choices for the update parameter β_k in standard conjugate gradient methods are equivalent. For non-quadratic objective functions, each choice for β_k leads to algorithms with different numerical performances (number of iterations, number of function and its gradient evaluations or CPU time).

Other methods proposed are known as the *hybrid conjugate gradient* methods and the *parameterized conjugate gradient* methods. The hybrid methods are based on the *projection* idea: (Touati-Ahmen & Storey, 1990), (Hu & Storey, 1991), (Gilbert & Nocedal, 1992), (Dai & Yuan, 2001) and (Yabe & Takano, 2004). Others are based on the idea of *convex combination* of the standard conjugate gradient methods: (Andrei, 2008b, 2008d, 2008e, 2009a, 2010), (Liu & Li, 2014).

New conjugate gradient methods were suggested as being based on different ingredients in order to improve their convergence to the solution and to reduce jamming. Thus, the following methods can be mentioned:

- *modified of the standard schemes* (Andrei, 2009b), (Dai & Liao, 2001), (Hager & Zhang, 2005) and (Dai & Kou, 2013);
- *memoryless conjugate gradient methods BFGS preconditioned* (Perry, 1976), (Shanno, 1978a, 1978b) and (Andrei, 2007a, 2017a);
- *conjugate gradient method with guaranteed descent* (Hager & Zhang, 2005, 2006a);
- *spectral conjugate gradient methods* (Birgin & Martínez, 2001);
- *scaled conjugate gradient methods* (Andrei, 2007a, 2007b, 2007c, 2008a);
- *three-term conjugate gradient methods* (Beale, 1972), (Nazareth, 1977), (Andrei, 2007a, 2011, 2013a, 2013b, 2015), (Babaie-Kafaki & Ghanbari, 2014);
- *conjugate gradient methods with sufficient descent* (Andrei, 2008c);
- *conjugate gradient methods with modified secant equation* (Andrei, 2008b);
- *conjugate gradient method with finite difference Hessian/vector product approximation* (Andrei, 2008f, 2009c).

Intensive numerical experiments proved that the conjugate gradient algorithms are reliable for solving large-scale unconstrained optimization problems. The best performances are obtained by the modified of the standard schemes CONMIN given by Shanno (1978a, 1983), SCALCG proposed by Andrei (2007a, 2007b, 2007c, 2008a), CG-DESCENT elaborated by Hager and Shanno (2005), DESCON proposed by Andrei, (2013c), etc.

In this Technical Report we are interested to present the performances of a conjugate gradient algorithm proposed by Dai and Kou (2013) subject to the procedures for the stepsize computation. The conjugate gradient algorithm by Dai and Kou is a simple projection of the self-scaling memoryless BFGS direction by Perry and Shanno into the one-dimensional manifold

$$S_{k+1}=\{-g_{k+1}+\beta d_k : \beta\in\mathbb{R}\}.$$

The obtained conjugate gradient parameter β_k corresponding to this algorithm differs from the Hager and Zhang algorithm only in a constant coefficient in the second term.

2. Conjugate gradient method closest to scaled memoryless BFGS

A new family of conjugate gradient methods was derived by Dai and Kou (2013). They suggested using the self-scaling memoryless BFGS method by Perry and Shanno in which the search direction is defined as

$$d_{k+1} = -H_{k+1} g_{k+1}, \tag{6}$$

where

$$H_{k+1} = \frac{1}{\tau_k}\left(I - \frac{s_k y_k^T + y_k s_k^T}{y_k^T s_k} \right) + \left(1 + \frac{1}{\tau_k} \frac{\|y_k\|^2}{y_k^T s_k} \right) \frac{s_k s_k^T}{y_k^T s_k} \tag{7}$$

and τ_k is a scaling parameter. Observe that the matrix Hessian H_{k+1} in (6) is obtained by setting $H_k = (1/\tau_k) I,$ namely the scaled identity matrix, in the BFGS update to the inverse Hessian:

$$H_{k+1} = H_k - \frac{s_k y_k^T H_k + H_k y_k s_k^T}{y_k^T s_k} + \left(1 + \frac{y_k^T H_k y_k}{y_k^T s_k} \right) \frac{s_k s_k^T}{y_k^T s_k}. \tag{8}$$

Now, substituting (7) in (6) leads to the Perry-Shanno search direction with a multiplier difference

$$d_{k+1}^{PS} = -g_{k+1} + \left(\frac{y_k^T g_{k+1}}{y_k^T s_k} - \left(\tau_k + \frac{\|y_k\|^2}{y_k^T s_k} \right) \frac{s_k^T g_{k+1}}{y_k^T s_k} \right) s_k + \frac{s_k^T g_{k+1}}{y_k^T s_k} y_k. \tag{9}$$

By simple deletion of the last term in (9), Dai and Kou obtained a conjugate gradient algorithm

$$d_{k+1}^{DK} = -g_{k+1} + \beta_k^{DK}(\tau_k) s_k, \tag{10}$$

where

$$\beta_k^{DK}(\tau_k) = \frac{y_k^T g_{k+1}}{y_k^T s_k} - \left(\tau_k + \frac{\|y_k\|^2}{y_k^T s_k} \right) \frac{s_k^T g_{k+1}}{y_k^T s_k}. \tag{11}$$

As suggested by Oren and Spedicato (1976), if τ_k in (11) is chosen as

$$\tau_k^H = \frac{\|y_k\|^2}{(y_k^T s_k)}, \tag{12}$$

then (11) corresponds to the Hager and Zhang formula for updating the conjugate gradient parameter. Other values for the scaling parameter τ_k in (11) were proposed as follows. Oren (1974) and Oren and Luenberger (1974) proposed for τ_k the value $y_k^T s_k / (s_k^T B_k s_k)$ with $B_k = H_k^{-1}.$ If H_k is the identity matrix, then this value reduces to

$$\tau_k^B = \frac{y_k^T s_k}{\|s_k\|^2}. \tag{13}$$

Al-Baali (1998) suggested the following two choices:

$$\bar{\tau}_k^H = \min\left\{1, \frac{\|y_k\|^2}{y_k^T s_k}\right\} \quad \text{and} \quad \bar{\tau}_k^B = \min\left\{1, \frac{y_k^T s_k}{\|s_k\|^2}\right\}. \tag{14}$$

However, a more reasonable way to deal with the last term in (9) instead of its simple deletion was proposed by Dai and Kou (2013). They suggested projecting the self-scaling memoryless BFGS direction by Perry and Shanno into the one-dimensional manifold

$$S_{k+1} = \{-g_{k+1} + \beta d_k : \beta \in \mathbb{R}\}.$$

Specifically, the search direction from S_{k+1} is chosen to be closest to the Perry/Shanno direction d_{k+1}^{PS}:

$$d_{k+1}^{DK} = \arg\min\{\|d - d_{k+1}^{PS}\|_2 : d \in S_{k+1}\}. \tag{15}$$

Thus, a relatively simple formula for the conjugate gradient parameter was obtained as

$$\beta_k^{DK}(\tau_k) = \frac{y_k^T g_{k+1}}{s_k^T y_k} - \left(\tau_k + \frac{\|y_k\|^2}{s_k^T y_k} - \frac{s_k^T y_k}{\|s_k\|^2}\right)\frac{s_k^T g_{k+1}}{s_k^T y_k}. \tag{16}$$

If the line search is exact, $s_k^T g_{k+1} = 0,$ then the second term in (16) is missing and the search direction reduces to the HS formula.

It is worth seeing that (16) with (12) or (13) or (14) is exactly the Dai and Liao formula

$$\beta_k^{DL} = \frac{g_{k+1}^T y_k - t g_{k+1}^T s_k}{y_k^T d_k}, \tag{17}$$

where in this case t is adaptively modified as $t = \tau_k - \|y_k\|^2 / (y_k^T s_k),$ where τ_k is given by (12), (13) or (14).

Similarly to Gilbert and Nocedal (1992), who proved the convergence of the PRP method for general functions by restricting $\beta_k \geq 0,$ Dai and Kou defined a truncation form of the conjugate gradient parameter from (16) as:

$$\beta_k^{DK+}(\tau_k) = \max\left\{\beta_k^{DK}(\tau_k), \eta \frac{s_k^T g_{k+1}}{\|s_k\|^2}\right\}, \quad \eta \in [0,1). \tag{18}$$

Hence, the Dai-Kou conjugate gradient algorithm belongs to the same family of conjugate gradient methods obtained from the memoryless BFGS method by Perry and Shanno. Under the

assumption *(i)* and *(ii)*, the family of conjugate gradient methods (10) and (16) generate sufficient descent directions. Dai and Kou (2013) noticed that: *numerical experiments with this family of conjugate gradient algorithms showed that the most efficient is the one where the parameter* τ_k *is given by (13).* Therefore, substituting this choice of τ_k in (16) and (18) we get

$$\beta_k^{DK} = \frac{y_k^T g_{k+1}}{s_k^T y_k} - \frac{\|y_k\|^2}{s_k^T y_k} \frac{s_k^T g_{k+1}}{s_k^T y_k} \tag{19}$$

and

$$\beta_k^{DK+} = \max\left\{ \frac{y_k^T g_{k+1}}{s_k^T y_k} - \frac{\|y_k\|^2}{s_k^T y_k} \frac{s_k^T g_{k+1}}{s_k^T y_k}, \eta \frac{s_k^T g_{k+1}}{\|s_k\|^2} \right\}, \tag{20}$$

where $\eta \in [0,1)$, respect tively. ($\eta = 0.5$ in our experiments.) Observe that (19) is exactly the Dai-Liao conjugate gradient algorithm with $t = \|y_k\|^2 / (s_k^T y_k)$. Again, note that (19) differs from the Hager and Zhang algorithm only in a constant coefficient in the second term. Under the classical assumptions *(i)* and *(ii)*, Dai and Kou (2013) proved the convergence of this computational scheme.

3. Numerical experiments

In the following, let us present the performances of the conjugate gradient algorithm

$$x_{k+1} = x_k + \alpha_k d_k^{DK+}, \qquad k = 0,1,\ldots, \tag{21}$$

where the search direction is given by

$$d_{k+1}^{DK+} = -g_{k+1} + \beta_k^{DK+}(\tau_k) s_k, \tag{22}$$

and β_k^{DK+} is computed as in (20). The algorithm (21), (22) with (20) is called DK+. In the following we shall consider some variants of this algorithm according to the procedure for the stepsize α_k computation in (21).

The algorithms were tested using the unconstrained optimization problems from UOP collection described in Andrei (2018). The UOP collection includes 80 unconstrained optimization problems with different structures and complexities. In this collection, some problems are quadratic and some of them are highly nonlinear. The problems are presented in extended (separable) or generalized (chained) form. The Hessian for the problems in extended form has a block-diagonal structure. On the other hand, the Hessian for the problems in generalized form has a banded structure with small bandwidth, often being tri- or penta- diagonal. For some other optimization problems from this set, the corresponding Hessian has a sparse structure or it is a dense (full) matrix. For each test problem, 10 numerical experiments were run with the number of variables: $n = 1000,\ldots,10,000$. Thus in each numerical experiment 800 have been solved.

All algorithms have been coded in double precision Fortran and compiled with f77 (default compiler settings) and run on an Intel Pentium 4, 1.8 GHz workstation. The algorithms we compare in these numerical experiments find local solutions. Therefore, the comparisons of the algorithms are given in the following context. Let f_i^{ALG1} and f_i^{ALG2} be the optimal value found by ALG1 and ALG2 for problem $i = 1,\ldots,800$, respectively. We say that, in the particular problem i, the performance of ALG1 was better than the performance of ALG2 if:

$$\left| f_i^{ALG1} - f_i^{ALG2} \right| < 10^{-3} \tag{23}$$

and if the number of iterations (#iter), or the number of function-gradient evaluations (#fg), or the CPU time of ALG1 was less than the number of iterations, or the number of function-gradient evaluations, or the CPU time corresponding to ALG2, respectively. The standard Wolfe line search (4) and (5) are implemented with $\rho = 0.0001$ and $\sigma = 0.8$. The initial value for the stepsize is computed as $\alpha_k^0 = \alpha_{k-1} \|d_{k-1}\| / \|d_k\|$. The iterations are stopped if the inequality $\|g_k\|_\infty \le 10^{-6}$ is satisfied, where $\|.\|_\infty$ is the maximum absolute component of a vector. The maximum number of iterations was limited to 2000. To compare the performances of algorithms, the Dolan and Moré (2002) performance profiles are used. Appendixes 1-3 contain the results of the Dai-Kou algorithm with standard Wolfe, approximate Wolfe and improved Wolfe line search, respectively.

3.1. DK+ with standard Wolfe line search

In this section we present the numerical performances of DK+ with the standard Wolfe line search (4) and (5), denoted as DK+w, for solving 800 problems from the UOP collection. Figure 1 presents the Dolan and Moré performance profiles of DK+w with standard Wolfe line search versus CONMIN, SCALCG (spectral), CG-DESCENT and DESCONa. The algorithms we compare versus DK+w are as follows. CONMIN is a conjugate gradient memoryless BFGS preconditioned (Shanno, 1983). SCALCG is a scaling conjugate gradient memoryless BFGS preconditioned (Andrei, 2007a, 2007b, 2007c, 2008a). CG-DESCENT is a conjugate gradient algorithm with guaranteed descent (Hager and Zhang, 2005). DESCONa is an accelerated conjugate gradient algorithm with guaranteed descent and conjugacy conditions and a modified Wolfe line search (Andrei, 2013c).

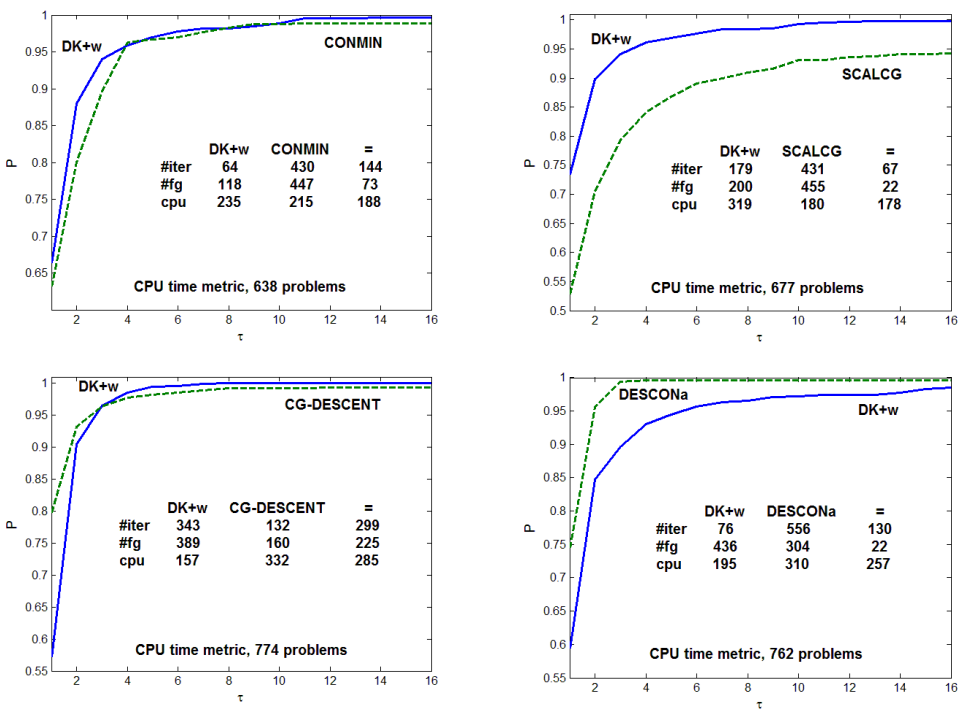

Fig. 1 Performance profiles of DK+w versus CONMIN, SCALCG (spectral), CG-DESCENT and DESCONa

From Figure 1 we see that among these algorithms, DK+w is top performer versus CONMIN and SCALCG. However, it is less efficient than CG-DESCENT. Observe that DESCONa is way more efficient and more robust than DK+w.

3.2. DK+ with approximate Wolfe line search

To compare the algorithms we manufactured a new code implementing (20), (21) and (22), where the stepsize is determined by the approximate Wolfe conditions

$$\sigma d_k^T g_k \le d_k^T g_{k+1} \le (2\rho - 1) d_k^T g_k, \tag{24}$$

introuced by Hager and Zhang (2005), where $0 < \rho < 1/2$ and $\rho < \sigma < 1$. The first inequality in (24) is the same as (5). When f is quadratic, the second inequality in (24) is equivalent to (4). In general, when $\varphi_k(\alpha) = f(x_k + \alpha d_k)$ is replaced by a quadratic interpolant $q(.)$ that matches $\varphi_k(\alpha)$ at $\alpha = 0$ and $\varphi_k'(\alpha)$ at $\alpha = 0$ and $\alpha = \alpha_k$, (4) reduces to the second inequality in (24). As shown by Hager and Zhang (2005), the first Wolfe condition (4) limits the accuracy of a conjugate gradient method to the order of the square root of the machine precision, while with the approximate Wolfe line search we can achieve accuracy to the order of the machine precision. Hager and Zhang (2006a) showed that faster convergence is often obtained when using the approximate Wolfe line search in conjugate gradient methods, since a local minimizer of $\varphi_k(\alpha)$ satisfies (24), while a point satisfying the standard or the strong Wolfe line search is obtained by computing a local minimizer of the approximating function $\phi_k(\alpha) = \varphi_k(\alpha) - \varphi_k(0) - \alpha\rho\varphi_k'(0)$ introduced by Moré and Thuente (1994). The difference is significant. When the approximate Wolfe line search is used, the function f is minimized along the search direction d_k rather than an approximation $\phi_k(\alpha)$ to f (Hager & Zhang, 2006b).

The algorithm using (21) and (22) implementing the approximate Wolfe line search (24) is called DK+aw (DK+ with approximate Wolfe line search). Let us denote CG-DESCENTaw the CG-DESCENT algorithm with approximate Wolfe line search (Hager and Zhang, 2005).

Figure 2 shows the performance profiles of DK+aw versus CONMIN, SCALCG (spectral), CG-DESCENTaw and DESCONa. From Figure 2 we see that DK+aw is more efficient and more robust than CONMIN and SCALCG used in this comparison. Observe that CG-DESCENTaw is top performer versus DK+aw, being slightly more robust. From Figure 2 we see that DESCONa which is an accelerated conjugate gradient algorithm with guaranteed descent and conjugacy conditions which implements a modified Wolfe line search is top performer versus DK+aw. The modified Wolfe line search used in DESCONa consists in modifying the curvature condition (5) where the parameter σ is not maintained at a fixed value along the iterations. Instead, at each iteration it is computed as:

$$\sigma_k = \frac{\|g_{k+1}\|^2}{|y_k^T g_{k+1}| + \|g_{k+1}\|^2}. \tag{25}$$

There is a difference between the performances of DK+w (DK+ with standard Wolfe line search) versus CONMIN, SCALCG, CG-DESCENT and DESCONa (see Figure 1) and the performances of DK+aw (DK+ with approximate Wolfe line search) versus the algorithms mentioned above (see Figure 2). These numerical experiments emphasize the importance of the line search in conjugate gradient methods. We see that conjugate gradient methods are very sensitive to the procedures for stepsize computation. This is in sharp contrast with the Newton or quasi-Newton methods for which, for the vast majority of iterations, the stepsize is equal to 1.

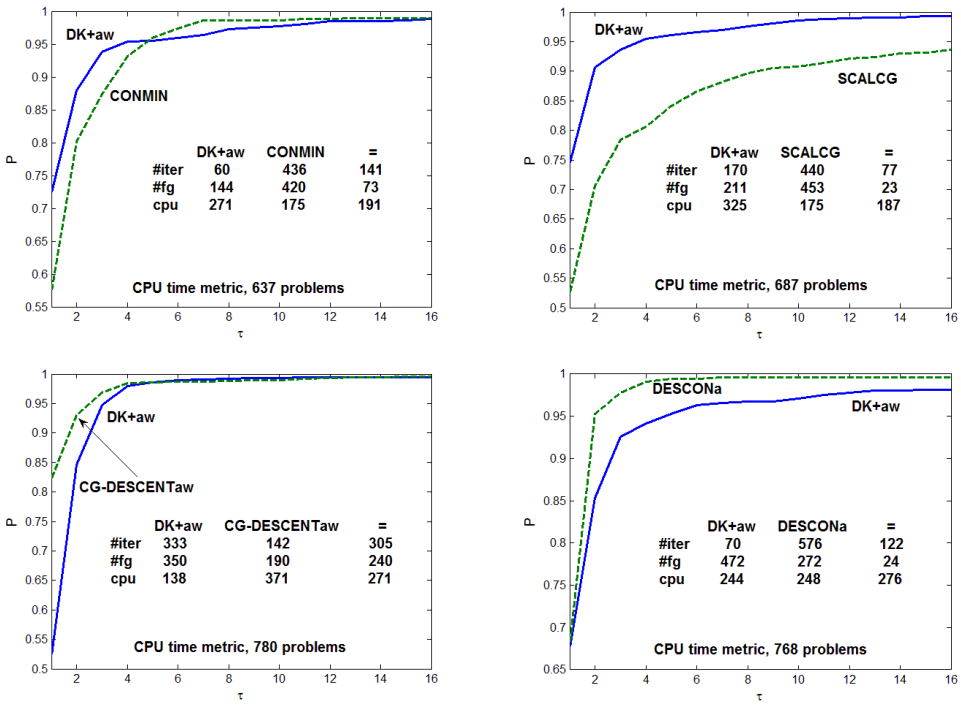

Fig. 2 Performance profiles of DK+aw versus CONMIN, SCALCG (spectral), CG-DESCENTaw and DESCONa

3.3. DK+ with improved Wolfe line search

The improved Wolfe line search was introduced by Dai and Kou (2013). This new line search consists in the modified first Wolfe condition as

$$\phi_k(\alpha) \le \phi_k(0) + \min\{\varepsilon|\phi_k'(0)|, \rho\alpha\phi_k'(0) + \eta_k\}, \tag{26}$$

where $\phi_k(\alpha) = f(x_k + \alpha d_k)$, $\varepsilon > 0$ is a given constant and $\{\eta_k\}$ is a positive sequence satisfying $\sum_{k\ge1} \eta_k < \infty$, and in the second Wolfe line search

$$\phi_k'(\alpha) \ge \sigma\phi_k'(0), \tag{27}$$

where $0 < \rho < \sigma < 1$. Observe that $\phi'(0) = g_{k+1}^T d_k$, where $g_{k+1} = \nabla f(x_{k+1})$.

The improved Wolfe line search (26) is implemented with $\varepsilon = 10^{-6}$ and $\eta_k = 1/(k^2)$, where k is the iteration number. The algorithm given by (21), (22) with (20) and implementing the improved Wolfe line search (26) and (27) is called DK+iw (DK+ with improved Wolfe line search). To see the performances of DK+iw we manufactured a new code implemented (20), (21) and (22) with improved line search (26) and (27). Figure 3 shows the Dolan and Moré performance profiles of DK+iw versus DK+w (DK+ with standard Wolfe line search) and versus DK+aw (DK+ with approximate Wolfe line search of Hager and Zhang).

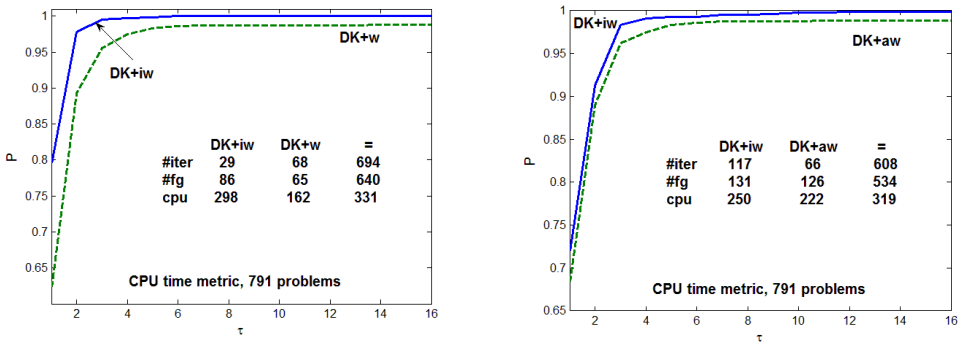

Fig. 3 Performance profiles of DK+iw versus DK+w and versus DK+aw

We see that DK+iw is more efficient and more robust than DK+w. In other words, in the context of the conjugate gradient algorithms, the improved line search (26) represents a real improvement versus the standard Wolfe line search. Also, from Figure 3 we have the computational evidence that DK+iw is more efficient and more robust than DK+aw.

In Figure 4 we present the performance profiles of DK+iw versus CONMIN, SCALCG (spectral), CG-DESCENTaw and DESCONa. We see that this variant of DK+ with improved Wolfe line search is less efficient than CG-DESCENTaw (CG-DESCENT with approximate Wolfe line search). Observe that DESCONa which is a conjugate gradient algorithm with guaranteed descent and conjugacy conditions and with a modified Wolfe line search is top performer, being more efficient and more robust than DK+iw.

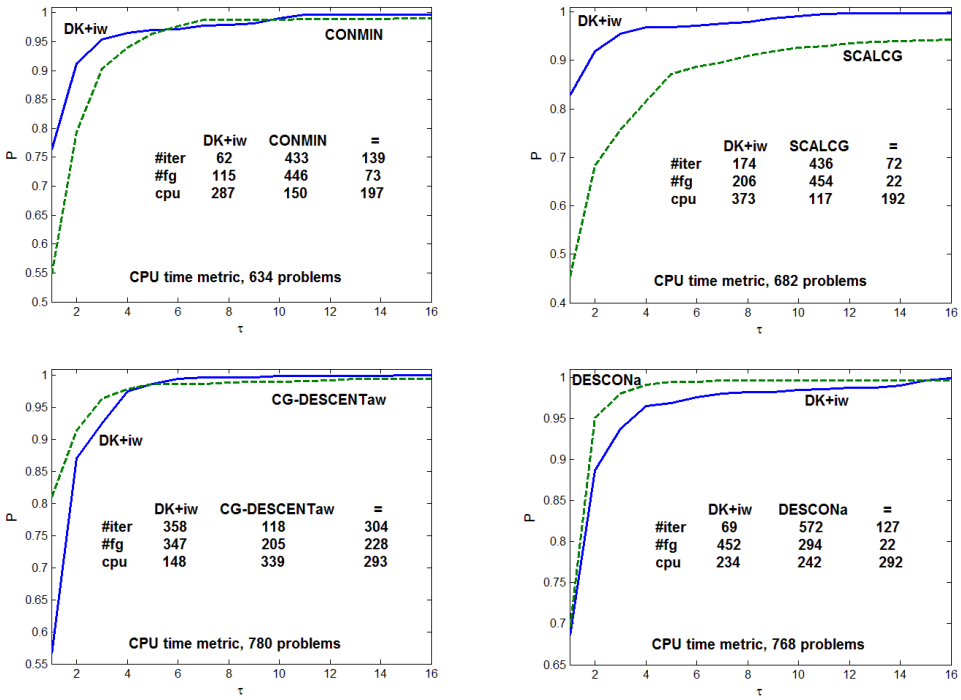

Fig. 4 Performance profiles of DK+iw versus CONMIN, SCALCG (spectral), CG-DESCENT and DESCONa

By comparing Figures 2 and 4 we see that DK+iw has better performances than DK+aw. Both the approximate Wolfe line search condition (24) and the improved Wolfe line search condition (26) have been introduced to avoid the numerical drawback of the first Wolfe line search condition responsible with the sufficient reduction of the objective function values. The second inequality in (24) is an approximation to the first Wolfe condition. When the iterates are near a local minimum, this approximation can be evaluated with greater accuracy than the original condition, since the approximate Wolfe conditions are expressed in terms of the derivative of function $\phi_k(\alpha) = f(x_k + \alpha d_k)$, not as the difference of function values which numerically may be rather unstable. On the other hand, since the improved Wolfe line search (26) is expressed as the difference of the function values $\phi_k(\alpha) - \phi_k(0)$, the numerical instabilities are not avoided. The extra positive term η_k in (26) which allows a slight increase in the function value compensates this deficiency. Observe that the improved Wolfe line search conditions (26) and (27) are weaker that the standard Wolfe line search (4) and (5).

In the second set of numerical experiments we present the performances of DK+w and DK+aw for solving five applications from the MINPACK-2 collection. These applications are taken from Averick, Carter, & Moré, (1991) and from Averick, Carter, Moré, & Xue, (1992). Table 1 presents the characteristics of the applications.

Table 1 Characteristics of the MINPACK-2 applications

	Applications	Parameters
A1	Elastic plastic torsion	$c = 5$
A2	Pressure distribution in a journal bearing	$b = 10,\ \ \varepsilon = 0.1$
A3	Optimal design with composite materials	$\lambda = 0.008$
A4	Steady-state combustion	$\lambda = 5$
A5	Minimal surface with Enneper conditions	-

Table 2 presents the performances of DK+w and DK+aw, for solving these applications, each of them with 40,000 variables ($nx = 200$ and $ny = 200$), where *n* is the number of variables, *#iter* is the number of iterations, *#fg* is the number of function and its gradient evaluations and *cpu* is CPU time to get a solution.

Table 2 Performances of DK+w and DK+aw for solving five applications from the MINPACK-2 collection

		DK+w			DK+aw		
	n	*#iter*	*#fg*	*cpu*	*#iter*	*#fg*	*cpu*
A1	40000	406	637	7.92	323	647	7.17
A2	40000	940	1508	20.54	791	1583	32.64
A3	40000	4001	6260	182.63	987	1976	65.87
A4	40000	670	1065	84.22	435	871	80.42
A5	40000	417	654	8.80	289	580	9.54
Total	-	6434	10124	304.11	2825	5657	195.64

From Table 2 we see that DK+aw (DK+ with approximate Wolfe line search) is top performer versus DK+w (DK+ with Wolfe line search). We see that DK+aw is 1.5 times faster than DK+w.

Table 3 contains the total performances of LBFGS ($m = 5$) (Liu & Nocedal, 1989), of TN (Nash, 1985), of CONMIN (Shanno, 1983), of SCALCG (θ_k - anticipative), of SCALCG (θ_k - spectral) (Andrei, 2007a, 2007b, 2007c, 2008a), of CG-DESCENTaw (Hager and Zhang, 2005), DESCONa (Andrei, 2013c), DK+w and DK+aw (see Table 2) for solving all five applications from the MINPACK-2 collection, each of them with 40,000 variables. See (Andrei, 2019).

Table 3 The total performances of LBFGS ($m = 5$), TN, CONMIN, SCALCG, CG-DESCENTaw, DESCONa, DK+w and DK+aw for solving five applications from the MINPACK-2 collection with 40,000 variables

	#iter	*#fg*	*cpu*
LBFGS ($m = 5$)	4842	4987	102.92
TN	153	3714	104.57
CONMIN	3022	6117	120.17
SCALCG (spectral)	3821	4865	105.47
SCALCG (anticipative)	4090	5212	107.14
CG-DESCENTaw	2875	5756	186.34
DESCONa	2485	5115	78.99
DK+w	6434	10124	304.11
DK+aw	2825	5657	195.64

From Table 3, by comparing these algorithms we see that DESCONa (the accelerated version of DESCON) is top performer.

In the following, let us see the performances of the above described conjugate gradient algorithms for solving large-scale applications from the MINPACK-2 collection, each with 250,000 variables ($nx = 500$ and $ny = 500$). The performances of DK+w and of DK+aw, for solving the five applications from MINPACK-2 collection, each of them with 250,000 variables are presented in Table 4.

Table 4 Performances of DK+w and DK+aw for solving five large-scale applications from the MINPACK-2 collection

		DK+w			DK+aw		
	n	*#iter*	*#fg*	*cpu*	*#iter*	*#fg*	*cpu*
A1	250000	693	1093	107.58	613	1227	145.21
A2	250000	2299	3650	413.70	1762	3525	457.22
A3	250000	4001	6257	1048.29	2354	4710	1007.74
A4	250000	1396	2211	846.27	923	1847	937.49
A5	250000	931	1455	227.40	622	1245	202.24
Total	-	9320	14666	2643.24	6274	12554	2749.90

Table 5 presents the performances of LBFGS ($m = 5$), of TN, of CONMIN, of SCALCG (θ_k -spectral), of SCALCGa (θ_k -spectral and accelerated), of CG-DESCENTaw, DESCONa, DK+w and DK+aw (see Table 4) for solving all five applications from the MINPACK-2 collection, each of them with 250,000 variables. See (Andrei, 2019).

Table 5 The total performances of LBFGS ($m = 5$), TN, CONMIN, SCALCG, CG-DESCENTaw, DESCONa, DK+w and DK+aw for solving five applications from the MINPACK-2 collection with 250,000 variables

	#iter	*#fg*	*cpu*
LBFGS ($m = 5$)	11295	11581	2455.07
TN	252	8433	2068.62
CONMIN	7227	14666	2600.36
SCALCG (spectral)	9691	12287	2473.99
SCALCGa (accelerated)	6091	12358	2258.38
CG-DESCENTaw	6292	12590	2686.67
DESCONa	5810	11780	1537.55
DK+w	9320	14666	2643.24
DK+aw	6274	12554	2749.90

Observe that both DK+w and DK+aw are less efficient versus these conjugate gradient algorithms. The fastest is DESCONa which is a conjugate gradient algorithm with guaranteed sufficient descent and conjugacy condition which implements a modified Wolfe line search condition (25). More details on the performances of conjugate gradient algorithms and comparisons may be found in Andrei (2019).

4. Conclusion

Our interests were to see the performances of a recent conjugate gradient algorithm proposed by Dai and Kou (2013), subject to the procedure for stepsize computation. It is known that conjugate gradient algorithms are very sensitive to the stepsize computation. To improve the performances the following two procedures for stepsize computation have been proposed: the approximate Wolfe line search of Hager and Zhang (2005), given by (24) and the improved Wolfe line search of Dai and Kou (2013), given by (26) and (27). In order to see the performances of these two procedures for stepsize computation we considered the conjugate gradient algorithm DK+ of Dai and Kou (2013). We observed that the improved Wolfe line search represents an improvement of the performances of conjugate gradient algorithm DK+ versus the approximate Wolfe line search. The improved Wolfe line search does not avoid the numerical difficulties of the first Wolfe condition responsible with the reduction of the function values. However by including a parameter η_k it compensates the numerical difficulties associated to the first Wiolfe condition.

As an additional comment, observe that the DK+ algorithm given by (21) and (22) is not obtained from a principle. Instead, it is derived from (16) using a particular value for parameter τ_k obtained from trial and error. Specifically, the search direction in DK+ is chosen to be closest to the Perry/Shanno direction. For the scaling parameter τ_k many formulae may be proposed. From numerical experiments, Dai and Kou (2013) arrived at the value given by (13) as a good one, but not necessarily the best. It is remarkable that the parameter β_k in the search direction of DK is a simple modification with a constant coefficient of the second term of the Hager and Zhang algorithm. Another point which must be emphasized is that Dai and Kou introduced a special truncation procedure which improves the performances of the algorithm.

The conclusion of the numerical experiments we presented (see the Appendixes) is that the performances of the conjugate gradient methods are less dependent by the formula for computation the conjugate gradient parameter β_k, they are strongly dependent by the procedure for stepsize computation, and possible by some other extra ingredients like: initial estimation of the stepsize, restart strategy, truncation of β_k procedures, stopping criteria, etc. Anyway, the critical (weak) point in conjugate gradient methods remains the stepsize computation.

