Numerical experiments with DESCON for solving 14 applications of unconstrained optimization

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This technical report presents the numerical performances of DESCON for solving the following applications of unconstrained optimization:

1. Weber Function [1, pp. 58]

$$f(x) = 2\sqrt{(x_1-2)^2+(x_2-42)^2} + 4\sqrt{(x_1-90)^2+(x_2-11)^2} + 5\sqrt{(x_1-43)^2+(x_2-88)^2}.$$

2. Enzyme reaction [1, pp. 62]

$$f(x) = \sum_{i=1}^{11}\left(y_i - \frac{x_1(u_i^2+u_i x_2)}{u_i^2+u_i x_3+x_4}\right)^2,$$

where y_i and u_i have the following values:

i	y_i	u_i	i	y_i	u_i
1	0.1957	4.000	7	0.0456	0.125
2	0.1947	2.000	8	0.0342	0.100
3	0.1735	1.000	9	0.0323	0.0833
4	0.1600	0.500	10	0.0235	0.0714
5	0.0844	0.250	11	0.0246	0.0625
6	0.0627	0.167			

3. Solution of a chemical reactor [2, pp. 1455-1481]

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$$
\begin{aligned}
f(x) = {} & (1 - x_1 - k_1 x_1 x_6 + r_1 x_4)^2 \\
& + (1 - x_2 - k_2 x_2 x_6 + r_2 x_5)^2 \\
& + (-x_3 + 2k_3 x_4 x_5)^2 \\
& + (k_1 x_1 x_6 - r_1 x_4 - k_3 x_4 x_5)^2 \\
& + (1{,}5(k_2 x_2 x_6 - r_2 x_5) - k_3 x_4 x_5)^2 \\
& + (1 - x_4 - x_5 - x_6)^2
\end{aligned}
$$

where: $k_1 = 31{,}24 \quad k_2 = 0{,}272 \quad k_3 = 303{,}03 \quad r_1 = 2{,}062 \quad r_2 = 0{,}02$.

4. Robot kinematics problem [3, pp. 152-157], [4, pp. 101-103], [5, pp. 329-331]

$$
\begin{aligned}
f(x) = {} & (4{,}731 \cdot 10^{-3} x_1 x_3 - 0{,}3578 x_2 x_3 - \\
& \quad 0{,}1238 x_1 + x_7 - 1{,}637 \cdot 10^{-3} x_2 - 0{,}9338 x_4 - 0{,}3571)^2 \\
& + (0{,}2238 x_1 x_3 + 0{,}7623 x_2 x_3 \\
& \quad + 0{,}2638 x_1 - x_7 - 0{,}07745 x_2 - 0{,}6734 x_4 - 0{,}6022)^2 \\
& + (x_6 x_8 + 0{,}3578 x_1 + 4{,}731 \cdot 10^{-3} x_2)^2 \\
& + (-0{,}7623 x_1 + 0{,}2238 x_2 + 0{,}3461)^2 \\
& + (x_1^2 + x_2^2 - 1)^2 \\
& + (x_3^2 + x_4^2 - 1)^2 \\
& + (x_5^2 + x_6^2 - 1)^2 \\
& + (x_7^2 + x_8^2 - 1)^2 .
\end{aligned}
$$

5. Solar Spectroscopy [1, p. 68]

$$
f(x) = \sum_{i=1}^{13} \left(x_1 + x_2 \exp\left(-\frac{(i + x_3)^2}{x_4} \right) - y_i \right)^2 ,
$$

where $y_i, i = 1, \ldots, 13$ are as in the below table

i	y_i	i	y_i
1	0.5	8	2.5
2	0.8	9	1.6
3	1	10	1.3
4	1.4	11	0.7
5	2	12	0.4
6	2.4	13	0.3
7	2.7		

6. Estimation of parameters [6, p. 430]

$$f(x)=\sum_{i=1}^{7}\left(\frac{x_1^2+a_i x_2^2+a_i^2 x_3^2}{(1+a_i x_4^2)b_i}-1\right)^2,$$

where the parameters $a_i, b_i,\ \ i=1,\ldots,7$ have the following values:

i	a_i	b_i
1	0.0	7.391
2	0.000428	11.18
3	0.0010	16.44
4	0.00161	16.20
5	0.00209	22.20
6	0.00348	24.02
7	0.00525	31.32

