

Numerical Experiments with Relaxed Gradient Descent with Backtracking for Unconstrained Optimization

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In this work we present the numerical results with RELGRAD package which implements the *relaxed gradient descent algorithm* with backtracking. A number of 430 unconstrained optimization problems have been considered in this experiment. The Fortran code of RELGRAD is authored by Andrei.

The problems are placed at the web page: www.ici.ro/camo/neculai/ansoft. You should go to this address and hit RELGRAD.

The criteria for stopping the iterations are:

$$\|\nabla f(x_k)\|_\infty \leq \varepsilon_g \quad \text{or} \quad t_k |g_k^T d_k| \leq \varepsilon_f |f(x_{k+1})|,$$

where $\varepsilon_g = 10^{-6}$ and $\varepsilon_f = 10^{-20}$.

The code implements the Armijo line search in a backtracking manner with $\alpha = 0.0001$ and $\beta = 0.8$.

The following tables give the results of the optimization process, where:

- n = dimension of the problem (number of variables),
- iter = number of iterations,
- fgcnt = number of function and its gradient evaluations,
- time(c) = cpu time in centeseconds
- fxnew = function value in optimal point, $f(x^*)$,
- ginf = infinite norm of the gradient in optimal point, $\|\nabla f(x^*)\|_\infty$.

1 Relaxed Gradient Algorithm: Extended Freudenstein & Roth					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	619	19778	252	.1767071327215E-13	.2490011753825E-06
200	619	19778	505	.3534142654431E-13	.2490011753825E-06
300	619	19778	764	.5301213981646E-13	.2490011753825E-06
400	619	19778	1021	.7068285308862E-13	.2490011753825E-06
500	619	19778	1269	.8835356636077E-13	.2490011753825E-06
600	619	19778	1532	.1060242796329E-12	.2490011753825E-06
700	619	19778	1785	.1236949929051E-12	.2490011753825E-06
800	619	19778	2038	.1413657061772E-12	.2490011753825E-06
900	619	19778	2296	.1590364194494E-12	.2490011753825E-06
1000	619	19778	2549	.1767071327215E-12	.2490011753825E-06

TOTAL	6190	197780	140.11 (seconds)		

2 Relaxed Gradient Algorithm: Extended Trigonometric					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	40	267	11	.7638933573391E-12	.4952934243503E-06
200	51	332	27	.3732891144827E-10	.9513826590770E-06
300	540	1368	154	.1418059573916E-06	.9736954478866E-06
400	48	417	66	.2309488313481E-12	.1727220011435E-06
500	47	428	82	.6517169442640E-12	.5972096866323E-06
600	69	450	99	.6470754693887E-10	.9941697901707E-06
700	124	613	165	.2912724895708E-05	.9728758549023E-06
800	39	440	132	.3326858889982E-12	.7623272466012E-06
900	42	490	170	.4814917964184E-13	.2593073640758E-06
1000	254	880	335	.3238072475813E-07	.9323390005796E-06
TOTAL	1254	5685	12.41(seconds)		

3 Relaxed Gradient Algorithm: Extended Rosenbrock					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	1943	51677	1011	.6801678981815E-10	.7831886472036E-06
200	1943	51677	2010	.1360335796363E-09	.7831886472036E-06
300	1943	51677	3026	.2040503694545E-09	.7831886472036E-06
400	1943	51677	4032	.2720671592726E-09	.7831886472036E-06
500	1943	51677	5036	.3400839490908E-09	.7831886472036E-06
600	1943	51677	6048	.4081007389089E-09	.7831886472036E-06
700	1943	51677	7063	.4761175287271E-09	.7831886472036E-06
800	1943	51677	8063	.5441343185452E-09	.7831886472036E-06
900	1943	51677	9173	.6121511083634E-09	.7831886472036E-06
1000	1943	51677	10084	.6801678981815E-09	.7831886472036E-06
TOTAL	19430	516770	555.46(seconds)		

4 Relaxed Gradient Algorithm: Extended White & Holst					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	4812	138931	2746	.1371069917103E-09	.8672058661127E-06
200	4812	138931	5493	.2742139834205E-09	.8672058661127E-06
300	4812	138931	8244	.4113209751308E-09	.8672058661127E-06
400	4812	138931	10996	.5484279668411E-09	.8672058661127E-06
500	4812	138931	13748	.6855349585514E-09	.8672058661127E-06
600	4812	138931	16588	.8226419502616E-09	.8672058661127E-06
700	4812	138931	19257	.9597489419719E-09	.8672058661127E-06
800	4812	138931	22003	.1096855933682E-08	.8672058661127E-06
900	4812	138931	24711	.1233962925392E-08	.8672058661127E-06
1000	4812	138931	27446	.1371069917103E-08	.8672058661127E-06
TOTAL	48120	1389310	1512.32(seconds)		

