Unconstrained Optimization
Test Functions

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Extended Freudenstein & Roth Function

$$f(x)=\sum_{i=1}^{n/2}\left(-13+x_{2i-1}+((5-x_{2i})x_{2i}-2)x_{2i}\right)^2$$
$$+\left(-29+x_{2i-1}+((x_{2i}+1)x_{2i}-14)x_{2i}\right)^2,$$

$$x_0=[0.5,-2,0.5,-2,\ldots,0.5,-2].$$

Extended Trigonometric Function

$$f(x)=\sum_{i=1}^{n}\left(\left(n-\sum_{j=1}^{n}\cos x_j\right)+i(1-\cos x_i)-\sin x_i\right)^2,$$

$$x_0=[0.2,0.2,\ldots,0.2].$$

Extended Rosenbrock Function

$$f(x)=\sum_{i=1}^{n/2}c\left(x_{2i}-x_{2i-1}^2\right)^2+\left(1-x_{2i-1}\right)^2,$$

$$x_0=[-1.2,1,\ldots,-1.2,1]. \qquad c=100$$

Extended White & Holst Function

$$f(x)=\sum_{i=1}^{n/2}c\left(x_{2i}-x_{2i-1}^3\right)^2+\left(1-x_{2i-1}\right)^2,$$

$$x_0=[-1.2,1,\ldots,-1.2,1], \qquad c=100.$$

Extended Beale Function

$$f(x)=\sum_{i=1}^{n/2}\left(1.5-x_{2i-1}\left(1-x_{2i}\right)\right)^2+\left(2.25-x_{2i-1}\left(1-x_{2i}^2\right)\right)^2+\left(2.625-x_{2i-1}\left(1-x_{2i}^3\right)\right)^2,$$

$$x_0=[1,0.8,\ldots,1,0.8].$$

Extended Penalty Function

$$f(x)=\sum_{i=1}^{n-1}\left(x_i-1\right)^2+\left(\sum_{j=1}^{n}x_j^2-0.25\right)^2,$$

$$x_0=[1,2,\ldots,n].$$

Perturbed Quadratic Function

$$f(x)=\sum_{i=1}^{n}ix_i^2+\frac{1}{100}\left(\sum_{i=1}^{n}x_i\right)^2,$$

$$x_0=[0.5,0.5,\ldots,0.5]$$

Raydan 1 Function

$$f(x)=\sum_{i=1}^{n}\frac{i}{10}\left(\exp(x_i)-x_i\right),$$

$$x_0=[1,1,\ldots,1].$$

Raydan 2 Function

$$f(x)=\sum_{i=1}^{n}\left(\exp(x_i)-x_i\right),$$

$$x_0=[1,1,\ldots,1].$$

Diagonal1 Function

$$f(x)=\sum_{i=1}^{n}(\exp(x_i)-ix_i),$$

$$x_0=[1/n,1/n,\ldots,1/n].$$

Diagonal2 Function

$$f(x)=\sum_{i=1}^{n}\left(\exp(x_i)-\frac{x_i}{i}\right),$$

$$x_0=[1/1,1/2,\ldots,1/n].$$

Diagonal3 Function

$$f(x) = \sum_{i=1}^{n} \left(\exp(x_i) - i \sin(x_i) \right),$$

$$x_0 = [1,1,\ldots,1].$$

Hager Function

$$f(x) = \sum_{i=1}^{n} \left(\exp(x_i) - \sqrt{i} x_i \right),$$

$$x_0 = [1,1,\ldots,1].$$

Generalized Tridiagonal-1 Function

$$f(x) = \sum_{i=1}^{n-1} \left(x_i + x_{i+1} - 3 \right)^2 + \left(x_i - x_{i+1} + 1 \right)^4,$$

$$x_0 = [2,2,\ldots,2].$$

Extended Tridiagonal-1 Function

$$f(x) = \sum_{i=1}^{n/2} \left(x_{2i-1} + x_{2i} - 3 \right)^2 + \left(x_{2i-1} - x_{2i} + 1 \right)^4,$$

$$x_0 = [2,2,\ldots,2].$$

Extended Three Exponential Terms Function

$$f(x) = \sum_{i=1}^{n/2} \left(\exp(x_{2i-1} + 3x_{2i} - 0.1) + \exp(x_{2i-1} - 3x_{2i} - 0.1) + \exp(-x_{2i-1} - 0.1) \right),$$

