Numerical Experiments with a variant of Barzilai-Borwein Method for Unconstrained Optimization

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In this work we present the numerical results with ASA package (Anticipative Scalar Approximation) which implements a gradient descent algorithm with backtracking. The corresponding algorithm selects the steplength, according to a backtracking procedure, along the negative gradient, in which the initial stepsize is determined using the *Barzilai-Borwein method*. This initial steplength is used in a backtracking procedure which implements the Armijo rule in a monotone manner. The algorithm does not have any protection to negative or too small values for initial stepsize. In fact this is the main drawback of the Barzilai and Borwein algorithm. The problems presenting such phenomena have been rejected from my experiment.

A number of 310 unconstrained optimization problems have been considered in this experiment. The Fortran code is authored by Andrei.

The criteria for stopping the iterations are: $\|g_{k+1}\|_\infty \le \varepsilon_g$ or $t_{k+1}|g_{k+1}^T g_{k+1}| \le \varepsilon_f |f_{k+1}|$, where $\varepsilon_g = 10^{-6}$ and $\varepsilon_f = 10^{-20}$. The code considers backtracking using Armijo line search implemented in the following manner:

Step 1. Set: $t = \bar{t}_{k+1}$ and $f_{\min} = min\{f(x_j), j = 0,1,\ldots,k+1\}$.
Setp 2. While $f(x_{k+1} - tg_{k+1}) > f_{\min} - \alpha t g_{k+1}^T g_{k+1}$, set $t = t\beta$.
Step 3. Set $t_{k+1} = t$.

in which $\alpha = 0.0001$ and $\beta = 0.8$.

Here $\bar{t}_{k+1} = \dfrac{1}{\gamma_{k+1}^{BB}}$, where $\gamma_k^{BB} = \dfrac{s_k^T y_k}{s_k^T s_k}$, $s_k = x_k - x_{k-1}$ and $y_k = \nabla f(x_k) - \nabla f(x_{k-1})$.

The following tables give the results of ASA package, where:

n	dimension of the problem (number of variables),
iter	number of iterations,
fgcnt	number of function and its gradient evaluations,
time(c)	cpu time in centeseconds,
fxnew	function value in (local) optimal point, $f(x^*)$,
ginf	infinite norm ofthe gradient in optimal point, $\|\nabla f(x^*)\|_\infty$,
ngama	number of negative γ_{k+1},
stepinimin	$min\{\bar{t}_k, k = 0,1,\ldots, iter\}$
stepinimax	$max\{\bar{t}_k, k = 0,1,\ldots, iter\}$

1 Barzilai-Borwein Algorithm: Extended Freudenstein & Roth Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	218	1615	204	.1510921083162E-10	.4185494617559E-06	.2302933930214E-03	.3447761203517E+00
2000	240	1762	445	.6963702654747E-11	.2008047701452E-06	.2302933930214E-03	.3447761799570E+00
3000	240	1771	670	.7143723915238E-10	.5249604555502E-06	.2302933930214E-03	.3447758857537E+00
4000	145	1086	554	.2117430498084E-10	.2482136913784E-06	.2302933930214E-03	.3447761064814E+00
5000	138	1028	654	.8849204976498E-21	.2457767322994E-10	.2302933930214E-03	.3447761837342E+00
6000	285	2096	1615	.4393944390157E-09	.9232360866918E-06	.2302933930214E-03	.3447752099043E+00
7000	175	1302	1170	.3704493764789E-09	.7843487423997E-06	.2302933930214E-03	.3447758613977E+00
8000	295	2173	2246	.5017942384123E-09	.8557555517541E-06	.2302933930214E-03	.3447620884608E+00
9000	170	1254	1456	.5597188799591E-11	.8487927516398E-07	.2302933930214E-03	.3447762120313E+00
10000	140	1047	1356	.7918966144058E-10	.3026616504087E-06	.2302933930214E-03	.3447761204450E+00
TOTAL	2046	15134	103.70(seconds)				