5 Relaxed Gradient Algorithm: Extended Beale					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	365	4526	38	.7854296676923E-10	.8790670854967E-06
200	365	4526	82	.1570859335385E-09	.8790670854967E-06
300	365	4526	121	.2356289003077E-09	.8790670854967E-06
400	365	4526	170	.3141718670769E-09	.8790670854967E-06
500	365	4526	209	.3927148338461E-09	.8790670854967E-06
600	365	4526	247	.4712578006154E-09	.8790670854967E-06
700	365	4526	291	.5498007673846E-09	.8790670854967E-06
800	365	4526	329	.6283437341538E-09	.8790670854967E-06
900	365	4526	379	.7068867009230E-09	.8790670854967E-06
1000	365	4526	418	.7854296676923E-09	.8790670854967E-06
TOTAL	3650	45260	22.84(seconds)		

6 Relaxed Gradient Algorithm: Extended Penalty					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	33	725	11	.7500000000000E+02	.3031462427749E-06
200	35	868	27	.1602796621681E+03	.5979573481939E-06
300	33	912	44	.2478925765033E+03	.6509712742186E-06
400	32	964	66	.3368146699141E+03	.8569157508865E-06
500	35	1013	77	.4266194324845E+03	.9529546796117E-07
600	38	1137	110	.5170771098472E+03	.5280587294099E-06
700	34	1054	121	.6080459723100E+03	.9934367829048E-06
800	35	1130	148	.6994307914492E+03	.6682456491217E-06
900	35	1159	170	.7911636782044E+03	.1795714879016E-06
1000	38	1264	203	.8831940750670E+03	.4599488667517E-06
TOTAL	348	10226	9.77(seconds)		

7 Relaxed Gradient Algorithm: Perturbed Quadratic					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	222	4307	39	.1095885160961E-14	.6619603922807E-07
200	228	5100	88	.1236234819105E-12	.7505860556119E-06
300	466	11180	285	.1611147623337E-12	.8048037535325E-06
400	468	11791	401	.2201732424079E-12	.9421301636392E-06
500	529	13951	594	.7633911083178E-13	.5547152643773E-06
600	532	14431	736	.3345251155855E-12	.9770797659684E-06
700	491	13601	818	.2132184320461E-12	.9270003167988E-06
800	573	16157	1104	.3054428995537E-15	.2697767551251E-06
900	640	18532	1417	.2435897848099E-12	.9915674610953E-06
1000	831	24403	2164	.2468566292779E-12	.9981516262066E-06
TOTAL	4980	133453	76.46(seconds)		

8 Relaxed Gradient Algorithm: Raydan 1					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	371	2021	22	.5050000000000E+03	.9870055832164E-06
200	625	5349	126	.2010000000000E+04	.9388098975206E-06
300	776	7945	286	.4515000000000E+04	.9974529478380E-06
400	1038	12301	588	.8020000000000E+04	.9494710362903E-06
500	1215	15668	933	.1252500000000E+05	.9781165451942E-06
600	1318	17824	1280	.1803000000000E+05	.9403806340943E-06
700	1484	21121	1763	.2453500000000E+05	.9071988902213E-06
800	1631	24485	2346	.3204000000000E+05	.9776550297640E-06
900	1752	26976	2900	.4054500000000E+05	.9721018711603E-06
1000	1996	31794	3801	.5005000000000E+05	.9671440132266E-06
TOTAL	12206	165484	140.45(seconds)		

9 Relaxed Gradient Algorithm: Raydan 2					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	17	37	0	.1000000000000E+03	.9347658158236E-06
200	17	37	0	.2000000000001E+03	.9347658158236E-06
300	17	37	0	.3000000000001E+03	.9347658158236E-06
400	17	37	0	.4000000000002E+03	.9347658158236E-06
500	17	37	0	.5000000000002E+03	.9347658158236E-06
600	17	37	0	.6000000000003E+03	.9347658158236E-06
700	17	37	5	.7000000000003E+03	.9347658158236E-06
800	17	37	0	.8000000000004E+03	.9347658158236E-06
900	17	37	0	.9000000000004E+03	.9347658158236E-06
1000	17	37	5	.1000000000000E+04	.9347658158236E-06
TOTAL	170	370	.10(seconds)		

11 Relaxed Gradient Algorithm: Diagonal 2					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	1828	3659	44	.1574135370147E+02	.9945944489260E-06
200	3423	6849	175	.1985453540073E+02	.9979906050248E-06
300	4919	9841	374	.2248591724537E+02	.9984233963679E-06
400	6277	12557	631	.2445342429118E+02	.9983314042897E-06
500	7631	15265	956	.2603689737074E+02	.9994643053816E-06
600	8879	17761	1351	.2736784662032E+02	.9990518461723E-06
700	10154	20311	1802	.2851919946323E+02	.9999625354458E-06
800	11409	22821	2307	.2953583055504E+02	.9990303650910E-06
900	12615	25233	2850	.3044741583192E+02	.9992243885604E-06
1000	13826	27655	3483	.3127464993162E+02	.9998948500023E-06
TOTAL	80961	161952	139.73(seconds)		

