

# Numerical Experiments with Gradient Descent with Backtracking for Unconstrained Optimization

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In this work we present the numerical results with GRAD package which implements the *gradient descent algorithm* with backtracking. A number of 360 unconstrained optimization problems have been considered in this experiment. The Fortran code of GRAD is authored by Andrei.

The problems are placed at the web page: [www.ici.ro/camo/neculai/ansoft](http://www.ici.ro/camo/neculai/ansoft). You should go to this address and hit RELGRAD.

The criteria for stopping the iterations are:

$$\|\nabla f(x_k)\|_\infty \leq \varepsilon_g \quad \text{or} \quad t_k |g_k^T d_k| \leq \varepsilon_f |f(x_{k+1})|,$$

where  $\varepsilon_g = 10^{-6}$  and  $\varepsilon_f = 10^{-20}$ .

The code implements the Armijo line search in a backtracking manner with  $\alpha = 0.0001$  and  $\beta = 0.8$ .

The following tables give the results of the optimization process, where:

- n = dimension of the problem (number of variables),
- iter = number of iterations,
- fgcnt = number of function and its gradient evaluations,
- time(c) = cpu time in centeseconds
- fxnew = function value in optimal point,  $f(x^*)$ ,
- ginf = infinite norm of the gradient in optimal point,  $\|\nabla f(x^*)\|_\infty$ .

1 Gradient Algorithm: Extended Freudenstein & Roth				Function	
n	iter	fgcnt	time(c)	fxnew	ginf
100	11026	394407	5020	.1846648617071E-11	.9696454569623E-06
200	11026	394407	10079	.3693297234142E-11	.9696454569623E-06
300	11026	394407	15154	.5539945851212E-11	.9696454569623E-06
400	11026	394407	20207	.7386594468283E-11	.9696454569623E-06
500	11026	394407	25293	.9233243085354E-11	.9696454569623E-06
600	11026	394407	30396	.1107989170242E-10	.9696454569623E-06
700	11026	394407	35383	.1292654031950E-10	.9696454569623E-06
800	11026	394407	40540	.1477318893657E-10	.9696454569623E-06
900	11026	394407	45544	.1661983755364E-10	.9696454569623E-06
1000	11026	394407	50691	.1846648617071E-10	.9696454569623E-06
TOTAL	110260	3944070	2783.07 (seconds)		

<b>2 Gradient Algorithm: Extended Trigonometric</b>					<b>Function</b>
n	iter	fgcnt	time(c)	fxnew	ginf
100	45	448	22	.6618017280692E-12	.9174737240008E-06
200	69	598	44	.3404403367038E-11	.9790162799803E-06
300	304	1608	181	.3420266212925E-06	.8188542046528E-06
400	64	877	138	.1845817333740E-10	.9208949468352E-06
500	327	1786	340	.4415948689526E-07	.9684472826057E-06
600	73	1056	242	.8249855036381E-11	.8292179369130E-06
700	70	1202	318	.1320620432382E-10	.8921693250321E-06
800	80	1337	401	.1810843988958E-10	.9978685039894E-06
900	95	1505	517	.3983902334059E-11	.8991589341347E-06
1000	62	1052	401	.3769258056565E-11	.7052261092665E-06
TOTAL	1189	11469	26.04(seconds)		

<b>3 Gradient Algorithm: Extended Rosenbrock</b>					<b>Function</b>
n	iter	fgcnt	time(c)	fxnew	ginf
100	18778	566825	11029	.1854336188778E-10	.9975958267143E-06
200	18778	566825	22074	.3708672377556E-10	.9975958267143E-06
300	18778	566825	33159	.5563008566333E-10	.9975958267143E-06
400	18778	566825	44242	.7417344755111E-10	.9975958267143E-06
500	18778	566825	55250	.9271680943889E-10	.9975958267143E-06
600	18778	566825	66416	.1112601713267E-09	.9975958267143E-06
700	18778	566825	77549	.1298035332144E-09	.9975958267143E-06
800	18778	566825	88457	.1483468951022E-09	.9975958267143E-06
900	18778	566825	99460	.1668902569900E-09	.9975958267143E-06
1000	18778	566825	110773	.1854336188778E-09	.9975958267143E-06
TOTAL	187780	5668250	6084.09(seconds)		