Appendix 1

Dai-Kou conjugate gradient algorithm
Standard Wolfe line search (WLS=Wolfe line search)

1 Dai-Kou. Standard WLS. Function:Extended Freudenstein & Roth

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	12	124	115	1	0.2449212683962E+05	0.2747155019023E-06	4
2000	12	27	18	1	0.4898425367924E+05	0.5934663800389E-06	0
3000	12	125	116	2	0.7347638051886E+05	0.1322588740749E-06	4
4000	13	28	19	1	0.9796850735847E+05	0.5439626207249E-06	0
5000	11	25	16	1	0.1224606341981E+06	0.9960185707314E-06	0
6000	13	28	19	2	0.1469527610377E+06	0.1077915534609E-07	0
7000	14	130	122	7	0.1714448878773E+06	0.3552912630767E-08	4
8000	12	27	18	1	0.1959370147170E+06	0.8223763394710E-06	0
9000	12	29	19	3	0.2204291415566E+06	0.7852941763531E-06	0
10000	13	29	20	2	0.2449212683962E+06	0.4149569576839E-11	0

TOTAL 124 572 482 0.21(seconds)

2 Dai-Kou. Standard WLS. Function:Extended White & Holst

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	11	25	15	0	0.9044668587886E-12	0.8730601372086E-07	0
2000	11	25	15	0	0.1808936493812E-11	0.8730607766970E-07	0
3000	11	25	15	1	0.2713399640957E-11	0.8730599640138E-07	0
4000	11	25	15	0	0.3617867207512E-11	0.8730600705952E-07	0
5000	11	25	15	1	0.4522306573876E-11	0.8730574238235E-07	0
6000	11	25	15	1	0.5426825004142E-11	0.8730619268881E-07	0
7000	11	25	15	1	0.6331175912787E-11	0.8730536046563E-07	0
8000	11	25	15	1	0.7235638223072E-11	0.8730540309819E-07	0
9000	11	25	15	1	0.8140106425094E-11	0.8730546527068E-07	0
10000	11	25	15	1	0.9044544089235E-11	0.8730537512057E-07	0

TOTAL 110 250 150 0.07(seconds)

3 Dai-Kou. Standard WLS. Function:Tridiagonal White-Holst

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	2001	4050	2050	30	0.2729207083850E+03	0.2297806995082E+01	2
2000	2001	4052	2052	59	0.1055995942786E+04	0.2707425616411E+01	2
3000	2001	4054	2054	99	0.1837873373146E+04	0.2461529140088E+01	2
4000	2001	4044	2044	130	0.2620251767189E+04	0.2224944468043E+01	2
5000	2001	4039	2039	202	0.3401956452857E+04	0.1876237258858E+01	2
6000	2001	4049	2049	227	0.4184556302476E+04	0.2503371553904E+01	2
7000	2001	4050	2050	242	0.4966370513902E+04	0.2046859604720E+01	2
8000	2001	4061	2061	563	0.5748882325181E+04	0.2808190860854E+01	2
9000	2001	4046	2046	696	0.6532304924909E+04	0.2749547168603E+01	2
10000	2001	4052	2052	800	0.7315160550614E+04	0.2265569763122E+01	2

TOTAL 20010 40497 20497 30.48(seconds)

4 Dai-Kou. Standard WLS. Function:Extended Beale

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	12	25	15	0	0.4082404550275E-11	0.6146879953400E-07	0
2000	12	25	15	1	0.8172204810851E-11	0.6149757176792E-07	0
3000	12	25	15	2	0.1226816500740E-10	0.6152312807440E-07	0
4000	12	25	15	2	0.1633467089572E-10	0.6147863229338E-07	0
5000	12	25	15	3	0.2043371814360E-10	0.6150256048558E-07	0
6000	12	25	15	2	0.2461509230098E-10	0.6162511145208E-07	0
7000	12	25	15	3	0.2861445786226E-10	0.6151061931677E-07	0
8000	12	25	15	3	0.3273079679656E-10	0.6153838042897E-07	0
9000	12	25	15	4	0.3670202583870E-10	0.6143454801651E-07	0
10000	12	25	15	4	0.4077847215615E-10	0.6143333815100E-07	0

TOTAL 120 250 150 0.24(seconds)

5 Dai-Kou. Standard WLS. Function:Extended Powell

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	71	143	75	1	0.4367709467585E-07	0.7878972931806E-06	0
2000	71	143	75	3	0.8735200650442E-07	0.7886528161270E-06	0
3000	71	143	75	5	0.1310268837897E-06	0.7888530263845E-06	0
4000	71	143	75	8	0.1746933296312E-06	0.7904345675378E-06	0
5000	71	143	75	8	0.2183878965580E-06	0.7873500919611E-06	0
6000	71	143	75	10	0.2620396895302E-06	0.7904353829826E-06	0
7000	71	143	75	13	0.3057073125359E-06	0.7908906262255E-06	0
8000	71	143	75	13	0.3493146082686E-06	0.7960885269577E-06	0
9000	71	143	75	14	0.3931204671251E-06	0.7856421510523E-06	0
10000	71	143	75	16	0.4366673580713E-06	0.7946174432727E-06	0

TOTAL 710 1430 750 0.91(seconds)

6 Dai-Kou. Standard WLS. Function:Extended Maratos

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	9	26	19	0	-0.5280864426222E+03	0.1076241928910E-06	0

```
      2000        9       26       19        1     -0.1056172885244E+04   0.7941204449580E-07 0
      3000        9       26       19        1     -0.1584259327867E+04   0.6878391167753E-07 0
      4000        9       26       19        2     -0.2112345770489E+04   0.1181336408695E-06 0
      5000        9       26       19        1     -0.2640432213111E+04   0.5226534494085E-07 0
      6000        9       26       19        2     -0.3168518655733E+04   0.2864253581939E-07 0
      7000        9       26       19        2     -0.3696605098356E+04   0.1344662334812E-06 0
      8000        9       26       19        3     -0.4224691540978E+04   0.9533927913807E-07 0
      9000        9       26       19        3     -0.4752777983600E+04   0.2029684651461E-06 0
     10000        9       26       19        3     -0.5280864426222E+04   0.4138538512241E-07 0
-----------------------------------------------------------------------------------------------
TOTAL     90     260     190    0.18(seconds)

   7  Dai-Kou. Standard WLS. Function:Extended CLIFF

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        7       16       10        3      0.4232867951672E+03   0.1350742275807E-06 0
      2000        7       17       11        1      0.8465735906175E+03   0.4010118190845E-06 0
      3000        6       14        9        2      0.1269860385420E+04   0.1558767679377E-06 0
      4000        7       16       10        4      0.1693147180577E+04   0.3636214684910E-07 0
      5000        7       18       12        5      0.2116433978946E+04   0.5648424044047E-06 0
      6000        7       21       15        8      0.2539720771925E+04   0.6447629512518E-06 0
      7000        6       14        9        5      0.2963007578395E+04   0.7089088185364E-06 0
      8000        9       31       26       15      0.3386294361120E+04   0.3663834236001E-08 0
      9000       13       42       35       22      0.3809581156328E+04   0.2684434800004E-06 0
     10000        7       19       13       10      0.4232867956794E+04   0.9458281069463E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     76     208     150    0.75(seconds)

   8  Dai-Kou. Standard WLS. Function:Extended Wood

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000      220      476      265        6      0.4474028739085E-10   0.5949119893506E-06 0
      2000      120      284      182        9      0.8755547906121E-10   0.5958915069160E-06 0
      3000      287      613      337       25      0.1092906701489E-09   0.9030170471824E-06 0
      4000      177      382      218       21      0.1275985754437E-10   0.5966534371460E-06 0
      5000      229      490      279       33      0.2077952958196E-10   0.8321378861703E-06 0
      6000      295      616      328       46      0.1370954317710E-09   0.5862551046176E-06 0
      7000      128      286      168       24      0.3824334631098E-11   0.3845540215189E-07 0
      8000      132      312      201       30      0.1868760064966E-10   0.7487514948566E-07 0
      9000      147      329      193       41      0.1811097313612E-11   0.5856158877162E-06 0
     10000      105      238      141       29      0.3277217205789E-11   0.2543349655960E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   1840    4026    2312    2.64(seconds)

   9  Dai-Kou. Standard WLS. Function:Extended Hiebert

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000       39      110       82        2      0.9336029973771E-20   0.2372999574619E-09 0
      2000       38      108       82        2      0.5453191626599E-11   0.3349214264005E-07 0
      3000       44      122       92        4      0.6347411400534E-13   0.2512479115282E-08 0
      4000       41      112       82        6      0.3105696324978E-12   0.4962430460366E-08 0
      5000       39      113       84        6      0.8769710377109E-14   0.1198937837421E-08 0
      6000       41      116       89        7      0.1302753499861E-10   0.2535330184734E-07 0
      7000       40      119       92        8      0.8444944105696E-14   0.4393545793236E-07 0
      8000       46      124       95       11      0.2958948995041E-13   0.1967005794178E-06 0
      9000       46      128       97       19      0.7622704918162E-13   0.4007739207543E-06 0
     10000       46      129      101       14      0.2158998917984E-13   0.2078345708870E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    420    1181     896    0.79(seconds)

  10  Dai-Kou. Standard WLS. Function:Extended Rosenbrock                  10

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000       55      173      134        2      0.3027995798751E-09   0.6404716756947E-06 0
      2000       63      193      149        4      0.3041241027297E-12   0.3261146877661E-06 0
      3000       61      195      154        8      0.1639137504468E-15   0.4181896295902E-07 0
      4000       56      173      136        9      0.6291064264114E-09   0.7070184437907E-06 0
      5000       57      179      136        9      0.1633237812957E-08   0.6179257091767E-06 0
      6000       55      183      144       12      0.4111600777389E-09   0.3613354060406E-06 0
      7000       62      199      156       15      0.9622876613996E-15   0.1130117909832E-07 0
      8000       54      177      140       15      0.6110012611185E-12   0.9873879491806E-08 0
      9000       61      199      157       21      0.9006777421960E-16   0.1125766146970E-09 0
     10000       60      182      141       21      0.3131298798981E-09   0.2045319469346E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    584    1853    1447    1.16(seconds)

  11  Dai-Kou. Standard WLS. Function:Generalized Rosenbrock

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000     2001     4043     2054       52      0.1853453865440E-04   0.1228981742474E-02 2
      2000     2001     4039     2049      108      0.1581508645179E-04   0.1395611495992E-02 2
      3000     2001     4094     2118      170      0.4663889670490E-05   0.3277206765814E-04 2
      4000     2001     4027     2035      117      0.1565765440486E-04   0.2916684974851E-02 2
      5000     2001     4089     2101      130      0.6250015059003E-05   0.2943319961386E-03 2
      6000     2001     4051     2060      157      0.9603463190692E-05   0.7979019101110E-03 2
      7000     2001     4069     2084      184      0.5521206675885E-05   0.2910877611417E-03 2
      8000     2001     4049     2057      206      0.7352914697985E-05   0.6725318876271E-04 2
      9000     2001     4060     2075      244      0.1005545833505E-04   0.1619340323861E-02 2
     10000     2001     4031     2041      258      0.1718887718362E-04   0.8338954399296E-03 2
```

```
-----------------------------------------------------------------------------------------
TOTAL  20010   40552  20674   16.26(seconds)

  12  Dai-Kou. Standard WLS. Function:Extended Himmelblau

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000        8       20       13        0      0.1178921828895E-13   0.5189838424087E-07 0
     2000        8       20       13        0      0.2357832325816E-13   0.5189804495671E-07 0
     3000        8       20       13        0      0.3536773731041E-13   0.5189812844549E-07 0
     4000        8       20       13        0      0.4715689449195E-13   0.5189840910987E-07 0
     5000        8       20       13        1      0.5894590624754E-13   0.5189862049633E-07 0
     6000        8       20       13        0      0.7073524389900E-13   0.5189880879015E-07 0
     7000        8       20       13        1      0.8252446532644E-13   0.5189830075210E-07 0
     8000        8       20       13        1      0.9431350320936E-13   0.5189791705902E-07 0
     9000        8       20       13        1      0.1061028604153E-12   0.5189887273900E-07 0
    10000        8       20       13        2      0.1178921295258E-12   0.5189880168473E-07 0
-----------------------------------------------------------------------------------------
TOTAL     80     200    130    0.06(seconds)

  13  Dai-Kou. Standard WLS. Function:HIMMELBG

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000        1      109      109       14      0.0000000000000E+00   0.0000000000000E+00 4
     2000        1      109      109       32      0.0000000000000E+00   0.0000000000000E+00 4
     3000        1      109      109       44      0.0000000000000E+00   0.0000000000000E+00 4
     4000        1      109      109       60      0.0000000000000E+00   0.0000000000000E+00 4
     5000        1      109      109       75      0.0000000000000E+00   0.0000000000000E+00 4
     6000        1      109      109       88      0.0000000000000E+00   0.0000000000000E+00 4
     7000        1      109      109      107      0.0000000000000E+00   0.0000000000000E+00 4
     8000        1      109      109      117      0.0000000000000E+00   0.0000000000000E+00 4
     9000        1      109      109      135      0.0000000000000E+00   0.0000000000000E+00 4
    10000        1      109      109      148      0.0000000000000E+00   0.0000000000000E+00 4
-----------------------------------------------------------------------------------------
TOTAL     10    1090   1090    8.20(seconds)

  14  Dai-Kou. Standard WLS. Function:HIMMELBH

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000        5       11        6        0     -0.5000000000000E+03   0.8980697185912E-06 0
     2000        5       11        6        0     -0.9999999999999E+03   0.8813226255455E-06 0
     3000        5       11        6        0     -0.1500000000000E+04   0.8942482825347E-06 0
     4000        5       11        6        0     -0.2000000000000E+04   0.8813095480065E-06 0
     5000        5       11        6        0     -0.2500000000000E+04   0.8865594467444E-06 0
     6000        5       11        6        0     -0.3000000000000E+04   0.9106136196202E-06 0
     7000        5       11        6        1     -0.3500000000000E+04   0.9020986704300E-06 0
     8000        5       11        6        0     -0.4000000000000E+04   0.9107697289679E-06 0
     9000        5       11        6        1     -0.4500000000000E+04   0.9190630807510E-06 0
    10000        5       11        6        0     -0.5000000000000E+04   0.8939210611736E-06 0
-----------------------------------------------------------------------------------------
TOTAL     50     110     60    0.02(seconds)

  15  Dai-Kou. Standard WLS. Function:Extended Trigonometric ET1

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000       61      124       63        7      0.4416452165565E-06   0.7740098336651E-06 0
     2000       66      133       67       15      0.2105830840363E-06   0.9767606541527E-06 0
     3000       64      129       65       23      0.1396950410393E-06   0.9172299891618E-06 0
     4000       68      137       69       32      0.1031640051006E-06   0.9881656528216E-06 0
     5000       68      137       69       40      0.8391314399747E-07   0.7553436067101E-06 0
     6000       76      155       80       55      0.6775590597053E-07   0.7775652223570E-06 0
     7000       71      143       73       57      0.5780637210883E-07   0.9645324950195E-06 0
     8000       72      147       75       66      0.5116408858486E-07   0.4598535426653E-06 0
     9000       72      146       74       78      0.4619487639780E-07   0.6452111095535E-06 0
    10000       75      152       78       91      0.3934441707222E-07   0.9434758524149E-06 0
-----------------------------------------------------------------------------------------
TOTAL    693    1403    713    4.64(seconds)

  16  Dai-Kou. Standard WLS. Function:Extended Trigonometric ET2

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000      316      644      386       41      0.2823235951063E-12   0.9006671566202E-06 0
     2000      465      946      556      120      0.2286985703871E-12   0.9965578154492E-06 0
     3000      994     1995     1026      365      0.7474568886909E-12   0.9331373128890E-06 0
     4000      708     1445      886      388      0.1201895776627E-11   0.9357306125259E-06 0
     5000      868     1768     1050      578      0.6605791364981E-12   0.9600957626836E-06 0
     6000     1064     2161     1232      822      0.9167102839947E-12   0.6855381869987E-06 0
     7000     1034     2095     1244      947      0.6940731515719E-12   0.9186823214230E-06 0
     8000     1235     2501     1422     1270      0.7556519400270E-12   0.9381279367524E-06 0
     9000     1207     2479     1492     1449      0.1536828158478E-11   0.8867931292552E-06 0
    10000     1344     2755     1684     1810      0.1579400372170E-11   0.9035548894963E-06 0
-----------------------------------------------------------------------------------------
TOTAL   9235   18789  10978   77.90(seconds)

  17  Dai-Kou. Standard WLS. Function:Extended Block Diagonal BD1

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000        7       15        8        1      0.1323406312888E-10   0.4803343568016E-06 0
     2000        7       15        8        0      0.2646809882919E-10   0.4803341076488E-06 0
```

```
      3000        7       15        8        2      0.3970214632669E-10   0.4803340956403E-06 0
      4000        7       15        8        1      0.5293622555055E-10   0.4803342342266E-06 0
      5000        7       15        8        2      0.6617024873938E-10   0.4803341140532E-06 0
      6000        7       15        8        3      0.7940434946312E-10   0.4803342675781E-06 0
      7000        7       15        8        2      0.9263836295029E-10   0.4803341519763E-06 0
      8000        7       15        8        3      0.1058723803288E-09   0.4803340733499E-06 0
      9000        7       15        8        4      0.1191065221192E-09   0.4803342635959E-06 0
     10000        7       15        8        4      0.1323405614171E-09   0.4803342296553E-06 0
 ----------------------------------------------------------------------------------------------
 TOTAL     70     150     80    0.22(seconds)

   18  Dai-Kou. Standard WLS. Function:Extended Tridiagonal-1

         n     iter    nfunc    ngrad  time(c)               f                    gnorm         s
 ----------------------------------------------------------------------------------------------
      1000       21       44       27        0      0.1710290945233E-08   0.2485502917074E-06 0
      2000       21       44       27        1      0.3421377953530E-08   0.2482865674310E-06 0
      3000       21       44       27        1      0.5131738282246E-08   0.2483671645297E-06 0
      4000       21       44       27        1      0.6839745605890E-08   0.2487806853103E-06 0
      5000       21       44       27        2      0.8550104178802E-08   0.2487366361553E-06 0
      6000       21       44       27        2      0.1026405949455E-07   0.2482911486544E-06 0
      7000       21       44       27        2      0.1198209814825E-07   0.2475899777884E-06 0
      8000       21       44       27        2      0.1368690869070E-07   0.2481793990749E-06 0
      9000       21       44       27        3      0.1540037502797E-07   0.2479673456000E-06 0
     10000       21       44       27        3      0.1709294436957E-07   0.2492132762286E-06 0
 ----------------------------------------------------------------------------------------------
 TOTAL    210     440    270    0.17(seconds)

   19  Dai-Kou. Standard WLS. Function:Extended Three Exponential Terms

         n     iter    nfunc    ngrad  time(c)               f                    gnorm         s
 ----------------------------------------------------------------------------------------------
      1000        7       15        8        0      0.1279633348329E+04   0.7959519969791E-06 0
      2000        9       18       11        2      0.2559266696658E+04   0.4355410823109E-06 0
      3000       10       19       13        4      0.3838900044987E+04   0.9068301665138E-12 0
      4000        9       19       12        4      0.5118533393317E+04   0.1176969632866E-10 0
      5000        9       20       12        6      0.6398166741646E+04   0.4465776348717E-06 0
      6000        7       16        9        5      0.7677800089974E+04   0.5600590602395E-06 0
      7000        8       18       12        8      0.8957433438304E+04   0.9301427466787E-11 0
      8000        8       97       91       62      0.1023706678663E+05   0.5875370285777E-11 4
      9000        7       16       10        8      0.1151670013496E+05   0.1770247362032E-06 0
     10000        8       18       11       11      0.1279633348329E+05   0.1132718563390E-07 0
 ----------------------------------------------------------------------------------------------
 TOTAL     82     256    189    1.10(seconds)

   20  Dai-Kou. Standard WLS. Function:Generalized Tridiagonal-1           20

         n     iter    nfunc    ngrad  time(c)               f                    gnorm         s
 ----------------------------------------------------------------------------------------------
      1000       22       40       28        1      0.9972103074860E+03   0.9499208104558E-06 0
      2000       22      136      124        2      0.1997210307486E+04   0.9653037156276E-06 4
      3000       22       39       29        1      0.2997210307486E+04   0.9627719603778E-06 0
      4000       22       39       29        1      0.3997210307486E+04   0.9416307147347E-06 0
      5000       22       39       29        2      0.4997210307486E+04   0.9548193942166E-06 0
      6000       22       39       29        3      0.5997210307486E+04   0.9663752673639E-06 0
      7000       22       39       29        4      0.6997210307486E+04   0.9194185053296E-06 0
      8000       22      126      116       14      0.7997210307486E+04   0.9458110974414E-06 4
      9000       21      131      122       12      0.8997210307486E+04   0.2026078693262E-05 4
     10000       21      136      127       11      0.9997210307486E+04   0.2013567142800E-05 4
 ----------------------------------------------------------------------------------------------
 TOTAL    218     764    662    0.51(seconds)

   21  Dai-Kou. Standard WLS. Function:Generalized Tridiagonal-2

         n     iter    nfunc    ngrad  time(c)               f                    gnorm         s
 ----------------------------------------------------------------------------------------------
      1000       39       77       42        1      0.1114854332625E+01   0.8623950049682E-06 0
      2000       44       86       48        2      0.1114854332625E+01   0.6784155595015E-06 0
      3000       44       87       47        3      0.1114854332625E+01   0.7697441577088E-06 0
      4000       45       88       49        4      0.1114854332625E+01   0.7325318261713E-06 0
      5000       45       89       48        5      0.1114854332625E+01   0.9309803198046E-06 0
      6000       46       90       50        5      0.1114854332625E+01   0.8362278038976E-06 0
      7000       47       91       52        7      0.1114854332625E+01   0.8485323967044E-06 0
      8000       47       92       51        8      0.1114854332625E+01   0.9461517229692E-06 0
      9000       47       92       51        9      0.1114854332625E+01   0.9647650667824E-06 0
     10000       47       92       51        9      0.1114854332625E+01   0.9423957639407E-06 0
 ----------------------------------------------------------------------------------------------
 TOTAL    451     884    489    0.53(seconds)

   22  Dai-Kou. Standard WLS. Function:Tridiagonal Double Borded

         n     iter    nfunc    ngrad  time(c)               f                    gnorm         s
 ----------------------------------------------------------------------------------------------
      1000      448      897      619        8      0.1528391502750E-09   0.9613573713826E-06 0
      2000      460      921      628       16      0.1903658012739E-09   0.8729531995081E-06 0
      3000      325      651      430       17      0.3050518241796E-08   0.8946286995124E-06 0
      4000     1913     3827     2790      132      0.8583776920745E-10   0.9975889735852E-06 0
      5000     1980     3961     2693      171      0.1545633728505E-09   0.9937141816690E-06 0
      6000     2001     4001     2778      210      0.1631580745585E-08   0.8438104872606E-05 2
      7000     2001     4001     2007      214      0.1063894251775E-05   0.4337663919755E-03 2
      8000     2001     4001     2829      282      0.1000031051361E-06   0.1217082825293E-02 2
      9000     2001     4001     2701      303      0.1960114691869E-07   0.1428146464282E-04 2
     10000     2001     4001     2375      322      0.5104977356439E-06   0.4851898920690E-03 2
 ----------------------------------------------------------------------------------------------
```

```
TOTAL  15131   30262  19850   16.75(seconds)

  23  Dai-Kou. Standard WLS. Function:Broyden Pentadiagonal

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       38       77       39        1      0.7461227006094E-13   0.8821076348581E-06 0
     2000       40       81       41        1      0.5472083776458E-13   0.6398183516102E-06 0
     3000       40       81       41        3      0.9277297464218E-13   0.7786250212443E-06 0
     4000       40       81       41        2      0.9408818710153E-13   0.9424813307015E-06 0
     5000       40       81       41        4      0.6037018024351E-13   0.6988056446702E-06 0
     6000       39       79       40        5      0.7039044239927E-13   0.7851178551813E-06 0
     7000       38       77       39        4      0.6865125110153E-13   0.7762853829565E-06 0
     8000       37       75       38        5      0.6587812446165E-13   0.7542413521530E-06 0
     9000       36       73       37        5      0.9654096934473E-13   0.9790572706176E-06 0
    10000       36       73       37        6      0.6189057596778E-13   0.7594562195627E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    384     778    394    0.36(seconds)

  24  Dai-Kou. Standard WLS. Function:Extended PSC1

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       10       21       11        0      0.3865995282465E+03   0.7750709551946E-07 0
     2000       10       21       11        2      0.7731990564929E+03   0.7421207237623E-07 0
     3000       10       21       11        2      0.1159798584739E+04   0.8180339650110E-07 0
     4000       10       21       11        3      0.1546398112986E+04   0.8729345746250E-07 0
     5000       10       21       11        3      0.1932997641232E+04   0.1255222438767E-06 0
     6000       10       21       11        4      0.2319597169479E+04   0.6385597406400E-07 0
     7000       10       21       11        5      0.2706196697725E+04   0.1121266771653E-06 0
     8000       10       21       11        6      0.3092796225971E+04   0.7786351008399E-07 0
     9000       10       21       11        7      0.3479395754218E+04   0.3140296349069E-06 0
    10000       10       21       11        7      0.3865995282465E+04   0.9327316419228E-07 0
-----------------------------------------------------------------------------------------------
TOTAL    100     210    110    0.39(seconds)

  25  Dai-Kou. Standard WLS. Function:Perturbed Quadratic PQ1

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000      179      359      180        3      0.1671157287378E-12   0.9680375908780E-06 0
     2000      254      509      255        6      0.1675840389168E-12   0.9684290156498E-06 0
     3000      312      625      313       12      0.1516123931612E-12   0.9211687000507E-06 0
     4000      360      721      361       17      0.1666553061782E-12   0.9652068105196E-06 0
     5000      403      807      404       25      0.1587404985738E-12   0.9421594011434E-06 0
     6000      441      883      442       33      0.1734386389473E-12   0.9841041992678E-06 0
     7000      477      955      478       43      0.1614539709109E-12   0.9499462373797E-06 0
     8000      510     1021      511       54      0.1622466350354E-12   0.9521986955753E-06 0
     9000      541     1083      542       63      0.1627543529942E-12   0.9536220813917E-06 0
    10000      570     1141      571       70      0.1690528495127E-12   0.9715731145394E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   4047    8104   4057    3.26(seconds)

  26  Dai-Kou. Standard WLS. Function:Perturbed Quadratic PQ2

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000     2001     4012     2957       27      0.1854152687298E-01   0.3414118113872E+00 2
     2000     2001     4001     3001       55      0.1005695997463E+05   0.2365161222999E+03 2
     3000     2001     4025     3020       85      0.2128071839750E+02   0.8212759011889E+01 2
     4000     2001     4076     2109      101      0.1515072473714E+02   0.7491347009571E+01 2
     5000     2001     4033     2182      134      0.1081683459083E+01   0.1983665455935E+01 2
     6000     2001     4001     3000      168      0.6065178225916E+05   0.1438358666279E+04 2
     7000     2001     4001     2849      194      0.2987516677376E+04   0.1949298663881E+03 2
     8000     2001     4056     2172      208      0.6848976219771E+01   0.7018188303941E+02 2
     9000     2001     4069     3070      257      0.4719420952261E+03   0.3120771429297E+02 2
    10000     2001     4328     2426      269      0.1747496204659E+01   0.1508052051294E+02 2
-----------------------------------------------------------------------------------------------
TOTAL  20010   40602  26786   14.98(seconds)

  27  Dai-Kou. Standard WLS. Function:Almost Perturbed Quadratic

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000      175      351      176        2      0.1907481248696E-12   0.9833712570810E-06 0
     2000      249      499      250        5      0.1642303467783E-12   0.9185227971497E-06 0
     3000      305      611      306       10      0.1782071487878E-12   0.9500428573982E-06 0
     4000      352      705      353       15      0.1935143004613E-12   0.9830929003930E-06 0
     5000      394      789      395       20      0.1867682008045E-12   0.9680773817580E-06 0
     6000      432      865      433       29      0.1811142710268E-12   0.9552819597656E-06 0
     7000      466      933      467       34      0.1997841245930E-12   0.9953889573281E-06 0
     8000      499      999      500       43      0.1835117337954E-12   0.9602914351229E-06 0
     9000      529     1059      530       52      0.1915727752350E-12   0.9777500026117E-06 0
    10000      558     1117      559       62      0.1856458641079E-12   0.9647910846176E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   3959    7928   3969    2.72(seconds)

  28  Dai-Kou. Standard WLS. Function:Almost Perturbed Quartic

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       27       56       29        0      0.1137509969107E-08   0.7452517472568E-06 0
     2000       31       65       34        0      0.6339513692871E-09   0.2695826284186E-06 0
     3000       37       79       42        2      0.5498635368700E-09   0.4205454331185E-06 0
```

```
     4000       39       86       48        2      0.7516688815646E-08   0.4856249854558E-06 0
     5000       32       69       37        1      0.1961437660771E-08   0.9771179454373E-06 0
     6000       36       79       44        2      0.3538073857667E-08   0.6353712101886E-06 0
     7000       36       79       43        3      0.2401490008435E-08   0.2484038256657E-06 0
     8000       33       71       38        3      0.1107159453182E-07   0.7277307891703E-06 0
     9000       38       85       47        4      0.2300089444928E-08   0.5894858118769E-06 0
    10000       42       93       51        5      0.2506618827313E-08   0.5657734515372E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    351     762    413    0.22(seconds)

  29  Dai-Kou. Standard WLS. Function:Extended Penalty Function U52

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       21      141      124        1      0.8831940750670E+03   0.4127036544402E-06 4
     2000       21       43       24        1      0.1814063664869E+04   0.5399711130169E-06 0
     3000       24       48       28        1      0.2755973749503E+04   0.6376100952963E-06 0
     4000       27       53       32        1      0.3704070534948E+04   0.1639732435710E-06 0
     5000       25       49       28        2      0.4656333923744E+04   0.6584645984041E-06 0
     6000       25      127      106        6      0.5611676659140E+04   0.1086110759177E-05 4
     7000       25       50       27        2      0.6569428560737E+04   0.8777092397102E-06 0
     8000       27      145      124        6      0.7529139638522E+04   0.4975852277209E-09 4
     9000       28      141      119        7      0.8490489281459E+04   0.2857990840791E-05 4
    10000       30      158      134        9      0.9453238852842E+04   0.5191166191422E-07 4
-----------------------------------------------------------------------------------------------
TOTAL    253     955    746    0.36(seconds)

  30  Dai-Kou. Standard WLS. Function:TR-Summ of quadratics              30

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000     1009     2034     1029       14      0.1375184858438E-16   0.8517238546325E-06 0
     2000     2001     4030     2032       55      0.3123928302780E-12   0.3218872051488E-04 2
     3000     2001     4019     2021       85      0.3048544940733E-09   0.5830426105179E-03 2
     4000     2001     4012     2013      109      0.3571908701356E-09   0.5200966912439E-03 2
     5000     2001     4021     2023      134      0.1393118274641E-09   0.4345028813393E-03 2
     6000     2001     4013     2014      160      0.3710844148370E-09   0.5469381653183E-03 2
     7000     2001     4013     2014      193      0.1540170613906E-09   0.3463517000068E-03 2
     8000     2001     4013     2014      214      0.2442658324141E-09   0.3628177331144E-03 2
     9000      446      916      472       56      0.1128624692884E-13   0.9049025708915E-06 0
    10000     2001     4012     2013      282      0.2131845976491E-09   0.5279704370726E-03 2
-----------------------------------------------------------------------------------------------
TOTAL  17463   35083  17645   13.02(seconds)

  31  Dai-Kou. Standard WLS. Function:Quadratic Diagonal Perturbed

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000      101      203      122        2      0.3205264856237E-10   0.9926440547088E-06 0
     2000      142      285      173        4      0.3495251411631E-10   0.9758509842935E-06 0
     3000      169      339      202        9      0.3505367335610E-10   0.9633685083699E-06 0
     4000      198      397      239       11      0.3763064161148E-10   0.9777181142617E-06 0
     5000      218      437      262       17      0.3874251990775E-10   0.9799099044219E-06 0
     6000      239      479      293       21      0.4231475151418E-10   0.9981213929153E-06 0
     7000      252      505      304       25      0.4288064586029E-10   0.9939725389025E-06 0
     8000      270      541      325       31      0.4138990834222E-10   0.9768538461450E-06 0
     9000      289      579      354       42      0.4575860016552E-10   0.9967078933060E-06 0
    10000      298      597      359       43      0.4688464811175E-10   0.9960534784597E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   2176    4362   2633    2.05(seconds)

  32  Dai-Kou. Standard WLS. Function:Full Hessian FH1

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       29       59       31        9      0.1212871287129E+02   0.1527666881884E-12 0
     2000       33       67       34       19      0.1212871287129E+02   0.1510638121260E-06 0
     3000       36       73       38       33      0.1212871287129E+02   0.3055333763768E-12 0
     4000       38       76       40       44      0.1212871287129E+02   0.2220446049250E-12 0
     5000       39       79       41       56      0.1212871287129E+02   0.3250733016102E-12 0
     6000       40       81       42       69      0.1212871287129E+02   0.2824407374646E-12 0
     7000       41      162      122      195      0.1212871287129E+02   0.5861977570021E-13 4
     8000       41       83       42       91      0.1212871287129E+02   0.5968830123493E-06 0
     9000       43       87       45      113      0.1212871287129E+02   0.4707345624411E-12 0
    10000       44       88       46      127      0.1212871287129E+02   0.2287947609148E-11 0
-----------------------------------------------------------------------------------------------
TOTAL    384     855    481    7.56(seconds)

  33  Dai-Kou. Standard WLS. Function:Full Hessian FH2

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000     1148     2678     2123      103     -0.7078102012499E+03   0.6977735856465E-05 4
     2000     2001     4043     2329      279     -0.2829821020925E+04   0.1476296086840E-01 2
     3000        9      127      122       15     -0.9296965068582E+03   0.5396863471765E+01 4
     4000        9      123      118       19     -0.9497903742750E+03   0.7498471100670E+01 4
     5000     2001     4181     2467      729     -0.1768119862165E+05   0.4626794203979E-02 2
     6000        9      124      119       29     -0.5140791623633E+04   0.1190576188930E+02 4
     7000        9      123      118       34     -0.7031354301187E+04   0.1353006084134E+02 4
     8000     2001     4023     2060     1077     -0.4525164337981E+05   0.2772587767257E+00 2
     9000        9      126      121       46     -0.6054291403837E+04   0.5940479612231E+02 4
    10000        9      126      121       49     -0.7358619137551E+04   0.6728128168841E+02 4
-----------------------------------------------------------------------------------------------
TOTAL   7205   15674   9698   23.80(seconds)
```

34 Dai-Kou. Standard WLS. Function:Full Hessian FH3

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	17	43	27	1	0.4767296853771E+03	0.8417576680131E-07	0
2000	17	44	27	3	0.1905966240052E+04	0.5240953186991E-06	0
3000	17	43	27	5	0.4287709662222E+04	0.2201974087512E-06	0
4000	19	45	30	6	0.7621959951885E+04	0.2545041569342E-06	0
5000	19	45	30	7	0.1190871710904E+05	0.2066532276089E-06	0
6000	18	44	28	10	0.1714798113370E+05	0.7574807625588E-06	0
7000	18	145	130	37	0.2333975202584E+05	0.2031042720674E-06	4
8000	20	49	35	14	0.3048402978548E+05	0.3930001089003E-06	0
9000	19	45	30	14	0.3858081441262E+05	0.1030613905328E-06	0
10000	19	53	38	18	0.4763010590725E+05	0.7275395219608E-06	0

TOTAL 183 556 402 1.15(seconds)

35 Dai-Kou. Standard WLS. Function:Diagonal Full Borded

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	484	1151	792	42	0.2546912879882E-04	0.1542525637961E-06	0
2000	535	1278	893	91	0.2956063926340E-04	0.4823768502279E-06	0
3000	533	1248	816	126	0.6370502738504E-04	0.8434116450780E-06	0
4000	478	1204	805	170	0.3014981362666E-04	0.5167154943076E-06	0
5000	775	1929	1328	335	0.3364472517159E-04	0.5332672858303E-06	0
6000	483	1204	833	255	0.7007014571957E-04	0.5085608572335E-06	0
7000	904	2171	1477	526	0.5173436378144E-04	0.7425560816915E-06	0
8000	597	1775	1443	547	0.5837566143796E-04	0.6887033854328E-06	0
9000	604	1612	1167	521	0.5129942970010E-04	0.3704719159390E-06	0
10000	481	1274	914	458	0.7448165782244E-04	0.7047235060344E-06	0

TOTAL 5874 14846 10468 30.71(seconds)