12 Relaxed Gradient Algorithm: Diagonal 3					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	185	3179	71	-.4605795020600E+04	.6947755137076E-06
200	215	4163	187	-.1918247539294E+05	.7629829483971E-06
300	387	8287	560	-.4375601954925E+05	.2736891815620E-04
400	753	17476	1571	-.7832825022511E+05	.1074093848437E-04
500	749	17967	1999	-.1228997540954E+06	.7543551211125E-06
600	446	11100	1467	-.1774707956624E+06	.7148817053426E-04
700	443	10985	1686	-.2420415170304E+06	.5000286140769E-04
800	530	14280	2494	-.3166120034280E+06	.4340094158731E-03
900	433	11282	2208	-.4011823100174E+06	.3934591233948E-04
1000	703	19271	4174	-.4957524745606E+06	.1674767989807E-03
TOTAL	4844	117990	164.17(seconds)		

13 Relaxed Gradient Algorithm: Hager					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	57	395	5	-.6530786727331E+03	.4811466732931E-06
200	58	451	16	-.2493201505568E+04	.7923184276193E-06
300	65	593	33	-.5276871910453E+04	.6140203926679E-06
400	73	679	55	-.8886479558416E+04	.9188645125079E-06
500	69	672	66	-.1324635151502E+05	.7755684445165E-06
600	112	1250	148	-.1830180119599E+05	.6835885744284E-06
700	65	731	99	-.2401061200239E+05	.3496610384356E-05
800	108	1215	193	-.3033875790101E+05	.6579913674356E-06
900	125	1459	258	-.3725796386792E+05	.7574424126747E-06
1000	93	1135	225	-.4474419132154E+05	.1627814830292E-05
TOTAL	825	8580	10.98(seconds)		

14 Relaxed Gradient Algorithm: Generalized Tridiagonal 1					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	37	478	16	.9721030748599E+02	.8628796308585E-06
200	41	542	33	.1972103074860E+03	.8808060953935E-06
300	34	414	39	.2972103074860E+03	.6375566707906E-06
400	37	442	55	.3972103074860E+03	.8600947234427E-06
500	38	452	66	.4972103074860E+03	.7552267171640E-06
600	42	548	104	.5972103074860E+03	.8964722080229E-06
700	41	495	104	.6972103074860E+03	.4337229519980E-06
800	45	571	143	.7972103074860E+03	.7414635454506E-06
900	42	519	143	.8972103074860E+03	.6741214095829E-06
1000	52	698	209	.9972103074860E+03	.6684921172173E-06
TOTAL	409	5159	9.12(seconds)		

15 Relaxed Gradient Algorithm: Extended Tridiagonal 1 **Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	3148	6332	99	.7870319030167E-07	.9996011359510E-06
200	3148	6332	192	.1574063806033E-06	.9996011359510E-06
300	3148	6332	285	.2361095709050E-06	.9996011359510E-06
400	3148	6332	385	.3148127612067E-06	.9996011359510E-06
500	3148	6332	483	.3935159515083E-06	.9996011359510E-06
600	3148	6332	577	.4722191418100E-06	.9996011359510E-06
700	3148	6332	675	.5509223321117E-06	.9996011359510E-06
800	3148	6332	769	.6296255224133E-06	.9996011359510E-06
900	3148	6332	863	.7083287127150E-06	.9996011359510E-06
1000	3148	6332	966	.7870319030167E-06	.9996011359510E-06

TOTAL 31480 63320 52.94(seconds)

16 Relaxed Gradient Algorithm: Extended Three Expo Terms **Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	22	181	0	.1279633348329E+03	.8243936848729E-06
200	22	181	11	.2559266696658E+03	.8243936848729E-06
300	22	181	11	.3838900044987E+03	.8243936848729E-06
400	22	181	17	.5118533393317E+03	.8243936848729E-06
500	22	181	22	.6398166741646E+03	.8243936848729E-06
600	22	181	22	.7677800089975E+03	.8243936848729E-06
700	22	181	27	.8957433438304E+03	.8243936848729E-06
800	22	181	33	.1023706678663E+04	.8243936848729E-06
900	22	181	39	.1151670013496E+04	.8243936848729E-06
1000	22	181	38	.1279633348329E+04	.8243936848729E-06

TOTAL 220 1810 2.20(seconds)

18 Relaxed Gradient Algorithm: Diagonal 4 **Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	145	2351	17	.1283238334742E-12	.6551679562218E-06
200	145	2351	38	.2566476669485E-12	.6551679562218E-06
300	145	2351	55	.3849715004227E-12	.6551679562218E-06
400	145	2351	77	.5132953338970E-12	.6551679562218E-06
500	145	2351	93	.6416191673712E-12	.6551679562218E-06
600	145	2351	110	.7699430008454E-12	.6551679562218E-06
700	145	2351	126	.8982668343197E-12	.6551679562218E-06
800	145	2351	154	.1026590667794E-11	.6551679562218E-06
900	145	2351	164	.1154914501268E-11	.6551679562218E-06
1000	145	2351	187	.1283238334742E-11	.6551679562218E-06