<b>4 Gradient Algorithm: Extended White &amp; Holst</b>					<b>Function</b>
n	iter	fgcnt	time(c)	fxnew	ginf
100	25001	812538	16071	.1276257720789E-04	.7514981707990E-03
200	25001	812538	31995	.2552515441579E-04	.7514981707990E-03
300	25001	812538	48125	.3828773162368E-04	.7514981707990E-03
400	25001	812538	64060	.5105030883157E-04	.7514981707990E-03
500	25001	812538	80224	.6381288603947E-04	.7514981707990E-03
600	25001	812538	96169	.7657546324736E-04	.7514981707990E-03
700	25001	812538	112246	.8933804045526E-04	.7514981707990E-03
800	25001	812538	128339	.1021006176631E-03	.7514981707990E-03
900	25001	812538	144289	.1148631948710E-03	.7514981707990E-03
1000	25001	812538	160273	.1276257720789E-03	.7514981707990E-03
TOTAL	250010	8125380	8817.91(seconds)		

<b>5 Gradient Algorithm: Extended Beale</b>					<b>Function</b>
n	iter	fgcnt	time(c)	fxnew	ginf
100	973	15732	148	.1204420960859E-10	.9321646606083E-06
200	973	15732	285	.2408841921717E-10	.9321646606083E-06
300	973	15732	434	.3613262882576E-10	.9321646606083E-06
400	973	15732	577	.4817683843434E-10	.9321646606083E-06
500	973	15732	720	.6022104804293E-10	.9321646606083E-06
600	973	15732	862	.7226525765152E-10	.9321646606083E-06
700	973	15732	1011	.8430946726010E-10	.9321646606083E-06
800	973	15732	1153	.9635367686869E-10	.9321646606083E-06
900	973	15732	1302	.1083978864773E-09	.9321646606083E-06
1000	973	15732	1439	.1204420960859E-09	.9321646606083E-06
TOTAL	9730	157320	79.31(seconds)		

6 Gradient Algorithm: Extended Penalty					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	128	2584	44	.7500000000000E+02	.9085836880640E-06
200	157	3385	104	.1602796621681E+03	.9577533843785E-06
300	87	2378	115	.2478925765033E+03	.4595402645080E-06
400	187	4252	275	.3368146699141E+03	.9240776237540E-06
500	80	2696	214	.4266194324846E+03	.9905206159733E-06
600	97	2876	275	.5170771098472E+03	.1941084566415E-06
700	123	3344	379	.6080459723100E+03	.7829060841846E-06
800	250	5625	736	.6994307914492E+03	.3716105501095E-06
900	78	2798	406	.7911636782044E+03	.6421381253396E-06
1000	83	3076	495	.8831940750670E+03	.7566089852418E-06
TOTAL	1270	33014	30.43(seconds)		

7 Gradient Algorithm: Perturbed Quadratic					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	716	16257	137	.5030478740984E-13	.9746113311382E-06
200	1430	36857	632	.5286020729337E-13	.9990302720600E-06
300	2145	59133	1521	.4711307737665E-13	.9890491757629E-06
400	2865	82696	2834	.5300995319434E-13	.9788266137540E-06
500	3574	106717	4576	.5556551843965E-13	.9770450322270E-06
600	4289	131508	6810	.4777848763460E-13	.9820733312368E-06
700	5063	158708	9524	.3328995780443E-13	.9326472562905E-06
800	5741	183522	12589	.5533468601076E-13	.9989949935964E-06
900	6447	209325	16159	.4261074249560E-13	.9819312668431E-06
1000	7159	235991	20257	.5881213319954E-13	.9811213651388E-06
TOTAL	39429	1220714	750.39(seconds)		