36 Dai-Kou. Standard WLS. Function:Diagonal Double Borded Arrow Up

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	34	71	47	1	0.1338563836791E-12	0.6444438853017E-06	0
2000	34	71	49	1	0.1487303043794E-12	0.7178466143700E-07	0
3000	50	102	70	3	0.6613631404252E-11	0.3418175131546E-06	0
4000	43	90	63	2	0.1732539891849E-11	0.6249376484836E-07	0
5000	594	1192	784	45	0.1414796539219E-10	0.1769905884002E-06	0
6000	36	76	53	4	0.3705766605108E-12	0.1010668322598E-06	0
7000	163	330	180	16	0.3041797885967E-11	0.9990786493296E-06	0
8000	26	56	40	4	0.1461134968353E-10	0.9269873608526E-06	0
9000	30	64	44	4	0.3308569786259E-12	0.1165503910409E-06	0
10000	2001	4004	2106	289	0.1620226663601E-11	0.1422227209063E-04	2

TOTAL 3011 6056 3436 3.69(seconds)

37 Dai-Kou. Standard WLS. Function:QP1 Extended Quadratic Penalty

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	16	104	92	1	0.3990006250000E+04	0.1926782506108E-09	4
2000	15	32	19	1	0.7990003125000E+04	0.1120530841007E-08	0
3000	15	30	18	0	0.1199000208333E+05	0.8224738213537E-10	0
4000	15	31	18	1	0.1599000156250E+05	0.5161288534457E-06	0
5000	16	132	119	4	0.1999000125000E+05	0.1720203420870E-10	4
6000	16	104	91	4	0.2399000104167E+05	0.7467821250698E-10	4
7000	16	131	118	6	0.2799000089286E+05	0.2200267017263E-09	4
8000	16	135	122	6	0.3199000078125E+05	0.2917282368138E-05	4
9000	17	136	123	7	0.3599000069445E+05	0.3130933830340E-09	4
10000	18	35	21	2	0.3999000062499E+05	0.3008873303771E-06	0

TOTAL 160 870 741 0.32(seconds)

38 Dai-Kou. Standard WLS. Function:QP2 Extended Quadratic Penalty

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	40	106	74	4	0.1775561597831E-16	0.1645958036500E-06	0
2000	38	101	72	7	0.2116253478574E-15	0.2817444055737E-06	0
3000	42	110	80	12	0.9634512169793E-13	0.8362235348701E-07	0
4000	38	101	71	15	0.1778758748533E-15	0.4868383030340E-06	0
5000	41	112	81	20	0.3487396119224E-13	0.5282494491077E-08	0
6000	41	119	88	25	0.5749165085734E-15	0.9582311123506E-06	0
7000	44	101	67	25	0.3121345914156E-13	0.4223601504824E-08	0
8000	42	119	85	32	0.1998183654973E-13	0.3161038937774E-08	0
9000	41	111	82	36	0.1789854878403E-17	0.4016271759631E-07	0
10000	37	104	74	35	0.1736522871327E-20	0.1324451659457E-08	0

TOTAL 404 1084 774 2.11(seconds)

39 Dai-Kou. Standard WLS. Function:QP3 Extended Quadratic Penalty

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	20	48	31	1	-0.9990000000000E+03	0.1875739112468E-06	0
2000	20	45	29	0	-0.1999000000000E+04	0.3249971348004E-06	0
3000	21	47	32	1	-0.2999000000000E+04	0.6173728195374E-11	0
4000	21	48	29	2	-0.3999000000000E+04	0.4424175767966E-06	0

5000	23	49	32	2	-0.4999000000000E+04	0.8133249630000E-10	0
6000	22	49	33	2	-0.5999000000000E+04	0.2406119747947E-10	0
7000	20	46	28	2	-0.6999000000000E+04	0.9161728638185E-06	0
8000	22	53	37	3	-0.7999000000000E+04	0.1865381183201E-09	0
9000	22	151	135	9	-0.8999000000000E+04	0.7589306954894E-09	4
10000	24	59	40	4	-0.9999000000000E+04	0.5257314586437E-09	0

TOTAL 215 595 426 0.26(seconds)

40 Dai-Kou. Standard WLS. Function:STAIRCASE S1 40

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	1999	3999	2001	24	0.1683993776811E-16	0.1050239006872E-08	0
2000	2001	4001	2001	47	0.9354352673411E-04	0.4833615857740E-03	2
3000	2001	4001	2001	76	0.2128815959619E-03	0.4219839242978E-03	2
4000	2001	4001	2001	100	0.4223184738648E-03	0.3194427163180E-03	2
5000	2001	4001	2001	118	0.6367941057572E-03	0.4357694770079E-03	2
6000	2001	4001	2001	145	0.8463421241898E-03	0.1140723499702E-02	2
7000	2001	4001	2001	168	0.1149501657394E-02	0.1568335661432E-02	2
8000	2001	4001	2001	188	0.1499584428045E-02	0.2128301402990E-02	2
9000	2001	4001	2001	214	0.2327583348370E-02	0.6559592420672E-03	2
10000	2001	4001	2001	237	0.2341601560546E-02	0.2745450799921E-02	2

TOTAL 20008 40008 20010 13.17(seconds)

41 Dai-Kou. Standard WLS. Function:STAIRCASE S2

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	1999	3999	2001	24	0.2700222462687E-16	0.1018179318635E-08	0
2000	2001	4001	2001	47	0.9373942382170E-04	0.4576209168072E-03	2
3000	2001	4001	2001	71	0.2139290732117E-03	0.4120412070279E-03	2
4000	2001	4001	2001	96	0.3762554317334E-03	0.7622103539688E-03	2
5000	2001	4001	2001	120	0.5883166299121E-03	0.8833981364660E-03	2
6000	2001	4001	2001	142	0.1035155290527E-02	0.4411030568008E-03	2
7000	2001	4001	2001	166	0.1152509547157E-02	0.1332242685748E-02	2
8000	2001	4001	2001	195	0.1499250120119E-02	0.2056686249489E-02	2
9000	2001	4001	2001	212	0.2368642947602E-02	0.6582689820789E-03	2
10000	2001	4001	2001	236	0.2343947691016E-02	0.2606601643492E-02	2

TOTAL 20008 40008 20010 13.09(seconds)

42 Dai-Kou. Standard WLS. Function:STAIRCASE S3

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	1999	3999	2001	26	0.2251745310710E-13	0.1354515006824E-06	0
2000	2001	4001	2001	47	0.9420175875238E-04	0.1317227965784E-02	2
3000	2001	4001	2001	71	0.2134740571680E-03	0.4934394532938E-03	2
4000	2001	4001	2001	94	0.3758556311008E-03	0.1043195008606E-02	2
5000	2001	4001	2001	121	0.5945461844552E-03	0.7222737694974E-03	2
6000	2001	4001	2001	142	0.8724228668367E-03	0.6709562831020E-03	2
7000	2001	4001	2001	169	0.1152743024252E-02	0.1426449591236E-02	2
8000	2001	4001	2001	188	0.1503650978075E-02	0.1855556221926E-02	2
9000	2001	4001	2001	218	0.1907882804503E-02	0.1716009552183E-02	2
10000	2001	4001	2001	239	0.2350045959306E-02	0.2136219194654E-02	2

TOTAL 20008 40008 20010 13.15(seconds)

43 Dai-Kou. Standard WLS. Function:NONDQUAR

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	2001	4013	2445	34	0.3267336952662E-05	0.2988197471574E-04	2
2000	2001	4016	2399	68	0.4213673219581E-05	0.1718439421017E-04	2
3000	2001	4009	2391	105	0.3500511879531E-05	0.1894771683405E-04	2
4000	2001	4018	2383	138	0.3590390817347E-05	0.1230869405974E-04	2
5000	2001	4010	2218	168	0.3646128290280E-05	0.9925659939677E-05	2
6000	1804	3625	2078	181	0.4412430986060E-05	0.9195286703331E-06	0
7000	2001	4022	2274	237	0.4206366989015E-05	0.2337966925370E-04	2
8000	2001	4017	2278	272	0.4926126710811E-05	0.1274198915792E-04	2
9000	2001	4024	2492	317	0.4652024449155E-05	0.5787434319530E-05	2
10000	2001	4027	2431	347	0.4656437789317E-05	0.6024982777389E-05	2

TOTAL 19813 39781 23389 18.67(seconds)

44 Dai-Kou. Standard WLS. Function:TRIDIA

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	261	523	262	3	0.7639736673201E-15	0.7107047265129E-06	0
2000	371	743	372	10	0.1485652009186E-14	0.9575403116929E-06	0
3000	459	919	460	19	0.9568057410973E-15	0.9824189871520E-06	0
4000	532	1065	533	28	0.8740251355717E-15	0.8942329331931E-06	0
5000	594	1189	595	40	0.1192977220591E-14	0.8905753739718E-06	0
6000	653	1307	654	53	0.9910301630624E-15	0.9981651753768E-06	0
7000	704	1409	705	63	0.1302124332540E-14	0.9842440471197E-06	0
8000	756	1513	757	78	0.9275109056099E-15	0.8766630847566E-06	0
9000	801	1603	802	98	0.1093366142717E-14	0.9908236962860E-06	0
10000	847	1695	848	113	0.8596529755363E-15	0.9270479529008E-06	0

TOTAL 5978 11966 5988 5.05(seconds)

45 Dai-Kou. Standard WLS. Function:ARWHEAD

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	11	29	22	0	0.0000000000000E+00	0.1443235588448E-09	0
2000	9	23	17	0	0.0000000000000E+00	0.2257029062789E-06	0
3000	9	23	17	1	0.0000000000000E+00	0.5052917724453E-08	0
4000	9	23	17	1	0.0000000000000E+00	0.1933455002786E-08	0
5000	10	25	17	1	0.0000000000000E+00	0.5490545727915E-06	0
6000	12	35	27	1	0.0000000000000E+00	0.2825426351214E-07	0
7000	10	27	19	2	0.0000000000000E+00	0.1569494360436E-09	0
8000	11	26	19	1	0.0000000000000E+00	0.5395026688448E-09	0
9000	9	123	117	6	0.0000000000000E+00	0.7400829092094E-11	4
10000	8	115	109	7	0.0000000000000E+00	0.7211651231362E-08	4

TOTAL 98 449 381 0.20(seconds)

46 Dai-Kou. Standard WLS. Function:NONDIA

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	36	126	109	1	0.9898968740952E+00	0.1301911602432E-12	0
2000	36	134	112	1	0.9898970597803E+00	0.8503893225420E-06	0
3000	40	251	224	3	0.9898969096136E+00	0.3111134808748E-05	4
4000	51	316	290	6	0.9898968740952E+00	0.1603980837039E-11	4
5000	40	242	218	7	0.9898968787086E+00	0.5351566592788E-07	0
6000	50	156	126	6	0.9898968741496E+00	0.4843650492314E-08	0
7000	45	231	203	8	0.9898971124124E+00	0.4245673214387E-06	0
8000	57	217	186	10	0.9898968789503E+00	0.4964037507438E-07	0
9000	59	260	225	13	0.9898970938915E+00	0.8938950982751E-06	0
10000	46	182	152	11	0.9898983821157E+00	0.6570021233165E-06	0

TOTAL 460 2115 1845 0.66(seconds)

47 Dai-Kou. Standard WLS. Function:BDQRTIC

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	508	1137	686	11	0.3983817950583E+04	0.3573940311816E-03	4
2000	502	1128	758	26	0.7989427682699E+04	0.8566138779560E-03	4
3000	115	397	349	13	0.1199503741453E+05	0.9131919669027E-03	4
4000	339	838	594	38	0.1600064714647E+05	0.1356685094617E-02	4
5000	982	2139	1220	113	0.2000625687845E+05	0.1279861468644E-01	4
6000	600	1334	765	81	0.2401186661283E+05	0.1657610892048E-01	4
7000	1161	2484	1372	181	0.2801747634284E+05	0.4665816216418E-02	4
8000	958	2076	1184	173	0.3202308607471E+05	0.5093484455001E-01	4
9000	421	1047	731	101	0.3602869580825E+05	0.4979236069810E-02	4
10000	1138	2431	1349	251	0.4003430553930E+05	0.1961339563214E-02	4

TOTAL 6724 15011 9008 9.88(seconds)

48 Dai-Kou. Standard WLS. Function:DQDRTIC

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	7	15	9	0	0.1707992867039E-16	0.1072908380163E-06	0
2000	7	15	9	0	0.3321619487869E-14	0.1270722504919E-06	0
3000	7	15	9	1	0.2239457285124E-14	0.1387969760681E-06	0
4000	7	15	9	0	0.2766787894298E-14	0.1354243021668E-06	0
5000	7	15	9	0	0.3105616959169E-14	0.1385087563382E-06	0
6000	7	15	9	0	0.8183326196249E-13	0.7840625219890E-06	0
7000	7	15	9	1	0.4141659377473E-13	0.4069763453704E-06	0
8000	7	15	9	1	0.1432224829712E-12	0.7568630573762E-06	0
9000	10	21	12	1	0.1551710787516E-24	0.1680227479257E-10	0
10000	8	17	10	1	0.9864780356662E-13	0.6280518013333E-06	0

TOTAL 74 158 94 0.05(seconds)

49 Dai-Kou. Standard WLS. Function:EG2

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	13	128	127	4	-0.9989473933008E+03	0.1453883695581E-04	4
2000	8	116	115	7	-0.1999947392741E+04	0.9195025242865E-06	4
3000	8	24	20	2	-0.2999947392570E+04	0.7670660435426E-07	0
4000	7	23	21	3	-0.3999947392484E+04	0.6493400842103E-06	0
5000	5	13	11	2	-0.4999947392433E+04	0.1812874540128E-07	0
6000	6	15	13	3	-0.5999947392399E+04	0.1510723437345E-07	0
7000	5	14	12	2	-0.6999947392374E+04	0.1294902504191E-07	0
8000	5	12	10	3	-0.7999947392355E+04	0.1133037580009E-07	0
9000	5	16	14	4	-0.8999947392341E+04	0.1513699253535E-07	0
10000	7	23	20	6	-0.9999947392330E+04	0.4460701195583E-06	0

TOTAL 69 384 363 0.36(seconds)

50 Dai-Kou. Standard WLS. Function:EG3 50

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	44	154	133	4	-0.9995000000000E+03	0.2194322619734E-07	0
2000	101	412	368	23	-0.1999499999979E+04	0.1652789002388E-03	4
3000	70	313	274	27	-0.2999500000000E+04	0.7035186764171E-05	4
4000	62	196	164	23	-0.3999499999980E+04	0.5645604417768E-06	0
5000	111	381	296	51	-0.4999499998412E+04	0.5425158060633E-03	4

```
     6000       61      299      267       51     -0.5999499999618E+04   0.3027215376543E-02 4
     7000       85      368      323       71     -0.6999500000000E+04   0.5194654733994E-05 4
     8000       39      225      201       53     -0.7999499976734E+04   0.9104225953788E-04 4
     9000      128      461      384      116     -0.8999499990459E+04   0.4129474357111E-03 4
    10000       91      424      372      122     -0.9999499965451E+04   0.3061700269673E-02 4
-----------------------------------------------------------------------------------------------
TOTAL    792    3233   2782    5.41(seconds)

  51  Dai-Kou. Standard WLS. Function:EDENSCH

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       24      141      129        1      0.6003284592021E+04   0.5773764197858E-06 4
     2000       24      141      131        4      0.1200328459202E+05   0.7298591913063E-06 4
     3000       22      122      111        3      0.1800328459202E+05   0.4134734241212E-05 4
     4000       23      140      130        5      0.2400328459202E+05   0.1575436102962E-05 4
     5000       22      140      130        6      0.3000328459202E+05   0.2647169898487E-05 4
     6000       21      135      125        7      0.3600328459202E+05   0.4929974740196E-05 4
     7000       24      162      156       10      0.4200328459202E+05   0.5787739314478E-06 4
     8000       22      133      123       10      0.4800328459202E+05   0.3020055032721E-05 4
     9000       24       53       46        5      0.5400328459202E+05   0.8290914021991E-06 0
    10000       23      120      111       10      0.6000328459202E+05   0.1444557476082E-05 4
-----------------------------------------------------------------------------------------------
TOTAL    229    1287   1192    0.61(seconds)

  52  Dai-Kou. Standard WLS. Function:FLETCHCR

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       21       46       26        0      0.7625916916815E-15   0.8061751707191E-06 0
     2000       22       48       27        1      0.4924774666006E-15   0.9166746913542E-06 0
     3000       23       50       28        1      0.1366091036710E-15   0.4913514927261E-06 0
     4000       23       50       28        2      0.2551265805010E-15   0.6014723828628E-06 0
     5000       23       50       28        2      0.3581787393068E-15   0.6027222719351E-06 0
     6000       23       50       28        2      0.4327776201078E-15   0.5162530401602E-06 0
     7000       23       50       28        2      0.5247127519631E-15   0.6413574336194E-06 0
     8000       23       50       28        3      0.6530736585339E-15   0.7134611787776E-06 0
     9000       23       50       28        5      0.7885642740291E-15   0.7750306393829E-06 0
    10000       23       50       28        3      0.9010873199045E-15   0.8190867980757E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    227     494    277    0.21(seconds)

  53  Dai-Kou. Standard WLS. Function:ENGVAL1

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       23      125      112        2      0.1108194718785E+04   0.8170153256500E-06 4
     2000       21      139      124        2      0.2218313143943E+04   0.4823602348658E-05 4
     3000       21      133      120        3      0.3328431569101E+04   0.3411180342638E-05 4
     4000       22      138      127        4      0.4438549994258E+04   0.1550271002760E-05 4
     5000       24      137      126        6      0.5548668419415E+04   0.3822146297239E-06 4
     6000       18      137      121        6      0.6658786844577E+04   0.8476074831810E-04 4
     7000       22      137      125        8      0.7768905269731E+04   0.1956139202619E-05 4
     8000       20      137      125        9      0.8879023694889E+04   0.1243178322774E-04 4
     9000       21      139      127        9      0.9989142120047E+04   0.4829904788117E-05 4
    10000       20      138      125       11      0.1109926054521E+05   0.1386095220690E-04 4
-----------------------------------------------------------------------------------------------
TOTAL    212    1360   1232    0.60(seconds)

  54  Dai-Kou. Standard WLS. Function:DENSCHNA

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        9       19       10        1      0.7349139229309E-15   0.2410527003887E-08 0
     2000        9       19       10        1      0.1469828098174E-14   0.2410527182774E-08 0
     3000        9       19       10        2      0.2204741752877E-14   0.2410527008624E-08 0
     4000        9       19       10        2      0.2939656629611E-14   0.2410527247501E-08 0
     5000        9       19       10        2      0.3674569513369E-14   0.2410526974709E-08 0
     6000        9       19       10        3      0.4409483360199E-14   0.2410526952212E-08 0
     7000        9       19       10        3      0.5144399044859E-14   0.2410527342592E-08 0
     8000        9       19       10        5      0.5879311056623E-14   0.2410527041225E-08 0
     9000        9       19       10        4      0.6614223347532E-14   0.2410526751329E-08 0
    10000        9       19       10        5      0.7349142115747E-14   0.2410527518565E-08 0
-----------------------------------------------------------------------------------------------
TOTAL     90     190    100    0.28(seconds)

  55  Dai-Kou. Standard WLS. Function:DENSCHNB

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       11       23       12        0      0.1058489654893E-11   0.1301352306288E-06 0
     2000       11       23       12        0      0.2115640384446E-11   0.1300940724408E-06 0
     3000       11       23       12        0      0.3173467339971E-11   0.1300942109967E-06 0
     4000       11       23       12        1      0.4233707326119E-11   0.1301313688291E-06 0
     5000       11       23       12        1      0.5293659926912E-11   0.1301501253810E-06 0
     6000       11       23       12        1      0.6350420211765E-11   0.1301299264273E-06 0
     7000       11       23       12        1      0.7406830700130E-11   0.1301124239834E-06 0
     8000       11       23       12        1      0.8462803579320E-11   0.1300959322865E-06 0
     9000       11       23       12        1      0.9519099498875E-11   0.1300853114489E-06 0
    10000       11       23       12        2      0.1057580024020E-10   0.1300793038101E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    110     230    120    0.08(seconds)

  56  Dai-Kou. Standard WLS. Function:DENSCHNC
```

```
        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     2000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     3000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     4000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     5000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     6000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     7000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     8000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     9000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
    10000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
-----------------------------------------------------------------------------------------------
TOTAL      0      10     10    0.00(seconds)

  57  Dai-Kou. Standard WLS. Function:DENSCHNF

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       25       57       34        1      0.2279741433594E-15   0.1237525345092E-07 0
     2000       25       57       34        1      0.4559566753772E-15   0.1236901497430E-07 0
     3000       25       57       34        1      0.6839309033719E-15   0.1237298755395E-07 0
     4000       25       57       34        2      0.9119132531692E-15   0.1237389689244E-07 0
     5000       25       57       34        2      0.1139881793974E-14   0.1237431639782E-07 0
     6000       25       57       34        3      0.1367882096347E-14   0.1237318363865E-07 0
     7000       25       57       34        4      0.1595849731737E-14   0.1237265194965E-07 0
     8000       25       57       34        4      0.1823897464175E-14   0.1237466690065E-07 0
     9000       25       57       34        5      0.2051807617542E-14   0.1237410673979E-07 0
    10000       25       57       34        5      0.2279775220254E-14   0.1237231615603E-07 0
-----------------------------------------------------------------------------------------------
TOTAL    250      570    340    0.28(seconds)

  58  Dai-Kou. Standard WLS. Function:SINQUAD

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       77      234      175       10      0.1124740447730E-06   0.4546770474060E-07 0
     2000      392      957      698       69      0.3118783728310E-07   0.4525171028391E-06 0
     3000      277      721      557       79      0.1474898166250E-05   0.3811241347545E-06 0
     4000      451     1043      699      147      0.1303013816672E-04   0.3530122620215E-06 0
     5000      676     1551      970      260      0.2060613839626E-04   0.8755404593309E-06 0
     6000     1191     2700     1690      558      0.5041271554593E-05   0.2409467228503E-06 0
     7000      366      937      638      229      0.5298284980561E-06   0.4374294082427E-06 0
     8000      162      568      473      174      0.1937497887823E-04   0.6201184720799E-06 0
     9000      325      862      619      279      0.7541731964414E-05   0.5325862200612E-06 0
    10000       93      292      228       93      0.7214999063362E-04   0.8734189673748E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   4010    9865   6747   18.98(seconds)

  59  Dai-Kou. Standard WLS. Function:DIXON3DQ

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000     1000     2001     1001        9      0.9990009990010E-03   0.1849009834132E-10 0
     2000     2000     4001     2001       34      0.4997501249375E-03   0.9730882766235E-11 0
     3000     2001     4001     2001       54      0.8482225059693E-03   0.2028489409215E-02 2
     4000     2001     4001     2001       92      0.2802126025867E-02   0.6701169715576E-02 2
     5000     2001     4001     2001      108      0.2808595702149E-02   0.2098950524872E-02 2
     6000     2001     4001     2001      103      0.2808595702149E-02   0.2098950524872E-02 2
     7000     2001     4001     2001      117      0.2808595702149E-02   0.2098950524872E-02 2
     8000     2001     4001     2001      137      0.2808595702149E-02   0.2098950524872E-02 2
     9000     2001     4001     2001      153      0.2808595702149E-02   0.2098950524872E-02 2
    10000     2001     4001     2001      173      0.2808595702149E-02   0.2098950524872E-02 2
-----------------------------------------------------------------------------------------------
TOTAL  19008   38010  19010    9.80(seconds)

  60  Dai-Kou. Standard WLS. Function:BIGGSB1                            60

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000      500     1001      501        4      0.1549462877782E-15   0.2328511738980E-07 0
     2000     1000     2001     1001       17      0.1622873778237E-15   0.2315846470147E-07 0
     3000     1500     3001     1501       38      0.2078031304144E-15   0.2323047576525E-07 0
     4000     2000     4001     2001       68      0.5063448164185E-15   0.2367777796053E-07 0
     5000     2001     4001     2001       87      0.8095952023988E-03   0.8995502250102E-03 2
     6000     2001     4001     2001      101      0.8095952023988E-03   0.8995502250102E-03 2
     7000     2001     4001     2001      119      0.8095952023988E-03   0.8995502250102E-03 2
     8000     2001     4001     2001      137      0.8095952023988E-03   0.8995502250102E-03 2
     9000     2001     4001     2001      158      0.8095952023988E-03   0.8995502250102E-03 2
    10000     2001     4001     2001      174      0.8095952023988E-03   0.8995502250102E-03 2
-----------------------------------------------------------------------------------------------
TOTAL  17006   34010  17010    9.03(seconds)

  61  Dai-Kou. Standard WLS. Function:PRODsin

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        0        1        1        0      0.9989999999998E-12   0.2998999999999E-08 0
     2000        0        1        1        0      0.3998000000000E-11   0.5998999999999E-08 0
     3000        0        1        1        0      0.8996999999998E-11   0.8998999999997E-08 0
     4000        0        1        1        0      0.1599600000000E-10   0.1199900000000E-07 0
     5000        0        1        1        0      0.2499500000000E-10   0.1499900000000E-07 0
     6000        0        1        1        0      0.3599400000000E-10   0.1799900000000E-07 0
```

```
     7000        0        1        1        0      0.4899300000000E-10   0.2099900000000E-07 0
     8000        0        1        1        1      0.6399200000000E-10   0.2399900000000E-07 0
     9000        0        1        1        0      0.8099099999999E-10   0.2699899999999E-07 0
    10000        0        1        1        0      0.9998999999998E-10   0.2999899999999E-07 0
-----------------------------------------------------------------------------------------------
TOTAL      0      10     10    0.01(seconds)

  62  Dai-Kou. Standard WLS. Function:PROD1

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        2        5        3        0      0.1637470899020E-34   0.8093135113219E-17 0
     2000        2        5        3        0      0.2027987163540E-31   0.2848148285143E-15 0
     3000        2        5        3        1      0.1039211808735E-33   0.2038834773821E-16 0
     4000        2        5        3        0      0.9361408092150E-31   0.6119283648320E-15 0
     5000        2        5        3        0      0.5416583025153E-31   0.4654710742958E-15 0
     6000        2        5        3        0      0.1346916891038E-29   0.2321134973273E-14 0
     7000        2        5        3        0      0.1043357942124E-29   0.2042897884990E-14 0
     8000        2        5        3        0      0.4707489147354E-29   0.4339349788784E-14 0
     9000        2        5        3        1      0.1277853256892E-29   0.2260843432785E-14 0
    10000        2        5        3        0      0.4412006169173E-28   0.1328458681205E-13 0
-----------------------------------------------------------------------------------------------
TOTAL     20      50     30    0.02(seconds)

  63  Dai-Kou. Standard WLS. Function:PRODcos

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        4        9        5        0      0.2668568171659E-23   0.9285542614719E-10 0
     2000        4        9        5        1      0.1022890715782E-23   0.8548840635477E-10 0
     3000        4        9        5        0      0.6072922017321E-24   0.8212504087080E-10 0
     4000        4        9        5        1      0.4513461700027E-24   0.8250231787568E-10 0
     5000        4        9        5        1      0.3660126881783E-24   0.8352670862297E-10 0
     6000        4        9        5        1      0.3023312360363E-24   0.8347021921971E-10 0
     7000        4        9        5        1      0.2294636606925E-24   0.7875608349129E-10 0
     8000        4        9        5        1      0.2617671660481E-24   0.9010680572903E-10 0
     9000        4        9        5        2      0.1751692614650E-24   0.7830495472690E-10 0
    10000        4        9        5        1      0.1327902113200E-24   0.7195719622807E-10 0
-----------------------------------------------------------------------------------------------
TOTAL     40      90     50    0.09(seconds)

  64  Dai-Kou. Standard WLS. Function:PROD2

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        0        1        1        0      0.1004995000000E-16   0.4019980010000E-11 0
     2000        0        1        1        0      0.2019990000000E-16   0.8079960010000E-11 0
     3000        0        1        1        0      0.3044985000000E-16   0.1217994001000E-10 0
     4000        0        1        1        0      0.4079980000000E-16   0.1631992001000E-10 0
     5000        0        1        1        0      0.5124975000000E-16   0.2049990001000E-10 0
     6000        0        1        1        0      0.6179970000000E-16   0.2471988001000E-10 0
     7000        0        1        1        0      0.7244965000000E-16   0.2897986001000E-10 0
     8000        0        1        1        0      0.8319960000000E-16   0.3327984001000E-10 0
     9000        0        1        1        0      0.9404955000000E-16   0.3761982001000E-10 0
    10000        0        1        1        0      0.1049995000000E-15   0.4199980001000E-10 0
-----------------------------------------------------------------------------------------------
TOTAL      0      10     10    0.00(seconds)

  65  Dai-Kou. Standard WLS. Function:DIXMAANA

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        8       17        9        0      0.1000000000000E+01   0.6810895251955E-07 0
     2000        8       17        9        1      0.1000000000001E+01   0.6810883655927E-07 0
     3000        8       17        9        1      0.1000000000001E+01   0.6498578597812E-07 0
     4000        8       17        9        0      0.1000000000002E+01   0.6811024397548E-07 0
     5000        8       17        9        2      0.1000000000002E+01   0.6809368966984E-07 0
     6000        8       17        9        1      0.1000000000003E+01   0.6812233587944E-07 0
     7000        8       17        9        2      0.1000000000003E+01   0.7124166712158E-07 0
     8000        8       17        9        1      0.1000000000004E+01   0.6812178371916E-07 0
     9000        8       17        9        2      0.1000000000004E+01   0.6499410983907E-07 0
    10000        8       17        9        2      0.1000000000005E+01   0.6812209649783E-07 0
-----------------------------------------------------------------------------------------------
TOTAL     80     170     90    0.12(seconds)

  66  Dai-Kou. Standard WLS. Function:DIXMAANB

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        7       15        8        0      0.1000000000000E+01   0.2626149583226E-06 0
     2000        7       15        8        1      0.1000000000000E+01   0.2636252608177E-06 0
     3000        7       15        8        0      0.1000000000000E+01   0.2638147809329E-06 0
     4000        7       15        8        1      0.1000000000001E+01   0.2639920380912E-06 0
     5000        7       15        8        1      0.1000000000001E+01   0.2641548339231E-06 0
     6000        7       15        8        1      0.1000000000001E+01   0.2639730967009E-06 0
     7000        7       15        8        2      0.1000000000001E+01   0.2640434634587E-06 0
     8000        7       15        8        1      0.1000000000001E+01   0.2641067664488E-06 0
     9000        7       15        8        1      0.1000000000001E+01   0.2639822194530E-06 0
    10000        7       15        8        2      0.1000000000002E+01   0.2642928226693E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     70     150     80    0.10(seconds)

  67  Dai-Kou. Standard WLS. Function:DIXMAANC
```

```
         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        9       19       10        0      0.1000000000000E+01   0.1314014064258E-06 0
      2000        9       19       10        0      0.1000000000000E+01   0.8934527373806E-07 0
      3000        9       19       10        1      0.1000000000000E+01   0.8819123538326E-07 0
      4000        9       19       10        1      0.1000000000000E+01   0.1124420995931E-06 0
      5000        9       19       10        2      0.1000000000000E+01   0.9159251921392E-07 0
      6000        9       19       10        1      0.1000000000000E+01   0.9463951006667E-07 0
      7000        9       19       10        2      0.1000000000001E+01   0.1848002771070E-06 0
      8000        9       19       10        2      0.1000000000000E+01   0.1170135524566E-06 0
      9000        9       19       10        2      0.1000000000001E+01   0.2487819531421E-06 0
     10000        9       19       10        3      0.1000000000001E+01   0.1269696414305E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     90     190    100    0.14(seconds)

  68  Dai-Kou. Standard WLS. Function:DIXMAAND

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000       10       21       11        0      0.1000000000002E+01   0.8879014017100E-06 0
      2000       10       21       11        1      0.1000000000004E+01   0.8787469317526E-06 0
      3000       10       21       11        1      0.1000000000006E+01   0.8930886860198E-06 0
      4000       10       21       11        1      0.1000000000008E+01   0.8964143510518E-06 0
      5000       10       21       11        2      0.1000000000010E+01   0.8839282188482E-06 0
      6000       10       21       11        2      0.1000000000012E+01   0.8823756348420E-06 0
      7000       10       21       11        2      0.1000000000014E+01   0.8741660037675E-06 0
      8000       10       21       11        2      0.1000000000015E+01   0.8960670770560E-06 0
      9000       10       21       11        2      0.1000000000017E+01   0.8331163658828E-06 0
     10000       10       21       11        4      0.1000000000019E+01   0.8564145492559E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    100     210    110    0.17(seconds)

  69  Dai-Kou. Standard WLS. Function:DIXMAANL

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000      973     1947      974       27      0.1000000002440E+01   0.9301598604812E-06 0
      2000      539     1079      540       31      0.1000000119353E+01   0.9776913046399E-06 0
      3000      235      471      236       20      0.1000000108954E+01   0.9960578915824E-06 0
      4000      235      471      236       27      0.1000000072026E+01   0.9901065114811E-06 0
      5000      236      473      237       34      0.1000000056671E+01   0.9984365212190E-06 0
      6000      236      473      237       40      0.1000000050562E+01   0.9998824130652E-06 0
      7000      239      479      240       50      0.1000000046828E+01   0.9748918340208E-06 0
      8000      239      479      240       54      0.1000000046963E+01   0.9768804371667E-06 0
      9000      239      479      240       61      0.1000000048432E+01   0.9781948194580E-06 0
     10000      239      479      240       68      0.1000000050729E+01   0.9835112909178E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   3410    6830   3420    4.12(seconds)

  70  Dai-Kou. Standard WLS. Function:ARGLINB                              70

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        2        5        4        0      0.9090909090916E+00   0.5940819391981E-08 0
      2000        3        7        5        1      0.9090909090962E+00   0.1615489964024E-06 0
      3000        5       14       11        0      0.9090909090175E+00   0.9573159331921E-07 0
      4000        2        6        5        1      0.9090909090633E+00   0.1847674866440E-06 0
      5000        3      100       99        9      0.9090909090698E+00   0.2387241693214E-07 4
      6000        3        7        6        1      0.9090909090748E+00   0.9929972293321E-06 0
      7000        4       10        8        1      0.9090909091158E+00   0.3579152689781E-06 0
      8000        2        6        5        1      0.9090909092376E+00   0.3126478986815E-07 0
      9000        4       10        8        1      0.9090909085869E+00   0.8152201189660E-06 0
     10000        3       99       97       22      0.9090909098202E+00   0.1541317033116E-05 4
-----------------------------------------------------------------------------------------------
TOTAL     31     264    248    0.37(seconds)

  71  Dai-Kou. Standard WLS. Function:VARDIM

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        8       18       11        0      0.3388313703830E-20   0.1164153682343E-06 0
      2000       11       24       14        0      0.2816553413344E-24   0.4130029651606E-13 0
      3000       13       28       16        0      0.7377858593934E-25   0.1776356839400E-13 0
      4000       15       33       20        0      0.3185218916907E-23   0.9858780458671E-13 0
      5000       16      138      124        4      0.1113772990559E-27   0.8881784197001E-15 4
      6000       16       34       19        1      0.2053701794510E-23   0.6439293542826E-13 0
      7000       18      137      121        9      0.1430180169262E-27   0.1110223024625E-14 4
      8000       18      137      121        6      0.1699132434136E-27   0.8881784197001E-15 4
      9000       18       38       21        2      0.1305122862238E-22   0.1323385845353E-12 0
     10000       19       41       24        2      0.1112337322876E-21   0.3654854197066E-12 0
-----------------------------------------------------------------------------------------------
TOTAL    152     628    491    0.24(seconds)

  72  Dai-Kou. Standard WLS. Function:DIAG-AUP1

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000       19       39       24        0      0.5975878337922E-18   0.8519349137258E-07 0
      2000       18       37       21        1      0.1049132734927E-13   0.2061478086801E-07 0
      3000       17       35       21        1      0.4806947415471E-14   0.8413823377429E-08 0
      4000       15       32       19        0      0.1069301343986E-12   0.5640645042527E-07 0
      5000       20       43       28        2      0.1178442755446E-12   0.7124126307901E-06 0
      6000       16       34       20        1      0.7014534918691E-21   0.2094630247029E-08 0
      7000       16       34       19        2      0.3021187698969E-19   0.3259282799206E-07 0
```