$$x_0 = [0.1,0.1,\ldots,0.1].$$

Generalized Tridiagonal-2 Function

$$f(x) = \left((5 - 3x_1 - x_1^2) x_1 - 3x_2 + 1 \right)^2 +$$
$$\sum_{i=1}^{n-1} \left((5 - 3x_i - x_i^2) x_i - x_{i-1} - 3x_{i+1} + 1 \right)^2 + \left((5 - 3x_n - x_n^2) x_n - x_{n-1} + 1 \right)^2,$$

$$x_0 = [-1,-1,\ldots,-1].$$

Diagonal4 Function

$$f(x) = \sum_{i=1}^{n/2} \frac{1}{2} \left(x_{2i-1}^2 + c x_{2i}^2 \right),$$

$$x_0 = [1,1,\ldots,1], \qquad c = 100.$$

Diagonal5 Function

$$f(x) = \sum_{i=1}^{n} \log\left(\exp(x_i) + \exp(-x_i)\right),$$

$$x_0 = [1.1, 1.1, \ldots, 1.1].$$

Extended Himmelblau Function

$$f(x) = \sum_{i=1}^{n/2} \left(x_{2i-1}^2 + x_{2i} - 11\right)^2 + \left(x_{2i-1} + x_{2i}^2 - 7\right)^2,$$

$$x_0 = [1, 1, \ldots, 1].$$

Generalized Rosenbrock Function (c=100)

$$f(x) = \sum_{i=1}^{n-1} c\left(x_{i+1} - x_i^2\right)^2 + \left(1 - x_i\right)^2,$$

$$x_0 = [-1.2, 1\ldots., -1.2, 1], \qquad c = 100.$$

Generalized White & Holst Function (c=100)

$$f(x) = \sum_{i=1}^{n-1} c\left(x_{i+1} - x_i^3\right)^2 + \left(1 - x_i\right)^2,$$

$$x_0 = [-1.2, 1\ldots., -1.2, 1], \qquad c = 100.$$

Generalized PSC1 Function

$$f(x) = \sum_{i=1}^{n-1} \left(x_i^2 + x_{i+1}^2 + x_i x_{i+1}\right)^2 + \sin^2(x_i) + \cos^2(x_i),$$

$$x_0 = [3, 0.1, \ldots, 3, 0.1].$$

Extended PSC1 Function

$$f(x) = \sum_{i=1}^{n/2} \left(x_{2i-1}^2 + x_{2i}^2 + x_{2i-1} x_{2i}\right)^2 + \sin^2(x_{2i-1}) + \cos^2(x_{2i}),$$

$$x_0 = [3, 0.1, \ldots, 3, 0.1].$$

Extended Powell Function

$$f(x) = \sum_{i=1}^{n/4} \left(x_{4i-3} + 10x_{4i-2}\right)^2 + 5\left(x_{4i-1} - x_{4i}\right)^2 + \left(x_{4i-2} - 2x_{4i-1}\right)^4 + 10\left(x_{4i-3} - x_{4i}\right)^4,$$

$$x_0 = [3, -1, 0, 1, \ldots, 3, -1, 0.1].$$

Full Hessian FH1 Function

$$f(x) = (x_1 - 3)^2 + \sum_{i=2}^{n} \left(x_1 - 3 - 2(x_1 + x_2 + \cdots + x_i)^2 \right)^2,$$

$$x_0 = [0.01, 0.01, \ldots, 0.01].$$

Full Hessian FH2 Function

$$f(x) = (x_1 - 5)^2 + \sum_{i=2}^{n} \left(x_1 + x_2 + \cdots + x_i - 1 \right)^2,$$

$$x_0 = [0.01, 0.01, \ldots, 0.01].$$

Extended Block Diagonal BD1 Function

$$f(x) = \sum_{i=1}^{n/2} \left(x_{2i-1}^2 + x_{2i}^2 - 2 \right)^2 + \left(\exp(x_{2i-1} - 1) - x_{2i} \right)^2,$$

$$x_0 = [0.1, 0.1, \ldots, 0.1].$$

Extended Maratos Function (c=100)

$$f(x) = \sum_{i=1}^{n/2} x_{2i-1} + c\left(x_{2i-1}^2 + x_{2i}^2 - 1 \right)^2,$$