8 Gradient Algorithm: Raydan 1					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	603	5585	66	.5050000000000E+03	.8269267907229E-06
200	25001	307376	7321	.2010000000000E+04	.3445657002181E-05
300	25001	352793	12705	.4515000000000E+04	.7855544611395E-05
400	25001	384973	18273	.8020000000000E+04	.8990070468728E-05
500	25001	409972	24316	.1252500000000E+05	.1377393058475E-04
600	25001	430549	30769	.1803000000000E+05	.2272846744697E-04
700	25001	448215	37322	.2453500000000E+05	.2181942316012E-04
800	25001	462638	43924	.3204000000000E+05	.2235370829007E-04
900	25001	475260	51037	.4054500000000E+05	.4309441982539E-04
1000	25001	487637	57875	.5005000000000E+05	.4102737918597E-04
TOTAL	225612	3764998	2836.08(seconds)		

9 Gradient Algorithm: Raydan 2					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	4	11	0	.1000000000000E+03	.1899389957919E-07
200	4	11	0	.2000000000000E+03	.1899389957919E-07
300	4	11	0	.3000000000000E+03	.1899389957919E-07
400	4	11	0	.4000000000000E+03	.1899389957919E-07
500	4	11	6	.5000000000000E+03	.1899389957919E-07
600	4	11	0	.6000000000000E+03	.1899389957919E-07
700	4	11	0	.7000000000000E+03	.1899389957919E-07
800	4	11	0	.8000000000000E+03	.1899389957919E-07
900	4	11	0	.9000000000000E+03	.1899389957919E-07
1000	4	11	5	.1000000000000E+04	.1899389957919E-07
TOTAL	40	110	.11(seconds)		

**11 Gradient Algorithm: Diagonal 2****Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	912	1827	22	.1574135370147E+02	.9983393347542E-06
200	1695	3393	88	.1985453540073E+02	.9963716702061E-06
300	2425	4853	181	.2248591724537E+02	.9980202281652E-06
400	3121	6245	313	.2445342429119E+02	.9989426975453E-06
500	3792	7587	472	.2603689737073E+02	.9990008269751E-06
600	4443	8889	676	.2736784662031E+02	.9988466053557E-06
700	5077	10157	895	.2851919946320E+02	.9991296843234E-06
800	5697	11397	1148	.2953583055503E+02	.9989483053794E-06
900	6304	12611	1417	.3044741583192E+02	.9993488194126E-06
1000	6900	13803	1730	.3127464993159E+02	.9995550848393E-06
TOTAL	40366	80762	69.42(seconds)		

**12 Gradient Algorithm: Diagonal 3****Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	631	12476	280	-.4605795020600E+04	.4094366621150E-06
200	3376	76542	3466	-.1918247539294E+05	.2296096642779E-06
300	1908	47500	3230	-.4375601954925E+05	.1715263323047E-04
400	2116	55347	5014	-.7832825022511E+05	.7615396522341E-05
500	3025	81224	9118	-.1228997540954E+06	.3938661824938E-04
600	3130	86700	11567	-.1774707956624E+06	.8944688008280E-05
700	3777	107849	16692	-.2420415170304E+06	.1594740685436E-04
800	3689	106491	18790	-.3166120034280E+06	.6681105259683E-06
900	25001	729519	144070	-.4011823100174E+06	.1250663548010E-03
1000	4354	130013	28369	-.4957524745606E+06	.6843624528016E-06
TOTAL	51007	1433661	2405.96(seconds)		

**13 Gradient Algorithm: Hager****Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	69	676	11	-.6530786727331E+03	.7501207840467E-06
200	137	1525	60	-.2493201505568E+04	.3141457253705E-06
300	112	1345	82	-.5276871910453E+04	.6553510234850E-06
400	124	1643	132	-.8886479558416E+04	.4311245538381E-05
500	160	2160	214	-.1324635151502E+05	.1447803211055E-05
600	151	2118	248	-.1830180119599E+05	.9407463080764E-05
700	150	2134	296	-.2401061200239E+05	.1723970246090E-05
800	212	3039	483	-.3033875790101E+05	.3300028067440E-05
900	170	2624	473	-.3725796386792E+05	.1202237935884E-04
1000	170	2539	505	-.4474419132154E+05	.1374255224082E-04
TOTAL	1455	19803	25.04(seconds)		