TOTAL 1450 23510 10.21(seconds)

19 Relaxed Gradient Algorithm: Diagonal 5 **Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	11	25	6	.6931471805600E+02	.2599123568038E-06
200	11	25	0	.1386294361120E+03	.2599123568038E-06
300	11	25	5	.2079441541680E+03	.2599123568038E-06
400	11	25	0	.2772588722240E+03	.2599123568038E-06
500	11	25	6	.3465735902800E+03	.2599123568038E-06
600	11	25	5	.4158883083360E+03	.2599123568038E-06
700	11	25	11	.4852030263920E+03	.2599123568038E-06
800	11	25	6	.5545177444480E+03	.2599123568038E-06
900	11	25	11	.6238324625040E+03	.2599123568038E-06
1000	11	25	5	.6931471805600E+03	.2599123568038E-06

TOTAL 110 250 .55(seconds)

20 Relaxed Gradient Algorithm: Extended Himmelblau					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	21	430	6	.2670596951858E-12	.8753150978472E-06
200	21	430	5	.5341193903716E-12	.8753150978472E-06
300	21	430	11	.8011790855573E-12	.8753150978472E-06
400	21	430	17	.1068238780743E-11	.8753150978472E-06
500	21	430	16	.1335298475929E-11	.8753150978472E-06
600	21	430	22	.1602358171115E-11	.8753150978472E-06
700	21	430	27	.1869417866300E-11	.8753150978472E-06
800	21	430	28	.2136477561486E-11	.8753150978472E-06
900	21	430	33	.2403537256672E-11	.8753150978472E-06
1000	21	430	33	.2670596951858E-11	.8753150978472E-06
TOTAL	210	4300	1.98(seconds)		

22 Relaxed Gradient Algorithm: Extended PSC1					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	32	217	11	.3865995282465E+02	.4463467021543E-06
200	32	217	22	.7731990564930E+02	.4463467021543E-06
300	32	217	33	.1159798584739E+03	.4463467021543E-06
400	32	217	44	.1546398112986E+03	.4463467021543E-06
500	32	217	55	.1932997641232E+03	.4463467021543E-06
600	32	217	66	.2319597169479E+03	.4463467021543E-06
700	32	217	77	.2706196697725E+03	.4463467021543E-06
800	32	217	87	.3092796225972E+03	.4463467021543E-06
900	32	217	99	.3479395754218E+03	.4463467021543E-06
1000	32	217	110	.3865995282465E+03	.4463467021543E-06
TOTAL	320	2170	6.04(seconds)		

24 Relaxed Gradient Algorithm: Extended Block-Diagonal BD1					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	37	311	5	.1571964783001E-10	.8759500100396E-06
200	37	311	6	.3143929566002E-10	.8759500100396E-06
300	37	311	16	.4715894349002E-10	.8759500100396E-06
400	37	311	17	.6287859132003E-10	.8759500100396E-06
500	37	311	22	.7859823915004E-10	.8759500100396E-06
600	37	311	27	.9431788698005E-10	.8759500100396E-06
700	37	311	27	.1100375348101E-09	.8759500100396E-06
800	37	311	39	.1257571826401E-09	.8759500100396E-06
900	37	311	38	.1414768304701E-09	.8759500100396E-06
1000	37	311	44	.1571964783001E-09	.8759500100396E-06
TOTAL	370	3110	2.41(seconds)		

25 Relaxed Gradient Algorithm: Extended Maratos					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	608	15210	138	-.5003121103481E+02	.9748197237091E-06
200	608	15210	269	-.1000624220696E+03	.9748197237091E-06
300	602	15051	406	-.1500936331044E+03	.9977394428216E-06
400	602	15051	538	-.2001248441392E+03	.9977394428216E-06
500	602	15051	671	-.2501560551741E+03	.9977394428216E-06
600	616	15398	823	-.3001872662089E+03	.8621030779531E-06
700	616	15398	961	-.3502184772437E+03	.8621030779531E-06
800	616	15398	1099	-.4002496882785E+03	.8621030779531E-06
900	616	15398	1241	-.4502808993134E+03	.8621030779531E-06
1000	616	15398	1373	-.5003121103482E+03	.8621030779531E-06
TOTAL	6102	152563	75.19(seconds)		