$$x_0 = [1.1, 0.1, \ldots, 1.1, 0.1], \qquad c = 100.$$

Extended Cliff Function

$$f(x) = \sum_{i=1}^{n/2} \left(\frac{x_{2i-1} - 3}{100} \right)^2 - \left(x_{2i-1} - x_{2i} \right) + \exp\left(20(x_{2i-1} - x_{2i}) \right),$$

$$x_0 = [0, -1, \ldots, 0, -1].$$

Quadratic Diagonal Perturbed Function

$$f(x) = \left(\sum_{i=1}^{n} x_i \right)^2 + \sum_{i=1}^{n} \frac{i}{100} x_i^2,$$

$$x_0 = [0.5, 0.5, \ldots, 0.5].$$

Extended Wood Function

$$f(x) = \sum_{i=1}^{n/4} 100\left(x_{4i-3}^2 - x_{4i-2} \right)^2 + \left(x_{4i-3} - 1 \right)^2 + 90\left(x_{4i-1}^2 - x_{4i} \right)^2 + \left(1 - x_{4i-1} \right)^2 + 10.1\left\{ \left(x_{4i-2} - 1 \right)^2 + \left(x_{4i} - 1 \right)^2 \right\} + 19.8\left(x_{4i-2} - 1 \right)\left(x_{4i} - 1 \right),$$

$$x_0 = [-3, -1, -3, -1, \ldots, -3, -1, -3, -1].$$

Extended Hiebert Function

$$f(x)=\sum_{i=1}^{n/2}\left(x_{2i-1}-10\right)^2+\left(x_{2i-1}x_{2i}-50000\right)^2,$$

$$x_0=[0,0,\ldots,0].$$

Quadratic QF1 Function

$$f(x)=\frac{1}{2}\sum_{i=1}^{n}ix_i^2-x_n,$$

$$x_0=[1,1,\ldots,1].$$

Extended Quadratic Penalty QP1 Function

$$f(x)=\sum_{i=1}^{n-1}\left(x_i^2-2\right)^2+\left(\sum_{i=1}^{n}x_i^2-0.5\right)^2,$$

$$x_0=[1,1,\ldots,1].$$

Extended Quadratic Penalty QP2 Function

$$f(x)=\sum_{i=1}^{n-1}\left(x_i^2-\sin x_i\right)^2+\left(\sum_{i=1}^{n}x_i^2-100\right)^2,$$

$$x_0=[1,1,\ldots,1].$$

Quadratic Function QF2

$$f(x)=\frac{1}{2}\sum_{i=1}^{n}i\left(x_i^2-1\right)^2-x_n,$$

$$x_0=[0.5,0.5,\ldots,0.5].$$

Extended EP1 Function

$$f(x)=\sum_{i=1}^{n/2}\left(\exp(x_{2i-1}-x_{2i})-5\right)^2+\left(x_{2i-1}-x_{2i}\right)^2\left(x_{2i-1}-x_{2i}-11\right)^2,$$

$$x_0=[1.5,1.5,\ldots,1.5].$$

Extended Tridiagonal-2 Function

$$f(x)=\sum_{i=1}^{n-1}\left(x_ix_{i+1}-1\right)^2+c\left(x_i+1\right)\left(x_{i+1}+1\right),$$

$$x_0=[1.,1.,\ldots,1.].\qquad c=0.1$$

FLETCBV3 Function

$$f(x)=\frac{1}{2}p\left(x_1^2+x_n^2\right)+\sum_{i=1}^{n-1}\frac{p}{2}\left(x_i-x_{i+1}\right)^2-\sum_{i=1}^{n}\left(\frac{p(h^2+2)}{h^2}x_i+\frac{cp}{h^2}\cos(x_i)\right),$$

where:

$$p=1/10^8,\qquad h=1/(n+1),\qquad c=1,$$
$$x_0=[h,2h,\ldots,nh].$$

FLETCHCR Function

$$f(x)=\sum_{i=1}^{n-1}c\left(x_{i+1}-x_i+1-x_i^2\right)^2,$$

$$x_0=[0.,0.,\ldots,0.],\qquad c=100.$$

BDQRTIC Function

$$f(x)=\sum_{i=1}^{n-4}\left(-4x_i+3\right)^2+\left(x_i^2+2x_{i+1}^2+3x_{i+2}^2+4x_{i+3}^2+5x_n^2\right)^2,$$