**14 Gradient Algorithm: Generalized Tridiagonal 1****Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	63	1045	33	.9721030748599E+02	.8906935597963E-06
200	63	1045	66	.1972103074860E+03	.8906935597963E-06
300	69	1151	105	.2972103074860E+03	.8780649518592E-06
400	71	1185	148	.3972103074860E+03	.8230689019229E-06
500	71	1185	181	.4972103074860E+03	.8230689019229E-06
600	69	1156	214	.5972103074860E+03	.3804819039832E-06
700	69	1153	247	.6972103074860E+03	.5557665501144E-06
800	69	1153	286	.7972103074860E+03	.5557665501144E-06
900	69	1153	318	.8972103074860E+03	.5557665501144E-06
1000	69	1153	357	.9972103074860E+03	.5557665501144E-06
TOTAL	682	11379	19.55(seconds)		

15 Gradient Algorithm: Extended Tridiagonal 1					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	9344	47638	791	.8875024037377E-08	.9994046134655E-06
200	9344	47638	1577	.1775004807475E-07	.9994046134655E-06
300	9344	47638	2367	.2662507211213E-07	.9994046134655E-06
400	9344	47638	3158	.3550009614951E-07	.9994046134655E-06
500	9344	47638	3949	.4437512018689E-07	.9994046134655E-06
600	9344	47638	4740	.5325014422426E-07	.9994046134655E-06
700	9344	47638	5542	.6212516826164E-07	.9994046134655E-06
800	9344	47638	6333	.7100019229902E-07	.9994046134655E-06
900	9344	47638	7129	.7987521633640E-07	.9994046134655E-06
1000	9344	47638	7926	.8875024037377E-07	.9994046134655E-06
TOTAL	93440	476380	435.12(seconds)		

16 Gradient Algorithm: Extended Three Expo Terms					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	197	1986	44	.1279633348329E+03	.9748710043667E-06
200	197	1986	88	.2559266696658E+03	.9748710043667E-06
300	197	1986	132	.3838900044987E+03	.9748710043667E-06
400	197	1986	170	.5118533393317E+03	.9748710043667E-06
500	197	1986	220	.6398166741646E+03	.9748710043667E-06
600	197	1986	264	.7677800089975E+03	.9748710043667E-06
700	197	1986	302	.8957433438304E+03	.9748710043667E-06
800	197	1986	346	.1023706678663E+04	.9748710043667E-06
900	197	1986	396	.1151670013496E+04	.9748710043667E-06
1000	197	1986	434	.1279633348329E+04	.9748710043667E-06
TOTAL	1970	19860	23.96(seconds)		

18 Gradient Algorithm: Diagonal 4					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	727	14241	115	.4459476613499E-11	.9783704423160E-06
200	727	14241	225	.8918953226997E-11	.9783704423160E-06
300	727	14241	346	.1337842984050E-10	.9783704423160E-06
400	727	14241	456	.1783790645399E-10	.9783704423160E-06
500	727	14241	566	.2229738306749E-10	.9783704423160E-06
600	727	14241	687	.2675685968099E-10	.9783704423160E-06
700	727	14241	796	.3121633629449E-10	.9783704423160E-06
800	727	14241	917	.3567581290799E-10	.9783704423160E-06
900	727	14241	1027	.4013528952149E-10	.9783704423160E-06
1000	727	14241	1143	.4459476613499E-10	.9783704423160E-06
TOTAL	7270	142410	62.78(seconds)		

19 Gradient Algorithm: Diagonal 5					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	2	7	0	.6931471805600E+02	.2153657512743E-06
200	2	7	0	.1386294361120E+03	.2153657512743E-06
300	2	7	0	.2079441541680E+03	.2153657512743E-06
400	2	7	6	.2772588722240E+03	.2153657512743E-06
500	2	7	0	.3465735902800E+03	.2153657512743E-06
600	2	7	0	.4158883083360E+03	.2153657512743E-06
700	2	7	5	.4852030263920E+03	.2153657512743E-06
800	2	7	0	.5545177444480E+03	.2153657512743E-06
900	2	7	0	.6238324625040E+03	.2153657512743E-06
1000	2	7	6	.6931471805600E+03	.2153657512743E-06
TOTAL	20	70	.17(seconds)		