27 Relaxed Gradient Algorithm: Quadratic Diagonal Perturbed Function

n	iter	fgcnt	time(c)	fxnew	ginf
100	2433	46047	401	.5683539221834E-10	.9355639178406E-06
200	3756	82685	1434	.6379378203105E-10	.9894884940244E-06
300	4602	109932	2856	.6429316130629E-10	.9951091985126E-06
400	4829	120626	4174	.5675493835486E-10	.8841271999552E-06
500	5759	150192	6492	.5501750298831E-10	.8801731908348E-06
600	6135	164934	8558	.7375444332293E-10	.9939404690747E-06
700	6732	185874	11248	.4920729450620E-10	.9014769086355E-06
800	7895	222438	15396	.5756872587763E-10	.8655304977737E-06
900	7775	223424	17433	.6909435732739E-10	.9945131537332E-06
1000	8313	242423	21042	.6125910779920E-10	.8902012241019E-06
TOTAL	58229	1548575	890.34(seconds)		

28 Relaxed Gradient Algorithm: Extended Wood Function

n	iter	fgcnt	time(c)	fxnew	ginf
100	1677	44770	1055	.3517420435220E-10	.9896717421376E-06
200	1677	44770	2131	.7034840870440E-10	.9896717421376E-06
300	1677	44770	3164	.1055226130566E-09	.9896717421376E-06
400	1677	44770	4218	.1406968174088E-09	.9896717421376E-06
500	1677	44770	5273	.1758710217610E-09	.9896717421376E-06
600	1677	44770	6333	.2110452261132E-09	.9896717421376E-06
700	1677	44770	7480	.2462194304654E-09	.9896717421376E-06
800	1677	44770	8437	.2813936348176E-09	.9896717421376E-06
900	1677	44770	9491	.3165678391698E-09	.9896717421376E-06
1000	1677	44770	10540	.3517420435220E-09	.9896717421376E-06
TOTAL	16770	447700	581.22(seconds)		

29 Relaxed Gradient Algorithm: Extended Hiebert Function

n	iter	fgcnt	time(c)	fxnew	ginf
100	21996	734229	6042	.4116129628811E-09	.9849432541842E-06
200	21996	734229	12084	.8232259257623E-09	.9849432541842E-06
300	21996	734229	18164	.1234838888643E-08	.9849432541842E-06
400	21996	734229	24288	.1646451851525E-08	.9849432541842E-06
500	21996	734229	30214	.2058064814406E-08	.9849432541842E-06
600	21996	734229	36300	.2469677777287E-08	.9849432541842E-06
700	21996	734229	42408	.2881290740168E-08	.9849432541842E-06
800	21996	734229	48423	.3292903703049E-08	.9849432541842E-06
900	21996	734229	54606	.3704516665930E-08	.9849432541842E-06
1000	21996	734229	60490	.4116129628811E-08	.9849432541842E-06
TOTAL	219960	7342290	3330.19(seconds)		

30 Relaxed Gradient Algorithm: Quadratic QF1 Function

n	iter	fgcnt	time(c)	fxnew	ginf
100	178	2943	5	-.4999999999812E-02	.6100352179698E-06
200	268	5188	6	-.2499999999725E-02	.9487487144799E-06
300	406	8504	11	-.1666666666657E-02	.1804716502907E-06
400	512	11378	22	-.1249999999500E-02	.9998711701298E-06
500	485	11296	22	-.999999997053E-03	.7668445121092E-06
600	619	14809	44	-.833333328632E-03	.9695358933807E-06
700	537	13287	44	-.7142857138240E-03	.9884077837265E-06
800	573	14458	55	-.624999998906E-03	.4678165517201E-06
900	726	18774	77	-.555555555469E-03	.8567672855236E-06
1000	828	21725	98	-.499999999950E-03	.2949087916093E-06
TOTAL	5132	122362	3.84(seconds)		

31 Relaxed Gradient Algorithm: Extended Quadratic Penalty QP1 Function

n	iter	fgcnt	time(c)	fxnew	ginf
100	24	307	6	.3900625000000E+03	.7684480517054E-06
200	27	382	5	.7900312500000E+03	.6834550718105E-06
300	26	356	11	.1190020833333E+04	.9462301849410E-06
400	37	468	22	.1590015625000E+04	.2949967532791E-07
500	22	363	17	.1990012500000E+04	.1617378005969E-06
600	29	396	22	.2390010416667E+04	.9147227355197E-06
700	34	434	33	.2790008928571E+04	.2339321625300E-06
800	27	366	27	.3190007812500E+04	.2257109581888E-06
900	38	541	50	.3590006944444E+04	.9573594847203E-06
1000	44	682	66	.3990006250000E+04	.8618342122210E-06

TOTAL	308	4295	2.59(seconds)		

32 Relaxed Gradient Algorithm: Extended Quadratic Penalty QP2 Function

n	iter	fgcnt	time(c)	fxnew	ginf
100	1179	29657	703	.1593635297289E-10	.9533038770970E-06
200	431	10894	521	.4243947853435E-10	.9236105080739E-06
300	2199	55498	3928	.1345581384227E+02	.3631396993837E-05
400	3682	92494	8941	.2098686696678E+02	.5201150859255E-05
500	8012	200714	24245	.2129391846412E-10	.5982553829660E-06
600	1619	40861	5904	.5434386894715E-11	.6680761544533E-06
700	1176	29701	4954	.3166329193109E-10	.9786498366049E-06
800	706	17732	3378	.1094563282433E-10	.4205145387238E-06
900	822	20573	4416	.4474121571722E-10	.4461738573130E-06
1000	808	20191	4872	.8751652994889E-10	.8280534301824E-06