$$x_0=[1.,1.,\ldots,1.].$$

TRIDIA Function

$$f(x)=\gamma\left(\delta x_1-1\right)^2+\sum_{i=2}^{n}i\left(\alpha x_i-\beta x_{i-1}\right)^2,$$

$$\alpha=2,\quad \beta=1,\quad \gamma=1,\quad \delta=1,$$

$$x_0=[1,1,\ldots,1].$$

ARGLINB (m=20) Function

$$f(x)=\sum_{i=1}^{m}\left(\sum_{j=1}^{n}ijx_j-1\right)^2,$$

$$x_0=[1,1,\ldots,1].$$

ARWHEAD Function

$$f(x)=\sum_{i=1}^{n-1}\left(-4x_i+3\right)+\sum_{i=1}^{n-1}\left(x_i^2+x_n^2\right)^2,$$

$$x_0=[1.,1.,\ldots,1.].$$

NONDIA (Shanno-78) Function

$$f(x)=(x_1-1)^2+\sum_{i=2}^{n}100\left(x_1-x_{i-1}^2\right)^2,$$

$$x_0=[-1.,-1.,\ldots,-1.].$$

NONDQUAR Function

$$f(x) = (x_1 - x_2)^2 + \sum_{i=1}^{n-2} \left(x_i + x_{i+1} + x_n \right)^4 + (x_{n-1} + x_n)^2,$$

$$x_0 = [1.,-1.,\ldots,1.,-1.,].$$

DQDRTIC Function

$$f(x) = \sum_{i=1}^{n-2} \left(x_i^2 + c x_{i+1}^2 + d x_{i+2}^2 \right),$$

$$c = 100., \quad d = 100.,$$

$$x_0 = [3.,3.,\ldots,3.].$$

EG2 Function

$$f(x) = \sum_{i=1}^{n-1} \sin(x_1 + x_i^2 - 1) + \frac{1}{2} \sin(x_n^2),$$

$$x_0 = [1.,1.,\ldots,1.].$$

CURLY20 Function

$$f(x) = \sum_{i=1}^{n} q_i^4 - 20 q_i^2 - 0.1 q_i,$$

where

$$q_i = \begin{cases} x_i + x_{i+1} + \cdots + x_{i+k}, & i \le n-k, \\ x_i + x_{i+1} + \cdots + x_n, & i > n-k, \end{cases} \qquad k = 20,$$

$$x_0 = [0.001/(n+1),\ldots,0.001/(n+1)].$$

DIXMAANA Function

$$f(x) = 1 + \sum_{i=1}^{n} \alpha\, x_i^2 \left(\frac{i}{n} \right)^{k1} + \sum_{i=1}^{n-1} \beta\, x_i^2 \left(x_{i+1} + x_{i+1}^2 \right)^2 \left(\frac{i}{n} \right)^{k2} + \sum_{i=1}^{2m} \gamma\, x_i^2 x_{i+m}^4 \left(\frac{i}{n} \right)^{k3} + \sum_{i=1}^{m} \delta\, x_i x_{i+2m} \left(\frac{i}{n} \right)^{k4},$$

$$m = n/3,$$

$$\alpha = 1, \quad \beta = 0., \quad \gamma = 0.125, \quad \delta = 0.125, \quad k1 = 0, \quad k2 = 0, \quad k3 = 0, \quad k4 = 0,$$

$$x_0 = [2.,2.,\ldots,2.].$$

DIXMAANB Function

$$f(x) = 1 + \sum_{i=1}^{n} \alpha\, x_i^2 \left(\frac{i}{n}\right)^{k1} + \sum_{i=1}^{n-1} \beta\, x_i^2 \left(x_{i+1} + x_{i+1}^2\right)^2 \left(\frac{i}{n}\right)^{k2} + \sum_{i=1}^{2m} \gamma\, x_i^2\, x_{i+m}^4 \left(\frac{i}{n}\right)^{k3} + \sum_{i=1}^{m} \delta\, x_i x_{i+2m} \left(\frac{i}{n}\right)^{k4},$$