2 Barzilai-Borwein Algorithm: Extended Trigonometric Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	44	148	61	.3882863440038E-12	.2626278158725E-06	.1042175124638E-04	.3077104506215E+01
2000	30	122	99	.1754216887907E-11	.3561085127810E-06	.2308093089399E-05	.3212389751518E+01
3000	30	124	148	.7296398907745E-11	.3493768707124E-06	.1220725215775E-05	.2356644974903E+01
4000	30	124	198	.3095855790788E-12	.2069734084766E-06	.7545172929379E-06	.9743933760699E+00
5000	31	128	258	.6658507416368E-12	.2377315306177E-06	.4826903073387E-06	.1141684739770E+01
6000	32	132	318	.6491312033809E-11	.2048825494858E-06	.2774931459698E-06	.1157656004931E+01
7000	32	133	379	.1044511451245E-12	.7021574030098E-07	.2456373754165E-06	.1014582153940E+01
8000	32	135	434	.4055246054244E-11	.1980470878310E-06	.1818903649102E-06	.1435695862625E+01
9000	31	133	484	.1223118026369E-10	.3687882101223E-06	.1420133615362E-06	.9104349416387E+00
10000	32	136	549	.1069083630259E-09	.9622009930323E-06	.1163805853137E-06	.1174915710810E+01
TOTAL	324	1315	29.28(seconds)				

3 Barzilai-Borwein Algorithm: Extended Rosenbrock Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	248	1981	390	.4375079385561E-14	.2364332698807E-08	.5271859877171E-03	.5260777239755E+01
2000	162	1339	533	.9884605768020E-09	.7931941124859E-06	.5271859877171E-03	.5260776046972E+01
3000	109	887	533	.5306787674591E-13	.4752161294683E-08	.5271859877171E-03	.5260775105154E+01
4000	103	844	686	.1579433648245E-08	.7091162060541E-06	.5271859877171E-03	.5260775122143E+01
5000	703	5544	5531	.9758149550813E-09	.5287300941688E-06	.5271859877171E-03	.5260777023286E+01
6000	137	1109	1341	.8693508380235E-09	.4297548312766E-06	.5271859877171E-03	.5260776238628E+01
7000	193	1569	2197	.2365420745319E-12	.6572755848703E-08	.5271859877171E-03	.5260777675123E+01
8000	171	1337	2158	.1179925639686E-12	.4339756668175E-08	.5271859877172E-03	.5260775502588E+01
9000	408	3183	5866	.6977288638312E-08	.9088257733295E-06	.5271859877171E-03	.5260777478125E+01
10000	527	4193	8398	.3112854056912E-08	.6540225697170E-06	.5271859877171E-03	.5260777099097E+01
TOTAL	2761	21986	276.33(seconds)				

4 Barzilai-Borwein Algorithm: Extended White & Holst Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	82	644	127	.1806396909117E-09	.3605435630713E-06	.4995517887394E-03	.4995119093783E+01
2000	82	644	258	.3612794257130E-09	.3605435630930E-06	.4995517887381E-03	.4995119073752E+01
3000	82	644	384	.5419192111277E-09	.3605436075236E-06	.4995517887389E-03	.4995119083205E+01
4000	82	644	511	.7225534433416E-09	.3605422514144E-06	.4995517887343E-03	.4995119122447E+01
5000	82	644	648	.9031917449461E-09	.3605422069838E-06	.4995517887337E-03	.4995119114090E+01
6000	82	644	775	.1083835816608E-08	.3605431851184E-06	.4995517887363E-03	.4995119080185E+01
7000	82	644	906	.1264478377159E-08	.3605436519759E-06	.4995517887383E-03	.4995119071144E+01
8000	82	644	1038	.1445114455441E-08	.3605431407204E-06	.4995517887376E-03	.4995119094517E+01
9000	82	644	1170	.1625749984533E-08	.3605427627350E-06	.4995517887356E-03	.4995119098305E+01
10000	82	644	1302	.1806387326629E-08	.3605425849150E-06	.4995517887358E-03	.4995119116729E+01
TOTAL	820	6440	71.19(seconds)				