7. Propan combustion in air [7, p. 143-151], [8, pp.18-19], [4, pp. 54-56], [5, p. 327]

$$\begin{aligned}
f(x)=&\left(x_1x_2+x_1-3x_5\right)^2+\\
&\left(2x_1x_2+x_1+2R_{10}x_2^2+x_2x_3^2+R_7x_2x_3+R_9x_2x_4+R_8x_2-Rx_5\right)^2+\\
&\left(2x_2x_3^2+R_7x_2x_3+2R_5x_3^2+R_6x_3-8x_5\right)^2+\\
&\left(R_9x_2x_4+2x_4^2-4Rx_5\right)^2+\\
&\left(x_1x_2+x_1+R_{10}x_2^2+x_2x_3^2+R_7x_2x_3+R_9x_2x_4+R_8x_2+R_5x_3^2+R_6x_3+x_4^2-1\right)^2
\end{aligned}$$

where:

$R_5=0{,}193$ $\quad R_6=0.4106217541E-3$ $\quad R_7=0.5451766686E-3$

$R_8=0.44975E-6$ $\quad R_9=0.3407354178E-4$ $\quad R_{10}=0.9615E-6$

$R=10$

8. **Gear train with minimum inertia** [9], [10, Problem 328, p. 149]

$$f(x)=0.1(12+x_1^2+(1+x_2^2)/x_1^2+(x_1^2x_2^2+100)/x_1^4x_2^4).$$

9. Human Heart Dipole. [1, p. 65], [8, p. 17], [4, pp. 51-54], [11, pp. 817-823]

$$\begin{aligned}
f(x)=&\left(x_1+x_2-s_{mx}\right)^2+\\
&\left(x_3+x_4-s_{my}\right)^2+\\
&\left(x_1x_5+x_2x_6-x_3x_7-x_4x_8-s_A\right)^2+\\
&\left(x_1x_7+x_2x_8+x_3x_5+x_4x_6-s_B\right)^2+\\
&\left(x_1(x_5^2-x_7^2)-2x_3x_5x_7+x_2(x_6^2-x_8^2)-2x_4x_6x_8-s_C\right)^2+\\
&\left(x_3(x_5^2-x_7^2)+2x_1x_5x_7+x_4(x_6^2-x_8^2)+2x_2x_6x_8-s_D\right)^2+
\end{aligned}$$

$$\left(x_1x_5(x_5^2-3x_7^2)+x_3x_7(x_7^2-3x_5^2)+x_2x_6(x_6^2-3x_8^2)+x_4x_8(x_8^2-3x_6^2)-s_E\right)^2+$$
$$\left(x_3x_5(x_5^2-3x_7^2)-x_1x_7(x_7^2-3x_5^2)+x_4x_6(x_6^2-3x_8^2)-x_2x_8(x_8^2-3x_6^2)-s_F\right)^2$$

where:

$s_{mx}=0{,}485$ $\quad s_A=-0{,}0581$ $\quad s_C=0{,}105$ $\quad s_E=0{,}167$

$s_{my}=-0{,}0019$ $\quad s_B=0{,}015$ $\quad s_D=0{,}0406$ $\quad s_F=-0{,}399$.

10. Neurophysiology [4, pp. 57-61], [12, pp. 915-930]

$$\begin{aligned}f(x)=&(x_1^2+x_3^2-1)^2+(x_2^2+x_4^2-1)^2\\&+(x_5x_3^3+x_6x_4^3-1)^2+(x_5x_1^3+x_6x_2^3-2)^2\\&+(x_5x_1x_3^2+x_6x_2x_4^2-1)^2+(x_5x_3x_1^2+x_6x_4x_2^2-4)^2.\end{aligned}$$