20 Gradient Algorithm: Extended Himmelblau					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	110	2105	17	.3517743009580E-12	.9941131315617E-06
200	110	2105	33	.7035486019161E-12	.9941131315617E-06
300	110	2105	55	.1055322902874E-11	.9941131315617E-06
400	110	2105	66	.1407097203832E-11	.9941131315617E-06
500	110	2105	88	.1758871504790E-11	.9941131315617E-06
600	110	2105	104	.2110645805748E-11	.9941131315617E-06
700	110	2105	126	.2462420106706E-11	.9941131315617E-06
800	110	2105	138	.2814194407664E-11	.9941131315617E-06
900	110	2105	153	.3165968708622E-11	.9941131315617E-06
1000	110	2105	176	.3517743009580E-11	.9941131315617E-06

TOTAL 1100 21050 9.56(seconds)

22 Gradient Algorithm: Extended PSC1					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	128	935	50	.3865995282465E+02	.9177406540273E-06
200	128	935	93	.7731990564931E+02	.9177406540273E-06
300	128	935	143	.1159798584740E+03	.9177406540273E-06
400	128	935	186	.1546398112986E+03	.9177406540273E-06
500	128	935	242	.1932997641233E+03	.9177406540273E-06
600	128	935	286	.2319597169479E+03	.9177406540273E-06
700	128	935	329	.2706196697726E+03	.9177406540273E-06
800	128	935	379	.3092796225972E+03	.9177406540273E-06
900	128	935	429	.3479395754219E+03	.9177406540273E-06
1000	128	935	478	.3865995282465E+03	.9177406540273E-06

TOTAL 1280 9350 26.15(seconds)

24 Gradient Algorithm: Extended Block-Diagonal BD1					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	98	1180	16	.1140641819101E-11	.8821350535615E-06
200	98	1180	33	.2281283638202E-11	.8821350535615E-06
300	98	1180	50	.3421925457303E-11	.8821350535615E-06
400	98	1180	66	.4562567276404E-11	.8821350535615E-06
500	98	1180	87	.5703209095505E-11	.8821350535615E-06
600	98	1180	99	.6843850914606E-11	.8821350535615E-06
700	98	1180	116	.7984492733707E-11	.8821350535615E-06
800	98	1180	131	.9125134552808E-11	.8821350535615E-06
900	98	1180	154	.1026577637191E-10	.8821350535615E-06
1000	98	1180	165	.1140641819101E-10	.8821350535615E-06

TOTAL 980 11800 9.17(seconds)

25 Gradient Algorithm: Extended Maratos					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	4987	143964	1291	-.5003121103483E+02	.9653968546797E-06
200	5084	146763	2636	-.1000624220697E+03	.9425877456337E-06
300	5084	146763	3955	-.1500936331045E+03	.9425864990753E-06
400	5084	146763	5273	-.2001248441393E+03	.9425864990753E-06
500	5084	146763	6585	-.2501560551742E+03	.9425864990753E-06
600	5187	149735	8068	-.3001872662090E+03	.9716160418449E-06
700	5187	149735	9415	-.3502184772438E+03	.9716160418449E-06
800	5187	149735	10847	-.4002496882787E+03	.9716160418449E-06
900	5187	149735	12111	-.4502808993135E+03	.9716160418449E-06
1000	5187	149735	13512	-.5003121103484E+03	.9716160418449E-06

TOTAL 51258 1479691 736.93(seconds)

27 Gradient Algorithm: Quadratic Diagonal Perturbed					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	11054	250286	2159	.4848065076104E-10	.9989280896840E-06
200	22980	591753	10227	.3458755222521E-10	.9753617254219E-06
300	25001	688992	17856	.1439693285690E-09	.2081273106908E-05
400	25001	721609	24898	.4037552605184E-09	.3583609811410E-05
500	25001	746610	32280	.7782659366383E-09	.4994147408980E-05
600	25001	766746	39650	.1244068562339E-08	.5910924266960E-05
700	25001	783909	47401	.1820628317523E-08	.6642134272391E-05
800	25001	799505	55134	.2569862905753E-08	.8026855501683E-05
900	25001	812047	63121	.3272518669092E-08	.8330726027622E-05
1000	25001	824506	71161	.4266775620768E-08	.1010065355192E-04
TOTAL	234042	6985963	3638.87(seconds)		