8000	16	34	20	2	0.1100666505644E-18	0.1095778960902E-06	0
9000	18	45	32	2	0.4514494016323E-19	0.8063244116840E-07	0
10000	16	36	22	2	0.1071187833023E-10	0.1930100416538E-06	0

TOTAL 171 369 226 0.13(seconds)

73 Dai-Kou. Standard WLS. Function:ENGVAL8

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	22	130	119	1	-0.1175249106591E+05	0.3746532813764E-05	4
2000	20	139	129	2	-0.2351469422181E+05	0.1970032267096E-05	4
3000	22	144	136	2	-0.3527689737772E+05	0.7655069906724E-05	4
4000	21	126	118	2	-0.4703910053362E+05	0.1545352898091E-05	4
5000	20	142	132	4	-0.5880130368952E+05	0.6299486969397E-05	4
6000	21	131	122	4	-0.7056350684544E+05	0.1498862786775E-05	4
7000	19	140	128	5	-0.8232571000132E+05	0.4803484483595E-04	4
8000	17	137	126	6	-0.9408791315724E+05	0.9210452517294E-04	4
9000	18	135	125	5	-0.1058501163131E+06	0.3586635569075E-04	4
10000	18	129	119	6	-0.1176123194690E+06	0.4411244399449E-04	4

TOTAL 198 1353 1254 0.37(seconds)

74 Dai-Kou. Standard WLS. Function:QUARTIC

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	13	27	14	0	0.4280478150116E-06	0.3764256462865E-06	0
2000	13	27	14	0	0.8560956291180E-06	0.3764256459879E-06	0
3000	13	27	14	0	0.1284143444995E-05	0.3764256462776E-06	0
4000	13	27	14	1	0.1712191260935E-05	0.3764256464330E-06	0
5000	13	27	14	0	0.2140239077192E-05	0.3764256465679E-06	0
6000	13	27	14	0	0.2568286893362E-05	0.3764256466484E-06	0
7000	13	27	14	1	0.2996334708889E-05	0.3764256466452E-06	0
8000	13	27	14	1	0.3424382525005E-05	0.3764256466914E-06	0
9000	13	27	14	1	0.3852430340552E-05	0.3764256466857E-06	0
10000	13	27	14	1	0.4280478158679E-05	0.3764256468512E-06	0

TOTAL 130 270 140 0.05(seconds)

75 Dai-Kou. Standard WLS. Function:LIARWHD

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	23	48	30	1	0.5942979905026E-22	0.2640625496042E-09	0
2000	20	44	27	0	0.1838929153425E-14	0.3830090081150E-08	0
3000	25	54	35	1	0.1136987361148E-14	0.2424794563254E-08	0
4000	18	39	23	1	0.6590934680830E-19	0.3767025003043E-07	0
5000	26	55	37	2	0.7936815872194E-14	0.3144292862837E-06	0
6000	19	43	29	1	0.2014445373911E-12	0.2791844430421E-06	0
7000	22	50	34	2	0.3109041763848E-13	0.9092071948023E-06	0
8000	24	59	41	2	0.1884743916723E-18	0.1552555661810E-06	0
9000	17	44	31	3	0.1664491098729E-11	0.6280498843688E-06	0
10000	21	47	32	3	0.1276462857207E-09	0.4770880689504E-06	0

TOTAL 215 483 319 0.16(seconds)

76 Dai-Kou. Standard WLS. Function:NONSCOMP

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	33	67	34	0	0.6566071765718E-13	0.8364840360324E-06	0
2000	31	63	32	1	0.1046003917401E-12	0.6863853250465E-06	0
3000	32	65	33	1	0.1365739677496E-12	0.7857595022481E-06	0
4000	32	65	33	1	0.4324326273691E-13	0.5017064761249E-06	0
5000	32	65	33	2	0.1102075813851E-12	0.8146770306822E-06	0
6000	34	69	35	2	0.2073798680734E-13	0.3264571207257E-06	0
7000	32	65	33	2	0.1518987641544E-12	0.9800339276511E-06	0
8000	34	69	35	3	0.2865634240332E-13	0.4671917193178E-06	0
9000	33	67	34	3	0.1035137471332E-12	0.9525664586732E-06	0
10000	33	67	34	4	0.1047246912896E-12	0.9831362288973E-06	0

TOTAL 326 662 336 0.19(seconds)

77 Dai-Kou. Standard WLS. Function:Linear perturbed

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	183	367	184	1	-0.1871367713288E-03	0.9828654752662E-06	0
2000	260	521	261	5	-0.2044592024171E-03	0.9498648586657E-06	0
3000	319	639	320	9	-0.2145937470793E-03	0.9396793927696E-06	0
4000	369	739	370	13	-0.2217847573354E-03	0.9084746985348E-06	0
5000	412	825	413	17	-0.2273627211437E-03	0.9738770732539E-06	0
6000	452	905	453	24	-0.2319203434350E-03	0.9357654285162E-06	0
7000	488	977	489	31	-0.2357738128185E-03	0.9616552664616E-06	0
8000	522	1045	523	37	-0.2391118744338E-03	0.9497582902258E-06	0
9000	553	1107	554	47	-0.2420562767041E-03	0.9992513153294E-06	0
10000	583	1167	584	53	-0.2446901507113E-03	0.9998714318327E-06	0

TOTAL 4141 8292 4151 2.37(seconds)

78 Dai-Kou. Standard WLS. Function:CUBE

n	iter	nfunc	ngrad	time(c)	f	gnorm	s

1000	2001	4158	2213	19	0.4404555259295E-05	0.3112196153935E-03	2
2000	2001	4096	2137	51	0.5185128105732E-05	0.1173220487280E-03	2
3000	2001	4037	2051	155	0.9604528881397E-05	0.6692210195309E-03	2
4000	2001	4142	2177	229	0.6154091028030E-05	0.1140392571308E-02	2
5000	2001	4152	2197	272	0.3207560690818E-05	0.1782555980521E-03	2
6000	2001	4094	2123	324	0.7534075515563E-05	0.1430616887957E-02	2
7000	2001	4011	2013	377	0.3869322607476E-04	0.4172269919958E-02	2
8000	2001	4090	2132	431	0.7882653512469E-05	0.1828879647013E-02	2
9000	2001	4103	2144	477	0.8293114681790E-05	0.8631241781080E-03	2
10000	2001	4058	2069	526	0.1577961887531E-04	0.1746228331519E-03	2

TOTAL 20010 40941 21256 28.61(seconds)

79 Dai-Kou. Standard WLS. Function:HARKERP

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	2	5	3	0	-0.2501250625312E+00	0.1247676081938E-07	0
2000	2	5	3	1	-0.2500625156289E+00	0.1241222940906E-06	0
3000	2	5	3	0	-0.2500416736122E+00	0.3739377381660E-08	0
4000	2	5	3	0	-0.2500312539067E+00	0.7322543882826E-08	0
5000	2	5	3	1	-0.2500250025002E+00	0.1234972146384E-07	0
6000	2	5	3	0	-0.2500208350695E+00	0.2781046905907E-08	0
7000	2	5	3	1	-0.2500178584184E+00	0.2522843742996E-08	0
8000	2	5	3	1	-0.2500156259765E+00	0.3954938431464E-09	0
9000	2	5	3	0	-0.2500138896604E+00	0.2622662159129E-08	0
10000	2	5	3	1	-0.2500125006250E+00	0.2386322046521E-08	0

TOTAL 20 50 30 0.05(seconds)

80 Dai-Kou. Standard WLS. Function:QUARTICM

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	29	59	30	0	0.4696217672714E-06	0.4038285681118E-06	0
2000	30	61	31	2	0.2861613779183E-05	0.9309120661459E-06	0
3000	31	63	32	2	0.4139913236970E-05	0.9058808347146E-06	0
4000	32	65	33	3	0.3302757244739E-05	0.6162457379808E-06	0
5000	33	67	34	4	0.1911811238434E-05	0.3459245594764E-06	0
6000	33	67	34	4	0.4745304408588E-05	0.5966215045683E-06	0
7000	33	67	34	5	0.1023722696811E-04	0.9460623191902E-06	0
8000	34	69	35	7	0.3789269394350E-05	0.4061625063651E-06	0
9000	34	69	35	7	0.6791040298400E-05	0.5759257986319E-06	0
10000	34	69	35	8	0.1157700770525E-04	0.7939438828669E-06	0

TOTAL 323 656 333 0.42(seconds)

Appendix 2

Dai-Kou conjugate gradient algorithm
Approximate Wolfe line search (WLS=Wolfe line search)

1 Dai-Kou. Approximate WLS. Function:Extended Freudenstein & Roth

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	12	27	18	1	0.2449212683962E+05	0.2747155640748E-06	0
2000	12	27	18	0	0.4898425367924E+05	0.5934663800389E-06	0
3000	12	27	18	1	0.7347638051886E+05	0.1322588580877E-06	0
4000	13	28	19	1	0.9796850735847E+05	0.5439626207249E-06	0
5000	11	25	16	1	0.1224606341981E+06	0.9960185707314E-06	0
6000	13	28	19	1	0.1469527610377E+06	0.1077915534609E-07	0
7000	13	33	24	2	0.1714448878773E+06	0.3598330522436E-08	0
8000	12	27	18	1	0.1959370147170E+06	0.8223763394710E-06	0
9000	13	28	19	2	0.2204291415566E+06	0.7886822270819E-06	0
10000	13	29	20	3	0.2449212683962E+06	0.4149569576839E-11	0

TOTAL 124 279 189 0.13(seconds)

2 Dai-Kou. Approximate WLS. Function:Extended White & Holst

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	11	25	15	0	0.9044668587886E-12	0.8730601372086E-07	0
2000	11	25	15	0	0.1808936493812E-11	0.8730607766970E-07	0
3000	11	25	15	1	0.2713399640957E-11	0.8730599640138E-07	0
4000	11	25	15	0	0.3617867207512E-11	0.8730600705952E-07	0
5000	11	25	15	1	0.4522306573876E-11	0.8730574238235E-07	0
6000	11	25	15	1	0.5426825004142E-11	0.8730619268881E-07	0
7000	11	25	15	1	0.6331175912787E-11	0.8730536046563E-07	0
8000	11	25	15	1	0.7235638223072E-11	0.8730540309819E-07	0
9000	11	25	15	1	0.8140106425094E-11	0.8730546527068E-07	0
10000	11	25	15	2	0.9044544089235E-11	0.8730537512057E-07	0

TOTAL 110 250 150 0.08(seconds)

3 Dai-Kou. Approximate WLS. Function:Tridiagonal White-Holst

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	2001	4140	2140	29	0.2790978732451E+03	0.2467531602720E+01	2
2000	2001	4122	2122	57	0.1061090681170E+04	0.2343586058084E+01	2

```
      3000     2001     4145     2145       98      0.1846833230222E+04   0.2638131064197E+01 2
      4000     2001     4134     2134      121      0.2626694832170E+04   0.2656675939575E+01 2
      5000     2001     4133     2133      142      0.3409174417076E+04   0.2704949640521E+01 2
      6000     2001     4134     2134      171      0.4190906736573E+04   0.3405729456804E+01 2
      7000     2001     4132     2132      199      0.4972765142351E+04   0.2745698044908E+01 2
      8000     2001     4128     2128      423      0.5753440590978E+04   0.2906603208002E+01 2
      9000     2001     4111     2111      522      0.6536983735045E+04   0.2672520240695E+01 2
     10000     2001     4118     2118      581      0.7317877313738E+04   0.2359348224555E+01 2
-----------------------------------------------------------------------------------------------
TOTAL  20010   41297  21297   23.43(seconds)

   4  Dai-Kou. Approximate WLS. Function:Extended Beale

        n     iter    nfunc    ngrad  time(c)               f                    gnorm         s
-----------------------------------------------------------------------------------------------
     1000       12       25       15        1      0.4082404550275E-11   0.6146879953400E-07 0
     2000       12       25       15        1      0.8172204810851E-11   0.6149757176792E-07 0
     3000       12       25       15        1      0.1226816500740E-10   0.6152312807440E-07 0
     4000       12       25       15        2      0.1633467089572E-10   0.6147863229338E-07 0
     5000       12       25       15        2      0.2043371814360E-10   0.6150256048558E-07 0
     6000       12       25       15        2      0.2461509230098E-10   0.6162511145208E-07 0
     7000       12       25       15        4      0.2861445786226E-10   0.6151061931677E-07 0
     8000       12       25       15        3      0.3273079679656E-10   0.6153838042897E-07 0
     9000       12       25       15        4      0.3670202583870E-10   0.6143454801651E-07 0
    10000       12       25       15        4      0.4077847215615E-10   0.6143333815100E-07 0
-----------------------------------------------------------------------------------------------
TOTAL    120      250    150    0.24(seconds)

   5  Dai-Kou. Approximate WLS. Function:Extended Powell

        n     iter    nfunc    ngrad  time(c)               f                    gnorm         s
-----------------------------------------------------------------------------------------------
     1000      487      977      548       11      0.8247833943480E-08   0.9544788104147E-06 0
     2000      491      985      588       22      0.1793301849008E-07   0.9862543559483E-06 0
     3000      489      981      566       35      0.2400736548647E-07   0.9855812907199E-06 0
     4000      487      977      560       45      0.3090271457341E-07   0.9688371991278E-06 0
     5000      487      977      564       57      0.3926540965930E-07   0.9942016415614E-06 0
     6000      491      985      554       68      0.4361359249168E-07   0.9178349881033E-06 0
     7000      429      861      514       70      0.1977107101442E-06   0.9783576561166E-06 0
     8000      589     1181      758      113      0.6820018699529E-07   0.9992522319985E-06 0
     9000      541     1085      682      116      0.8614827137096E-07   0.9635934824157E-06 0
    10000      113      233      136       27      0.2473850029513E-05   0.9302348890533E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   4604    9242   5470    5.64(seconds)

   6  Dai-Kou. Approximate WLS. Function:Extended Maratos

        n     iter    nfunc    ngrad  time(c)               f                    gnorm         s
-----------------------------------------------------------------------------------------------
     1000        9       26       19        0     -0.5280864426222E+03   0.1076241928910E-06 0
     2000        9       26       19        0     -0.1056172885244E+04   0.7941204449580E-07 0
     3000        9       26       19        0     -0.1584259327867E+04   0.6878391167753E-07 0
     4000        9       26       19        2     -0.2112345770489E+04   0.1181336408695E-06 0
     5000        9       26       19        1     -0.2640432213111E+04   0.5226534494085E-07 0
     6000        9       26       19        2     -0.3168518655733E+04   0.2864253581939E-07 0
     7000        9       26       19        1     -0.3696605098356E+04   0.1344662334812E-06 0
     8000        9       26       19        2     -0.4224691540978E+04   0.9533927913807E-07 0
     9000        9       26       19        2     -0.4752777983600E+04   0.2029684651461E-06 0
    10000        9       26       19        3     -0.5280864426222E+04   0.4138538512241E-07 0
-----------------------------------------------------------------------------------------------
TOTAL     90      260    190    0.13(seconds)

   7  Dai-Kou. Approximate WLS. Function:Extended CLIFF

        n     iter    nfunc    ngrad  time(c)               f                    gnorm         s
-----------------------------------------------------------------------------------------------
     1000        7       16       10        1      0.4232867951672E+03   0.1350742275807E-06 0
     2000        7       17       11        1      0.8465735906175E+03   0.4010118190845E-06 0
     3000        6       14        9        2      0.1269860385420E+04   0.1558767679377E-06 0
     4000        7       16       10        3      0.1693147180577E+04   0.3636214684910E-07 0
     5000        7       18       12        5      0.2116433978946E+04   0.5648424044047E-06 0
     6000        7       21       15        6      0.2539720771925E+04   0.6447629512518E-06 0
     7000        6       14        9        5      0.2963007578395E+04   0.7089088185364E-06 0
     8000        9       31       26       13      0.3386294361120E+04   0.3663834236001E-08 0
     9000       13       42       35       19      0.3809581156328E+04   0.2684434800004E-06 0
    10000        7       19       13        9      0.4232867956794E+04   0.9458281069463E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     76      208    150    0.64(seconds)

   8  Dai-Kou. Approximate WLS. Function:Extended Wood

        n     iter    nfunc    ngrad  time(c)               f                    gnorm         s
-----------------------------------------------------------------------------------------------
     1000      225      509      294        6      0.3597551303495E-10   0.9470559562367E-06 0
     2000      189      408      230       10      0.1467381407357E-11   0.5651897636404E-06 0
     3000      131      306      191       11      0.6091785944337E-09   0.9085714801849E-06 0
     4000      352      744      400       37      0.3118339407705E-10   0.9614829099489E-06 0
     5000      405      840      449       57      0.9848324547775E-10   0.8393133939633E-06 0
     6000      338      706      374       52      0.7042293620124E-10   0.8915897260291E-06 0
     7000      280      599      332       53      0.1074014467991E-09   0.8092819872075E-06 0
     8000      140      323      205       31      0.3753996341840E-10   0.6648066546600E-06 0
     9000      130      298      191       34      0.2273050058076E-10   0.8395890387191E-06 0
    10000      127      294      183       36      0.3967230567741E-10   0.3091945664613E-06 0
-----------------------------------------------------------------------------------------------
```

```
TOTAL   2317    5027   2849    3.27(seconds)

   9  Dai-Kou. Approximate WLS. Function:Extended Hiebert

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       44      132       97        2      0.9215863802089E-15   0.1355910193023E-06 0
     2000       48      142      106        3      0.5142665931811E-11   0.2755996300287E-07 0
     3000       46      132       96        4      0.5552773260828E-15   0.5891667953595E-07 0
     4000       41      127       96        5      0.3938293569370E-09   0.1706746312940E-06 0
     5000       46      136      100        7      0.4441593105574E-14   0.3043712836564E-08 0
     6000       49      145      107        9      0.7031459561446E-14   0.9965265946821E-08 0
     7000       45      145      111       11      0.6441741941053E-13   0.4289530686160E-06 0
     8000       46      132       97       11      0.7782190230846E-13   0.4383864222528E-06 0
     9000       45      139      103       12      0.3503195656759E-13   0.2784078212361E-06 0
    10000       43      133      100       13      0.1736071151798E-12   0.5891606882450E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    453    1363   1013    0.77(seconds)

  10  Dai-Kou. Approximate WLS. Function:Extended Rosenbrock                 10

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       62      204      160        2      0.5242493581132E-11   0.7835021520464E-07 0
     2000       56      175      136        4      0.1352506849413E-08   0.8556770847434E-06 0
     3000       59      201      158        6      0.2773938020067E-09   0.3965374695269E-06 0
     4000       54      165      127        6      0.1343111379881E-13   0.3256041153363E-06 0
     5000       54      182      143       10      0.1490418992852E-09   0.2098787810212E-06 0
     6000       55      182      143       11      0.9939264202694E-09   0.5958562532271E-06 0
     7000       59      205      164       14      0.9893797981678E-12   0.3100741885476E-07 0
     8000       58      188      146       16      0.2734837507106E-09   0.2300368784347E-06 0
     9000       59      167      127       16      0.3791141487306E-13   0.3253804167185E-07 0
    10000       55      170      128       17      0.1116355612264E-09   0.1135558314047E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    571    1839   1432    1.02(seconds)

  11  Dai-Kou. Approximate WLS. Function:Generalized Rosenbrock

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000     2001     4054     2064       52      0.1118026703258E-04   0.1897689124593E-02 2
     2000     2001     4114     2133      105      0.7796889349536E-05   0.4841400842199E-03 2
     3000     2001     4108     2129      159      0.6216424046050E-05   0.1406677525723E-02 2
     4000     2001     4060     2068       87      0.1089399843901E-04   0.5290663157367E-03 2
     5000     2001     4043     2052       96      0.2074751517980E-04   0.1874235849200E-02 2
     6000     2001     4039     2046      115      0.1536790826561E-04   0.4595860504770E-02 2
     7000     2001     4052     2063      135      0.1305125552792E-04   0.4376132128738E-03 2
     8000     2001     4044     2053      152      0.1529358480023E-04   0.8232972945380E-03 2
     9000     2001     4041     2048      172      0.1277907302296E-04   0.5073162470696E-03 2
    10000     2001     4053     2061      191      0.1048781550056E-04   0.4730830343656E-03 2
-----------------------------------------------------------------------------------------------
TOTAL  20010   40608  20717   12.64(seconds)

  12  Dai-Kou. Approximate WLS. Function:Extended Himmelblau

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        8       20       13        0      0.1178921828895E-13   0.5189838424087E-07 0
     2000        8       20       13        1      0.2357832325816E-13   0.5189804495671E-07 0
     3000        8       20       13        0      0.3536773731041E-13   0.5189812844549E-07 0
     4000        8       20       13        0      0.4715689449195E-13   0.5189840910987E-07 0
     5000        8       20       13        1      0.5894590624754E-13   0.5189862049633E-07 0
     6000        8       20       13        0      0.7073524389900E-13   0.5189880879015E-07 0
     7000        8       20       13        1      0.8252446532644E-13   0.5189830075210E-07 0
     8000        8       20       13        0      0.9431350320936E-13   0.5189791705902E-07 0
     9000        8       20       13        1      0.1061028604153E-12   0.5189887273900E-07 0
    10000        8       20       13        1      0.1178921295258E-12   0.5189880168473E-07 0
-----------------------------------------------------------------------------------------------
TOTAL     80     200    130    0.05(seconds)

  13  Dai-Kou. Approximate WLS. Function:HIMMELBG

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        1       10       10        1      0.0000000000000E+00   0.0000000000000E+00 0
     2000        1       10       10        0      0.0000000000000E+00   0.0000000000000E+00 0
     3000        1       10       10        1      0.0000000000000E+00   0.0000000000000E+00 0
     4000        1       10       10        1      0.0000000000000E+00   0.0000000000000E+00 0
     5000        1       10       10        1      0.0000000000000E+00   0.0000000000000E+00 0
     6000        1       10       10        1      0.0000000000000E+00   0.0000000000000E+00 0
     7000        1       10       10        2      0.0000000000000E+00   0.0000000000000E+00 0
     8000        1       10       10        2      0.0000000000000E+00   0.0000000000000E+00 0
     9000        1       10       10        2      0.0000000000000E+00   0.0000000000000E+00 0
    10000        1       10       10        2      0.0000000000000E+00   0.0000000000000E+00 0
-----------------------------------------------------------------------------------------------
TOTAL     10     100    100    0.13(seconds)

  14  Dai-Kou. Approximate WLS. Function:HIMMELBH

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        5       11        6        0     -0.5000000000000E+03   0.8980697185912E-06 0
     2000        5       11        6        0     -0.9999999999999E+03   0.8813226255455E-06 0
     3000        5       11        6        0     -0.1500000000000E+04   0.8942482825347E-06 0
```

```
     4000        5       11        6        0     -0.2000000000000E+04   0.8813095480065E-06 0
     5000        5       11        6        0     -0.2500000000000E+04   0.8865594467444E-06 0
     6000        5       11        6        1     -0.3000000000000E+04   0.9106136196202E-06 0
     7000        5       11        6        0     -0.3500000000000E+04   0.9020986704300E-06 0
     8000        5       11        6        0     -0.4000000000000E+04   0.9107697289679E-06 0
     9000        5       11        6        1     -0.4500000000000E+04   0.9190630807510E-06 0
    10000        5       11        6        0     -0.5000000000000E+04   0.8939210611736E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     50     110     60    0.02(seconds)

  15  Dai-Kou. Approximate WLS. Function:Extended Trigonometric ET1

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       61      124       63        6      0.4416452165565E-06   0.7740098336651E-06 0
     2000       66      133       67       11      0.2105830840363E-06   0.9767606541527E-06 0
     3000       64      129       65       16      0.1396950410393E-06   0.9172299891618E-06 0
     4000       65      132       67       26      0.1070225903247E-06   0.8953544747121E-06 0
     5000       67      137       70       29      0.8263874221174E-07   0.9566624731315E-06 0
     6000       71      145       76       37      0.7072948401874E-07   0.9336965103348E-06 0
     7000       71      143       73       42      0.5780637210883E-07   0.9645324950195E-06 0
     8000       71      145       74       48      0.5117037210382E-07   0.8203938963260E-06 0
     9000       76      158       84       61      0.4603144272058E-07   0.9913353447109E-06 0
    10000       76      158       83       67      0.3934415033819E-07   0.6768285681163E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    688    1404    722    3.43(seconds)

  16  Dai-Kou. Approximate WLS. Function:Extended Trigonometric ET2

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000      316      646      386       30      0.4560922427882E-12   0.9272057365550E-06 0
     2000      605     1224      702      115      0.4585209320856E-12   0.9718966423141E-06 0
     3000      994     1995     1026      265      0.7474568886909E-12   0.9331373128890E-06 0
     4000      850     1727      976      320      0.9172971367063E-12   0.7359775486143E-06 0
     5000      992     2007     1097      459      0.4866740001732E-12   0.9580575322713E-06 0
     6000     1181     2386     1309      895      0.2293374565301E-12   0.6852229179423E-06 0
     7000     1034     2095     1244     1902      0.6940731515719E-12   0.9186823214230E-06 0
     8000     1235     2501     1422     1401      0.7556519400270E-12   0.9381279367524E-06 0
     9000     1416     2917     1698     1234      0.2773192452523E-12   0.5530619560319E-06 0
    10000     2001     4078     2298     3267      0.4347313615282E-12   0.6285610627671E-05 2
-----------------------------------------------------------------------------------------------
TOTAL  10624   21576  12158   98.88(seconds)

  17  Dai-Kou. Approximate WLS. Function:Extended Block Diagonal BD1

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        7       15        8        1      0.1323406312888E-10   0.4803343568016E-06 0
     2000        7       15        8        1      0.2646809882919E-10   0.4803341076488E-06 0
     3000        7       15        8        2      0.3970214632669E-10   0.4803340956403E-06 0
     4000        7       15        8        3      0.5293622555055E-10   0.4803342342266E-06 0
     5000        7       15        8        4      0.6617024873938E-10   0.4803341140532E-06 0
     6000        7       15        8        4      0.7940434946312E-10   0.4803342675781E-06 0
     7000        7       15        8        5      0.9263836295029E-10   0.4803341519763E-06 0
     8000        7       15        8        6      0.1058723803288E-09   0.4803340733499E-06 0
     9000        7       15        8        7      0.1191065221192E-09   0.4803342635959E-06 0
    10000        7       15        8        7      0.1323405614171E-09   0.4803342296553E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     70     150     80    0.40(seconds)

  18  Dai-Kou. Approximate WLS. Function:Extended Tridiagonal-1

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       21       44       27        0      0.1710290945233E-08   0.2485502917074E-06 0
     2000       21       44       27        1      0.3421377953530E-08   0.2482865674310E-06 0
     3000       21       44       27        1      0.5131738282246E-08   0.2483671645297E-06 0
     4000       21       44       27        2      0.6839745605890E-08   0.2487806853103E-06 0
     5000       21       44       27        2      0.8550104178802E-08   0.2487366361553E-06 0
     6000       21       44       27        3      0.1026405949455E-07   0.2482911486544E-06 0
     7000       21       44       27        4      0.1198209814825E-07   0.2475899777884E-06 0
     8000       21       44       27        5      0.1368690869070E-07   0.2481793990749E-06 0
     9000       21       44       27        5      0.1540037502797E-07   0.2479673456000E-06 0
    10000       21       44       27        6      0.1709294436957E-07   0.2492132762286E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    210     440    270    0.29(seconds)

  19  Dai-Kou. Approximate WLS. Function:Extended Three Exponential Terms

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        7       15        8        2      0.1279633348329E+04   0.7959519969791E-06 0
     2000        9       18       11        4      0.2559266696658E+04   0.4355410823109E-06 0
     3000       10       19       13        8      0.3838900044987E+04   0.9068301665138E-12 0
     4000        9       19       12       10      0.5118533393317E+04   0.1176969632866E-10 0
     5000        9       20       12       12      0.6398166741646E+04   0.4465776348717E-06 0
     6000        7       16        9       11      0.7677800089974E+04   0.5600590602395E-06 0
     7000        8       18       12       16      0.8957433438304E+04   0.9301427466787E-11 0
     8000        8       18       12       19      0.1023706678663E+05   0.5913503302486E-11 0
     9000        7       16       10       17      0.1151670013496E+05   0.1770247362032E-06 0
    10000        8       18       11       21      0.1279633348329E+05   0.1132718563390E-07 0
-----------------------------------------------------------------------------------------------
TOTAL     82     177    110    1.20(seconds)
```

20 Dai-Kou. Approximate WLS. Function:Generalized Tridiagonal-1 20

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	22	40	28	1	0.9972103074860E+03	0.9499208104558E-06	0
2000	22	40	28	1	0.1997210307486E+04	0.9653036925350E-06	0
3000	22	39	29	2	0.2997210307486E+04	0.9627719603778E-06	0
4000	22	39	29	3	0.3997210307486E+04	0.9416307147347E-06	0
5000	22	39	29	4	0.4997210307486E+04	0.9548193942166E-06	0
6000	22	39	29	4	0.5997210307486E+04	0.9663752673639E-06	0
7000	22	39	29	6	0.6997210307486E+04	0.9194185053296E-06	0
8000	22	39	29	6	0.7997210307486E+04	0.9608585185283E-06	0
9000	22	38	30	7	0.8997210307486E+04	0.9502179691978E-06	0
10000	22	38	30	8	0.9997210307486E+04	0.9566738903288E-06	0

TOTAL 220 390 290 0.42(seconds)

21 Dai-Kou. Approximate WLS. Function:Generalized Tridiagonal-2

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	39	77	42	2	0.1114854332625E+01	0.8623950049682E-06	0
2000	44	86	48	4	0.1114854332625E+01	0.6784155595015E-06	0
3000	44	87	47	5	0.1114854332625E+01	0.7697441577088E-06	0
4000	45	88	49	8	0.1114854332625E+01	0.7325318261713E-06	0
5000	45	89	48	9	0.1114854332625E+01	0.9309803198046E-06	0
6000	46	90	50	11	0.1114854332625E+01	0.8362278038976E-06	0
7000	47	91	52	13	0.1114854332625E+01	0.8485323967044E-06	0
8000	47	92	51	16	0.1114854332625E+01	0.9461517229692E-06	0
9000	47	92	51	17	0.1114854332625E+01	0.9647650667824E-06	0
10000	47	92	51	18	0.1114854332625E+01	0.9423957639407E-06	0

TOTAL 451 884 489 1.03(seconds)

22 Dai-Kou. Approximate WLS. Function:Tridiagonal Double Borded

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	448	897	619	15	0.1528391502750E-09	0.9613573713826E-06	0
2000	460	921	628	31	0.1903658012739E-09	0.8729531995081E-06	0
3000	325	651	430	32	0.3050518241796E-08	0.8946286995124E-06	0
4000	1913	3827	2790	265	0.8583776920745E-10	0.9975889735852E-06	0
5000	1980	3961	2693	203	0.1545633728505E-09	0.9937141816690E-06	0
6000	2001	4001	2778	151	0.1631580745585E-08	0.8438104872606E-05	2
7000	2001	4001	2007	157	0.1063894251775E-05	0.4337663919755E-03	2
8000	2001	4001	2829	204	0.1000031051361E-06	0.1217082825293E-02	2
9000	2001	4001	2701	224	0.1960114691869E-07	0.1428146464282E-04	2
10000	2001	4001	2375	237	0.5104977356439E-06	0.4851898920690E-03	2

TOTAL 15131 30262 19850 15.19(seconds)

23 Dai-Kou. Approximate WLS. Function:Broyden Pentadiagonal

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	38	77	39	0	0.7461227006094E-13	0.8821076348581E-06	0
2000	40	81	41	1	0.5472083776458E-13	0.6398183516102E-06	0
3000	40	81	41	2	0.9277297464218E-13	0.7786250212443E-06	0
4000	40	81	41	2	0.9408818710153E-13	0.9424813307015E-06	0
5000	40	81	41	2	0.6037018024351E-13	0.6988056446702E-06	0
6000	39	79	40	3	0.7039044239927E-13	0.7851178551813E-06	0
7000	38	77	39	5	0.6865125110153E-13	0.7762853829565E-06	0
8000	37	75	38	3	0.6587812446165E-13	0.7542413521530E-06	0
9000	36	73	37	5	0.9654096934473E-13	0.9790572706176E-06	0
10000	36	73	37	4	0.6189057596778E-13	0.7594562195627E-06	0

TOTAL 384 778 394 0.27(seconds)

24 Dai-Kou. Approximate WLS. Function:Extended PSC1

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	10	21	11	1	0.3865995282465E+03	0.7750709551946E-07	0
2000	10	21	11	1	0.7731990564929E+03	0.7421207237623E-07	0
3000	10	21	11	1	0.1159798584739E+04	0.8180339650110E-07	0
4000	10	21	11	2	0.1546398112986E+04	0.8729345746250E-07	0
5000	10	21	11	2	0.1932997641232E+04	0.1255222438767E-06	0
6000	10	21	11	3	0.2319597169479E+04	0.6385597406400E-07	0
7000	10	21	11	4	0.2706196697725E+04	0.1121266771653E-06	0
8000	10	21	11	4	0.3092796225971E+04	0.7786351008399E-07	0
9000	10	21	11	5	0.3479395754218E+04	0.3140296349069E-06	0
10000	10	21	11	5	0.3865995282465E+04	0.9327316419228E-07	0

TOTAL 100 210 110 0.28(seconds)