TOTAL	20634	518315	618.62(seconds)		

33 Relaxed Gradient Algorithm: Quadratic QF2 Function

n	iter	fgcnt	time(c)	fxnew	ginf
100	209	4695	105	-.1001246890523E+01	.9084932242254E-06
200	311	7912	346	-.1000624220697E+01	.8725786186936E-06
300	311	8467	549	-.1000416320022E+01	.7385187583952E-06
400	635	18031	1527	-.1000312304931E+01	.6935357172280E-06
500	583	17228	1823	-.1000249875125E+01	.9859908065568E-06
600	714	21597	2714	-.1000208246600E+01	.6256538924915E-06
700	583	17861	2685	.9998213647503E+00	.7586051218311E-06
800	779	24528	4125	-.1000156201202E+01	.7245345495647E-06
900	667	21348	4059	-.1000138850330E+01	.9861976331737E-06
1000	750	24405	5130	-.1000124968766E+01	.9532667438368E-06

TOTAL	5542	166072	230.63(seconds)		

34 Relaxed Gradient Algorithm: Extended EP1 Function

n	iter	fgcnt	time(c)	fxnew	ginf
100	16	459	0	.7931762881473E+03	.2057261242021E-06
200	16	459	11	.1586352576295E+04	.2057261242021E-06
300	16	459	6	.2379528864442E+04	.2057261242021E-06
400	16	459	16	.3172705152589E+04	.2057261242021E-06
500	16	459	11	.3965881440736E+04	.2057261242021E-06
600	16	459	22	.4759057728884E+04	.2057261242021E-06
700	16	459	22	.5552234017031E+04	.2057261242021E-06
800	16	459	22	.6345410305178E+04	.2057261242021E-06
900	16	459	28	.7138586593325E+04	.2057261242021E-06
1000	16	459	33	.7931762881473E+04	.2057261242021E-06

TOTAL	160	4590	1.71(seconds)		

35 Relaxed Gradient Algorithm: Extended Tridiagonal 2 **Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	104	492	5	.3858318323642E+02	.9692380972259E-06
200	103	494	6	.7755609559644E+02	.9484809239957E-06
300	105	508	11	.1165290079564E+03	.9731504643229E-06
400	101	486	22	.1555019203164E+03	.9056559935605E-06
500	94	421	16	.1944748326764E+03	.9539864084807E-06
600	100	470	22	.2334477450364E+03	.9809112179182E-06
700	96	418	27	.2724206573964E+03	.9080302800224E-06
800	94	403	28	.3113935697565E+03	.8849382056308E-06
900	99	452	33	.3503664821165E+03	.9806144362356E-06
1000	92	395	33	.3893393944765E+03	.9451169927788E-06
TOTAL	988	4539	2.03(seconds)		

36 Relaxed Gradient Algorithm: BDQRTIC (CUTE) **Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	776	23129	1192	.3787691918087E+03	.8903592806793E-06
200	2786	160335	16752	.7793301650051E+03	.8457634706739E-06
300	1088	38023	6020	.1179891138202E+04	.1039560637080E-04
400	852	30494	6465	.1580452111398E+04	.2822572753854E-04
500	1235	45589	12078	.1981013084595E+04	.1091334499721E-03
600	1487	58670	18746	.2381574057791E+04	.1348428116599E-05
700	5300	335478	124631	.2782135030987E+04	.2111205978972E-04
800	1837	71928	30681	.3182696004184E+04	.2069584152275E-06
900	1319	52143	25052	.3583256977380E+04	.3866979396919E-04
1000	1568	62735	33455	.3983817950577E+04	.1209887215390E-03
TOTAL	18248	878524	2750.72(seconds)		

37 Relaxed Gradient Algorithm: TRIDIA (CUTE) **Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	750	21471	181	.4602456067653E-13	.9592622543662E-06
200	833	26367	445	.2054884192665E-12	.6376388324014E-06
300	1376	46341	1181	.3420095477107E-12	.9956667456889E-06
400	2074	72601	2466	.5071317773673E-12	.9997190908440E-06
500	2180	78321	3329	.1731917427616E-12	.5841999106693E-06
600	2598	95398	4861	.4690502092992E-12	.9615153326870E-06
700	2862	107055	6371	.3648582803076E-12	.8479229252067E-06
800	2304	87516	5959	.1203787817058E-14	.5748813298283E-06
900	2276	87948	6740	.1778072147312E-12	.5910041331880E-06
1000	3135	122291	10496	.4311555208766E-12	.9216731677864E-06
TOTAL	20388	745309	420.29(seconds)		