$$m = n/3,$$

$$\alpha = 1, \quad \beta = 0.0625, \quad \gamma = 0.0625, \quad \delta = 0.0625, \quad k1 = 0, \quad k2 = 0, \quad k3 = 0, \quad k4 = 0,$$

$$x_0 = [2., 2., \ldots, 2.].$$

DIXMAANC Function

$$f(x) = 1 + \sum_{i=1}^{n} \alpha\, x_i^2 \left(\frac{i}{n}\right)^{k1} + \sum_{i=1}^{n-1} \beta\, x_i^2 \left(x_{i+1} + x_{i+1}^2\right)^2 \left(\frac{i}{n}\right)^{k2} + \sum_{i=1}^{2m} \gamma\, x_i^2\, x_{i+m}^4 \left(\frac{i}{n}\right)^{k3} + \sum_{i=1}^{m} \delta\, x_i x_{i+2m} \left(\frac{i}{n}\right)^{k4},$$

$$m = n/3,$$

$$\alpha = 1, \quad \beta = 0.125, \quad \gamma = 0.125, \quad \delta = 0.125, \quad k1 = 0, \quad k2 = 0, \quad k3 = 0, \quad k4 = 0,$$

$$x_0 = [2., 2., \ldots, 2.].$$

DIXMAANE Function

$$f(x) = 1 + \sum_{i=1}^{n} \alpha\, x_i^2 \left(\frac{i}{n}\right)^{k1} + \sum_{i=1}^{n-1} \beta\, x_i^2 \left(x_{i+1} + x_{i+1}^2\right)^2 \left(\frac{i}{n}\right)^{k2} + \sum_{i=1}^{2m} \gamma\, x_i^2\, x_{i+m}^4 \left(\frac{i}{n}\right)^{k3} + \sum_{i=1}^{m} \delta\, x_i x_{i+2m} \left(\frac{i}{n}\right)^{k4},$$

$$m = n/3,$$

$$\alpha = 1, \quad \beta = 0., \quad \gamma = 0.125, \quad \delta = 0.125, \quad k1 = 1, \quad k2 = 0, \quad k3 = 0, \quad k4 = 1,$$

$$x_0 = [2., 2., \ldots, 2.].$$

DIXMAANJ Function

$$f(x) = 1 + \sum_{i=1}^{n} \alpha\, x_i^2 \left(\frac{i}{n}\right)^{k1} + \sum_{i=1}^{n-1} \beta\, x_i^2 \left(x_{i+1} + x_{i+1}^2\right)^2 \left(\frac{i}{n}\right)^{k2} + \sum_{i=1}^{2m} \gamma\, x_i^2\, x_{i+m}^4 \left(\frac{i}{n}\right)^{k3} + \sum_{i=1}^{m} \delta\, x_i x_{i+2m} \left(\frac{i}{n}\right)^{k4},$$

$$m = n/3,$$

$$\alpha = 1, \quad \beta = 0.0625, \quad \gamma = 0.0625, \quad \delta = 0.0625, \quad k1 = 2, \quad k2 = 0, \quad k3 = 0, \quad k4 = 2,$$

$$x_0 = [2., 2., \ldots, 2.].$$

Partial Perturbed Quadratic

$$f(x) = x_1^2 + \sum_{i=1}^{n}\left(ix_i^2 + \frac{1}{100}\left(x_1 + x_2 + \cdots + x_i\right)^2 \right)$$

$$x_0 = [0.5, 0.5, \ldots, 0.5].$$

Broyden Tridiagonal

$$f(x) = (3x_1 - 2x_1^2)^2 + \sum_{i=2}^{n-1}(3x_i - 2x_i^2 - x_{i-1} - 2x_{i+1} + 1)^2 + (3x_n - 2x_n^2 - x_{n-1} + 1)^2,$$

$$x_0 = [-1, -1, \ldots, -1].$$

Almost Perturbed Quadratic

$$f(x) = \sum_{i=1}^{n} ix_i^2 + \frac{1}{100}(x_1 + x_n)^2,$$

$$x_0 = [0.5, 0.5, \ldots, 0.5].$$

Tridiagonal Perturbed Quadratic

$$f(x) = x_1^2 + \sum_{i=2}^{n-1} ix_i^2 + (x_{i-1} + x_i + x_{i+1})^2,$$

$$x_0 = [0.5, 0.5, \ldots, 0.5].$$

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