28 Gradient Algorithm: Extended Wood					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	10756	325030	7596	.6707268250726E-11	.9782397666918E-06
200	10756	325030	15198	.1341453650145E-10	.9782397666918E-06
300	10756	325030	22810	.2012180475218E-10	.9782397666918E-06
400	10756	325030	30473	.2682907300290E-10	.9782397666918E-06
500	10756	325030	38019	.3353634125363E-10	.9782397666918E-06
600	10756	325030	45720	.4024360950436E-10	.9782397666918E-06
700	10756	325030	53349	.4695087775508E-10	.9782397666918E-06
800	10756	325030	60918	.5365814600581E-10	.9782397666918E-06
900	10756	325030	68487	.6036541425654E-10	.9782397666918E-06
1000	10756	325030	76176	.6707268250726E-10	.9782397666918E-06
TOTAL	107560	3250300	4187.46(seconds)		

30 Gradient Algorithm: Quadratic QF1					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	727	14241	17	-.4999999999911E-02	.9685867381481E-06
200	1452	32901	82	-.2499999999908E-02	.9421179957281E-06
300	2186	53465	379	-.16666666666587E-02	.9963016044390E-06
400	2900	74672	1412	-.1249999999900E-02	.9889823148201E-06
500	3615	96681	3054	-.9999999998927E-03	.9805778689032E-06
600	4350	119846	4981	-.8333333332426E-03	.9771232374334E-06
700	5170	146018	6965	-.7142857142287E-03	.9735163679636E-06
800	5799	167305	14324	-.6249999998918E-03	.9686949822285E-06
900	6571	192940	15462	-.5555555554809E-03	.9704373264974E-06
1000	7238	216041	26815	-.4999999998875E-03	.9781215514204E-06
TOTAL	40008	1114110	734.91(seconds)		

31 Gradient Algorithm: Extended Quadratic Penalty QP1					Function
n	iter	fgcnt	time(c)	fxnew	ginf
100	48	659	6	.3900625000000E+03	.7371897439157E-06
200	51	731	16	.7900312500000E+03	.7830601712611E-06
300	50	723	22	.1190020833333E+04	.4207875088369E-06
400	59	899	33	.1590015625000E+04	.4054211873994E-06
500	54	817	44	.1990012500000E+04	.4955384760308E-06
600	54	856	50	.2390010416667E+04	.4573505709704E-06
700	49	719	49	.2790008928571E+04	.1057079346833E-06
800	61	868	71	.3190007812500E+04	.7735522132846E-06
900	57	877	77	.3590006944444E+04	.9322434779135E-06
1000	60	887	88	.3990006250000E+04	.6405762654760E-06
TOTAL	543	8036	4.56(seconds)		

**33 Gradient Algorithm: Quadratic QF2****Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	823	21258	451	-.1001246890523E+01	.9342908389165E-06
200	1639	47356	2010	-.1000624220697E+01	.9758265328008E-06
300	2459	75446	4767	-.1000416320022E+01	.9912708709425E-06
400	3338	106747	9063	-.1000312304931E+01	.9850881123041E-06
500	4180	137833	14632	-.1000249875125E+01	.9965439530246E-06
600	4922	166242	21070	-.1000208246600E+01	.9230380553227E-06
700	5770	198783	29775	-.1000178507699E+01	.9689859243878E-06
800	6957	243909	41040	.9998437011413E+00	.9581568118922E-06
900	7348	261381	49944	.9998610725094E+00	.9612676463711E-06
1000	8765	316058	66613	.9998749687344E+00	.8048233034463E-06

TOTAL 46201 1575013 2393.65(seconds)

**34 Gradient Algorithm: Extended EPl****Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	25001	659321	4356	.7931762881473E+03	.1963173289952E-05
200	25001	659322	8728	.1586352576295E+04	.2013977706949E-05
300	25001	659322	13105	.2379528864442E+04	.2013977706949E-05
400	25001	659322	17417	.3172705152589E+04	.2013977706949E-05
500	25001	659322	21805	.3965881440736E+04	.2013977706949E-05
600	25001	659322	26189	.4759057728884E+04	.2013977706949E-05
700	25001	659322	30538	.5552234017031E+04	.2013977706949E-05
800	25001	659322	34773	.6345410305178E+04	.2013977706949E-05
900	25001	659322	38937	.7138586593325E+04	.2013977706949E-05
1000	25001	659322	43287	.7931762881473E+04	.2013977706949E-05