11. Combustion application [17], [18, pp. 61-63]

$$\begin{aligned}f(x)=&(x_2+2x_6+x_9+2x_{10}-10^{-5})^2+\\&(x_3+x_8-3\cdot10^{-5})^2+\\&(x_1+x_3+2x_5+2x_8+x_9+x_{10}-5\cdot10^{-5})^2+\\&(x_4+2x_7-10^{-5})^2+\\&(0.5140437\cdot10^{-7}x_5-x_1^2)^2+\\&(0.1006932\cdot10^{-6}x_6-2x_2^2)^2+\\&(0.7816278\cdot10^{-15}x_7-x_4^2)^2+\\&(0.1496236\cdot10^{-6}x_8-x_1x_3)^2+\\&(0.6194411\cdot10^{-7}x_9-x_1x_2)^2+\\&(0.2089296\cdot10^{-14}x_{10}-x_1x_2^2)^2.\end{aligned}$$

12. Thermistor [13, pp.722-723]

$$f(x)=\sum_{i=1}^{16}\left(y_i-x_1\exp\left(\frac{x_2}{45+5i+x_3}\right)\right)^2$$

where

i	y_i	i	y_i
1	34780	9	8261
2	28610	10	7030
3	23650	11	6005
4	19630	12	5147
5	16370	13	4427
6	13720	14	3820
7	11540	15	3307
8	9744	16	2872

13. Optimal design of a Gear Train [14, pp. 95-105], [4, p. 79]

$$f(x)=\left(\frac{1}{6.931}-\frac{x_1x_2}{x_3x_4}\right)^2.$$

14. Circuit design [15, p. 501], [13, pp.243-244], [16, pp.367-370]

$$f(x)=(x_1x_3-x_2x_4)^2+\sum_{k=1}^{4}(a_k^2+b_k^2),$$

where

$$\begin{aligned}
a_k &= (1-x_1x_2)x_3\left\{\exp\left[x_5(g_{1k}-g_{3k}x_7\cdot 10^{-3}-g_{5k}x_8\cdot 10^{-3})\right]-1\right\}\\
&\quad +g_{4k}x_2-g_{5k},\quad k=1,\ldots,4,\\
b_k &= (1-x_1x_2)x_4\left\{\exp\left[x_6(g_{1k}-g_{2k}-g_{3k}x_7\cdot 10^{-3}-g_{4k}x_9\cdot 10^{-3})\right]-1\right\}\\
&\quad +g_{4k}-g_{5k}x_1,\quad k=1,\ldots,4,
\end{aligned}$$

$$g=\begin{bmatrix} 0.4850 & 0.7520 & 0.8690 & 0.9820\\ 0.3690 & 1.2540 & 0.7030 & 1.4550\\ 5.2095 & 10.0677 & 22.9274 & 20.2153\\ 23.3037 & 101.7790 & 111.4610 & 191.2670\\ 28.5132 & 111.8467 & 134.3884 & 211.4823 \end{bmatrix},$$

Solution of these applications, given by DESCON, are as follows:

```
   ***************************************************
   * DESCON                                      ***
   * Another Accelerated Conjugate Gradient with   ***
   * Guaranteed Descent and Conjugacy conditions   ***
   * --------------------------------------------- ***
   * Project: FCGA                               ***
   * The Fastest Conjugate Gradient Algorithm      ***
   *                                             ***
   * Dr. Neculai Andrei                          ***
   * Research Institute for Informatics            ***
   * Bucharest - Romania                         ***
   ***************************************************

   Parameters: Descent= 0.8750000000000E+00    Conjugacy= 0.5000000000000E-01

   Date: ---  Month: 6  Day: 3  Year: 2020

   DESCON. Powell restart.

   DESCON. variant with acceleration.

   ib = number of iterations in which beta=0
   it = number of iterations in which theta=1
-------------------------------------------------------------------------------------------------

     1    DESCON Function:1. Weber Function (Andrei, U71)
          stoptest= 1

      n   iter   irs  fgcnt lscnt   time(c)        fxnew              gnorm           ib    it
-------------------------------------------------------------------------------------------------
      2   1878  1785  10001  1873         0-0.2644531414650E+03 0.8208740831576E+00 1683  1683
-------------------------------------------------------------------------------------------------
  TOTAL   1878  1785  10001  1873      0.00 (seconds)    proc=  95.05%

      1      0.9000000000000E+02
      2      0.1100000000000E+02

                                   f(x0) = -0.3747313737140E+02
                                   f(x*) = -0.2644531414650E+03
```