38 Relaxed Gradient Algorithm: ARWHEAD (CUTE) **Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	100	2317	121	.3891467226583E-12	.3058517785703E-06
200	184	4773	522	.7869924330273E-12	.9404776126908E-06
300	190	5322	873	.5150288507304E-11	.6420803355667E-06
400	137	3999	868	.9954525159901E-11	.7709700229341E-06
500	247	7313	1994	.1050492483799E-10	.7085736015711E-06
600	229	7020	2301	.1610149405300E-11	.2545488486660E-06
700	294	9104	3482	.2584369242047E-11	.8199819669595E-06
800	306	9717	4246	.9622004473568E-11	.9774225332807E-06
900	329	10640	5229	.3584840865996E-11	.8983041363469E-06
1000	182	5921	3246	.3526577955441E-11	.8748488223143E-06
TOTAL	2198	66126	228.82(seconds)		

39 Relaxed Gradient Algorithm: NONDIA (CUTE)					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	667	26609	868	.1336011647012E-13	.7180010795835E-06
200	1380	59051	3905	.1002635852269E-10	.6838396288993E-06
300	2340	104471	10299	.3308812273429E-10	.7683870052430E-06
400	2614	119817	15697	.2946212934974E-10	.5464372132553E-06
500	4563	213276	35581	.6171740402956E-10	.6557032042544E-06
600	3787	180406	35542	.3863301687848E-10	.4155605491939E-06
700	4355	209943	49021	.3308699288689E-10	.3903611855623E-06
800	9673	472569	126192	.3533082648229E-09	.9929003832271E-06
900	6939	342362	103029	.3953499463452E-09	.8877902822113E-06
1000	9337	464892	155148	.2470917269536E-09	.6334072522977E-06
TOTAL	45655	2193396	5352.82(seconds)		

41 Relaxed Gradient Algorithm: DQDRTIC (CUTE)					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	334	7458	170	.2472148429812E-12	.9320203135775E-06
200	222	4949	231	.1282764772209E-12	.7160158955344E-06
300	325	7212	510	.2650194768595E-12	.8442506839548E-06
400	282	6267	583	.2524109180873E-12	.9927292880236E-06
500	348	7792	911	.2510393101434E-12	.9917243855729E-06
600	411	9145	1286	.2377072614376E-12	.9649389535361E-06
700	363	8040	1318	.2853531089237E-12	.9850586361024E-06
800	322	7161	1346	.2805166162663E-12	.6165040733006E-06
900	383	8564	1812	.1616521687316E-12	.3644542406714E-06
1000	325	7286	1709	.2486246981263E-12	.9669422112913E-06
TOTAL	3315	73874	98.76(seconds)		

43 Relaxed Gradient Algorithm: DIXMAANA (CUTE)					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	16	64	6	.1000000000001E+01	.2488190189113E-06
200	16	64	11	.1000000000001E+01	.2488190189113E-06
300	16	64	16	.1000000000002E+01	.2488190189113E-06
400	16	64	17	.1000000000002E+01	.2488190189113E-06
500	16	64	27	.1000000000003E+01	.2488190189113E-06
600	16	64	33	.1000000000004E+01	.2488190189113E-06
700	16	64	39	.1000000000004E+01	.2488190189113E-06
800	16	64	38	.1000000000005E+01	.2488190189113E-06
900	16	64	50	.1000000000005E+01	.2488190189113E-06
1000	16	64	49	.1000000000006E+01	.2488190189113E-06
TOTAL	160	640	2.86(seconds)		

44 Relaxed Gradient Algorithm: DIXMAANB (CUTE)					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	15	61	6	.1000000000002E+01	.8183376190092E-06
200	15	61	11	.1000000000004E+01	.8183440697829E-06
300	15	61	16	.1000000000006E+01	.8177302570701E-06
400	15	61	17	.1000000000007E+01	.8183376190092E-06
500	15	61	27	.1000000000009E+01	.8183440697829E-06
600	15	61	27	.1000000000011E+01	.8177302570701E-06
700	15	61	39	.1000000000013E+01	.8183376190092E-06
800	15	61	38	.1000000000015E+01	.8183440697829E-06
900	15	61	44	.1000000000017E+01	.8177302570701E-06
1000	15	61	50	.1000000000018E+01	.8183376190092E-06
TOTAL	150	610	2.75(seconds)		

45 Relaxed Gradient Algorithm: DIXMAANC (CUTE)					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	16	79	5	.1000000000001E+01	.3813382265849E-06
200	16	79	11	.1000000000002E+01	.4044138761262E-06
300	16	79	16	.1000000000002E+01	.4421814694780E-06
400	16	79	28	.1000000000003E+01	.3813382265849E-06
500	16	79	27	.1000000000004E+01	.4044138761262E-06
600	16	79	44	.1000000000005E+01	.4421814694780E-06
700	16	79	44	.1000000000005E+01	.3813382265849E-06
800	16	79	49	.1000000000006E+01	.4044138761262E-06
900	16	79	55	.1000000000007E+01	.4421814694780E-06
1000	16	79	66	.1000000000008E+01	.3813382265849E-06
TOTAL	160	790	3.45(seconds)		