25 Dai-Kou. Approximate WLS. Function:Perturbed Quadratic PQ1

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	179	359	180	1	0.1671157287378E-12	0.9680375908780E-06	0
2000	254	509	255	5	0.1675840389168E-12	0.9684290156498E-06	0
3000	312	625	313	9	0.1516123931612E-12	0.9211687000507E-06	0
4000	360	721	361	13	0.1666553061782E-12	0.9652068105196E-06	0

```
     5000      403      807      404       19      0.1587404985738E-12   0.9421594011434E-06 0
     6000      441      883      442       24      0.1734386389473E-12   0.9841041992678E-06 0
     7000      477      955      478       31      0.1614539709109E-12   0.9499462373797E-06 0
     8000      510     1021      511       38      0.1622466350354E-12   0.9521986955753E-06 0
     9000      541     1083      542       45      0.1627543529942E-12   0.9536220813917E-06 0
    10000      570     1141      571       52      0.1690528495127E-12   0.9715731145394E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   4047    8104   4057    2.37(seconds)

  26  Dai-Kou. Approximate WLS. Function:Perturbed Quadratic PQ2

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000     2001     4012     2957       21      0.1854152687298E-01   0.3414118113872E+00 2
     2000     2001     4001     3001       41      0.1005695997463E+05   0.2365161222999E+03 2
     3000     2001     4025     3020       61      0.2128071839750E+02   0.8212759011889E+01 2
     4000     2001     4076     2109       74      0.1515072473714E+02   0.7491347009571E+01 2
     5000     2001     4033     2182       94      0.1081683459083E+01   0.1983665455935E+01 2
     6000     2001     4001     3000      122      0.6065178225916E+05   0.1438358666279E+04 2
     7000     2001     4001     2849      142      0.2987516677376E+04   0.1949298663881E+03 2
     8000     2001     4056     2172      149      0.6848976219771E+01   0.7018188303941E+02 2
     9000     2001     4069     3070      187      0.4719420952261E+03   0.3120771429297E+02 2
    10000     2001     4328     2426      195      0.1747496204659E+01   0.1508052051294E+02 2
-----------------------------------------------------------------------------------------------
TOTAL  20010   40602  26786   10.86(seconds)

  27  Dai-Kou. Approximate WLS. Function:Almost Perturbed Quadratic

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000      175      351      176        1      0.1907481248696E-12   0.9833712570810E-06 0
     2000      249      499      250        4      0.1642303467783E-12   0.9185227971497E-06 0
     3000      305      611      306        7      0.1782071487878E-12   0.9500428573982E-06 0
     4000      352      705      353       11      0.1935143004613E-12   0.9830929003930E-06 0
     5000      394      789      395       15      0.1867682008045E-12   0.9680773817580E-06 0
     6000      432      865      433       21      0.1811142710268E-12   0.9552819597656E-06 0
     7000      466      933      467       25      0.1997841245930E-12   0.9953889573281E-06 0
     8000      499      999      500       31      0.1835117337954E-12   0.9602914351229E-06 0
     9000      529     1059      530       38      0.1915727752350E-12   0.9777500026117E-06 0
    10000      558     1117      559       43      0.1856458641079E-12   0.9647910846176E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   3959    7928   3969    1.96(seconds)

  28  Dai-Kou. Approximate WLS. Function:Almost Perturbed Quartic

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       26       54       28        0      0.3690298440774E-09   0.5582991970629E-06 0
     2000       34       74       40        0      0.1401603566930E-08   0.3838569419025E-06 0
     3000       46      102       58        2      0.8788483523087E-09   0.3778552915307E-06 0
     4000       36       77       41        1      0.5404266689621E-09   0.9338697252557E-06 0
     5000       38       84       47        1      0.2638259281846E-09   0.4295470401537E-06 0
     6000       36       79       44        2      0.3538073857667E-08   0.6353712101886E-06 0
     7000       36       79       43        2      0.2401490008435E-08   0.2484038256657E-06 0
     8000       33       71       38        3      0.1107159453182E-07   0.7277307891703E-06 0
     9000       36       82       46        2      0.7289331895294E-08   0.6610530995147E-06 0
    10000       49      108       59        4      0.1835857188938E-09   0.6989127565109E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    370     810    444    0.17(seconds)

  29  Dai-Kou. Approximate WLS. Function:Extended Penalty Function U52

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       21       41       24        0      0.8831940750670E+03   0.4127046314518E-06 0
     2000       21       43       24        0      0.1814063664869E+04   0.5399711130169E-06 0
     3000       24       48       28        1      0.2755973749503E+04   0.6376100952963E-06 0
     4000       27       53       32        1      0.3704070534948E+04   0.1639732435710E-06 0
     5000       25       49       28        1      0.4656333923744E+04   0.6584645984041E-06 0
     6000       26       52       32        2      0.5611676659140E+04   0.6727572073166E-09 0
     7000       25       50       27        1      0.6569428560737E+04   0.8777092397102E-06 0
     8000       27       53       32        2      0.7529139638522E+04   0.4976260957006E-09 0
     9000       29       55       34        2      0.8490489281459E+04   0.9176646237133E-06 0
    10000       30       59       35        3      0.9453238852842E+04   0.5191164658636E-07 0
-----------------------------------------------------------------------------------------------
TOTAL    255     503    296    0.13(seconds)

  30  Dai-Kou. Approximate WLS. Function:TR-Summ of quadratics              30

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000     1004     2036     1034       10      0.2426163883776E-16   0.9851220067727E-08 0
     2000     2001     4030     2034       40      0.5748256711307E-13   0.8798033868467E-05 2
     3000     2001     4035     2039       61      0.9949307053550E-10   0.3918191497639E-03 2
     4000     2001     4018     2019       79      0.7603928429482E-11   0.1352912004014E-03 2
     5000     2001     4015     2016      100      0.3854601769868E-09   0.5899570677380E-03 2
     6000     2001     4022     2025      241      0.7424621261526E-10   0.2431242286016E-03 2
     7000     2001     4021     2023      381      0.9021297908595E-10   0.3317950513993E-03 2
     8000     2001     4026     2028      435      0.1178701563195E-10   0.1335845889463E-03 2
     9000     2001     4017     2018      492      0.6001403172900E-11   0.1318082252333E-03 2
    10000     2001     4026     2028      544      0.1454771719646E-09   0.6460640346071E-03 2
-----------------------------------------------------------------------------------------------
TOTAL  19013   38246  19264   23.83(seconds)
```

31 Dai-Kou. Approximate WLS. Function:Quadratic Diagonal Perturbed

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	101	203	122	3	0.3205264856237E-10	0.9926440547088E-06	0
2000	142	285	173	8	0.3495251411631E-10	0.9758509842935E-06	0
3000	169	339	202	15	0.3505367335610E-10	0.9633685083699E-06	0
4000	198	397	239	23	0.3763064161148E-10	0.9777181142617E-06	0
5000	218	437	262	32	0.3874251990775E-10	0.9799099044219E-06	0
6000	239	479	293	42	0.4231475151418E-10	0.9981213929153E-06	0
7000	252	505	304	51	0.4288064586029E-10	0.9939725389025E-06	0
8000	270	541	325	63	0.4138990834222E-10	0.9768538461450E-06	0
9000	289	579	354	75	0.4575860016552E-10	0.9967078933060E-06	0
10000	298	597	359	87	0.4688464811175E-10	0.9960534784597E-06	0

TOTAL 2176 4362 2633 3.99(seconds)

32 Dai-Kou. Approximate WLS. Function:Full Hessian FH1

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	29	59	31	17	0.1212871287129E+02	0.1527666881884E-12	0
2000	33	67	34	37	0.1212871287129E+02	0.1510638121260E-06	0
3000	36	73	38	61	0.1212871287129E+02	0.3055333763768E-12	0
4000	38	76	40	85	0.1212871287129E+02	0.2220446049250E-12	0
5000	39	79	41	111	0.1212871287129E+02	0.3250733016102E-12	0
6000	40	81	42	137	0.1212871287129E+02	0.2824407374646E-12	0
7000	41	83	43	128	0.1212871287129E+02	0.1687538997430E-12	0
8000	41	83	42	68	0.1212871287129E+02	0.5968830123493E-06	0
9000	43	87	45	81	0.1212871287129E+02	0.4707345624411E-12	0
10000	44	88	46	91	0.1212871287129E+02	0.2287947609148E-11	0

TOTAL 384 776 402 8.16(seconds)

33 Dai-Kou. Approximate WLS. Function:Full Hessian FH2

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	2001	4099	2367	103	-0.7078137471459E+03	0.1306603204575E-02	2
2000	2001	4082	2211	201	-0.2829830493161E+04	0.1507611747417E-01	2
3000	2001	4052	2627	323	-0.6366060667982E+04	0.9109463368985E-02	2
4000	2001	4472	3589	511	-0.1131653692153E+05	0.2880591221772E-03	2
5000	2001	4055	2285	507	-0.1768117703439E+05	0.1896787876677E-01	2
6000	2001	4022	2077	590	-0.2545731744052E+05	0.2788263394941E+00	2
7000	2001	4092	2366	1102	-0.3465315894246E+05	0.1687909552303E-01	2
8000	2001	4039	2570	2316	-0.4526031613953E+05	0.3534022623440E-01	2
9000	2001	4315	2868	1427	-0.5728196831876E+05	0.2112729682055E-02	2
10000	2001	4061	2328	1376	-0.7071759405249E+05	0.3788673432419E-01	2

TOTAL 20010 41289 25288 84.56(seconds)

34 Dai-Kou. Approximate WLS. Function:Full Hessian FH3

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	18	47	29	1	0.4767296853771E+03	0.8636939561671E-06	0
2000	19	48	31	3	0.1905966240052E+04	0.2718032121596E-06	0
3000	18	47	30	4	0.4287709662222E+04	0.8798986748104E-06	0
4000	20	49	33	7	0.7621959951885E+04	0.1269145600282E-06	0
5000	18	48	30	7	0.1190871710904E+05	0.6273232510717E-06	0
6000	19	49	32	10	0.1714798113370E+05	0.1808815226489E-06	0
7000	19	48	32	11	0.2333975202584E+05	0.8995612654772E-06	0
8000	20	50	35	14	0.3048402978548E+05	0.9840199766131E-07	0
9000	20	50	34	15	0.3858081441262E+05	0.7963290141581E-06	0
10000	19	48	31	15	0.4763010590725E+05	0.4894612040118E-06	0

TOTAL 190 484 317 0.87(seconds)

35 Dai-Kou. Approximate WLS. Function:Diagonal Full Borded

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	273	716	515	25	0.2592698336851E-04	0.6186546696016E-06	0
2000	528	1276	867	87	0.3404382791124E-04	0.8607490931081E-06	0
3000	392	1007	732	106	0.5470161000861E-04	0.3055979811548E-06	0
4000	712	1832	1323	256	0.3611230970776E-04	0.5925562184739E-06	0
5000	539	1331	913	227	0.6732136582145E-04	0.7400028953657E-06	0
6000	1112	2646	1723	524	0.4201438032299E-04	0.7476342490312E-06	0
7000	880	2162	1489	523	0.6519964449111E-04	0.4853920618921E-06	0
8000	515	1480	1167	434	0.7888063396079E-04	0.6788298127088E-06	0
9000	245	756	603	252	0.1172736497938E-03	0.5315522334043E-06	0
10000	1370	3209	2076	1055	0.6410561301882E-04	0.8876593167492E-06	0

TOTAL 6566 16415 11408 34.89(seconds)

36 Dai-Kou. Approximate WLS. Function:Diagonal Double Borded Arrow Up

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	34	71	47	0	0.1338563836791E-12	0.6444438853017E-06	0
2000	34	71	49	1	0.1487303043794E-12	0.7178466143700E-07	0
3000	50	102	70	3	0.6613631404252E-11	0.3418175131546E-06	0
4000	43	90	63	3	0.1732539891849E-11	0.6249376484836E-07	0
5000	41	88	63	3	0.7743868806577E-10	0.5105648003817E-06	0

```
      6000       52      110       79        5      0.1394392909996E-13   0.3979808053216E-07 0
      7000       46       98       71        5      0.9211933633733E-11   0.4425821152810E-06 0
      8000     1758     3522     1927      199      0.2883438972736E-12   0.9681714291844E-06 0
      9000       98      202      142       14      0.1234668109821E-09   0.9408961845293E-06 0
     10000       40       86       61        6      0.2528461899758E-10   0.3634922365753E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   2196    4440   2572    2.39(seconds)

  37  Dai-Kou. Approximate WLS. Function:QP1 Extended Quadratic Penalty

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000       16       32       20        1      0.3990006250000E+04   0.1828855703304E-09 0
      2000       15       32       19        0      0.7990003125000E+04   0.1120530841007E-08 0
      3000       15       30       18        1      0.1199000208333E+05   0.8224738213537E-10 0
      4000       15       31       18        1      0.1599000156250E+05   0.5161288534457E-06 0
      5000       16       32       19        1      0.1999000125000E+05   0.8342009776736E-10 0
      6000       16       33       20        1      0.2399000104167E+05   0.1678770082034E-10 0
      7000       16       32       19        2      0.2799000089286E+05   0.2262987965606E-09 0
      8000       16       32       19        2      0.3199000078125E+05   0.1338561676555E-08 0
      9000       17       35       22        2      0.3599000069445E+05   0.3412990690715E-09 0
     10000       18       35       21        3      0.3999000062499E+05   0.3008873303771E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    160     324    195    0.14(seconds)

  38  Dai-Kou. Approximate WLS. Function:QP2 Extended Quadratic Penalty

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000       40      110       77        4      0.1668797393902E-18   0.1615774181122E-07 0
      2000       40      121       90        8      0.8086672865438E-17   0.1126068127648E-06 0
      3000       40      125       95       14      0.4277954428025E-16   0.2365925411211E-06 0
      4000       38      107       75       14      0.1558784185028E-09   0.8220257540474E-06 0
      5000       41      137      105       24      0.5580274718914E-16   0.2951367150781E-06 0
      6000       41      122       88       25      0.7135570693684E-14   0.2181247359987E-08 0
      7000       43      111       79       26      0.7465919326165E-15   0.8182593091151E-06 0
      8000       39      114       83       31      0.6062690222849E-14   0.8947012019143E-06 0
      9000       43      129       94       40      0.7687586570857E-12   0.1848535802224E-07 0
     10000       42      122       90       42      0.3889968796815E-12   0.1247454630995E-07 0
-----------------------------------------------------------------------------------------------
TOTAL    407    1198    876    2.28(seconds)

  39  Dai-Kou. Approximate WLS. Function:QP3 Extended Quadratic Penalty

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000       22       56       38        0     -0.9990000000000E+03   0.6300831519750E-06 0
      2000       22       54       39        0     -0.1999000000000E+04   0.5456080032248E-11 0
      3000       21       47       32        1     -0.2999000000000E+04   0.6173728195374E-11 0
      4000       21       48       29        2     -0.3999000000000E+04   0.4424175767966E-06 0
      5000       24       54       36        2     -0.4999000000000E+04   0.7307043858926E-11 0
      6000       22       49       33        2     -0.5999000000000E+04   0.2406119747947E-10 0
      7000       21       49       34        2     -0.6999000000000E+04   0.8543610264430E-10 0
      8000       22       55       39        3     -0.7999000000000E+04   0.1309816699718E-09 0
      9000       21       52       36        4     -0.8999000000000E+04   0.3555118423136E-09 0
     10000       30       78       53        5     -0.9999000000000E+04   0.4585429815733E-09 0
-----------------------------------------------------------------------------------------------
TOTAL    226     542    369    0.21(seconds)

  40  Dai-Kou. Approximate WLS. Function:STAIRCASE S1                        40

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000     1999     3999     2001       24      0.1683993776811E-16   0.1050239006872E-08 0
      2000     2001     4001     2001       46      0.9354352673411E-04   0.4833615857740E-03 2
      3000     2001     4001     2001       71      0.2128815959619E-03   0.4219839242978E-03 2
      4000     2001     4001     2001       93      0.4223184738648E-03   0.3194427163180E-03 2
      5000     2001     4001     2001      122      0.6367941057572E-03   0.4357694770079E-03 2
      6000     2001     4001     2001      140      0.8463421241898E-03   0.1140723499702E-02 2
      7000     2001     4001     2001      162      0.1149501657394E-02   0.1568335661432E-02 2
      8000     2001     4001     2001      187      0.1499584428045E-02   0.2128301402990E-02 2
      9000     2001     4001     2001      210      0.2327583348370E-02   0.6559592420672E-03 2
     10000     2001     4001     2001      236      0.2341601560546E-02   0.2745450799921E-02 2
-----------------------------------------------------------------------------------------------
TOTAL  20008   40008  20010   12.91(seconds)

  41  Dai-Kou. Approximate WLS. Function:STAIRCASE S2

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000     1999     3999     2001       23      0.2700222462687E-16   0.1018179318635E-08 0
      2000     2001     4001     2001       47      0.9373942382170E-04   0.4576209168072E-03 2
      3000     2001     4001     2001       70      0.2139290732117E-03   0.4120412070279E-03 2
      4000     2001     4001     2001       93      0.3762554317334E-03   0.7622103539688E-03 2
      5000     2001     4001     2001      116      0.5883166299121E-03   0.8833981364660E-03 2
      6000     2001     4001     2001      139      0.1035155290527E-02   0.4411030568008E-03 2
      7000     2001     4001     2001      162      0.1152509547157E-02   0.1332242685748E-02 2
      8000     2001     4001     2001      188      0.1499250120119E-02   0.2056686249489E-02 2
      9000     2001     4001     2001      208      0.2368642947602E-02   0.6582689820789E-03 2
     10000     2001     4001     2001      233      0.2343947691016E-02   0.2606601643492E-02 2
-----------------------------------------------------------------------------------------------
TOTAL  20008   40008  20010   12.79(seconds)

  42  Dai-Kou. Approximate WLS. Function:STAIRCASE S3
```

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	1999	3999	2001	23	0.2251745310710E-13	0.1354515006824E-06	0
2000	2001	4001	2001	47	0.9420175875238E-04	0.1317227965784E-02	2
3000	2001	4001	2001	70	0.2134740571680E-03	0.4934394532938E-03	2
4000	2001	4001	2001	93	0.3758556311008E-03	0.1043195008606E-02	2
5000	2001	4001	2001	116	0.5945461844552E-03	0.7222737694974E-03	2
6000	2001	4001	2001	139	0.8724228668367E-03	0.6709562831020E-03	2
7000	2001	4001	2001	163	0.1152743024252E-02	0.1426449591236E-02	2
8000	2001	4001	2001	192	0.1503650978075E-02	0.1855556221926E-02	2
9000	2001	4001	2001	210	0.1907882804503E-02	0.1716009552183E-02	2
10000	2001	4001	2001	233	0.2350045959306E-02	0.2136219194654E-02	2

TOTAL 20008 40008 20010 12.86(seconds)

43 Dai-Kou. Approximate WLS. Function:NONDQUAR

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	2001	4013	2460	33	0.3766227262017E-05	0.1341952292610E-04	2
2000	2001	4011	2400	66	0.3014920231893E-05	0.2387491409698E-04	2
3000	2001	4018	2457	102	0.3965664875420E-05	0.1884793628329E-04	2
4000	2001	4024	2436	135	0.4080314070344E-05	0.1641399706172E-04	2
5000	2001	4018	2431	171	0.4393665308850E-05	0.6915943295978E-05	2
6000	2001	4025	2457	202	0.5187422075224E-05	0.1173963064132E-04	2
7000	2001	4024	2347	233	0.4557210318012E-05	0.2340036628188E-04	2
8000	2001	4022	2305	263	0.4069769643717E-05	0.2267650638989E-04	2
9000	2001	4019	2241	294	0.3866227888703E-05	0.1839225794171E-04	2
10000	2001	4031	2417	337	0.4256113647454E-05	0.3469234353167E-04	2

TOTAL 20010 40205 23951 18.36(seconds)

44 Dai-Kou. Approximate WLS. Function:TRIDIA

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	261	523	262	3	0.7639736673201E-15	0.7107047265129E-06	0
2000	371	743	372	9	0.1485652009186E-14	0.9575403116929E-06	0
3000	459	919	460	17	0.9568057410973E-15	0.9824189871520E-06	0
4000	532	1065	533	28	0.8740251355717E-15	0.8942329331931E-06	0
5000	594	1189	595	38	0.1192977220591E-14	0.8905753739718E-06	0
6000	653	1307	654	50	0.9910301630624E-15	0.9981651753768E-06	0
7000	704	1409	705	63	0.1302124332540E-14	0.9842440471197E-06	0
8000	756	1513	757	77	0.9275109056099E-15	0.8766630847566E-06	0
9000	801	1603	802	93	0.1093366142717E-14	0.9908236962860E-06	0
10000	847	1695	848	108	0.8596529755363E-15	0.9270479529008E-06	0

TOTAL 5978 11966 5988 4.86(seconds)

45 Dai-Kou. Approximate WLS. Function:ARWHEAD

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	11	29	22	0	0.0000000000000E+00	0.1443235588448E-09	0
2000	9	23	17	0	0.0000000000000E+00	0.2257029062789E-06	0
3000	9	23	17	1	0.0000000000000E+00	0.5052917724453E-08	0
4000	9	23	17	0	0.0000000000000E+00	0.1933455002786E-08	0
5000	10	25	17	0	0.0000000000000E+00	0.5490545727915E-06	0
6000	12	35	27	2	0.0000000000000E+00	0.2825426351214E-07	0
7000	10	27	19	1	0.0000000000000E+00	0.1569494360436E-09	0
8000	11	26	19	2	0.0000000000000E+00	0.5395026688448E-09	0
9000	9	24	18	1	0.0000000000000E+00	0.7159110131155E-11	0
10000	8	17	11	1	0.0000000000000E+00	0.7211658896509E-08	0

TOTAL 98 252 184 0.08(seconds)

46 Dai-Kou. Approximate WLS. Function:NONDIA

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	29	111	92	1	0.9898968742048E+00	0.5933850982798E-07	0
2000	32	98	76	1	0.9898968741776E+00	0.1788724619804E-07	0
3000	39	175	150	3	0.9898968775080E+00	0.7671831586955E-07	0
4000	42	151	125	3	0.9898968937044E+00	0.3350141343089E-06	0
5000	50	224	186	6	0.9898968742075E+00	0.8347201013753E-08	0
6000	58	218	190	8	0.9898968740952E+00	0.1493361392879E-08	0
7000	56	197	165	8	0.9898968742137E+00	0.6896530235778E-08	0
8000	52	188	157	9	0.9898968740952E+00	0.8834981852579E-11	0
9000	61	275	235	14	0.9898969006722E+00	0.7138966168520E-07	0
10000	62	269	237	15	0.9898968740952E+00	0.5906673597733E-06	0

TOTAL 481 1906 1613 0.68(seconds)

47 Dai-Kou. Approximate WLS. Function:BDQRTIC

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	653	1331	1025	16	0.3983817950577E+04	0.9265735748367E-06	0
2000	2001	3252	4380	103	0.7989427682541E+04	0.1630533293228E-04	2
3000	144	374	355	13	0.1199503741450E+05	0.9638685717507E-06	0
4000	377	844	638	38	0.1600064714647E+05	0.7621027240390E-06	0
5000	1005	2098	1202	108	0.2000625687843E+05	0.5795898463390E-06	0
6000	656	1386	853	87	0.2401186661040E+05	0.8078533174529E-06	0

```
     7000     1201     2506     1429      181      0.2801747634236E+05   0.5629579275808E-06 0
     8000     1006     2111     1259      176      0.3202308607432E+05   0.9352491796172E-06 0
     9000      971     2223     2445      267      0.3602869580629E+05   0.9815959032267E-06 0
    10000     1189     2482     1443      258      0.4003430553825E+05   0.7478085102264E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   9203   18607  15029   12.47(seconds)

  48  Dai-Kou. Approximate WLS. Function:DQDRTIC

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        7       15        9        0      0.1707992867039E-16   0.1072908380163E-06 0
     2000        7       15        9        0      0.3321619487869E-14   0.1270722504919E-06 0
     3000        7       15        9        0      0.2239457285124E-14   0.1387969760681E-06 0
     4000        7       15        9        1      0.2766787894298E-14   0.1354243021668E-06 0
     5000        7       15        9        0      0.3105616959169E-14   0.1385087563382E-06 0
     6000        7       15        9        1      0.8183326196249E-13   0.7840625219890E-06 0
     7000        7       15        9        0      0.4141659377473E-13   0.4069763453704E-06 0
     8000        7       15        9        1      0.1432224829712E-12   0.7568630573762E-06 0
     9000       10       21       12        1      0.1551710787516E-24   0.1680227479257E-10 0
    10000        8       17       10        1      0.9864780356662E-13   0.6280518013333E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     74     158     94    0.05(seconds)

  49  Dai-Kou. Approximate WLS. Function:EG2

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       14       37       37        2     -0.9989473933009E+03   0.1169314160206E-08 0
     2000        8       24       23        1     -0.1998947392741E+04   0.9195085817561E-06 0
     3000        8       24       20        2     -0.2998947392570E+04   0.7670660435426E-07 0
     4000        7       23       21        3     -0.3998947392484E+04   0.6493400842103E-06 0
     5000        5       13       11        2     -0.4998947392433E+04   0.1812874540128E-07 0
     6000        6       15       13        2     -0.5998947392399E+04   0.1510723437345E-07 0
     7000        5       14       12        3     -0.6998947392374E+04   0.1294902504191E-07 0
     8000        5       12       10        2     -0.7998947392355E+04   0.1133037580009E-07 0
     9000        5       16       14        4     -0.8998947392341E+04   0.1513699253535E-07 0
    10000        7       23       20        7     -0.9998947392330E+04   0.4460701195583E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     70     201    181    0.28(seconds)

  50  Dai-Kou. Approximate WLS. Function:EG3                                50

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       56      231      206        7     -0.9995000000000E+03   0.8253080776413E-07 0
     2000       37      146      130        8     -0.1999500000000E+04   0.1019251163882E-06 0
     3000       82      251      229       22     -0.2999500000000E+04   0.4726153086218E-06 0
     4000      120      317      282       36     -0.3999500000000E+04   0.3755073631900E-06 0
     5000       42      155      133       21     -0.4999500000000E+04   0.6125781994746E-08 0
     6000       57      177      168       31     -0.5999500000000E+04   0.2918741719028E-06 0
     7000       80      222      244       50     -0.6999499999999E+04   0.5447181834438E-06 0
     8000       37      145      126       32     -0.7999499999985E+04   0.4665944529326E-06 0
     9000       59      198      167       49     -0.8999500000000E+04   0.9735508842783E-09 0
    10000      110      354      306       99     -0.9999500000000E+04   0.6704148894340E-07 0
-----------------------------------------------------------------------------------------------
TOTAL    680    2196   1991    3.55(seconds)

  51  Dai-Kou. Approximate WLS. Function:EDENSCH

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       24       44       32        1      0.6003284592021E+04   0.5773764955030E-06 0
     2000       24       43       33        1      0.1200328459202E+05   0.7298591857552E-06 0
     3000       24       42       33        1      0.1800328459202E+05   0.3670166763836E-06 0
     4000       24       43       34        2      0.2400328459202E+05   0.7158880439562E-06 0
     5000       24       44       36        2      0.3000328459202E+05   0.4710786261075E-06 0
     6000       24       42       35        3      0.3600328459202E+05   0.4503740163031E-06 0
     7000       23       41       34        3      0.4200328459202E+05   0.8494690519889E-06 0
     8000       24       41       33        3      0.4800328459202E+05   0.3873994538495E-06 0
     9000       24       43       36        4      0.5400328459202E+05   0.5542013430393E-06 0
    10000       24       41       33        4      0.6000328459202E+05   0.5192776708629E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    239     424    339    0.24(seconds)

  52  Dai-Kou. Approximate WLS. Function:FLETCHCR

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       21       46       26        0      0.7625916916815E-15   0.8061751707191E-06 0
     2000       22       48       27        1      0.4924774666006E-15   0.9166746913542E-06 0
     3000       23       50       28        1      0.1366091036710E-15   0.4913514927261E-06 0
     4000       23       50       28        1      0.2551265805010E-15   0.6014723828628E-06 0
     5000       23       50       28        2      0.3581787393068E-15   0.6027222719351E-06 0
     6000       23       50       28        2      0.4327776201078E-15   0.5162530401602E-06 0
     7000       23       50       28        3      0.5247127519631E-15   0.6413574336194E-06 0
     8000       23       50       28        3      0.6530736585339E-15   0.7134611787776E-06 0
     9000       23       50       28        3      0.7885642740291E-15   0.7750306393829E-06 0
    10000       23       50       28        4      0.9010873199045E-15   0.8190867980757E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    227     494    277    0.20(seconds)

  53  Dai-Kou. Approximate WLS. Function:ENGVAL1
```

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	23	42	29	0	0.1108194718785E+04	0.8170153157305E-06	0
2000	23	43	30	1	0.2218313143943E+04	0.5688838046689E-06	0
3000	23	42	31	1	0.3328431569101E+04	0.9085942276678E-06	0
4000	24	42	33	2	0.4438549994258E+04	0.5221358795605E-06	0
5000	24	43	32	2	0.5548668419415E+04	0.3822146190657E-06	0
6000	23	42	31	2	0.6658786844573E+04	0.3727925066421E-06	0
7000	23	42	31	3	0.7768905269733E+04	0.5542782712809E-06	0
8000	24	42	34	3	0.8879023694889E+04	0.6687315510234E-06	0
9000	24	43	34	4	0.9989142120047E+04	0.7966644792745E-06	0
10000	25	44	36	4	0.1109926054520E+05	0.5052589595600E-06	0

TOTAL 236 425 321 0.22(seconds)

54 Dai-Kou. Approximate WLS. Function:DENSCHNA

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	9	19	10	1	0.7349139229309E-15	0.2410527003887E-08	0
2000	9	19	10	1	0.1469828098174E-14	0.2410527182774E-08	0
3000	9	19	10	1	0.2204741752877E-14	0.2410527008624E-08	0
4000	9	19	10	2	0.2939656629611E-14	0.2410527247501E-08	0
5000	9	19	10	3	0.3674569513369E-14	0.2410526974709E-08	0
6000	9	19	10	2	0.4409483360199E-14	0.2410526952212E-08	0
7000	9	19	10	4	0.5144399044859E-14	0.2410527342592E-08	0
8000	9	19	10	4	0.5879311056623E-14	0.2410527041225E-08	0
9000	9	19	10	4	0.6614223347532E-14	0.2410526751329E-08	0
10000	9	19	10	4	0.7349142115747E-14	0.2410527518565E-08	0

TOTAL 90 190 100 0.26(seconds)

55 Dai-Kou. Approximate WLS. Function:DENSCHNB

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	11	23	12	1	0.1058489654893E-11	0.1301352306288E-06	0
2000	11	23	12	0	0.2115640384446E-11	0.1300940724408E-06	0
3000	11	23	12	0	0.3173467339971E-11	0.1300942109967E-06	0
4000	11	23	12	1	0.4233707326119E-11	0.1301313688291E-06	0
5000	11	23	12	1	0.5293659926912E-11	0.1301501253810E-06	0
6000	11	23	12	1	0.6350420211765E-11	0.1301299264273E-06	0
7000	11	23	12	0	0.7406830700130E-11	0.1301124239834E-06	0
8000	11	23	12	1	0.8462803579320E-11	0.1300959322865E-06	0
9000	11	23	12	1	0.9519099498875E-11	0.1300853114489E-06	0
10000	11	23	12	1	0.1057580024020E-10	0.1300793038101E-06	0

TOTAL 110 230 120 0.07(seconds)

56 Dai-Kou. Approximate WLS. Function:DENSCHNC

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	0	1	1	0	0.0000000000000E+00	0.0000000000000E+00	0
2000	0	1	1	0	0.0000000000000E+00	0.0000000000000E+00	0
3000	0	1	1	0	0.0000000000000E+00	0.0000000000000E+00	0
4000	0	1	1	0	0.0000000000000E+00	0.0000000000000E+00	0
5000	0	1	1	0	0.0000000000000E+00	0.0000000000000E+00	0
6000	0	1	1	0	0.0000000000000E+00	0.0000000000000E+00	0
7000	0	1	1	0	0.0000000000000E+00	0.0000000000000E+00	0
8000	0	1	1	0	0.0000000000000E+00	0.0000000000000E+00	0
9000	0	1	1	1	0.0000000000000E+00	0.0000000000000E+00	0
10000	0	1	1	0	0.0000000000000E+00	0.0000000000000E+00	0

TOTAL 0 10 10 0.01(seconds)

57 Dai-Kou. Approximate WLS. Function:DENSCHNF

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	25	57	34	1	0.2279741433594E-15	0.1237525345092E-07	0
2000	25	57	34	1	0.4559566753772E-15	0.1236901497430E-07	0
3000	25	57	34	1	0.6839309033719E-15	0.1237298755395E-07	0
4000	25	57	34	2	0.9119132531692E-15	0.1237389689244E-07	0
5000	25	57	34	3	0.1139881793974E-14	0.1237431639782E-07	0
6000	25	57	34	3	0.1367882096347E-14	0.1237318363865E-07	0
7000	25	57	34	3	0.1595849731737E-14	0.1237265194965E-07	0
8000	25	57	34	4	0.1823897464175E-14	0.1237466690065E-07	0
9000	25	57	34	4	0.2051807617542E-14	0.1237410673979E-07	0
10000	25	57	34	5	0.2279775220254E-14	0.1237231615603E-07	0

TOTAL 250 570 340 0.27(seconds)