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    2    DESCON Function:2. Enzyme reaction  (Andrei, U79) (A)
         stoptest= 1

     n   iter   irs  fgcnt lscnt   time(c)        fxnew             gnorm          ib    it
---------------------------------------------------------------------------------------------
     4     48    12    143    42         1 0.3075056038514E-03 0.6886760990097E-08    1     1
---------------------------------------------------------------------------------------------
  TOTAL     48    12    143    42      0.01 (seconds)    proc=  25.00%

     1      0.1928069272220E+00
     2      0.1912816426491E+00
     3      0.1230559555238E+00
     4      0.1360620711174E+00

                               f(x0) =  0.5313172272109E-02
                               f(x*) =  0.3075056038514E-03

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    3    DESCON Function:3. Solution of a chemical reactor (A)
         stoptest= 1

     n   iter   irs  fgcnt lscnt   time(c)        fxnew             gnorm          ib    it
---------------------------------------------------------------------------------------------
     6     85    21    264    82         0 0.9665994663683E-15 0.4231231612938E-07    1     1
---------------------------------------------------------------------------------------------
  TOTAL     85    21    264    82      0.00 (seconds)    proc=  24.71%

     1      0.1030427701246E+01
     2      0.1020285140411E+01
     3     -0.6085536173458E-01
     4     -0.1003059545714E-03
     5      0.1001051960999E+01
     6     -0.9516617348223E-03

                               f(x0) =  0.1961733675060E+08
                               f(x*) =  0.9665994663683E-15

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    4    DESCON Function:4. Robot kinematics problem (A)
         stoptest= 1

     n   iter   irs  fgcnt lscnt   time(c)        fxnew             gnorm          ib    it
---------------------------------------------------------------------------------------------
     8   1843  1837  10006  1843         1 0.5463981044793E-05 0.1781782618260E-02 1835  1835
---------------------------------------------------------------------------------------------
  TOTAL   1843  1837  10006  1843      0.01 (seconds)    proc=  99.67%

     1      0.1645311866901E+00
     2     -0.9859963001813E+00
     3     -0.9464279151135E+00
     4     -0.3218090482133E+00
     5     -0.9982311597268E+00
     6      0.5941082491929E-01
     7      0.4089584958004E+00
     8     -0.9124474122066E+00

                               f(x0) =  0.1069004717719E-01
                               f(x*) =  0.5463981044793E-05

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    5    DESCON Function:5. Solar Spectroscopy (A)
         stoptest= 1

     n   iter   irs  fgcnt lscnt   time(c)        fxnew             gnorm          ib    it
---------------------------------------------------------------------------------------------
     4     12     5     38    11         1 0.8312307692553E+01 0.8585722387648E-07    0     0
---------------------------------------------------------------------------------------------
  TOTAL     12     5     38    11      0.01 (seconds)    proc=  41.67%

     1      0.1353846157041E+01
     2      0.9009710377047E+00
```

```
      3      0.1490308027760E+01
      4      0.2749133874479E+00

                                  f(x0) =  0.9958700480657E+01
                                  f(x*) =  0.8312307692553E+01

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     6    DESCON Function:6. Estimation of parameters (A)
          stoptest= 1

      n   iter   irs  fgcnt lscnt   time(c)        fxnew              gnorm           ib    it
-----------------------------------------------------------------------------------------------
      4     46    14    150    38         0 0.3185717881375E-01 0.6936429307668E-08    0     0
-----------------------------------------------------------------------------------------------
  TOTAL     46    14    150    38      0.00 (seconds)    proc=  30.43%

      1      0.2714380870286E+01
      2      0.1403920454259E+03
      3      0.1706253406595E+04
      4      0.3149541622005E+02

                                  f(x0) =  0.2905300235663E+01
                                  f(x*) =  0.3185717881375E-01