TOTAL 250010 6593219 2391.35(seconds)

**35 Gradient Algorithm: Extended Tridiagonal 2****Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	115	908	6	.3858318323643E+02	.9929589871727E-06
200	115	909	16	.7755609559644E+02	.9770416392052E-06
300	115	909	22	.1165290079564E+03	.9770416367627E-06
400	115	909	33	.1555019203165E+03	.9770416061206E-06
500	115	909	39	.1944748326765E+03	.9770415416166E-06
600	115	909	44	.2334477450365E+03	.9770415605181E-06
700	115	909	55	.2724206573965E+03	.9770415139720E-06
800	115	910	60	.3113935697565E+03	.9839592834682E-06
900	115	910	72	.3503664821165E+03	.9839594087013E-06
1000	115	910	76	.3893393944765E+03	.9839594050653E-06

TOTAL 1150 9092 4.23(seconds)

**41 Gradient Algorithm: DQDRTIC (CUTE)****Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	1558	40194	923	.2554719783949E-12	.9991461594240E-06
200	1558	40194	1862	.2616164746678E-12	.9991461594240E-06
300	1559	40221	2807	.2523378617739E-12	.9920742948975E-06
400	1559	40221	3751	.2544611760053E-12	.9920742948975E-06
500	1559	40221	4691	.2565844902366E-12	.9920742948975E-06
600	1559	40222	5635	.2526483196803E-12	.9944447494851E-06
700	1559	40222	6575	.2535574186292E-12	.9944447494851E-06
800	1559	40222	7524	.2544665175781E-12	.9944447494851E-06
900	1559	40222	8470	.2553756165270E-12	.9944447494851E-06
1000	1559	40222	9409	.2562847154760E-12	.9944447494851E-06

TOTAL 15588 402161 516.47(seconds)



43 Gradient Algorithm: DIXMAANA (CUTE)				Function	
n	iter	fgcnt	time(c)	fxnew	ginf
100	48	164	11	.1000000000008E+01	.7178955169162E-06
200	48	164	27	.1000000000016E+01	.7178955169162E-06
300	48	164	38	.1000000000024E+01	.7178955169162E-06
400	48	164	55	.1000000000032E+01	.7178955169162E-06
500	48	164	66	.1000000000040E+01	.7178955169162E-06
600	48	164	83	.1000000000048E+01	.7178955169162E-06
700	48	164	93	.1000000000057E+01	.7178955169162E-06
800	48	164	104	.1000000000065E+01	.7178955169162E-06
900	48	164	121	.1000000000073E+01	.7178955169162E-06
1000	48	164	132	.1000000000081E+01	.7178955169162E-06
TOTAL	480	1640	7.30(seconds)		

44 Gradient Algorithm: DIXMAANB (CUTE)				Function	
n	iter	fgcnt	time(c)	fxnew	ginf
100	46	206	16	.1000000000002E+01	.7010157076320E-06
200	46	206	33	.1000000000006E+01	.7010157076320E-06
300	46	206	49	.1000000000010E+01	.7010157076320E-06
400	46	206	66	.1000000000014E+01	.7010157076320E-06
500	46	206	83	.1000000000018E+01	.7010157076320E-06
600	46	206	99	.1000000000022E+01	.7010157076320E-06
700	46	206	115	.1000000000026E+01	.7010157076320E-06
800	46	206	132	.1000000000030E+01	.7010157076320E-06
900	46	206	148	.1000000000035E+01	.7010157076320E-06
1000	46	206	164	.1000000000039E+01	.7010157076320E-06
TOTAL	460	2060	9.05(seconds)		