58 Dai-Kou. Approximate WLS. Function:SINQUAD

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	168	466	345	17	0.2377945779771E-06	0.4379957392101E-06	0
2000	530	1257	818	84	0.1172861509059E-10	0.1021260303539E-06	0
3000	331	797	512	78	0.1670114848963E-06	0.8129850888523E-07	0
4000	604	1421	939	190	0.6306056240798E-06	0.1918390588823E-07	0
5000	275	684	468	115	0.1364677949814E-04	0.8636896876853E-06	0
6000	358	917	644	189	0.1282978108736E-03	0.9891981384081E-06	0
7000	338	858	606	207	0.1726070646471E-04	0.2435424352374E-06	0

```
     8000      837     1899     1165      494      0.3351796583396E-04   0.7184752722096E-06 0
     9000       94      301      235       96      0.8149974338258E-04   0.8731187508037E-06 0
    10000      524     1283      852      430      0.5019730355414E-04   0.3290977474509E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   4059    9883   6584   19.00(seconds)

  59  Dai-Kou. Approximate WLS. Function:DIXON3DQ

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000     1000     2001     1001       11      0.9990009990010E-03   0.1849009834132E-10 0
     2000     2000     4001     2001       44      0.4997501249375E-03   0.9730882766235E-11 0
     3000     2001     4001     2001       67      0.8482225059693E-03   0.2028489409215E-02 2
     4000     2001     4001     2001       89      0.2802126025867E-02   0.6701169715576E-02 2
     5000     2001     4001     2001      111      0.2808595702149E-02   0.2098950524872E-02 2
     6000     2001     4001     2001      133      0.2808595702149E-02   0.2098950524872E-02 2
     7000     2001     4001     2001      155      0.2808595702149E-02   0.2098950524872E-02 2
     8000     2001     4001     2001      178      0.2808595702149E-02   0.2098950524872E-02 2
     9000     2001     4001     2001      200      0.2808595702149E-02   0.2098950524872E-02 2
    10000     2001     4001     2001      222      0.2808595702149E-02   0.2098950524872E-02 2
-----------------------------------------------------------------------------------------------
TOTAL  19008   38010  19010   12.10(seconds)

  60  Dai-Kou. Approximate WLS. Function:BIGGSB1                            60

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000      500     1001      501        6      0.1549462877782E-15   0.2328511738980E-07 0
     2000     1000     2001     1001       22      0.1622873778237E-15   0.2315846470147E-07 0
     3000     1500     3001     1501       49      0.2078031304144E-15   0.2323047576525E-07 0
     4000     2000     4001     2001       88      0.5063448164185E-15   0.2367777796053E-07 0
     5000     2001     4001     2001      110      0.8095952023988E-03   0.8995502250102E-03 2
     6000     2001     4001     2001      131      0.8095952023988E-03   0.8995502250102E-03 2
     7000     2001     4001     2001      153      0.8095952023988E-03   0.8995502250102E-03 2
     8000     2001     4001     2001      176      0.8095952023988E-03   0.8995502250102E-03 2
     9000     2001     4001     2001      198      0.8095952023988E-03   0.8995502250102E-03 2
    10000     2001     4001     2001      219      0.8095952023988E-03   0.8995502250102E-03 2
-----------------------------------------------------------------------------------------------
TOTAL  17006   34010  17010   11.52(seconds)

  61  Dai-Kou. Approximate WLS. Function:PRODsin

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        0        1        1        0      0.9989999999998E-12   0.2998999999999E-08 0
     2000        0        1        1        0      0.3998000000000E-11   0.5998999999999E-08 0
     3000        0        1        1        0      0.8996999999998E-11   0.8998999999997E-08 0
     4000        0        1        1        0      0.1599600000000E-10   0.1199900000000E-07 0
     5000        0        1        1        0      0.2499500000000E-10   0.1499900000000E-07 0
     6000        0        1        1        0      0.3599400000000E-10   0.1799900000000E-07 0
     7000        0        1        1        1      0.4899300000000E-10   0.2099900000000E-07 0
     8000        0        1        1        0      0.6399200000000E-10   0.2399900000000E-07 0
     9000        0        1        1        0      0.8099099999999E-10   0.2699899999999E-07 0
    10000        0        1        1        1      0.9998999999998E-10   0.2999899999999E-07 0
-----------------------------------------------------------------------------------------------
TOTAL      0      10     10    0.02(seconds)

  62  Dai-Kou. Approximate WLS. Function:PROD1

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        2        5        3        0      0.1637470899020E-34   0.8093135113219E-17 0
     2000        2        5        3        0      0.2027987163540E-31   0.2848148285143E-15 0
     3000        2        5        3        0      0.1039211808735E-33   0.2038834773821E-16 0
     4000        2        5        3        0      0.9361408092150E-31   0.6119283648320E-15 0
     5000        2        5        3        0      0.5416583025153E-31   0.4654710742958E-15 0
     6000        2        5        3        0      0.1346916891038E-29   0.2321134973273E-14 0
     7000        2        5        3        0      0.1043357942124E-29   0.2042897884990E-14 0
     8000        2        5        3        0      0.4707489147354E-29   0.4339349788784E-14 0
     9000        2        5        3        0      0.1277853256892E-29   0.2260843432785E-14 0
    10000        2        5        3        0      0.4412006169173E-28   0.1328458681205E-13 0
-----------------------------------------------------------------------------------------------
TOTAL     20      50     30    0.00(seconds)

  63  Dai-Kou. Approximate WLS. Function:PRODcos

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        4        9        5        0      0.2668568171659E-23   0.9285542614719E-10 0
     2000        4        9        5        1      0.1022890715782E-23   0.8548840635477E-10 0
     3000        4        9        5        1      0.6072922017321E-24   0.8212504087080E-10 0
     4000        4        9        5        0      0.4513461700027E-24   0.8250231787568E-10 0
     5000        4        9        5        1      0.3660126881783E-24   0.8352670862297E-10 0
     6000        4        9        5        1      0.3023312360363E-24   0.8347021921971E-10 0
     7000        4        9        5        1      0.2294636606925E-24   0.7875608349129E-10 0
     8000        4        9        5        2      0.2617671660481E-24   0.9010680572903E-10 0
     9000        4        9        5        2      0.1751692614650E-24   0.7830495472690E-10 0
    10000        4        9        5        2      0.1327902113200E-24   0.7195719622807E-10 0
-----------------------------------------------------------------------------------------------
TOTAL     40      90     50    0.11(seconds)

  64  Dai-Kou. Approximate WLS. Function:PROD2

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
```

```
-----------------------------------------------------------------------------------------
     1000        0        1        1        0      0.1004995000000E-16   0.4019980010000E-11 0
     2000        0        1        1        0      0.2019990000000E-16   0.8079960010000E-11 0
     3000        0        1        1        0      0.3044985000000E-16   0.1217994001000E-10 0
     4000        0        1        1        0      0.4079980000000E-16   0.1631992001000E-10 0
     5000        0        1        1        0      0.5124975000000E-16   0.2049990001000E-10 0
     6000        0        1        1        0      0.6179970000000E-16   0.2471988001000E-10 0
     7000        0        1        1        0      0.7244965000000E-16   0.2897986001000E-10 0
     8000        0        1        1        0      0.8319960000000E-16   0.3327984001000E-10 0
     9000        0        1        1        0      0.9404955000000E-16   0.3761982001000E-10 0
    10000        0        1        1        0      0.1049995000000E-15   0.4199980001000E-10 0
-----------------------------------------------------------------------------------------
TOTAL      0      10     10    0.00(seconds)

  65  Dai-Kou. Approximate WLS. Function:DIXMAANA

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000        8       17        9        1      0.1000000000000E+01   0.6810895251955E-07 0
     2000        8       17        9        0      0.1000000000001E+01   0.6810883655927E-07 0
     3000        8       17        9        1      0.1000000000001E+01   0.6498578597812E-07 0
     4000        8       17        9        1      0.1000000000002E+01   0.6811024397548E-07 0
     5000        8       17        9        2      0.1000000000002E+01   0.6809368966984E-07 0
     6000        8       17        9        1      0.1000000000003E+01   0.6812233587944E-07 0
     7000        8       17        9        2      0.1000000000003E+01   0.7124166712158E-07 0
     8000        8       17        9        2      0.1000000000004E+01   0.6812178371916E-07 0
     9000        8       17        9        3      0.1000000000004E+01   0.6499410983907E-07 0
    10000        8       17        9        3      0.1000000000005E+01   0.6812209649783E-07 0
-----------------------------------------------------------------------------------------
TOTAL     80     170     90    0.16(seconds)

  66  Dai-Kou. Approximate WLS. Function:DIXMAANB

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000        7       15        8        0      0.1000000000000E+01   0.2626149583226E-06 0
     2000        7       15        8        1      0.1000000000000E+01   0.2636252608177E-06 0
     3000        7       15        8        0      0.1000000000000E+01   0.2638147809329E-06 0
     4000        7       15        8        1      0.1000000000001E+01   0.2639920380912E-06 0
     5000        7       15        8        2      0.1000000000001E+01   0.2641548339231E-06 0
     6000        7       15        8        1      0.1000000000001E+01   0.2639730967009E-06 0
     7000        7       15        8        2      0.1000000000001E+01   0.2640434634587E-06 0
     8000        7       15        8        2      0.1000000000001E+01   0.2641067664488E-06 0
     9000        7       15        8        2      0.1000000000001E+01   0.2639822194530E-06 0
    10000        7       15        8        3      0.1000000000002E+01   0.2642928226693E-06 0
-----------------------------------------------------------------------------------------
TOTAL     70     150     80    0.14(seconds)

  67  Dai-Kou. Approximate WLS. Function:DIXMAANC

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000        9       19       10        0      0.1000000000000E+01   0.1314014064258E-06 0
     2000        9       19       10        1      0.1000000000000E+01   0.8934527373806E-07 0
     3000        9       19       10        1      0.1000000000000E+01   0.8819123538326E-07 0
     4000        9       19       10        1      0.1000000000000E+01   0.1124420995931E-06 0
     5000        9       19       10        2      0.1000000000000E+01   0.9159251921392E-07 0
     6000        9       19       10        1      0.1000000000000E+01   0.9463951006667E-07 0
     7000        9       19       10        3      0.1000000000001E+01   0.1848002771070E-06 0
     8000        9       19       10        2      0.1000000000000E+01   0.1170135524566E-06 0
     9000        9       19       10        3      0.1000000000001E+01   0.2487819531421E-06 0
    10000        9       19       10        3      0.1000000000001E+01   0.1269696414305E-06 0
-----------------------------------------------------------------------------------------
TOTAL     90     190    100    0.17(seconds)

  68  Dai-Kou. Approximate WLS. Function:DIXMAAND

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000       10       21       11        1      0.1000000000002E+01   0.8879014017100E-06 0
     2000       10       21       11        0      0.1000000000004E+01   0.8787469317526E-06 0
     3000       10       21       11        1      0.1000000000006E+01   0.8930886860198E-06 0
     4000       10       21       11        2      0.1000000000008E+01   0.8964143510518E-06 0
     5000       10       21       11        2      0.1000000000010E+01   0.8839282188482E-06 0
     6000       10       21       11        2      0.1000000000012E+01   0.8823756348420E-06 0
     7000       10       21       11        2      0.1000000000014E+01   0.8741660037675E-06 0
     8000       10       21       11        3      0.1000000000015E+01   0.8960670770560E-06 0
     9000       10       21       11        3      0.1000000000017E+01   0.8331163658828E-06 0
    10000       10       21       11        3      0.1000000000019E+01   0.8564145492559E-06 0
-----------------------------------------------------------------------------------------
TOTAL    100     210    110    0.19(seconds)

  69  Dai-Kou. Approximate WLS. Function:DIXMAANL

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000      973     1947      974       38      0.1000000002440E+01   0.9301598604812E-06 0
     2000      539     1079      540       37      0.1000000119353E+01   0.9776913046399E-06 0
     3000      235      471      236       24      0.1000000108954E+01   0.9960578915824E-06 0
     4000      235      471      236       33      0.1000000072026E+01   0.9901065114811E-06 0
     5000      236      473      237       41      0.1000000056671E+01   0.9984365212190E-06 0
     6000      236      473      237       50      0.1000000050562E+01   0.9998824130652E-06 0
     7000      239      479      240       58      0.1000000046828E+01   0.9748918340208E-06 0
     8000      239      479      240       67      0.1000000046963E+01   0.9768804371667E-06 0
```

```
     9000      239      479      240       76      0.1000000048432E+01   0.9781948194580E-06 0
    10000      239      479      240       86      0.1000000050729E+01   0.9835112909178E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   3410    6830   3420    5.10(seconds)

  70  Dai-Kou. Approximate WLS. Function:ARGLINB                          70

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        2        5        4        1      0.9090909090916E+00   0.5940819391981E-08 0
     2000        3        7        5        0      0.9090909090962E+00   0.1615489964024E-06 0
     3000        5       13       10        1      0.9090909090254E+00   0.2316864993190E-06 0
     4000        2        6        5        0      0.9090909090633E+00   0.1847674866440E-06 0
     5000        4       10        9        1      0.9090909090741E+00   0.3906570782419E-06 0
     6000        3        7        6        1      0.9090909090748E+00   0.9929972293321E-06 0
     7000        4       10        8        2      0.9090909091158E+00   0.3579152689781E-06 0
     8000        2        6        5        0      0.9090909092376E+00   0.3126478986815E-07 0
     9000        6       15       14        2      0.9090909086330E+00   0.8911920303944E-06 0
    10000        5       14       12        3      0.9090909098043E+00   0.6579648470506E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     36      93     78    0.11(seconds)

  71  Dai-Kou. Approximate WLS. Function:VARDIM

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        8       18       11        0      0.3388313703830E-20   0.1164153682343E-06 0
     2000       11       24       14        0      0.2816553413344E-24   0.4130029651606E-13 0
     3000       13       28       16        0      0.7377858593934E-25   0.1776356839400E-13 0
     4000       15       33       20        0      0.3185218916907E-23   0.9858780458671E-13 0
     5000       16       38       24        2      0.1109458907483E-27   0.8881784197001E-15 0
     6000       16       34       19        1      0.2053701794510E-23   0.6439293542826E-13 0
     7000       18       42       26        2      0.1422168300694E-27   0.1110223024625E-14 0
     8000       18       40       24        2      0.8410011351378E-23   0.1127986593019E-12 0
     9000       18       38       21        2      0.1305122862238E-22   0.1323385845353E-12 0
    10000       19       41       24        3      0.1112337322876E-21   0.3654854197066E-12 0
-----------------------------------------------------------------------------------------------
TOTAL    152     336    199    0.12(seconds)

  72  Dai-Kou. Approximate WLS. Function:DIAG-AUP1

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       19       39       24        1      0.5975878337922E-18   0.8519349137258E-07 0
     2000       18       37       21        0      0.1049132734927E-13   0.2061478086801E-07 0
     3000       17       35       21        1      0.4806947415471E-14   0.8413823377429E-08 0
     4000       15       32       19        1      0.1069301343986E-12   0.5640645042527E-07 0
     5000       22       47       31        2      0.6917977112447E-17   0.3015738770150E-09 0
     6000       16       34       20        2      0.7014534918691E-21   0.2094630247029E-08 0
     7000       16       34       19        2      0.3021187698969E-19   0.3259282799206E-07 0
     8000       16       34       20        2      0.1100666505644E-18   0.1095778960902E-06 0
     9000       18       45       32        3      0.4514494016323E-19   0.8063244116840E-07 0
    10000       16       36       22        2      0.1071187833023E-10   0.1930100416538E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    173     373    229    0.16(seconds)

  73  Dai-Kou. Approximate WLS. Function:ENGVAL8

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       26       54       42        0     -0.1175249106591E+05   0.6984152101097E-06 0
     2000       26       52       42        1     -0.2351469422181E+05   0.7082269668501E-06 0
     3000       25       51       40        1     -0.3527689737772E+05   0.4935231405767E-06 0
     4000       25       51       40        2     -0.4703910053363E+05   0.5240505895898E-06 0
     5000       27       53       46        2     -0.5880130368953E+05   0.5062596493133E-06 0
     6000       26       52       43        2     -0.7056350684544E+05   0.8046108579407E-06 0
     7000       24       50       39        3     -0.8232571000134E+05   0.8459477243150E-06 0
     8000       27       53       46        3     -0.9408791315726E+05   0.3466739233460E-06 0
     9000       26       52       44        4     -0.1058501163132E+06   0.9200630302075E-06 0
    10000       27       54       45        5     -0.1176123194690E+06   0.3962913854849E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    259     522    427    0.23(seconds)

  74  Dai-Kou. Approximate WLS. Function:QUARTIC

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       13       27       14        0      0.4280478150116E-06   0.3764256462865E-06 0
     2000       13       27       14        0      0.8560956291180E-06   0.3764256459879E-06 0
     3000       13       27       14        1      0.1284143444995E-05   0.3764256462776E-06 0
     4000       13       27       14        0      0.1712191260935E-05   0.3764256464330E-06 0
     5000       13       27       14        1      0.2140239077192E-05   0.3764256465679E-06 0
     6000       13       27       14        1      0.2568286893362E-05   0.3764256466484E-06 0
     7000       13       27       14        1      0.2996334708889E-05   0.3764256466452E-06 0
     8000       13       27       14        1      0.3424382525005E-05   0.3764256466914E-06 0
     9000       13       27       14        2      0.3852430340552E-05   0.3764256466857E-06 0
    10000       13       27       14        1      0.4280478158679E-05   0.3764256468512E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    130     270    140    0.08(seconds)

  75  Dai-Kou. Approximate WLS. Function:LIARWHD

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
```

```
     1000       23       48       30        1      0.5942979905026E-22   0.2640625496042E-09 0
     2000       22       49       30        1      0.6062672043957E-12   0.8840502694704E-06 0
     3000       22       53       36        1      0.2210074729835E-19   0.3257155278824E-07 0
     4000       18       39       23        1      0.6590934680830E-19   0.3767025003043E-07 0
     5000       20       44       28        1      0.1149235931494E-14   0.7939045043453E-06 0
     6000       24       55       37        3      0.1780272381307E-14   0.7376873820869E-06 0
     7000       23       56       38        3      0.6083252015621E-16   0.4008116055726E-09 0
     8000       26       57       38        3      0.3841122499356E-20   0.2132128251731E-08 0
     9000       19       49       34        3      0.1605545775938E-11   0.5207674018708E-07 0
    10000       21       47       32        3      0.1276462857207E-09   0.4770880689504E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    218     497    326    0.20(seconds)

  76  Dai-Kou. Approximate WLS. Function:NONSCOMP

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       33       67       34        0      0.6566071765718E-13   0.8364840360324E-06 0
     2000       31       63       32        1      0.1046003917401E-12   0.6863853250465E-06 0
     3000       32       65       33        1      0.1365739677496E-12   0.7857595022481E-06 0
     4000       32       65       33        1      0.4324326273691E-13   0.5017064761249E-06 0
     5000       32       65       33        1      0.1102075813851E-12   0.8146770306822E-06 0
     6000       34       69       35        3      0.2073798680734E-13   0.3264571207257E-06 0
     7000       32       65       33        3      0.1518987641544E-12   0.9800339276511E-06 0
     8000       34       69       35        3      0.2865634240332E-13   0.4671917193178E-06 0
     9000       33       67       34        4      0.1035137471332E-12   0.9525664586732E-06 0
    10000       33       67       34        4      0.1047246912896E-12   0.9831362288973E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    326     662    336    0.21(seconds)

  77  Dai-Kou. Approximate WLS. Function:Linear perturbed

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000      183      367      184        2     -0.1871367713288E-03   0.9828654752662E-06 0
     2000      260      521      261        6     -0.2044592024171E-03   0.9498648586657E-06 0
     3000      319      639      320       11     -0.2145937470793E-03   0.9396793927696E-06 0
     4000      369      739      370       16     -0.2217847573354E-03   0.9084746985348E-06 0
     5000      412      825      413       24     -0.2273627211437E-03   0.9738770732539E-06 0
     6000      452      905      453       30     -0.2319203434350E-03   0.9357654285162E-06 0
     7000      488      977      489       39     -0.2357738128185E-03   0.9616552664616E-06 0
     8000      522     1045      523       47     -0.2391118744338E-03   0.9497582902258E-06 0
     9000      553     1107      554       56     -0.2420562767041E-03   0.9992513153294E-06 0
    10000      583     1167      584       67     -0.2446901507113E-03   0.9998714318327E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   4141    8292   4151    2.98(seconds)

  78  Dai-Kou. Approximate WLS. Function:CUBE

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000     2001     4167     2201       26      0.1906029093855E-05   0.1396366680428E-02 2
     2000     2001     4124     2153       50      0.7659720265409E-05   0.2024459493073E-02 2
     3000     2001     4152     2188       76      0.2005377604006E-05   0.1278622989527E-02 2
     4000     2001     4079     2105      100      0.1157920209519E-04   0.3442971389455E-03 2
     5000     2001     4019     2020      123      0.5533501784568E-04   0.6842604609894E-02 2
     6000     2001     4094     2128      150      0.8016233513260E-05   0.2088342019313E-03 2
     7000     2001     4120     2152      176      0.5673204688503E-05   0.4865566020799E-03 2
     8000     2001     4181     2224      205      0.4317312214537E-05   0.3086459265135E-02 2
     9000     2001     4112     2144      226      0.6587022860002E-05   0.2514987359321E-02 2
    10000     2001     4141     2171      252      0.5650928622900E-05   0.6024142129120E-03 2
-----------------------------------------------------------------------------------------------
TOTAL  20010   41189  21486   13.84(seconds)

  79  Dai-Kou. Approximate WLS. Function:HARKERP

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        2        5        3        0     -0.2501250625312E+00   0.1247676081938E-07 0
     2000        2        5        3        0     -0.2500625156289E+00   0.1241222940906E-06 0
     3000        2        5        3        0     -0.2500416736122E+00   0.3739377381660E-08 0
     4000        2        5        3        0     -0.2500312539067E+00   0.7322543882826E-08 0
     5000        2        5        3        0     -0.2500250025002E+00   0.1234972146384E-07 0
     6000        2        5        3        1     -0.2500208350695E+00   0.2781046905907E-08 0
     7000        2        5        3        0     -0.2500178584184E+00   0.2522843742996E-08 0
     8000        2        5        3        0     -0.2500156259765E+00   0.3954938431464E-09 0
     9000        2        5        3        1     -0.2500138896604E+00   0.2622662159129E-08 0
    10000        2        5        3        0     -0.2500125006250E+00   0.2386322046521E-08 0
-----------------------------------------------------------------------------------------------
TOTAL     20      50     30    0.02(seconds)

  80  Dai-Kou. Approximate WLS. Function:QUARTICM

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       29       59       30        0      0.4696217672714E-06   0.4038285681118E-06 0
     2000       30       61       31        0      0.2861613779183E-05   0.9309120661459E-06 0
     3000       31       63       32        1      0.4139913236970E-05   0.9058808347146E-06 0
     4000       32       65       33        2      0.3302757244739E-05   0.6162457379808E-06 0
     5000       33       67       34        1      0.1911811238434E-05   0.3459245594764E-06 0
     6000       33       67       34        2      0.4745304408588E-05   0.5966215045683E-06 0
     7000       33       67       34        2      0.1023722696811E-04   0.9460623191902E-06 0
     8000       34       69       35        3      0.3789269394350E-05   0.4061625063651E-06 0
     9000       34       69       35        3      0.6791040298400E-05   0.5759257986319E-06 0
```

```
    10000       34       69       35        3      0.1157700770525E-04   0.7939438828669E-06 0
 ---------------------------------------------------------------------------------------------
 TOTAL    323     656    333    0.17(seconds)
```

Appendix 3

```
Dai-Kou conjugate gradient algorithm
Improved Wolfe line search (WLS=Wolfe line search)

   1  Dai-Kou. Improved WLS. Function:Extended Freudenstein & Roth

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
 ---------------------------------------------------------------------------------------------
     1000       12      124      115        0      0.2449212683962E+05   0.2747155019023E-06 4
     2000       12       27       18        1      0.4898425367924E+05   0.5934663800389E-06 0
     3000       12      125      116        2      0.7347638051886E+05   0.1322588740749E-06 4
     4000       13       28       19        0      0.9796850735847E+05   0.5439626207249E-06 0
     5000       11       25       16        1      0.1224606341981E+06   0.9960185707314E-06 0
     6000       13       28       19        1      0.1469527610377E+06   0.1077915534609E-07 0
     7000       14      130      122        4      0.1714448878773E+06   0.3552912630767E-08 4
     8000       12       27       18        2      0.1959370147170E+06   0.8223763394710E-06 0
     9000       13       28       19        1      0.2204291415566E+06   0.7886822270819E-06 0
    10000       13       29       20        2      0.2449212683962E+06   0.4149569576839E-11 0
 ---------------------------------------------------------------------------------------------
 TOTAL    125     571    482    0.14(seconds)

   2  Dai-Kou. Improved WLS. Function:Extended White & Holst

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
 ---------------------------------------------------------------------------------------------
     1000       11       25       15        0      0.9044668587886E-12   0.8730601372086E-07 0
     2000       11       25       15        0      0.1808936493812E-11   0.8730607766970E-07 0
     3000       11       25       15        1      0.2713399640957E-11   0.8730599640138E-07 0
     4000       11       25       15        0      0.3617867207512E-11   0.8730600705952E-07 0
     5000       11       25       15        0      0.4522306573876E-11   0.8730574238235E-07 0
     6000       11       25       15        1      0.5426825004142E-11   0.8730619268881E-07 0
     7000       11       25       15        1      0.6331175912787E-11   0.8730536046563E-07 0
     8000       11       25       15        1      0.7235638223072E-11   0.8730540309819E-07 0
     9000       11       25       15        1      0.8140106425094E-11   0.8730546527068E-07 0
    10000       11       25       15        1      0.9044544089235E-11   0.8730537512057E-07 0
 ---------------------------------------------------------------------------------------------
 TOTAL    110     250    150    0.06(seconds)

   3  Dai-Kou. Improved WLS. Function:Tridiagonal White-Holst

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
 ---------------------------------------------------------------------------------------------
     1000     2001     4041     2041       21      0.2713285955949E+03   0.3002477930240E+01 2
     2000     2001     4034     2034       52      0.1053660993458E+04   0.3178143562634E+01 2
     3000     2001     4039     2039       66      0.1836104354426E+04   0.2267108356159E+01 2
     4000     2001     4041     2041       86      0.2618893864752E+04   0.2123999298154E+01 2
     5000     2001     4034     2034      146      0.3400963182100E+04   0.3345783742672E+01 2
     6000     2001     4033     2033      178      0.4181870115295E+04   0.2377090298954E+01 2
     7000     2001     4033     2033      202      0.4964329700307E+04   0.2724236945253E+01 2
     8000     2001     4037     2037      240      0.5746983636172E+04   0.2946136043339E+01 2
     9000     2001     4035     2035      200      0.6531947025997E+04   0.3658099424890E+01 2
    10000     2001     4036     2036      302      0.7312913252551E+04   0.2765758366919E+01 2
 ---------------------------------------------------------------------------------------------
 TOTAL  20010   40363  20363   14.93(seconds)

   4  Dai-Kou. Improved WLS. Function:Extended Beale

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
 ---------------------------------------------------------------------------------------------
     1000       12       25       15        0      0.4082404550275E-11   0.6146879953400E-07 0
     2000       12       25       15        0      0.8172204810851E-11   0.6149757176792E-07 0
     3000       12       25       15        1      0.1226816500740E-10   0.6152312807440E-07 0
     4000       12       25       15        1      0.1633467089572E-10   0.6147863229338E-07 0
     5000       12       25       15        0      0.2043371814360E-10   0.6150256048558E-07 0
     6000       12       25       15        1      0.2461509230098E-10   0.6162511145208E-07 0
     7000       12       25       15        1      0.2861445786226E-10   0.6151061931677E-07 0
     8000       12       25       15        1      0.3273079679656E-10   0.6153838042897E-07 0
     9000       12       25       15        1      0.3670202583870E-10   0.6143454801651E-07 0
    10000       12       25       15        1      0.4077847215615E-10   0.6143333815100E-07 0
 ---------------------------------------------------------------------------------------------
 TOTAL    120     250    150    0.07(seconds)

   5  Dai-Kou. Improved WLS. Function:Extended Powell

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
 ---------------------------------------------------------------------------------------------
     1000       71      143       75        1      0.4367709467585E-07   0.7878972931806E-06 0
     2000       71      143       75        1      0.8735200650442E-07   0.7886528161270E-06 0
     3000       71      143       75        2      0.1310268837897E-06   0.7888530263845E-06 0
     4000       71      143       75        2      0.1746933296312E-06   0.7904345675378E-06 0
     5000       71      143       75        3      0.2183878965580E-06   0.7873500919611E-06 0
     6000       71      143       75        4      0.2620396895302E-06   0.7904353829826E-06 0
     7000       71      143       75        4      0.3057073125359E-06   0.7908906262255E-06 0
     8000       71      143       75        5      0.3493146082686E-06   0.7960885269577E-06 0
     9000       71      143       75        5      0.3931204671251E-06   0.7856421510523E-06 0
    10000       71      143       75        6      0.4366673580713E-06   0.7946174432727E-06 0
```

```
-----------------------------------------------------------------------------------------
TOTAL    710    1430    750    0.33(seconds)

   6  Dai-Kou. Improved WLS. Function:Extended Maratos

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000        9       26       19        0     -0.5280864426222E+03   0.1076241928910E-06 0
     2000        9       26       19        1     -0.1056172885244E+04   0.7941204449580E-07 0
     3000        9       26       19        0     -0.1584259327867E+04   0.6878391167753E-07 0
     4000        9       26       19        0     -0.2112345770489E+04   0.1181336408695E-06 0
     5000        9       26       19        1     -0.2640432213111E+04   0.5226534494085E-07 0
     6000        9       26       19        0     -0.3168518655733E+04   0.2864253581939E-07 0
     7000        9       26       19        1     -0.3696605098356E+04   0.1344662334812E-06 0
     8000        9       26       19        1     -0.4224691540978E+04   0.9533927913807E-07 0
     9000        9       26       19        1     -0.4752777983600E+04   0.2029684651461E-06 0
    10000        9       26       19        1     -0.5280864426222E+04   0.4138538512241E-07 0
-----------------------------------------------------------------------------------------
TOTAL     90     260    190    0.06(seconds)

   7  Dai-Kou. Improved WLS. Function:Extended CLIFF

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000        7       16       10        0      0.4232867951672E+03   0.1350742275807E-06 0
     2000        7       17       11        1      0.8465735906175E+03   0.4010118190845E-06 0
     3000        6       14        9        1      0.1269860385420E+04   0.1558767679377E-06 0
     4000        7       16       10        1      0.1693147180577E+04   0.3636214684910E-07 0
     5000        7       18       12        1      0.2116433978946E+04   0.5648424044047E-06 0
     6000        7       21       15        3      0.2539720771925E+04   0.6447629512518E-06 0
     7000        6       14        9        1      0.2963007578395E+04   0.7089088185364E-06 0
     8000        9       31       26        5      0.3386294361120E+04   0.3663834236001E-08 0
     9000       13       42       35        7      0.3809581156328E+04   0.2684434800004E-06 0
    10000        7       19       13        4      0.4232867956794E+04   0.9458281069463E-06 0
-----------------------------------------------------------------------------------------
TOTAL     76     208    150    0.24(seconds)

   8  Dai-Kou. Improved WLS. Function:Extended Wood

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000      290      607      328        3      0.5679302646653E-11   0.8999453665393E-06 0
     2000      442      925      497        8      0.1871616308183E-10   0.9965859195712E-06 0
     3000      190      416      239        5      0.5187582238296E-11   0.7925585206114E-06 0
     4000      177      382      218        6      0.1275985754437E-10   0.5966534371460E-06 0
     5000      188      407      237       10      0.7902573468493E-11   0.8631396922266E-06 0
     6000      230      486      267       13      0.2876770970080E-09   0.9752349271228E-06 0
     7000      128      286      168        9      0.3824334631098E-11   0.3845540215189E-07 0
     8000      132      312      201       11      0.1868760064966E-10   0.7487514948566E-07 0
     9000      147      329      193       14      0.1811097313612E-11   0.5856158877162E-06 0
    10000      122      276      171       12      0.1662504342794E-10   0.4863364115426E-06 0
-----------------------------------------------------------------------------------------
TOTAL   2046    4426   2519    0.91(seconds)

   9  Dai-Kou. Improved WLS. Function:Extended Hiebert

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000       41      109       81        0      0.1044044608066E-12   0.8653964290974E-06 0
     2000       38      108       82        1      0.5453191626599E-11   0.3349214264005E-07 0
     3000       44      122       92        2      0.6347411400534E-13   0.2512479115282E-08 0
     4000       40      109       84        1      0.1763697805008E-13   0.1132320903371E-08 0
     5000       37      107       79        2      0.2290272104965E-11   0.1167904880380E-07 0
     6000       41      116       89        3      0.1302753499861E-10   0.2535330184734E-07 0
     7000       40      119       92        4      0.8444944105696E-14   0.4393545793236E-07 0
     8000       46      124       95        4      0.2958948995041E-13   0.1967005794178E-06 0
     9000       46      128       97        4      0.7622704918162E-13   0.4007739207543E-06 0
    10000       46      129      101        5      0.2158998917984E-13   0.2078345708870E-06 0
-----------------------------------------------------------------------------------------
TOTAL    419    1171    892    0.26(seconds)

  10  Dai-Kou. Improved WLS. Function:Extended Rosenbrock                 10

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000       55      182      145        0      0.4635575387349E-09   0.9204135231755E-06 0
     2000       55      180      143        2      0.2236324798821E-09   0.3650653113141E-06 0
     3000       62      196      153        2      0.2394512432518E-08   0.8799724615359E-06 0
     4000       57      193      156        3      0.8275933647170E-09   0.7379356061311E-06 0
     5000       61      183      142        3      0.2469293443197E-11   0.2512301477964E-07 0
     6000       60      183      139        5      0.8520358377122E-14   0.2131829517459E-06 0
     7000       52      170      132        4      0.1233030287948E-13   0.2370767533718E-06 0
     8000       59      179      140        9      0.2883424301228E-08   0.8391883859432E-06 0
     9000       56      188      151        6      0.2062373479360E-09   0.1504947277908E-06 0
    10000       58      193      155        7      0.1316508178233E-09   0.1124793591600E-06 0
-----------------------------------------------------------------------------------------
TOTAL    575    1847   1456    0.41(seconds)

  11  Dai-Kou. Improved WLS. Function:Generalized Rosenbrock

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000     2001     4100     2123       19      0.4891843185846E-05   0.8135973101612E-03 2
     2000     2001     4039     2045       38      0.1335977284075E-04   0.4179982841039E-04 2
```

```
      3000      2001      4079      2101        58       0.8607401924428E-05   0.8307051875438E-03 2
      4000      2001      4012      2014        76       0.2455786254620E-04   0.1666913902179E-02 2
      5000      2001      4096      2109        97       0.7324598566370E-05   0.2046249414235E-02 2
      6000      2001      4075      2093       115       0.6548175617245E-05   0.2172365683922E-03 2
      7000      2001      4030      2038       133       0.1544708950599E-04   0.1679267519191E-02 2
      8000      2001      4030      2038       152       0.1312545545286E-04   0.4451845220637E-02 2
      9000      2001      4123      2152       174       0.5579848445291E-05   0.7873573807594E-03 2
     10000      2001      4031      2041       192       0.1718887718362E-04   0.8338954399296E-03 2
-----------------------------------------------------------------------------------------------
TOTAL  20010   40615  20754   10.54(seconds)

  12  Dai-Kou. Improved WLS. Function:Extended Himmelblau

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        8       20       13        0       0.1178921828895E-13   0.5189838424087E-07 0
      2000        8       20       13        0       0.2357832325816E-13   0.5189804495671E-07 0
      3000        8       20       13        0       0.3536773731041E-13   0.5189812844549E-07 0
      4000        8       20       13        1       0.4715689449195E-13   0.5189840910987E-07 0
      5000        8       20       13        0       0.5894590624754E-13   0.5189862049633E-07 0
      6000        8       20       13        1       0.7073524389900E-13   0.5189880879015E-07 0
      7000        8       20       13        0       0.8252446532644E-13   0.5189830075210E-07 0
      8000        8       20       13        1       0.9431350320936E-13   0.5189791705902E-07 0
      9000        8       20       13        1       0.1061028604153E-12   0.5189887273900E-07 0
     10000        8       20       13        1       0.1178921295258E-12   0.5189880168473E-07 0
-----------------------------------------------------------------------------------------------
TOTAL     80     200    130    0.05(seconds)

  13  Dai-Kou. Improved WLS. Function:HIMMELBG

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        1       10       10        0       0.0000000000000E+00   0.0000000000000E+00 0
      2000        1       10       10        1       0.0000000000000E+00   0.0000000000000E+00 0
      3000        1       10       10        0       0.0000000000000E+00   0.0000000000000E+00 0
      4000        1       10       10        1       0.0000000000000E+00   0.0000000000000E+00 0
      5000        1       10       10        1       0.0000000000000E+00   0.0000000000000E+00 0
      6000        1       10       10        2       0.0000000000000E+00   0.0000000000000E+00 0
      7000        1       10       10        1       0.0000000000000E+00   0.0000000000000E+00 0
      8000        1       10       10        2       0.0000000000000E+00   0.0000000000000E+00 0
      9000        1       10       10        2       0.0000000000000E+00   0.0000000000000E+00 0
     10000        1       10       10        2       0.0000000000000E+00   0.0000000000000E+00 0
-----------------------------------------------------------------------------------------------
TOTAL     10     100    100    0.12(seconds)

  14  Dai-Kou. Improved WLS. Function:HIMMELBH

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        5       11        6        1      -0.5000000000000E+03   0.8980697185912E-06 0
      2000        5       11        6        0      -0.9999999999999E+03   0.8813226255455E-06 0
      3000        5       11        6        0      -0.1500000000000E+04   0.8942482825347E-06 0
      4000        5       11        6        0      -0.2000000000000E+04   0.8813095480065E-06 0
      5000        5       11        6        0      -0.2500000000000E+04   0.8865594467444E-06 0
      6000        5       11        6        1      -0.3000000000000E+04   0.9106136196202E-06 0
      7000        5       11        6        0      -0.3500000000000E+04   0.9020986704300E-06 0
      8000        5       11        6        0      -0.4000000000000E+04   0.9107697289679E-06 0
      9000        5       11        6        1      -0.4500000000000E+04   0.9190630807510E-06 0
     10000        5       11        6        0      -0.5000000000000E+04   0.8939210611736E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     50     110     60    0.03(seconds)

  15  Dai-Kou. Improved WLS. Function:Extended Trigonometric ET1

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000       63      128       65        6       0.4211254960573E-06   0.9677471934220E-06 0
      2000       66      133       67       11       0.2105830840363E-06   0.9767606541527E-06 0
      3000       64      129       65       16       0.1396950410393E-06   0.9172299891618E-06 0
      4000       68      137       69       23       0.1031640051006E-06   0.9881656528216E-06 0
      5000       68      137       69       28       0.8391314399747E-07   0.7553436067101E-06 0
      6000       76      155       80       39       0.6775590597053E-07   0.7775652223570E-06 0
      7000       71      143       73       42       0.5780637210883E-07   0.9645324950195E-06 0
      8000       71      144       73      123       0.5118804436990E-07   0.9136691404755E-06 0
      9000       69      139       70      141       0.4487952205407E-07   0.7570149206842E-06 0
     10000       75      152       78      172       0.3934441707222E-07   0.9434758524149E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    691    1397    709    6.01(seconds)

  16  Dai-Kou. Improved WLS. Function:Extended Trigonometric ET2

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000      316      644      386       83       0.2823235951063E-12   0.9006671566202E-06 0
      2000      465      946      556      244       0.2286985703871E-12   0.9965578154492E-06 0
      3000      987     1978     1014      719       0.1512030307338E-11   0.9903481902401E-06 0
      4000      758     1537      921      795       0.1051961746064E-11   0.9578889498241E-06 0
      5000     1019     2062     1143     1000       0.1683764322299E-11   0.9284499844155E-06 0
      6000     1039     2110     1264      805       0.1252789394962E-11   0.7808203170272E-06 0
      7000     1163     2366     1416     1057       0.4335700701098E-12   0.9970242180698E-06 0
      8000     1194     2436     1428     1232       0.1038060563175E-11   0.9438415928464E-06 0
      9000     1382     2821     1696     1623       0.3810420361960E-12   0.7961166164592E-06 0
     10000     1403     2863     1737     1833       0.8530478136508E-12   0.7873011636819E-06 0
-----------------------------------------------------------------------------------------------
```

```
TOTAL   9726   19763  11561   93.91(seconds)

  17  Dai-Kou. Improved WLS. Function:Extended Block Diagonal BD1

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        7       15        8        0      0.1323406312888E-10   0.4803343568016E-06 0
     2000        7       15        8        1      0.2646809882919E-10   0.4803341076488E-06 0
     3000        7       15        8        1      0.3970214632669E-10   0.4803340956403E-06 0
     4000        7       15        8        2      0.5293622555055E-10   0.4803342342266E-06 0
     5000        7       15        8        1      0.6617024873938E-10   0.4803341140532E-06 0
     6000        7       15        8        3      0.7940434946312E-10   0.4803342675781E-06 0
     7000        7       15        8        2      0.9263836295029E-10   0.4803341519763E-06 0
     8000        7       15        8        3      0.1058723803288E-09   0.4803340733499E-06 0
     9000        7       15        8        3      0.1191065221192E-09   0.4803342635959E-06 0
    10000        7       15        8        4      0.1323405614171E-09   0.4803342296553E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     70     150     80    0.20(seconds)

  18  Dai-Kou. Improved WLS. Function:Extended Tridiagonal-1

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       21       44       27        1      0.1710290945233E-08   0.2485502917074E-06 0
     2000       21       44       27        0      0.3421377953530E-08   0.2482865674310E-06 0
     3000       21       44       27        1      0.5131738282246E-08   0.2483671645297E-06 0
     4000       21       44       27        1      0.6839745605890E-08   0.2487806853103E-06 0
     5000       21       44       27        2      0.8550104178802E-08   0.2487366361553E-06 0
     6000       21       44       27        2      0.1026405949455E-07   0.2482911486544E-06 0
     7000       21       44       27        2      0.1198209814825E-07   0.2475899777884E-06 0
     8000       21       44       27        2      0.1368690869070E-07   0.2481793990749E-06 0
     9000       21       44       27        3      0.1540037502797E-07   0.2479673456000E-06 0
    10000       21       44       27        2      0.1709294436957E-07   0.2492132762286E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    210     440    270    0.16(seconds)

  19  Dai-Kou. Improved WLS. Function:Extended Three Exponential Terms

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        7       15        8        1      0.1279633348329E+04   0.7959519969791E-06 0
     2000        9       18       11        2      0.2559266696658E+04   0.4355410823109E-06 0
     3000       10       19       13        4      0.3838900044987E+04   0.9068301665138E-12 0
     4000        9       19       12        4      0.5118533393317E+04   0.1176969632866E-10 0
     5000        9       20       12        5      0.6398166741646E+04   0.4465776348717E-06 0
     6000        7       16        9        5      0.7677800089974E+04   0.5600590602395E-06 0
     7000        8       18       12        8      0.8957433438304E+04   0.9301427466787E-11 0
     8000        8       19       13        9      0.1023706678663E+05   0.5875370285777E-11 0
     9000        7       16       10        8      0.1151670013496E+05   0.1770247362032E-06 0
    10000        8       18       11       11      0.1279633348329E+05   0.1132718563390E-07 0
-----------------------------------------------------------------------------------------------
TOTAL     82     178    111    0.57(seconds)

  20  Dai-Kou. Improved WLS. Function:Generalized Tridiagonal-1           20

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       22       40       28        1      0.9972103074860E+03   0.9499208104558E-06 0
     2000       22      136      124        2      0.1997210307486E+04   0.9653037156276E-06 4
     3000       22       39       29        1      0.2997210307486E+04   0.9627719603778E-06 0
     4000       22       39       29        2      0.3997210307486E+04   0.9416307147347E-06 0
     5000       22       39       29        2      0.4997210307486E+04   0.9548193942166E-06 0
     6000       22       39       29        2      0.5997210307486E+04   0.9663752673639E-06 0
     7000       22       39       29        3      0.6997210307486E+04   0.9194185053296E-06 0
     8000       22       39       29        3      0.7997210307486E+04   0.9608585185283E-06 0
     9000       21      131      122       10      0.8997210307486E+04   0.2026078693262E-05 4
    10000       22      116      108       10      0.9997210307486E+04   0.9566739098688E-06 4
-----------------------------------------------------------------------------------------------
TOTAL    219     657    556    0.36(seconds)

  21  Dai-Kou. Improved WLS. Function:Generalized Tridiagonal-2

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       39       77       42        1      0.1114854332625E+01   0.8623950049682E-06 0
     2000       44       86       48        1      0.1114854332625E+01   0.6784155595015E-06 0
     3000       44       87       47        3      0.1114854332625E+01   0.7697441577088E-06 0
     4000       45       88       49        4      0.1114854332625E+01   0.7325318261713E-06 0
     5000       45       89       48        4      0.1114854332625E+01   0.9309803198046E-06 0
     6000       46       90       50        5      0.1114854332625E+01   0.8362278038976E-06 0
     7000       47       91       52        7      0.1114854332625E+01   0.8485323967044E-06 0
     8000       47       92       51        8      0.1114854332625E+01   0.9461517229692E-06 0
     9000       47       92       51        8      0.1114854332625E+01   0.9647650667824E-06 0
    10000       47       92       51        9      0.1114854332625E+01   0.9423957639407E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    451     884    489    0.50(seconds)

  22  Dai-Kou. Improved WLS. Function:Tridiagonal Double Borded

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000      448      897      619        8      0.1528391502750E-09   0.9613573713826E-06 0
     2000      460      921      628       15      0.1903658012739E-09   0.8729531995081E-06 0
     3000      325      651      430       16      0.3050518241796E-08   0.8946286995124E-06 0
```

```
     4000     1913     3827     2790      131      0.8583776920745E-10   0.9975889735852E-06 0
     5000     1980     3961     2693      165      0.1545633728505E-09   0.9937141816690E-06 0
     6000     2001     4001     2778      206      0.1631580745585E-08   0.8438104872606E-05 2
     7000     2001     4001     2007      210      0.1063894251775E-05   0.4337663919755E-03 2
     8000     2001     4001     2829      269      0.1000031051361E-06   0.1217082825293E-02 2
     9000     2001     4001     2701      298      0.1960114691869E-07   0.1428146464282E-04 2
    10000     2001     4001     2375      319      0.5104977356439E-06   0.4851898920690E-03 2
-----------------------------------------------------------------------------------------------
TOTAL  15131   30262  19850   16.37(seconds)

  23  Dai-Kou. Improved WLS. Function:Broyden Pentadiagonal

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       38       77       39        1      0.7461227006094E-13   0.8821076348581E-06 0
     2000       40       81       41        2      0.5472083776458E-13   0.6398183516102E-06 0
     3000       40       81       41        2      0.9277297464218E-13   0.7786250212443E-06 0
     4000       40       81       41        2      0.9408818710153E-13   0.9424813307015E-06 0
     5000       40       81       41        4      0.6037018024351E-13   0.6988056446702E-06 0
     6000       39       79       40        4      0.7039044239927E-13   0.7851178551813E-06 0
     7000       38       77       39        4      0.6865125110153E-13   0.7762853829565E-06 0
     8000       37       75       38        5      0.6587812446165E-13   0.7542413521530E-06 0
     9000       36       73       37        6      0.9654096934473E-13   0.9790572706176E-06 0
    10000       36       73       37        6      0.6189057596778E-13   0.7594562195627E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    384     778    394    0.36(seconds)

  24  Dai-Kou. Improved WLS. Function:Extended PSC1

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       10       21       11        1      0.3865995282465E+03   0.7750709551946E-07 0
     2000       10       21       11        1      0.7731990564929E+03   0.7421207237623E-07 0
     3000       10       21       11        2      0.1159798584739E+04   0.8180339650110E-07 0
     4000       10       21       11        3      0.1546398112986E+04   0.8729345746250E-07 0
     5000       10       21       11        3      0.1932997641232E+04   0.1255222438767E-06 0
     6000       10       21       11        4      0.2319597169479E+04   0.6385597406400E-07 0
     7000       10       21       11        5      0.2706196697725E+04   0.1121266771653E-06 0
     8000       10       21       11        5      0.3092796225971E+04   0.7786351008399E-07 0
     9000       10       21       11        5      0.3479395754218E+04   0.3140296349069E-06 0
    10000       10       21       11        6      0.3865995282465E+04   0.9327316419228E-07 0
-----------------------------------------------------------------------------------------------
TOTAL    100     210    110    0.35(seconds)

  25  Dai-Kou. Improved WLS. Function:Perturbed Quadratic PQ1

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000      179      359      180        3      0.1671157287378E-12   0.9680375908780E-06 0
     2000      254      509      255        6      0.1675840389168E-12   0.9684290156498E-06 0
     3000      312      625      313       11      0.1516123931612E-12   0.9211687000507E-06 0
     4000      360      721      361       18      0.1666553061782E-12   0.9652068105196E-06 0
     5000      403      807      404       25      0.1587404985738E-12   0.9421594011434E-06 0
     6000      441      883      442       33      0.1734386389473E-12   0.9841041992678E-06 0
     7000      477      955      478       41      0.1614539709109E-12   0.9499462373797E-06 0
     8000      510     1021      511       50      0.1622466350354E-12   0.9521986955753E-06 0
     9000      541     1083      542       60      0.1627543529942E-12   0.9536220813917E-06 0
    10000      570     1141      571       69      0.1690528495127E-12   0.9715731145394E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   4047    8104   4057    3.16(seconds)

  26  Dai-Kou. Improved WLS. Function:Perturbed Quadratic PQ2

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000     2001     4010     2536       26      0.4748685751489E+00   0.1168917912108E+01 2
     2000     2001     4001     3001       54      0.1005695997463E+05   0.2365161222999E+03 2
     3000     2001     4025     3020       82      0.2128071839750E+02   0.8212759011889E+01 2
     4000     2001     4076     2109       98      0.1515072473714E+02   0.7491347009571E+01 2
     5000     2001     4033     2182      125      0.1081683459083E+01   0.1983665455935E+01 2
     6000     2001     4001     3000      163      0.6065178225916E+05   0.1438358666279E+04 2
     7000     2001     4001     2849      189      0.2987516677376E+04   0.1949298663881E+03 2
     8000     2001     4056     2172      199      0.6848976219771E+01   0.7018188303941E+02 2
     9000     2001     4069     3070      248      0.4719420952261E+03   0.3120771429297E+02 2
    10000     2001     4328     2426      266      0.1747496204659E+01   0.1508052051294E+02 2
-----------------------------------------------------------------------------------------------
TOTAL  20010   40600  26365   14.50(seconds)

  27  Dai-Kou. Improved WLS. Function:Almost Perturbed Quadratic

        n     iter    nfunc    ngrad  time(c)               f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000      175      351      176        2      0.1907481248696E-12   0.9833712570810E-06 0
     2000      249      499      250        5      0.1642303467783E-12   0.9185227971497E-06 0
     3000      305      611      306       10      0.1782071487878E-12   0.9500428573982E-06 0
     4000      352      705      353       14      0.1935143004613E-12   0.9830929003930E-06 0
     5000      394      789      395       21      0.1867682008045E-12   0.9680773817580E-06 0
     6000      432      865      433       27      0.1811142710268E-12   0.9552819597656E-06 0
     7000      466      933      467       34      0.1997841245930E-12   0.9953889573281E-06 0
     8000      499      999      500       42      0.1835117337954E-12   0.9602914351229E-06 0
     9000      529     1059      530       49      0.1915727752350E-12   0.9777500026117E-06 0
    10000      558     1117      559       59      0.1856458641079E-12   0.9647910846176E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   3959    7928   3969    2.63(seconds)
```