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     7    DESCON Function:7. Propan combustion in air (A)
          stoptest= 1

      n   iter   irs  fgcnt lscnt   time(c)        fxnew              gnorm           ib    it
-----------------------------------------------------------------------------------------------
      5    724    96   2246   638         0 0.1224151943762E-06 0.8166044868424E-07    0     0
-----------------------------------------------------------------------------------------------
  TOTAL    724    96   2246   638      0.00 (seconds)    proc=  13.26%

      1      0.2991238843895E-02
      2      0.3619141085805E+02
      3     -0.6390900043838E-01
      4      0.8597065757677E+00
      5      0.3698126569313E-01

                                  f(x0) =  0.3312269269234E+08
                                  f(x*) =  0.1224151943762E-06

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     8    DESCON Function:8. Gear train with minimum inertia (A)
          stoptest= 1

      n   iter   irs  fgcnt lscnt   time(c)        fxnew              gnorm           ib    it
-----------------------------------------------------------------------------------------------
      2     14    10    154    13         0 0.1751192213346E+01 0.7760986494009E-07    0     0
-----------------------------------------------------------------------------------------------
  TOTAL     14    10    154    13      0.00 (seconds)    proc=  71.43%

      1      0.1624580581149E+01
      2      0.2076167371521E+01

                                  f(x0) =  0.2563325000000E+04
                                  f(x*) =  0.1751192213346E+01

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     9    DESCON Function:9. Human Heart Dipole. Andrei U84, pp.65
          stoptest= 1

      n   iter   irs  fgcnt lscnt   time(c)        fxnew              gnorm           ib    it
-----------------------------------------------------------------------------------------------
      8   1916   616  10002  1878         2 0.1120571259805E-01 0.1686188462847E-03    0     0
-----------------------------------------------------------------------------------------------
  TOTAL   1916   616  10002  1878      0.02 (seconds)    proc=  32.15%

      1      0.4819570059723E+00
      2      0.5871119384741E-04
      3     -0.3626351208397E-02
      4     -0.9150411651716E-04
```

```
     5     -0.1854011414573E+00
     6      0.1581555729485E+02
     7     -0.1186591332395E-01
     8     -0.8057330241911E+00

                                f(x0) =  0.1905692553768E+00
                                f(x*) =  0.1120571259805E-01

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    10    DESCON Function:10. Neurophysiology (A)
          stoptest= 1

     n   iter   irs  fgcnt lscnt   time(c)        fxnew             gnorm          ib    it
---------------------------------------------------------------------------------------------
     6     93    77    632    92         0 0.4539057615171E+01 0.4484463269794E-07    0     0
---------------------------------------------------------------------------------------------
  TOTAL     93    77    632    92      0.00 (seconds)    proc=  82.80%

     1     -0.7862416508770E+00
     2     -0.7862416508770E+00
     3     -0.6179190986623E+00
     4     -0.6179190986623E+00
     5     -0.2875512435197E+01
     6     -0.2875512435197E+01

                                f(x0) =  0.2399999199998E+02
                                f(x*) =  0.4539057615171E+01

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    11    DESCON Function:11. Combustion application (A)
          stoptest= 1

     n   iter   irs  fgcnt lscnt   time(c)        fxnew             gnorm          ib    it
---------------------------------------------------------------------------------------------
    10     51     9    142    39         0 0.6898812079492E-10 0.8219103504792E-07    0     0
---------------------------------------------------------------------------------------------
  TOTAL     51     9    142    39      0.00 (seconds)    proc=  17.65%

     1      0.8874504900626E-07
     2     -0.1316651516732E-02
     3      0.1505497783061E+00
     4      0.2748793273342E-02
     5      0.1267056825669E+00
     6      0.1802818644441E+00
     7     -0.1369400788225E-02
     8     -0.1505197758118E+00
     9      0.1534937104895E+00
    10     -0.2563653927753E+00

                                f(x0) =  0.1219988990749E+03
                                f(x*) =  0.6898812079492E-10

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    12    DESCON Function:12. Thermistor (A)
          stoptest= 1

     n   iter   irs  fgcnt lscnt   time(c)        fxnew             gnorm          ib    it
---------------------------------------------------------------------------------------------
     3   1839  1824  10005  1839        10 0.1726024568705E+03 0.1334938231384E+02 1818  1818
---------------------------------------------------------------------------------------------
  TOTAL   1839  1824  10005  1839      0.10 (seconds)    proc=  99.18%