45 Gradient Algorithm: DIXMAANC (CUTE)				Function	
n	iter	fgcnt	time(c)	fxnew	ginf
100	40	163	11	.1000000000009E+01	.8966322783867E-06
200	40	163	28	.1000000000018E+01	.8966322788534E-06
300	40	163	38	.1000000000027E+01	.8943464870911E-06
400	40	163	55	.1000000000036E+01	.8966322783867E-06
500	40	163	66	.1000000000045E+01	.8966322788534E-06
600	42	168	82	.1000000000032E+01	.7042946818264E-06
700	42	168	94	.1000000000037E+01	.7060947114649E-06
800	42	168	104	.1000000000042E+01	.7060947118324E-06
900	42	168	121	.1000000000047E+01	.7042946818264E-06
1000	42	168	131	.1000000000053E+01	.7060947114649E-06
TOTAL	410	1655	7.30(seconds)		

46 Gradient Algorithm: DIXMAANE (CUTE)				Function	
n	iter	fgcnt	time(c)	fxnew	ginf
100	715	1507	165	.1000000000000E+01	.9930254824913E-06
200	1365	2843	604	.1000000000000E+01	.7285558767972E-06
300	1843	3849	1263	.1000000000000E+01	.7692397971517E-06
400	2571	5274	2499	.1000000000000E+01	.8985429829013E-06
500	3160	6463	4054	.1000000000000E+01	.6985837937585E-06
600	3450	7067	5426	.1000000000000E+01	.8066437327997E-06
700	4360	9782	8959	.1000000000000E+01	.6244430901084E-06
800	4865	9873	11056	.1000000000000E+01	.9291099087366E-06
900	4974	10498	13111	.1000000000001E+01	.9012390307397E-06
1000	5825	11962	17334	.1000000000000E+01	.9906928913347E-06
TOTAL	33128	69118	644.71(seconds)		

**48 Gradient Algorithm: Broyden Tridiagonal****Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	553	11916	11	.3970671034881E+00	.9811087653411E-06
200	553	11917	22	.3970671034880E+00	.9876014548382E-06
300	553	11917	38	.3970671034880E+00	.9876406161791E-06
400	418	9011	33	.1985771489360E+01	.9959766484879E-06
500	418	9011	44	.1985771489360E+01	.9959766484879E-06
600	420	9053	55	.1985771489360E+01	.9820269536442E-06
700	420	9053	66	.1985771489360E+01	.9821480068117E-06
800	420	9053	77	.1985771489360E+01	.9821480068117E-06
900	420	9053	82	.1985771489360E+01	.9821480068117E-06
1000	953	20504	209	.2790664991066E+01	.9889522222806E-06
TOTAL	5128	110488	6.37(seconds)		

**49 Gradient Algorithm: Almost Perturbed Quadratic****Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	721	16370	17	.4474391436491E-13	.9434740731331E-06
200	1437	37037	110	.5049718504909E-13	.9595135769904E-06
300	2160	59546	456	.4261460871709E-13	.9218269413054E-06
400	2870	82840	1724	.5619323338421E-13	.9786745946300E-06
500	3585	107045	3757	.5691117358226E-13	.9834246274556E-06
600	4294	131661	5960	.5186273365366E-13	.9931198472078E-06
700	5064	158739	8090	.3728046496746E-13	.9994364602683E-06
800	5758	184065	16884	.5731054709119E-13	.9907379921752E-06
900	6465	209909	18043	.4440380268381E-13	.9899272894370E-06
1000	7196	237210	31236	.5738349310429E-13	.9893064105535E-06
TOTAL	39550	1224422	862.77(seconds)		

**50 Gradient Algorithm: Tridiagonal Perturbed Quadratic****Function**

n	iter	fgcnt	time(c)	fxnew	ginf
100	751	17222	143	.1478851885358E-12	.9394211314714E-06
200	1446	37438	632	.1609303999892E-12	.9895608942467E-06
300	2139	59129	1505	.1452127225029E-12	.9915972566034E-06
400	2849	82404	3861	.1574251088118E-12	.9855326609442E-06
500	3539	105841	7838	.1672164096072E-12	.9695612922951E-06
600	4225	129709	13341	.1529738450347E-12	.9737969695195E-06
700	4970	155947	22970	.1108164787154E-12	.9648121532072E-06
800	5644	180593	28050	.1764946984787E-12	.9959428203062E-06
900	6331	205716	33362	.1328611950340E-12	.9832462634875E-06
1000	7066	233097	39360	.1591928943690E-12	.9924902365437E-06
TOTAL	38960	1207096	1510.62(seconds)		

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