46 Relaxed Gradient Algorithm: DIXMAANE (CUTE)					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	760	1538	176	.1000000000025E+01	.9988225202797E-06
200	1368	2754	665	.1000000000050E+01	.9997479114023E-06
300	1938	3894	1422	.1000000000074E+01	.9956360205531E-06
400	2472	4962	2400	.1000000000100E+01	.9985314328138E-06
500	2953	5924	3587	.1000000000124E+01	.9975609559123E-06
600	3468	6954	5081	.1000000000150E+01	.9988697495822E-06
700	3939	7896	6794	.1000000000175E+01	.9986592086537E-06
800	4391	8800	8733	.1000000000200E+01	.9989654229407E-06
900	4845	9708	10952	.1000000000225E+01	.9991812827197E-06
1000	5239	10496	13265	.1000000000250E+01	.9997264750610E-06
TOTAL	31373	62926	530.75(seconds)		

47 Relaxed Gradient Algorithm: Partial Perturbed Quadratic					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	179	3505	50	.2131815045961E-12	.9671139479499E-06
200	227	5293	291	.7446984886837E-13	.6714715114956E-06
300	342	8921	1076	.2351314687451E-13	.2086389813398E-06
400	567	16152	3449	.4388289374595E-12	.9525016634819E-06
500	617	18623	6224	.1204213836316E-12	.7596089346621E-06
600	449	14136	6766	.1872297525045E-12	.8692863941362E-06
700	864	28419	18439	.2568357189396E-12	.9349884799446E-06
800	690	23464	19828	.1146278808188E-12	.7404077727076E-06
900	659	23065	24662	.9913307037616E-13	.7630436749172E-06
1000	1063	38147	50536	.2488258476242E-12	.7436709236698E-06
TOTAL	5657	179725	1313.21(seconds)		

48 Relaxed Gradient Algorithm: Broyden Tridiagonal					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	138	2458	0	.7125279095866E+00	.9798117837079E-06
200	126	2270	6	.7125279095861E+00	.8072863417752E-06
300	174	3161	11	.7125279095864E+00	.8395257989902E-06
400	172	3049	11	.3970671034880E+00	.9553695614617E-06
500	165	3003	16	.3970671034880E+00	.9924827925056E-06
600	165	3003	17	.3970671034880E+00	.9924827925056E-06
700	165	3003	22	.3970671034880E+00	.9924827925056E-06
800	170	3085	27	.3970671034876E+00	.8248922599718E-06
900	170	3085	28	.3970671034876E+00	.8248922599718E-06
1000	171	3125	33	.3970671034876E+00	.6308532221996E-06
TOTAL	1616	29242	1.71(seconds)		

49 Relaxed Gradient Algorithm: Almost Perturbed Quadratic Function

n	iter	fgcnt	time(c)	fxnew	ginf
100	222	4281	5	.4001260337372E-15	.3819234130920E-07
200	339	7546	6	.2419505345127E-12	.9884613035813E-06
300	410	9904	22	.4097173213673E-13	.3933290996210E-06
400	525	13309	33	.2008122222320E-12	.9001393948390E-06
500	557	14644	44	.1096596552478E-12	.7578948234002E-06
600	481	13064	43	.6131066804714E-13	.4969009099427E-06
700	517	14411	61	.1743266518952E-12	.8345951079082E-06
800	598	16898	82	.6579930763471E-13	.9852033185198E-06
900	714	20696	105	.1282280079901E-12	.9784903634994E-06
1000	744	21779	131	.1738795115144E-12	.8381303009210E-06
TOTAL	5107	136532	5.32(seconds)		

50 Relaxed Gradient Algorithm: Tridiagonal Perturbed Quadratic Function

n	iter	fgcnt	time(c)	fxnew	ginf
100	198	3925	33	.1241239228072E-12	.8178390410906E-06
200	236	5221	88	.1455394093427E-12	.7560967198924E-06
300	387	9361	236	.1135214791286E-12	.7901074528936E-06
400	452	11446	390	.1758434909952E-12	.7987946824753E-06
500	437	11539	495	.1375328921673E-12	.6970966599343E-06
600	582	15846	807	.2415226116975E-12	.9774329868189E-06
700	690	19131	1143	.1816966339419E-12	.8906235793126E-06
800	538	15134	1032	.1883659277019E-12	.8546603025601E-06
900	965	27874	2444	.1835915805671E-12	.8561504364798E-06
1000	627	18353	1566	.7936386581727E-13	.5612140939011E-06
TOTAL	5112	137830	82.34(seconds)		

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