     1      0.6188737787853E-02
     2      0.6099708654159E+04
     3      0.3424663424626E+03

                                f(x0) =  0.2335910048036E+10
                                f(x*) =  0.1726024568705E+03

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    13    DESCON Function:13. Optimal design of a Gear Train (A)
          stoptest= 1
```

```
      n   iter   irs  fgcnt lscnt   time(c)        fxnew              gnorm           ib    it
-----------------------------------------------------------------------------------------------
      4   1842  1840  10004  1842         1 0.2387780742094E-02 0.8995056760928E-04 1838  1838
-----------------------------------------------------------------------------------------------
  TOTAL   1842  1840  10004  1842      0.01 (seconds)    proc=  99.89%

      1      0.1899575821949E+02
      2      0.1378341202816E+02
      3      0.3681847545387E+02
      4      0.3681847545387E+02

                                f(x0) =  0.1743308858486E-01
                                f(x*) =  0.2387780742094E-02

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    14    DESCON Function:14. Circuit design (A)
          stoptest= 1

      n   iter   irs  fgcnt lscnt   time(c)        fxnew              gnorm           ib    it
-----------------------------------------------------------------------------------------------
      9    739    37   2166   662         2 0.1454860731888E-13 0.1554041459749E-06    1     1
-----------------------------------------------------------------------------------------------
  TOTAL    739    37   2166   662      0.02 (seconds)    proc=   5.01%

      1      0.8999999528762E+00
      2      0.4499875500428E+00
      3      0.1000006568140E+01
      4      0.2000068529268E+01
      5      0.7999971269057E+01
      6      0.7999693548494E+01
      7      0.5000031053984E+01
      8      0.9999877276919E+00
      9      0.2000052299378E+01

                                f(x0) =  0.2964578187893E+04
                                f(x*) =  0.1454860731888E-13

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A synthesis of this numerical experiment is presented in Table 1, where *n* is the number of variables, *iter* is the number of iterations to get a solution, *fgcnt* is the number of function and its gradient evaluations, *time(c)* is the time in centeseconds, *fx** is the value of the minimizing function in optimal point and *gnorm* is the infinite norm of the gradient.

Table 1

Performances of DESCON

n	iter	fgcnt	time(c)	fx*	gnorm	Name of Application
2	1878	10001	0	-0.2644531414650E+03	0.8208740831576E+00	1. Weber Function (Andrei, U71)
4	48	143	0	0.3075056038514E-03	0.6886760990097E-08	2. Enzyme reaction (Andrei, U79) (A)
6	85	264	0	0.9665994663683E-15	0.4231231612938E-07	3. Solution of a chemical reactor (A)
8	1843	10006	2	0.5463981044793E-05	0.1781782618260E-02	4. Robot kinematics problem (A)
4	12	38	0	0.8312307692553E+01	0.8585722387648E-07	5. Solar Spectroscopy (A)
4	46	150	0	0.3185717881375E-01	0.6936429307668E-08	6. Estimation of parameters (A)
5	724	2246	0	0.1224151943762E-06	0.8166044868424E-07	7. Propan combustion in air (A)
2	14	154	0	0.1751192213346E+01	0.7760986494009E-07	8. Gear train with minimum inertia (A)
8	1916	10002	1	0.1120571259805E-01	0.1686188462847E-03	9. Human Heart Dipole. Andrei U84, pp.65
6	93	632	0	0.4539057615171E+01	0.4484463269794E-07	10. Neurophysiology (A)
10	51	142	0	0.6898812079492E-10	0.8219103504792E-07	11. Combustion application (A)
3	1839	10005	9	0.1726024568705E+03	0.1334938231384E+02	12. Thermistor (A)
4	1842	10004	0	0.2387780742094E-02	0.8995056760928E-04	13. Optimal design of a Gear Train (A)
9	739	2166	2	0.1454860731888E-13	0.1554041459749E-06	14. Circuit design (A)
TOTAL	11130	55953	14.00 centeseconds			

Date: --- Month: 6 Day: 3 Year: 2020

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