28 Dai-Kou. Improved WLS. Function:Almost Perturbed Quartic

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	27	56	29	0	0.1137509969107E-08	0.7452517472568E-06	0
2000	31	65	34	1	0.6339513692871E-09	0.2695826284186E-06	0
3000	37	79	42	1	0.5498635368700E-09	0.4205454331185E-06	0
4000	39	86	48	2	0.7516688815646E-08	0.4856249854558E-06	0
5000	32	69	37	2	0.1961437660771E-08	0.9771179454373E-06	0
6000	36	79	44	2	0.3538073857667E-08	0.6353712101886E-06	0
7000	36	79	43	3	0.2401490008435E-08	0.2484038256657E-06	0
8000	41	87	47	4	0.1180677411426E-08	0.4431728835773E-06	0
9000	39	83	44	4	0.1633951320462E-08	0.3827394204190E-06	0
10000	44	97	54	4	0.9217564933952E-09	0.1430573235812E-06	0

TOTAL 362 780 422 0.23(seconds)

29 Dai-Kou. Improved WLS. Function:Extended Penalty Function U52

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	23	48	33	0	0.8831940750670E+03	0.1631901944027E-06	0
2000	21	43	24	1	0.1814063664869E+04	0.5399711130169E-06	0
3000	24	48	28	1	0.2755973749503E+04	0.6376100952963E-06	0
4000	27	53	32	1	0.3704070534948E+04	0.1639732435710E-06	0
5000	25	49	28	2	0.4656333923744E+04	0.6584645984041E-06	0
6000	25	127	106	4	0.5611676659140E+04	0.1086110759177E-05	4
7000	25	50	27	2	0.6569428560737E+04	0.8777092397102E-06	0
8000	27	145	124	6	0.7529139638522E+04	0.4975852277209E-09	4
9000	29	55	34	3	0.8490489281459E+04	0.9176646237133E-06	0
10000	30	158	134	8	0.9453238852842E+04	0.5191166191422E-07	4

TOTAL 256 776 570 0.28(seconds)

30 Dai-Kou. Improved WLS. Function:TR-Summ of quadratics 30

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	1009	2034	1029	13	0.1375184858438E-16	0.8517238546325E-06	0
2000	2001	4021	2023	54	0.4278116896649E-11	0.5169452008010E-04	2
3000	2001	4019	2021	79	0.3048544940733E-09	0.5830426105179E-03	2
4000	2001	4012	2013	107	0.3571908701356E-09	0.5200966912439E-03	2
5000	2001	4014	2015	135	0.2371325392845E-09	0.3092189112764E-03	2
6000	2001	4013	2014	159	0.3710844148370E-09	0.5469381653183E-03	2
7000	2001	4013	2014	187	0.1540170613906E-09	0.3463517000068E-03	2
8000	2001	4013	2014	214	0.2442658324141E-09	0.3628177331144E-03	2
9000	2001	4018	2020	240	0.3263109398730E-11	0.7665308816179E-04	2
10000	2001	4012	2013	266	0.2131845976491E-09	0.5279704370726E-03	2

TOTAL 19018 38169 19176 14.54(seconds)

31 Dai-Kou. Improved WLS. Function:Quadratic Diagonal Perturbed

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	101	203	122	1	0.3205264856237E-10	0.9926440547088E-06	0
2000	142	285	173	4	0.3495251411631E-10	0.9758509842935E-06	0
3000	169	339	202	8	0.3505367335610E-10	0.9633685083699E-06	0
4000	198	397	239	11	0.3763064161148E-10	0.9777181142617E-06	0
5000	218	437	262	16	0.3874251990775E-10	0.9799099044219E-06	0
6000	239	479	293	20	0.4231475151418E-10	0.9981213929153E-06	0
7000	252	505	304	25	0.4288064586029E-10	0.9939725389025E-06	0
8000	270	541	325	30	0.4138990834222E-10	0.9768538461450E-06	0
9000	289	579	354	37	0.4575860016552E-10	0.9967078933060E-06	0
10000	298	597	359	43	0.4688464811175E-10	0.9960534784597E-06	0

TOTAL 2176 4362 2633 1.95(seconds)

32 Dai-Kou. Improved WLS. Function:Full Hessian FH1

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	29	59	31	8	0.1212871287129E+02	0.1527666881884E-12	0
2000	33	67	34	19	0.1212871287129E+02	0.1510638121260E-06	0
3000	36	73	38	30	0.1212871287129E+02	0.3055333763768E-12	0
4000	38	76	40	42	0.1212871287129E+02	0.2220446049250E-12	0
5000	39	79	41	55	0.1212871287129E+02	0.3250733016102E-12	0
6000	40	81	42	67	0.1212871287129E+02	0.2824407374646E-12	0
7000	41	162	122	192	0.1212871287129E+02	0.5861977570021E-13	4
8000	41	83	42	91	0.1212871287129E+02	0.5968830123493E-06	0
9000	43	87	45	108	0.1212871287129E+02	0.4707345624411E-12	0
10000	44	88	46	122	0.1212871287129E+02	0.2287947609148E-11	0

TOTAL 384 855 481 7.34(seconds)

33 Dai-Kou. Improved WLS. Function:Full Hessian FH2

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	2001	4037	2278	135	-0.7078092752696E+03	0.3666134938082E-02	2
2000	2001	4038	2191	269	-0.2829824326961E+04	0.7401403703646E-02	2
3000	2001	4074	2406	415	-0.6366032766467E+04	0.1145186016156E-01	2
4000	2001	4020	2143	528	-0.1131520428266E+05	0.9079591235768E-01	2

```
     5000     2001     4043     2266      626     -0.1768118431638E+05   0.2737356602462E-01 2
     6000     2001     4082     2330      614     -0.2546002024725E+05   0.3880156268047E-01 2
     7000     2001     4097     2279      713     -0.3465311270498E+05   0.1739542102701E-01 2
     8000     2001     4053     2161      796     -0.4526026386218E+05   0.4221927018542E-01 2
     9000     2001     4163     2519     1366     -0.5728196533114E+05   0.2651248606505E-02 2
    10000     2001     4087     2289     2782     -0.7071774522123E+05   0.3858287946691E-01 2
-----------------------------------------------------------------------------------------------
TOTAL  20010   40694  22862   82.44(seconds)

  34  Dai-Kou. Improved WLS. Function:Full Hessian FH3

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       17       43       27        3      0.4767296853771E+03   0.8417576680131E-07 0
     2000       17       44       27        5      0.1905966240052E+04   0.5240953186991E-06 0
     3000       17       43       27        9      0.4287709662222E+04   0.2201974087512E-06 0
     4000       19       45       30       11      0.7621959951885E+04   0.2545041569342E-06 0
     5000       19       45       30       14      0.1190871710904E+05   0.2066532276089E-06 0
     6000       18       44       28       17      0.1714798113370E+05   0.7574807625588E-06 0
     7000       18      145      130       73      0.2333975202584E+05   0.2031042720674E-06 4
     8000       20       49       35       27      0.3048402978548E+05   0.3930001089003E-06 0
     9000       19       45       30       28      0.3858081441262E+05   0.1030613905328E-06 0
    10000       19       53       38       36      0.4763010590725E+05   0.7275395219608E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    183     556    402    2.23(seconds)

  35  Dai-Kou. Improved WLS. Function:Diagonal Full Borded

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000      482     1222      925       90      0.2528751934203E-04   0.2593001874953E-06 0
     2000     1168     2682     1741      364      0.2435016447073E-04   0.9358157426495E-06 0
     3000      630     1600     1133      345      0.2670583714333E-04   0.4128765875829E-06 0
     4000      851     1924     1183      503      0.4330176925521E-04   0.8603207720692E-06 0
     5000     1139     2555     1613      841      0.7523254313373E-04   0.8395989920143E-06 0
     6000      592     1590     1202      392      0.4522091162662E-04   0.8667643283999E-07 0
     7000      954     2231     1439      383      0.6145058874565E-04   0.3884869466664E-06 0
     8000      457     1338     1068      294      0.5716558963197E-04   0.9824519954951E-06 0
     9000      604     1612     1167      377      0.5129942970010E-04   0.3704719159390E-06 0
    10000      542     1276      815      312      0.1179342490998E-03   0.6673003963847E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   7419   18030  12286   39.01(seconds)

  36  Dai-Kou. Improved WLS. Function:Diagonal Double Borded Arrow Up

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       34       71       47        0      0.1338563836791E-12   0.6444438853017E-06 0
     2000       34       71       49        1      0.1487303043794E-12   0.7178466143700E-07 0
     3000       50      102       70        2      0.6613631404252E-11   0.3418175131546E-06 0
     4000       43       90       63        2      0.1732539891849E-11   0.6249376484836E-07 0
     5000      594     1192      784       34      0.1414796539219E-10   0.1769905884002E-06 0
     6000       36       76       53        2      0.3705766605108E-12   0.1010668322598E-06 0
     7000      163      330      180       12      0.3041797885967E-11   0.9990786493296E-06 0
     8000       26       56       40        3      0.1461134968353E-10   0.9269873608526E-06 0
     9000       30       64       44        3      0.3308569786259E-12   0.1165503910409E-06 0
    10000     2001     4004     2106      209      0.1620226663601E-11   0.1422227209063E-04 2
-----------------------------------------------------------------------------------------------
TOTAL   3011    6056   3436    2.68(seconds)

  37  Dai-Kou. Improved WLS. Function:QP1 Extended Quadratic Penalty

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       16      104       92        1      0.3990006250000E+04   0.1926782506108E-09 4
     2000       15       32       19        0      0.7990003125000E+04   0.1120530841007E-08 0
     3000       15       30       18        0      0.1199000208333E+05   0.8224738213537E-10 0
     4000       15       31       18        0      0.1599000156250E+05   0.5161288534457E-06 0
     5000       16      132      119        3      0.1999000125000E+05   0.1720203420870E-10 4
     6000       16      104       91        3      0.2399000104167E+05   0.7467821250698E-10 4
     7000       16      131      118        4      0.2799000089286E+05   0.2200267017263E-09 4
     8000       16      135      122        5      0.3199000078125E+05   0.2917282368138E-05 4
     9000       17      136      123        5      0.3599000069445E+05   0.3130933830340E-09 4
    10000       18       35       21        2      0.3999000062499E+05   0.3008873303771E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    160     870    741    0.23(seconds)

  38  Dai-Kou. Improved WLS. Function:QP2 Extended Quadratic Penalty

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       39      112       84        3      0.1823160838027E-15   0.5269657777155E-06 0
     2000       41      126       97        7      0.1132009946032E-15   0.4255764451728E-06 0
     3000       42      110       80        8      0.9634512169793E-13   0.8362235348701E-07 0
     4000       41      105       75       11      0.5266941877208E-13   0.9145992407762E-07 0
     5000       40      106       75       14      0.3253297278946E-17   0.1123396486948E-07 0
     6000       44      123       89       18      0.2704725418110E-19   0.6572236088681E-08 0
     7000       42       99       68       17      0.7848403474261E-18   0.1499529389545E-07 0
     8000       43      117       86       24      0.2188276543814E-17   0.5218225851423E-09 0
     9000       41      111       82       26      0.1789854878403E-17   0.4016271759631E-07 0
    10000       37      104       74       27      0.1736522871327E-20   0.1324451659457E-08 0
-----------------------------------------------------------------------------------------------
TOTAL    410    1113    810    1.55(seconds)
```

39 Dai-Kou. Improved WLS. Function:QP3 Extended Quadratic Penalty

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	20	48	31	0	-0.9990000000000E+03	0.1875739112468E-06	0
2000	22	48	32	1	-0.1999000000000E+04	0.3835498585256E-10	0
3000	21	47	32	0	-0.2999000000000E+04	0.6173728195374E-11	0
4000	21	48	29	1	-0.3999000000000E+04	0.4424175767966E-06	0
5000	23	49	32	2	-0.4999000000000E+04	0.8133249630000E-10	0
6000	22	49	33	1	-0.5999000000000E+04	0.2406119747947E-10	0
7000	20	46	28	2	-0.6999000000000E+04	0.9161728638185E-06	0
8000	22	53	37	2	-0.7999000000000E+04	0.1865381183201E-09	0
9000	22	51	35	3	-0.8999000000000E+04	0.8064457983783E-09	0
10000	24	59	40	3	-0.9999000000000E+04	0.5257314586437E-09	0

TOTAL 217 498 329 0.15(seconds)

40 Dai-Kou. Improved WLS. Function:STAIRCASE S1 40

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	1999	3999	2001	18	0.1683993776811E-16	0.1050239006872E-08	0
2000	2001	4001	2001	34	0.9354352673411E-04	0.4833615857740E-03	2
3000	2001	4001	2001	53	0.2128815959619E-03	0.4219839242978E-03	2
4000	2001	4001	2001	69	0.4223184738648E-03	0.3194427163180E-03	2
5000	2001	4001	2001	87	0.6367941057572E-03	0.4357694770079E-03	2
6000	2001	4001	2001	104	0.8463421241898E-03	0.1140723499702E-02	2
7000	2001	4001	2001	121	0.1149501657394E-02	0.1568335661432E-02	2
8000	2001	4001	2001	139	0.1499584428045E-02	0.2128301402990E-02	2
9000	2001	4001	2001	156	0.2327583348370E-02	0.6559592420672E-03	2
10000	2001	4001	2001	173	0.2341601560546E-02	0.2745450799921E-02	2

TOTAL 20008 40008 20010 9.54(seconds)

41 Dai-Kou. Improved WLS. Function:STAIRCASE S2

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	1999	3999	2001	17	0.2700222462687E-16	0.1018179318635E-08	0
2000	2001	4001	2001	62	0.9373942382170E-04	0.4576209168072E-03	2
3000	2001	4001	2001	140	0.2139290732117E-03	0.4120412070279E-03	2
4000	2001	4001	2001	189	0.3762554317334E-03	0.7622103539688E-03	2
5000	2001	4001	2001	236	0.5883166299121E-03	0.8833981364660E-03	2
6000	2001	4001	2001	282	0.1035155290527E-02	0.4411030568008E-03	2
7000	2001	4001	2001	328	0.1152509547157E-02	0.1332242685748E-02	2
8000	2001	4001	2001	378	0.1499250120119E-02	0.2056686249489E-02	2
9000	2001	4001	2001	425	0.2368642947602E-02	0.6582689820789E-03	2
10000	2001	4001	2001	474	0.2343947691016E-02	0.2606601643492E-02	2

TOTAL 20008 40008 20010 25.31(seconds)

42 Dai-Kou. Improved WLS. Function:STAIRCASE S3

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	1999	3999	2001	48	0.2251745310710E-13	0.1354515006824E-06	0
2000	2001	4001	2001	95	0.9420175875238E-04	0.1317227965784E-02	2
3000	2001	4001	2001	142	0.2134740571680E-03	0.4934394532938E-03	2
4000	2001	4001	2001	188	0.3758556311008E-03	0.1043195008606E-02	2
5000	2001	4001	2001	235	0.5945461844552E-03	0.7222737694974E-03	2
6000	2001	4001	2001	281	0.8724228668367E-03	0.6709562831020E-03	2
7000	2001	4001	2001	331	0.1152743024252E-02	0.1426449591236E-02	2
8000	2001	4001	2001	375	0.1503650978075E-02	0.1855556221926E-02	2
9000	2001	4001	2001	428	0.1907882804503E-02	0.1716009552183E-02	2
10000	2001	4001	2001	469	0.2350045959306E-02	0.2136219194654E-02	2

TOTAL 20008 40008 20010 25.92(seconds)

43 Dai-Kou. Improved WLS. Function:NONDQUAR

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	2001	4013	2445	68	0.3267336952662E-05	0.2988197471574E-04	2
2000	2001	4016	2399	134	0.4213673219581E-05	0.1718439421017E-04	2
3000	2001	4009	2391	203	0.3500511879531E-05	0.1894771683405E-04	2
4000	2001	4016	2470	276	0.4033771282739E-05	0.1564324375019E-04	2
5000	2001	4010	2218	279	0.3646128290280E-05	0.9925659939677E-05	2
6000	1804	3625	2078	177	0.4412430986060E-05	0.9195286703331E-06	0
7000	2001	4022	2274	230	0.4206366989015E-05	0.2337966925370E-04	2
8000	2001	4017	2278	262	0.4926126710811E-05	0.1274198915792E-04	2
9000	2001	4024	2492	306	0.4652024449155E-05	0.5787434319530E-05	2
10000	1969	3959	2255	324	0.3897188901930E-05	0.8631621818101E-06	0

TOTAL 19781 39711 23300 22.59(seconds)

44 Dai-Kou. Improved WLS. Function:TRIDIA

n	iter	nfunc	ngrad	time(c)	f	gnorm	s
1000	261	523	262	3	0.7639736673201E-15	0.7107047265129E-06	0
2000	371	743	372	10	0.1485652009186E-14	0.9575403116929E-06	0
3000	459	919	460	18	0.9568057410973E-15	0.9824189871520E-06	0
4000	532	1065	533	29	0.8740251355717E-15	0.8942329331931E-06	0
5000	594	1189	595	38	0.1192977220591E-14	0.8905753739718E-06	0

```
     6000      653     1307      654       50      0.9910301630624E-15   0.9981651753768E-06 0
     7000      704     1409      705       63      0.1302124332540E-14   0.9842440471197E-06 0
     8000      756     1513      757       77      0.9275109056099E-15   0.8766630847566E-06 0
     9000      801     1603      802       92      0.1093366142717E-14   0.9908236962860E-06 0
    10000      847     1695      848      108      0.8596529755363E-15   0.9270479529008E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   5978   11966   5988    4.88(seconds)

  45  Dai-Kou. Improved WLS. Function:ARWHEAD

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       11       29       22        1      0.0000000000000E+00   0.1443235588448E-09 0
     2000        9       23       17        0      0.0000000000000E+00   0.2257029062789E-06 0
     3000        9       23       17        0      0.0000000000000E+00   0.5052917724453E-08 0
     4000        9       23       17        1      0.0000000000000E+00   0.1933455002786E-08 0
     5000       10       25       17        1      0.0000000000000E+00   0.5490545727915E-06 0
     6000       12       35       27        2      0.0000000000000E+00   0.2825426351214E-07 0
     7000       11       27       19        1      0.0000000000000E+00   0.1401368058979E-09 0
     8000       11       26       19        2      0.0000000000000E+00   0.5395026688448E-09 0
     9000        9       24       18        1      0.0000000000000E+00   0.7159110131155E-11 0
    10000        8       17       11        2      0.0000000000000E+00   0.7211658896509E-08 0
-----------------------------------------------------------------------------------------------
TOTAL     99     252    184    0.11(seconds)

  46  Dai-Kou. Improved WLS. Function:NONDIA

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       32      114       95        0      0.9898968741080E+00   0.1410078266766E-07 0
     2000       38      131      111        2      0.9898968740952E+00   0.8702104488112E-06 0
     3000       43      179      156        3      0.9898968740952E+00   0.1088343167488E-09 0
     4000       47      156      131        4      0.9898968742314E+00   0.8209669558057E-07 0
     5000       40      242      218        6      0.9898968787086E+00   0.5351566592788E-07 0
     6000       57      251      217        9      0.9898968740952E+00   0.1380647152160E-07 0
     7000       50      305      277       11      0.9898968836058E+00   0.1841530347840E-04 4
     8000       66      386      349       16      0.9898968740952E+00   0.1358774204263E-12 4
     9000       59      257      223       13      0.9898968911669E+00   0.5720660260956E-07 0
    10000       56      222      192       13      0.9898968740952E+00   0.7286961420257E-10 0
-----------------------------------------------------------------------------------------------
TOTAL    488    2243   1969    0.77(seconds)

  47  Dai-Kou. Improved WLS. Function:BDQRTIC

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000      508     1137      686       12      0.3983817950583E+04   0.3573940311816E-03 4
     2000      522     1159      809       25      0.7989427682686E+04   0.8168406638083E-03 4
     3000      115      397      349       13      0.1199503741453E+05   0.9131919669027E-03 4
     4000      651     1467      923       61      0.1600064714648E+05   0.3074729287948E-02 4
     5000      139      447      371       24      0.2000625687844E+05   0.5796322118562E-03 4
     6000      613     1377      811       84      0.2401186661129E+05   0.2108949145089E-02 4
     7000     1161     2484     1372      174      0.2801747634284E+05   0.4665816216418E-02 4
     8000      958     2076     1184      170      0.3202308607471E+05   0.5093484455001E-01 4
     9000      421     1047      731      100      0.3602869580825E+05   0.4979236069810E-02 4
    10000     1138     2431     1349      245      0.4003430553930E+05   0.1961339563214E-02 4
-----------------------------------------------------------------------------------------------
TOTAL   6226   14022   8585    9.08(seconds)

  48  Dai-Kou. Improved WLS. Function:DQDRTIC

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        7       15        9        0      0.1707992867039E-16   0.1072908380163E-06 0
     2000        7       15        9        0      0.3321619487869E-14   0.1270722504919E-06 0
     3000        7       15        9        0      0.2239457285124E-14   0.1387969760681E-06 0
     4000        7       15        9        1      0.2766787894298E-14   0.1354243021668E-06 0
     5000        7       15        9        0      0.3105616959169E-14   0.1385087563382E-06 0
     6000        7       15        9        1      0.8183326196249E-13   0.7840625219890E-06 0
     7000        7       15        9        0      0.4141659377473E-13   0.4069763453704E-06 0
     8000        7       15        9        1      0.1432224829712E-12   0.7568630573762E-06 0
     9000       10       21       12        1      0.1551710787516E-24   0.1680227479257E-10 0
    10000        8       17       10        1      0.9864780356662E-13   0.6280518013333E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     74     158     94    0.05(seconds)

  49  Dai-Kou. Improved WLS. Function:EG2

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       13      128      127        4     -0.9989473933008E+03   0.1453883695581E-04 4
     2000        8      116      115        7     -0.1998947392741E+04   0.9195025242865E-06 4
     3000        8       24       20        2     -0.2998947392570E+04   0.7670660435426E-07 0
     4000        7       23       21        3     -0.3998947392484E+04   0.6493400842103E-06 0
     5000        5       13       11        1     -0.4998947392433E+04   0.1812874540128E-07 0
     6000        6       15       13        2     -0.5998947392399E+04   0.1510723437345E-07 0
     7000        5       14       12        3     -0.6998947392374E+04   0.1294902504191E-07 0
     8000        5       12       10        3     -0.7998947392355E+04   0.1133037580009E-07 0
     9000        5       16       14        4     -0.8998947392341E+04   0.1513699253535E-07 0
    10000        7       23       20        6     -0.9998947392330E+04   0.4460701195583E-06 0
-----------------------------------------------------------------------------------------------
TOTAL     69     384    363    0.35(seconds)

  50  Dai-Kou. Improved WLS. Function:EG3                                 50
```

```
        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       44      154      133        5     -0.9995000000000E+03   0.2194322619734E-07 0
     2000      101      392      350       22     -0.1999499999999E+04   0.1717798109323E-05 4
     3000       85      273      225       22     -0.2999500000000E+04   0.1179157716715E-06 0
     4000       77      216      170       23     -0.3999499999999E+04   0.1558426479603E-06 0
     5000      111      381      296       50     -0.4999499998412E+04   0.5425158060633E-03 4
     6000       66      284      253       48     -0.5999499999997E+04   0.9706736388545E-04 4
     7000      171      571      452      105     -0.6999499999852E+04   0.4288439649434E-03 4
     8000       43      215      190       48     -0.7999499976806E+04   0.1169713732113E-02 4
     9000       46      167      153       43     -0.8999500000000E+04   0.1189618949677E-06 0
    10000      137      504      432      138     -0.9999499999957E+04   0.6087587649666E-03 4
-----------------------------------------------------------------------------------------------
TOTAL    881    3157   2654    5.04(seconds)

  51  Dai-Kou. Improved WLS. Function:EDENSCH

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       24       44       32        1      0.6003284592021E+04   0.5773764955030E-06 0
     2000       24      135      125        2      0.1200328459202E+05   0.4782344248944E-06 4
     3000       24      139      130        4      0.1800328459202E+05   0.3670166670577E-06 4
     4000       23      140      130        5      0.2400328459202E+05   0.1575436102962E-05 4
     5000       22      140      130        6      0.3000328459202E+05   0.2647169898487E-05 4
     6000       23      122      114        6      0.3600328459202E+05   0.1108512142522E-05 4
     7000       24      131      125        8      0.4200328459202E+05   0.9430769738472E-06 4
     8000       23      137      128        9      0.4800328459202E+05   0.1280337992959E-05 4
     9000       22      137      127       10      0.5400328459202E+05   0.1850916676371E-05 4
    10000       24      138      130       12      0.6000328459202E+05   0.1027316739677E-05 4
-----------------------------------------------------------------------------------------------
TOTAL    233    1263   1171    0.63(seconds)

  52  Dai-Kou. Improved WLS. Function:FLETCHCR

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       21       46       26        1      0.7625916916815E-15   0.8061751707191E-06 0
     2000       22       48       27        0      0.4924774666006E-15   0.9166746913542E-06 0
     3000       23       50       28        2      0.1366091036710E-15   0.4913514927261E-06 0
     4000       23       50       28        1      0.2551265805010E-15   0.6014723828628E-06 0
     5000       23       50       28        2      0.3581787393068E-15   0.6027222719351E-06 0
     6000       23       50       28        2      0.4327776201078E-15   0.5162530401602E-06 0
     7000       23       50       28        2      0.5247127519631E-15   0.6413574336194E-06 0
     8000       23       50       28        3      0.6530736585339E-15   0.7134611787776E-06 0
     9000       23       50       28        3      0.7885642740291E-15   0.7750306393829E-06 0
    10000       23       50       28        4      0.9010873199045E-15   0.8190867980757E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    227     494    277    0.20(seconds)

  53  Dai-Kou. Improved WLS. Function:ENGVAL1

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       23      125      112        1      0.1108194718785E+04   0.8170153256500E-06 4
     2000       21      139      124        2      0.2218313143943E+04   0.4823602348658E-05 4
     3000       21      133      120        3      0.3328431569101E+04   0.3411180342638E-05 4
     4000       22      138      127        4      0.4438549994258E+04   0.1550271002760E-05 4
     5000       24      137      126        5      0.5548668419415E+04   0.3822146297239E-06 4
     6000       21      144      130        7      0.6658786844575E+04   0.9026196113294E-05 4
     7000       22      137      125        7      0.7768905269731E+04   0.1956139202619E-05 4
     8000       20      137      125        8      0.8879023694889E+04   0.1243178322774E-04 4
     9000       21      139      127        9      0.9989142120047E+04   0.4829904788117E-05 4
    10000       22      140      129       11      0.1109926054521E+05   0.3343106491638E-05 4
-----------------------------------------------------------------------------------------------
TOTAL    217    1369   1245    0.57(seconds)

  54  Dai-Kou. Improved WLS. Function:DENSCHNA

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000        9       19       10        0      0.7349139229309E-15   0.2410527003887E-08 0
     2000        9       19       10        1      0.1469828098174E-14   0.2410527182774E-08 0
     3000        9       19       10        2      0.2204741752877E-14   0.2410527008624E-08 0
     4000        9       19       10        1      0.2939656629611E-14   0.2410527247501E-08 0
     5000        9       19       10        2      0.3674569513369E-14   0.2410526974709E-08 0
     6000        9       19       10        3      0.4409483360199E-14   0.2410526952212E-08 0
     7000        9       19       10        3      0.5144399044859E-14   0.2410527342592E-08 0
     8000        9       19       10        3      0.5879311056623E-14   0.2410527041225E-08 0
     9000        9       19       10        4      0.6614223347532E-14   0.2410526751329E-08 0
    10000        9       19       10        4      0.7349142115747E-14   0.2410527518565E-08 0
-----------------------------------------------------------------------------------------------
TOTAL     90     190    100    0.23(seconds)

  55  Dai-Kou. Improved WLS. Function:DENSCHNB

        n     iter    nfunc    ngrad  time(c)              f                  gnorm         s
-----------------------------------------------------------------------------------------------
     1000       11       23       12        0      0.1058489654893E-11   0.1301352306288E-06 0
     2000       11       23       12        0      0.2115640384446E-11   0.1300940724408E-06 0
     3000       11       23       12        0      0.3173467339971E-11   0.1300942109967E-06 0
     4000       11       23       12        0      0.4233707326119E-11   0.1301313688291E-06 0
     5000       11       23       12        1      0.5293659926912E-11   0.1301501253810E-06 0
     6000       11       23       12        1      0.6350420211765E-11   0.1301299264273E-06 0
```

```
     7000       11       23       12        1      0.7406830700130E-11   0.1301124239834E-06 0
     8000       11       23       12        1      0.8462803579320E-11   0.1300959322865E-06 0
     9000       11       23       12        1      0.9519099498875E-11   0.1300853114489E-06 0
    10000       11       23       12        1      0.1057580024020E-10   0.1300793038101E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    110     230    120    0.06(seconds)

  56  Dai-Kou. Improved WLS. Function:DENSCHNC

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     2000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     3000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     4000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     5000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     6000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     7000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     8000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
     9000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
    10000        0        1        1        0      0.0000000000000E+00   0.0000000000000E+00 0
-----------------------------------------------------------------------------------------------
TOTAL      0      10     10    0.00(seconds)

  57  Dai-Kou. Improved WLS. Function:DENSCHNF

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       25       57       34        1      0.2279741433594E-15   0.1237525345092E-07 0
     2000       25       57       34        1      0.4559566753772E-15   0.1236901497430E-07 0
     3000       25       57       34        1      0.6839309033719E-15   0.1237298755395E-07 0
     4000       25       57       34        2      0.9119132531692E-15   0.1237389689244E-07 0
     5000       25       57       34        3      0.1139881793974E-14   0.1237431639782E-07 0
     6000       25       57       34        3      0.1367882096347E-14   0.1237318363865E-07 0
     7000       25       57       34        3      0.1595849731737E-14   0.1237265194965E-07 0
     8000       25       57       34        4      0.1823897464175E-14   0.1237466690065E-07 0
     9000       25       57       34        5      0.2051807617542E-14   0.1237410673979E-07 0
    10000       25       57       34        4      0.2279775220254E-14   0.1237231615603E-07 0
-----------------------------------------------------------------------------------------------
TOTAL    250     570    340    0.27(seconds)

  58  Dai-Kou. Improved WLS. Function:SINQUAD

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000      120      340      255       12      0.6947237145880E-06   0.2974238408771E-06 0
     2000      428      954      619       63      0.5445644154606E-05   0.5134095303897E-06 0
     3000      195      496      373       54      0.4238201776966E-05   0.2059464320668E-06 0
     4000      346      902      660      126      0.1344940538591E-04   0.1869015052479E-06 0
     5000      435      996      614      158      0.1574488924663E-04   0.3986264030277E-06 0
     6000      646     1499      930      292      0.1676071545612E-04   0.2640619579939E-06 0
     7000      587     1283      753      286      0.1032634297312E-03   0.7393928047130E-06 0
     8000      883     2007     1197      504      0.1485891783903E-04   0.4176503421269E-06 0
     9000     1157     2726     1819      823      0.8455515162263E-06   0.7869189753707E-06 0
    10000      639     1542     1001      506      0.9092306850798E-05   0.2238439536535E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   5436   12745   8221   28.24(seconds)

  59  Dai-Kou. Improved WLS. Function:DIXON3DQ

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000     1000     2001     1001       11      0.9990009990010E-03   0.1849009834132E-10 0
     2000     2000     4001     2001       44      0.4997501249375E-03   0.9730882766235E-11 0
     3000     2001     4001     2001       66      0.8482225059693E-03   0.2028489409215E-02 2
     4000     2001     4001     2001       90      0.2802126025867E-02   0.6701169715576E-02 2
     5000     2001     4001     2001      111      0.2808595702149E-02   0.2098950524872E-02 2
     6000     2001     4001     2001      133      0.2808595702149E-02   0.2098950524872E-02 2
     7000     2001     4001     2001      154      0.2808595702149E-02   0.2098950524872E-02 2
     8000     2001     4001     2001      178      0.2808595702149E-02   0.2098950524872E-02 2
     9000     2001     4001     2001      199      0.2808595702149E-02   0.2098950524872E-02 2
    10000     2001     4001     2001      221      0.2808595702149E-02   0.2098950524872E-02 2
-----------------------------------------------------------------------------------------------
TOTAL  19008   38010  19010   12.07(seconds)

  60  Dai-Kou. Improved WLS. Function:BIGGSB1                            60

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000      500     1001      501        5      0.1549462877782E-15   0.2328511738980E-07 0
     2000     1000     2001     1001       22      0.1622873778237E-15   0.2315846470147E-07 0
     3000     1500     3001     1501       49      0.2078031304144E-15   0.2323047576525E-07 0
     4000     2000     4001     2001       88      0.5063448164185E-15   0.2367777796053E-07 0
     5000     2001     4001     2001      109      0.8095952023988E-03   0.8995502250102E-03 2
     6000     2001     4001     2001      131      0.8095952023988E-03   0.8995502250102E-03 2
     7000     2001     4001     2001      153      0.8095952023988E-03   0.8995502250102E-03 2
     8000     2001     4001     2001      175      0.8095952023988E-03   0.8995502250102E-03 2
     9000     2001     4001     2001      197      0.8095952023988E-03   0.8995502250102E-03 2
    10000     2001     4001     2001      218      0.8095952023988E-03   0.8995502250102E-03 2
-----------------------------------------------------------------------------------------------
TOTAL  17006   34010  17010   11.47(seconds)

  61  Dai-Kou. Improved WLS. Function:PRODsin
```

```
         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        0        1        1        0      0.9989999999998E-12   0.2998999999999E-08 0
      2000        0        1        1        0      0.3998000000000E-11   0.5998999999999E-08 0
      3000        0        1        1        0      0.8996999999998E-11   0.8998999999997E-08 0
      4000        0        1        1        0      0.1599600000000E-10   0.1199900000000E-07 0
      5000        0        1        1        0      0.2499500000000E-10   0.1499900000000E-07 0
      6000        0        1        1        0      0.3599400000000E-10   0.1799900000000E-07 0
      7000        0        1        1        1      0.4899300000000E-10   0.2099900000000E-07 0
      8000        0        1        1        0      0.6399200000000E-10   0.2399900000000E-07 0
      9000        0        1        1        0      0.8099099999999E-10   0.2699899999999E-07 0
     10000        0        1        1        1      0.9998999999998E-10   0.2999899999999E-07 0
-----------------------------------------------------------------------------------------------
TOTAL      0      10     10    0.02(seconds)

  62  Dai-Kou. Improved WLS. Function:PROD1

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        2        5        3        0      0.1637470899020E-34   0.8093135113219E-17 0
      2000        2        5        3        0      0.2027987163540E-31   0.2848148285143E-15 0
      3000        2        5        3        0      0.1039211808735E-33   0.2038834773821E-16 0
      4000        2        5        3        0      0.9361408092150E-31   0.6119283648320E-15 0
      5000        2        5        3        0      0.5416583025153E-31   0.4654710742958E-15 0
      6000        2        5        3        1      0.1346916891038E-29   0.2321134973723E-14 0
      7000        2        5        3        0      0.1043357942124E-29   0.2042897884990E-14 0
      8000        2        5        3        0      0.4707489147354E-29   0.4339349788784E-14 0
      9000        2        5        3        1      0.1277853256892E-29   0.2260843432785E-14 0
     10000        2        5        3        0      0.4412006169173E-28   0.1328458681205E-13 0
-----------------------------------------------------------------------------------------------
TOTAL     20      50     30    0.02(seconds)

  63  Dai-Kou. Improved WLS. Function:PRODcos

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        4        9        5        0      0.2668568171659E-23   0.9285542614719E-10 0
      2000        4        9        5        1      0.1022890715782E-23   0.8548840635477E-10 0
      3000        4        9        5        0      0.6072922017321E-24   0.8212504087080E-10 0
      4000        4        9        5        0      0.4513461700027E-24   0.8250231787568E-10 0
      5000        4        9        5        1      0.3660126881783E-24   0.8352670862297E-10 0
      6000        4        9        5        2      0.3023312360363E-24   0.8347021921971E-10 0
      7000        4        9        5        1      0.2294636606925E-24   0.7875608349129E-10 0
      8000        4        9        5        2      0.2617671660481E-24   0.9010680572903E-10 0
      9000        4        9        5        2      0.1751692614650E-24   0.7830495472690E-10 0
     10000        4        9        5        2      0.1327902113200E-24   0.7195719622807E-10 0
-----------------------------------------------------------------------------------------------
TOTAL     40      90     50    0.11(seconds)

  64  Dai-Kou. Improved WLS. Function:PROD2

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        0        1        1        0      0.1004995000000E-16   0.4019980010000E-11 0
      2000        0        1        1        0      0.2019990000000E-16   0.8079960010000E-11 0
      3000        0        1        1        0      0.3044985000000E-16   0.1217994001000E-10 0
      4000        0        1        1        0      0.4079980000000E-16   0.1631992001000E-10 0
      5000        0        1        1        0      0.5124975000000E-16   0.2049990001000E-10 0
      6000        0        1        1        0      0.6179970000000E-16   0.2471988001000E-10 0
      7000        0        1        1        0      0.7244965000000E-16   0.2897986001000E-10 0
      8000        0        1        1        0      0.8319960000000E-16   0.3327984001000E-10 0
      9000        0        1        1        0      0.9404955000000E-16   0.3761982001000E-10 0
     10000        0        1        1        0      0.1049995000000E-15   0.4199980001000E-10 0
-----------------------------------------------------------------------------------------------
TOTAL      0      10     10    0.00(seconds)

  65  Dai-Kou. Improved WLS. Function:DIXMAANA

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        8       17        9        0      0.1000000000000E+01   0.6810895251955E-07 0
      2000        8       17        9        1      0.1000000000001E+01   0.6810883655927E-07 0
      3000        8       17        9        1      0.1000000000001E+01   0.6498578597812E-07 0
      4000        8       17        9        1      0.1000000000002E+01   0.6811024397548E-07 0
      5000        8       17        9        1      0.1000000000002E+01   0.6809368966984E-07 0
      6000        8       17        9        2      0.1000000000003E+01   0.6812233587944E-07 0
      7000        8       17        9        2      0.1000000000003E+01   0.7124166712158E-07 0
      8000        8       17        9        2      0.1000000000004E+01   0.6812178371916E-07 0
      9000        8       17        9        3      0.1000000000004E+01   0.6499410983907E-07 0
     10000        8       17        9        2      0.1000000000005E+01   0.6812209649783E-07 0
-----------------------------------------------------------------------------------------------
TOTAL     80     170     90    0.15(seconds)

  66  Dai-Kou. Improved WLS. Function:DIXMAANB

         n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
      1000        7       15        8        0      0.1000000000000E+01   0.2626149583226E-06 0
      2000        7       15        8        0      0.1000000000000E+01   0.2636252608177E-06 0
      3000        7       15        8        0      0.1000000000000E+01   0.2638147809329E-06 0
      4000        7       15        8        1      0.1000000000001E+01   0.2639920380912E-06 0
      5000        7       15        8        1      0.1000000000001E+01   0.2641548339231E-06 0
      6000        7       15        8        1      0.1000000000001E+01   0.2639730967009E-06 0
      7000        7       15        8        2      0.1000000000001E+01   0.2640434634587E-06 0
```

```
     8000        7       15        8        2      0.1000000000001E+01   0.2641067664488E-06 0
     9000        7       15        8        2      0.1000000000001E+01   0.2639822194530E-06 0
    10000        7       15        8        2      0.1000000000002E+01   0.2642928226693E-06 0
-------------------------------------------------------------------------------------------
TOTAL     70     150     80    0.11(seconds)

  67  Dai-Kou. Improved WLS. Function:DIXMAANC

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-------------------------------------------------------------------------------------------
     1000        9       19       10        0      0.1000000000000E+01   0.1314014064258E-06 0
     2000        9       19       10        1      0.1000000000000E+01   0.8934527373806E-07 0
     3000        9       19       10        0      0.1000000000000E+01   0.8819123538326E-07 0
     4000        9       19       10        1      0.1000000000000E+01   0.1124420995931E-06 0
     5000        9       19       10        1      0.1000000000000E+01   0.9159251921392E-07 0
     6000        9       19       10        2      0.1000000000000E+01   0.9463951006667E-07 0
     7000        9       19       10        2      0.1000000000001E+01   0.1848002771070E-06 0
     8000        9       19       10        3      0.1000000000000E+01   0.1170135524566E-06 0
     9000        9       19       10        3      0.1000000000001E+01   0.2487819531421E-06 0
    10000        9       19       10        3      0.1000000000001E+01   0.1269696414305E-06 0
-------------------------------------------------------------------------------------------
TOTAL     90     190    100    0.16(seconds)

  68  Dai-Kou. Improved WLS. Function:DIXMAAND

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-------------------------------------------------------------------------------------------
     1000       10       21       11        0      0.1000000000002E+01   0.8879014017100E-06 0
     2000       10       21       11        1      0.1000000000004E+01   0.8787469317526E-06 0
     3000       10       21       11        1      0.1000000000006E+01   0.8930886860198E-06 0
     4000       10       21       11        2      0.1000000000008E+01   0.8964143510518E-06 0
     5000       10       21       11        1      0.1000000000010E+01   0.8839282188482E-06 0
     6000       10       21       11        2      0.1000000000012E+01   0.8823756348420E-06 0
     7000       10       21       11        3      0.1000000000014E+01   0.8741660037675E-06 0
     8000       10       21       11        2      0.1000000000015E+01   0.8960670770560E-06 0
     9000       10       21       11        3      0.1000000000017E+01   0.8331163658828E-06 0
    10000       10       21       11        4      0.1000000000019E+01   0.8564145492559E-06 0
-------------------------------------------------------------------------------------------
TOTAL    100     210    110    0.19(seconds)

  69  Dai-Kou. Improved WLS. Function:DIXMAANL

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-------------------------------------------------------------------------------------------
     1000      973     1947      974       33      0.1000000002440E+01   0.9301598604812E-06 0
     2000      539     1079      540       37      0.1000000119353E+01   0.9776913046399E-06 0
     3000      235      471      236       24      0.1000000108954E+01   0.9960578915824E-06 0
     4000      235      471      236       33      0.1000000072026E+01   0.9901065114811E-06 0
     5000      236      473      237       41      0.1000000056671E+01   0.9984365212190E-06 0
     6000      236      473      237       49      0.1000000050562E+01   0.9998824130652E-06 0
     7000      239      479      240       58      0.1000000046828E+01   0.9748918340208E-06 0
     8000      239      479      240       68      0.1000000046963E+01   0.9768804371667E-06 0
     9000      239      479      240       75      0.1000000048432E+01   0.9781948194580E-06 0
    10000      239      479      240       84      0.1000000050729E+01   0.9835112909178E-06 0
-------------------------------------------------------------------------------------------
TOTAL   3410    6830   3420    5.02(seconds)

  70  Dai-Kou. Improved WLS. Function:ARGLINB                            70

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-------------------------------------------------------------------------------------------
     1000        2        5        4        1      0.9090909090916E+00   0.5940819391981E-08 0
     2000        3        7        5        0      0.9090909090962E+00   0.1615489964024E-06 0
     3000        5       14       11        1      0.9090909090175E+00   0.9573159331921E-07 0
     4000        2        6        5        0      0.9090909090633E+00   0.1847674866440E-06 0
     5000        3      100       99       10      0.9090909090698E+00   0.2387241693214E-07 4
     6000        3        7        6        1      0.9090909090748E+00   0.9929972293321E-06 0
     7000        4       10        8        1      0.9090909091158E+00   0.3579152689781E-06 0
     8000        2        6        5        1      0.9090909092376E+00   0.3126478986815E-07 0
     9000        7       22       21        4      0.9090909086082E+00   0.9042341844179E-06 0
    10000        4       61       59       12      0.9090909098045E+00   0.6164700607769E-06 4
-------------------------------------------------------------------------------------------
TOTAL     35     238    223    0.31(seconds)

  71  Dai-Kou. Improved WLS. Function:VARDIM

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
-------------------------------------------------------------------------------------------
     1000        8       18       11        0      0.3388313703830E-20   0.1164153682343E-06 0
     2000       11       24       14        0      0.2816553413344E-24   0.4130029651606E-13 0
     3000       13       28       16        0      0.7377858593934E-25   0.1776356839400E-13 0
     4000       15       33       20        1      0.3185218916907E-23   0.9858780458671E-13 0
     5000       16       38       24        1      0.1109458907483E-27   0.8881784197001E-15 0
     6000       16       34       19        2      0.2053701794510E-23   0.6439293542826E-13 0
     7000       18       42       26        2      0.1422168300694E-27   0.1110223024625E-14 0
     8000       18       40       24        2      0.8410011351378E-23   0.1127986593019E-12 0
     9000       18       38       21        2      0.1305122862238E-22   0.1323385845353E-12 0
    10000       19       41       24        3      0.1112337322876E-21   0.3654854197066E-12 0
-------------------------------------------------------------------------------------------
TOTAL    152     336    199    0.13(seconds)

  72  Dai-Kou. Improved WLS. Function:DIAG-AUP1

        n     iter    nfunc    ngrad  time(c)              f                   gnorm         s
```

```
-----------------------------------------------------------------------------------------
     1000       19       39       24        0      0.5975878337922E-18   0.8519349137258E-07 0
     2000       18       37       21        1      0.1049132734927E-13   0.2061478086801E-07 0
     3000       17       35       21        1      0.4806947415471E-14   0.8413823377429E-08 0
     4000       15       32       19        1      0.1069301343986E-12   0.5640645042527E-07 0
     5000       22       47       31        2      0.6917977112447E-17   0.3015738770150E-09 0
     6000       16       34       20        1      0.7014534918691E-21   0.2094630247029E-08 0
     7000       16       34       19        2      0.3021187698969E-19   0.3259282799206E-07 0
     8000       16       34       20        2      0.1100666505644E-18   0.1095778960902E-06 0
     9000       18       45       32        3      0.4514494016323E-19   0.8063244116840E-07 0
    10000       16       36       22        3      0.1071187833023E-10   0.1930100416538E-06 0
-----------------------------------------------------------------------------------------
TOTAL    173     373    229    0.16(seconds)

  73  Dai-Kou. Improved WLS. Function:ENGVAL8

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000       22      130      119        1     -0.1175249106591E+05   0.3746532813764E-05 4
     2000       20      139      129        2     -0.2351469422181E+05   0.1970032267096E-05 4
     3000       22      144      136        2     -0.3527689737772E+05   0.7655069906724E-05 4
     4000       21      126      118        3     -0.4703910053362E+05   0.1545352898091E-05 4
     5000       20      142      132        5     -0.5880130368952E+05   0.6299486969397E-05 4
     6000       21      131      122        4     -0.7056350684544E+05   0.1498862786775E-05 4
     7000       19      140      128        6     -0.8232571000132E+05   0.4803484483595E-04 4
     8000       17      137      126        7     -0.9408791315724E+05   0.9210452517294E-04 4
     9000       18      135      125        7     -0.1058501163131E+06   0.3586635569075E-04 4
    10000       18      129      119        8     -0.1176123194690E+06   0.4411244399449E-04 4
-----------------------------------------------------------------------------------------
TOTAL    198    1353   1254    0.45(seconds)

  74  Dai-Kou. Improved WLS. Function:QUARTIC

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000       13       27       14        0      0.4280478150116E-06   0.3764256462865E-06 0
     2000       13       27       14        1      0.8560956291180E-06   0.3764256459879E-06 0
     3000       13       27       14        0      0.1284143444995E-05   0.3764256462776E-06 0
     4000       13       27       14        1      0.1712191260935E-05   0.3764256464330E-06 0
     5000       13       27       14        0      0.2140239077192E-05   0.3764256465679E-06 0
     6000       13       27       14        1      0.2568286893362E-05   0.3764256466484E-06 0
     7000       13       27       14        1      0.2996334708889E-05   0.3764256466452E-06 0
     8000       13       27       14        2      0.3424382525005E-05   0.3764256466914E-06 0
     9000       13       27       14        1      0.3852430340552E-05   0.3764256466857E-06 0
    10000       13       27       14        1      0.4280478158679E-05   0.3764256468512E-06 0
-----------------------------------------------------------------------------------------
TOTAL    130     270    140    0.08(seconds)

  75  Dai-Kou. Improved WLS. Function:LIARWHD

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000       23       48       30        0      0.5942979905026E-22   0.2640625496042E-09 0
     2000       20       44       27        1      0.1838929153425E-14   0.3830090081150E-08 0
     3000       25       54       35        1      0.1136987361148E-14   0.2424794563254E-08 0
     4000       18       39       23        1      0.6590934680830E-19   0.3767025003043E-07 0
     5000       26       55       37        2      0.7936815872194E-14   0.3144292862837E-06 0
     6000       19       43       29        2      0.2014445373911E-12   0.2791844430421E-06 0
     7000       22       50       34        2      0.3109041763848E-13   0.9092071948023E-06 0
     8000       26       57       38        3      0.3841122499356E-20   0.2132128251731E-08 0
     9000       17       44       31        3      0.1664491098729E-11   0.6280498843688E-06 0
    10000       21       47       32        3      0.1276462857207E-09   0.4770880689504E-06 0
-----------------------------------------------------------------------------------------
TOTAL    217     481    316    0.18(seconds)

  76  Dai-Kou. Improved WLS. Function:NONSCOMP

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000       33       67       34        0      0.6566071765718E-13   0.8364840360324E-06 0
     2000       31       63       32        1      0.1046003917401E-12   0.6863853250465E-06 0
     3000       32       65       33        1      0.1365739677496E-12   0.7857595022481E-06 0
     4000       32       65       33        2      0.4324326273691E-13   0.5017064761249E-06 0
     5000       32       65       33        2      0.1102075813851E-12   0.8146770306822E-06 0
     6000       34       69       35        2      0.2073798680734E-13   0.3264571207257E-06 0
     7000       32       65       33        3      0.1518987641544E-12   0.9800339276511E-06 0
     8000       34       69       35        3      0.2865634240332E-13   0.4671917193178E-06 0
     9000       33       67       34        4      0.1035137471332E-12   0.9525664586732E-06 0
    10000       33       67       34        4      0.1047246912896E-12   0.9831362288973E-06 0
-----------------------------------------------------------------------------------------
TOTAL    326     662    336    0.22(seconds)

  77  Dai-Kou. Improved WLS. Function:Linear perturbed

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------
     1000      183      367      184        2     -0.1871367713288E-03   0.9828654752662E-06 0
     2000      260      521      261        6     -0.2044592024171E-03   0.9498648586657E-06 0
     3000      319      639      320       11     -0.2145937470793E-03   0.9396793927696E-06 0
     4000      369      739      370       17     -0.2217847573354E-03   0.9084746985348E-06 0
     5000      412      825      413       24     -0.2273627211437E-03   0.9738770732539E-06 0
     6000      452      905      453       30     -0.2319203434350E-03   0.9357654285162E-06 0
     7000      488      977      489       39     -0.2357738128185E-03   0.9616552664616E-06 0
     8000      522     1045      523       47     -0.2391118744338E-03   0.9497582902258E-06 0
```

```
     9000      553     1107      554       56     -0.2420562767041E-03   0.9992513153294E-06 0
    10000      583     1167      584       66     -0.2446901507113E-03   0.9998714318327E-06 0
-----------------------------------------------------------------------------------------------
TOTAL   4141    8292   4151    2.98(seconds)

  78  Dai-Kou. Improved WLS. Function:CUBE

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000     2001     4166     2215       25      0.5496701407013E-05   0.5543745553812E-02 2
     2000     2001     4119     2171       50      0.4426960428836E-05   0.1560660229296E-03 2
     3000     2001     4133     2185       76      0.5154781049788E-05   0.4571033537559E-03 2
     4000     2001     4060     2090       99      0.1588515822428E-04   0.9960847100849E-03 2
     5000     2001     4145     2191      126      0.6441577257788E-05   0.3014594058406E-02 2
     6000     2001     4084     2107      149      0.8019159431023E-05   0.1496851951816E-02 2
     7000     2001     4103     2142      177      0.7021604947340E-05   0.9352012205337E-03 2
     8000     2001     4136     2185      203      0.7956137762610E-05   0.1584620205038E-02 2
     9000     2001     4001     2001      219      0.5709742433750E-04   0.6418721712872E-02 2
    10000     2001     4057     2069      247      0.1562364631313E-04   0.5053693376167E-02 2
-----------------------------------------------------------------------------------------------
TOTAL  20010   41004  21356   13.71(seconds)

  79  Dai-Kou. Improved WLS. Function:HARKERP

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000        2        5        3        0     -0.2501250625312E+00   0.1247676081938E-07 0
     2000        2        5        3        1     -0.2500625156289E+00   0.1241222940906E-06 0
     3000        2        5        3        0     -0.2500416736122E+00   0.3739377381660E-08 0
     4000        2        5        3        0     -0.2500312539067E+00   0.7322543882826E-08 0
     5000        2        5        3        0     -0.2500250025002E+00   0.1234972146384E-07 0
     6000        2        5        3        0     -0.2500208350695E+00   0.2781046905907E-08 0
     7000        2        5        3        0     -0.2500178584184E+00   0.2522843742996E-08 0
     8000        2        5        3        0     -0.2500156259765E+00   0.3954938431464E-09 0
     9000        2        5        3        0     -0.2500138896604E+00   0.2622662159129E-08 0
    10000        2        5        3        0     -0.2500125006250E+00   0.2386322046521E-08 0
-----------------------------------------------------------------------------------------------
TOTAL     20       50      30    0.01(seconds)

  80  Dai-Kou. Improved WLS. Function:QUARTICM

        n     iter    nfunc    ngrad  time(c)               f                   gnorm         s
-----------------------------------------------------------------------------------------------
     1000       29       59       30        0      0.4696217672714E-06   0.4038285681118E-06 0
     2000       30       61       31        1      0.2861613779183E-05   0.9309120661459E-06 0
     3000       31       63       32        1      0.4139913236970E-05   0.9058808347146E-06 0
     4000       32       65       33        1      0.3302757244739E-05   0.6162457379808E-06 0
     5000       33       67       34        2      0.1911811238434E-05   0.3459245594764E-06 0
     6000       33       67       34        2      0.4745304408588E-05   0.5966215045683E-06 0
     7000       33       67       34        3      0.1023722696811E-04   0.9460623191902E-06 0
     8000       34       69       35        3      0.3789269394350E-05   0.4061625063651E-06 0
     9000       34       69       35        3      0.6791040298400E-05   0.5759257986319E-06 0
    10000       34       69       35        4      0.1157700770525E-04   0.7939438828669E-06 0
-----------------------------------------------------------------------------------------------
TOTAL    323      656     333    0.20(seconds)
```

Legend:
n = the number of variables,
iter = the number of iterations,
nfunc = the number of minimizing function evaluations,
ngrad = the number of gradient of minimizing function evaluations,
time (c) = the CPU time in centeseconds,
f = the local optimal value of minimizing function,
gnorm = the infinite norm of the gradient of the minimizing function,
s = status: 0 (convergence tolerance satisfied),
2 (total number of iterations exceeded maxit, by default maxit = 2000),
4 (number secant iterations exceed nsecant in line search, by default nsecant = 50).

5. Additional discussions

As we have already seen the Dai-Kou conjugate gradient algorithm given by (20), (21) and (22) was obtained by projecting the self-scaling memoryless BFGS direction by Perry and Shanno into the one-dimensional manifold

$$S_{k+1} = \{-g_{k+1} + \beta d_k : \beta \in \mathbb{R}\}. \tag{28}$$

Therefore, solution of this projection leads us to $\beta_k^{DK}(\tau_k)$ given by (16). Dai and Kou (2013) say that numerical experiments with this family of conjugate gradient algorithms show that the most efficient is the one where the parameter τ_k^B given by (13).

In the following we give a *theoretical justification* of this selection of the parameter τ_k. Indeed, from (7) we have:

$$tr(H_{k+1}) = \frac{n-2}{\tau_k} + \left(1 + \frac{1}{\tau_k}\frac{\|y_k\|^2}{y_k^T s_k}\right)\frac{\|s_k\|^2}{y_k^T s_k}, \qquad \det(H_{k+1}) = \frac{1}{\tau_k^{n-1}}\frac{\|s_k\|^2}{y_k^T s_k}. \tag{29}$$

By the Wolfe line search conditions (4) and (5) we have that $y_k^T s_k > 0$. Therefore, the vectors y_k and s_k are nonzero vectors. Let V be the vector space spanned by $\{s_k, y_k\}$. Clearly, $\dim(V) \le 2$ and $\dim(V^\perp) \ge n-2$. Thus, there exists a set of mutually unit orthogonal vectors $\{u_k^i\}_{i=1}^{n-2} \subset V^\perp$ such that

$$s_k^T u_k^i = y_k^T u_k^i = 0, \;\; i = 1, \ldots, n-2,$$

which from (7) leads to

$$H_{k+1} u_k^i = \frac{1}{\tau_k} u_k^i, \;\; i = 1, \ldots, n-2.$$

Therefore, the matrix H_{k+1} has $n-2$ eigenvalues equal to $\frac{1}{\tau_k}$, which corresponds to $\{u_k^i\}_{i=1}^{n-2}$ as eigenvectors. Now, we are interested in finding the rest of the two remaining eigenvalues, denoted as λ_k^+ and λ_k^-, respectively. By the relationships between the determinant and the trace of H_{k+1} given by (29) and its eigenvalues, it follows that the other eigenvalues of H_{k+1} λ_k^+ and λ_k^- are the roots of the following quadratic polynomial

$$\lambda^2 - \left(1 + \frac{1}{\tau_k}\frac{\|y_k\|^2}{y_k^T s_k}\right)\frac{\|s_k\|^2}{y_k^T s_k}\lambda + \frac{1}{\tau_k}\frac{\|s_k\|^2}{y_k^T s_k} = 0. \tag{30}$$

After some simple algebraic manipulation we get:

$$\lambda_k^{\pm} = \frac{1}{2}\left[\left(1 + \frac{1}{\tau_k}\frac{\|y_k\|^2}{y_k^T s_k}\right)\frac{\|s_k\|^2}{y_k^T s_k} \pm \sqrt{\left(1 + \frac{1}{\tau_k}\frac{\|y_k\|^2}{y_k^T s_k}\right)^2 \frac{\|s_k\|^4}{(y_k^T s_k)^2} - \frac{4}{\tau_k}\frac{\|s_k\|^2}{y_k^T s_k}}\right]. \tag{31}$$

From (31) we ca write:

$$\begin{aligned}\lambda_k^{\pm} &= \frac{1}{2}\left[\left(\frac{\|s_k\|^2}{y_k^T s_k} + \frac{1}{\tau_k}\frac{\|y_k\|^2}{y_k^T s_k}\frac{\|s_k\|^2}{y_k^T s_k}\right) \pm \sqrt{\left(1 + \frac{1}{\tau_k}\frac{\|y_k\|^2}{y_k^T s_k}\right)^2 \left(\frac{\|s_k\|^2}{y_k^T s_k}\right)^2 - \frac{4}{\tau_k}\frac{\|s_k\|^2}{y_k^T s_k}}\right] \\ &= \frac{1}{2}\left[\left(\frac{\|s_k\|^2}{y_k^T s_k} + \frac{1}{\tau_k}\frac{\|y_k\|^2}{y_k^T s_k}\frac{\|s_k\|^2}{y_k^T s_k}\right) \pm \sqrt{\left(\frac{\|s_k\|^2}{y_k^T s_k} + \frac{1}{\tau_k}\frac{\|y_k\|^2}{y_k^T s_k}\frac{\|s_k\|^2}{y_k^T s_k}\right)^2 - \frac{4}{\tau_k}\frac{\|s_k\|^2}{y_k^T s_k}}\right].\end{aligned}$$

As we know

$$\frac{\|y_k\|^2}{y_k^T s_k}\frac{\|s_k\|^2}{y_k^T s_k} \geq 1.$$

In the following to get an approximation of the eigenvalues λ_k^+ and λ_k^- we may consider

$$\frac{\|y_k\|^2}{y_k^T s_k}\frac{\|s_k\|^2}{y_k^T s_k} \approx 1.$$

Therfore the eigenvalues λ_k^+ and λ_k^- may be approximated as:

$$\begin{aligned}\lambda_k^{\pm} &\approx \frac{1}{2}\left[\left(\frac{\|s_k\|^2}{y_k^T s_k}+\frac{1}{\tau_k}\right)\pm\sqrt{\left(\frac{\|s_k\|^2}{y_k^T s_k}+\frac{1}{\tau_k}\right)^2-\frac{4}{\tau_k}\frac{\|s_k\|^2}{y_k^T s_k}}\right]\\ &= \frac{1}{2}\left[\left(\frac{\|s_k\|^2}{y_k^T s_k}+\frac{1}{\tau_k}\right)\pm\left(\frac{\|s_k\|^2}{y_k^T s_k}-\frac{1}{\tau_k}\right)\right].\end{aligned}$$

Hence:

$$\lambda_k^+ = \frac{\|s_k\|^2}{y_k^T s_k},$$
$$\lambda_k^- = \frac{1}{\tau_k}.$$

In order to have all the eigenvalues of H_{k+1} clustered in a point, we should consider $\lambda_k^+ = 1/\tau_k$, i.e. $\tau_k = \tau_k^B$ as suggested by Oren and Luenberger (1976) and considered by Dai and Kou in their conjugate gradient algorithm (see Andrei, 2017b).

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