5 Barzilai-Borwein Algorithm: Extended Beale Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	46	219	16	.2199062934417E-16	.1505647828607E-09	.2041657374571E-01	.3316915514935E+01
2000	46	219	39	.4398133066924E-16	.1505653379722E-09	.2041657374536E-01	.3316915514752E+01
3000	46	219	60	.6597211165020E-16	.1505650465386E-09	.2041657374588E-01	.3316915514658E+01
4000	46	219	77	.8796281553361E-16	.1505650465386E-09	.2041657374612E-01	.3316915514789E+01
5000	46	219	99	.1099533394647E-15	.1505644914271E-09	.2041657374591E-01	.3316915514830E+01
6000	46	219	121	.1319439920077E-15	.1505653379722E-09	.2041657374589E-01	.3316915514805E+01
7000	46	219	142	.1539349271838E-15	.1505650465386E-09	.2041657374554E-01	.3316915514657E+01
8000	46	219	160	.1759256310672E-15	.1505650465386E-09	.2041657374530E-01	.3316915514632E+01
9000	46	219	181	.1979160340626E-15	.1505636448821E-09	.2041657374536E-01	.3316915514827E+01
10000	46	219	203	.2199062934417E-15	.1505647828607E-09	.2041657374542E-01	.3316915514946E+01
TOTAL	460	2190	10.98(seconds)				

6 Barzilai-Borwein Algorithm: Extended Penalty Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	44	185	33	.8831940750670E+03	.9712023696373E-07	.9792836120929E-09	.7087114119824E-01
2000	48	204	77	.1814063664869E+04	.1551146899814E-07	.1160269912607E-09	.9477570183363E-01
3000	45	205	110	.2755973749503E+04	.4324662421842E-06	.2987855138962E-10	.8572283226635E-01
4000	49	213	154	.3704070534948E+04	.2806095448894E-07	.1323952323352E-10	.6520999885712E-01
5000	49	218	197	.4656333923744E+04	.3276978204451E-07	.6779931839907E-11	.5366107260767E-01
6000	53	228	253	.5611676659140E+04	.1502476345561E-06	.4490624895257E-11	.3979591999291E-01
7000	54	232	297	.6569428560737E+04	.4691959827017E-07	.2795929286047E-11	.4003231224695E-01
8000	55	241	357	.7529139638522E+04	.1706743140219E-07	.1925222426468E-11	.4371553478825E-01
9000	58	255	417	.8490489281459E+04	.7974086170972E-06	.1200803238023E-11	.5634092316056E-01
10000	56	241	445	.9453238852842E+04	.2540421394236E-06	.9857669786694E-12	.3590000268113E-01
TOTAL	511	2222	23.40(seconds)				

7 Barzilai-Borwein Algorithm: Perturbed Quadratic Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	1353	6979	698	.2146236841254E-12	.9307792279493E-06	.5016242034536E-03	.4934853332118E+00
2000	2811	15053	3004	.2465136904370E-12	.9972947743391E-06	.2512395492543E-03	.4942392802468E+00
3000	3922	21766	6536	.2421019059741E-12	.9883940894079E-06	.1675557208859E-03	.4944825531887E+00
4000	5085	28963	11683	.2471532539904E-12	.9987695754804E-06	.1254086623495E-03	.4935275211970E+00
5000	6583	36455	18614	.2472235498569E-12	.9988684224607E-06	.1003374898375E-03	.4937377044807E+00
6000	7953	44979	27562	.2476761789768E-12	.9998141352773E-06	.8365558032562E-04	.4895696400952E+00
7000	9421	52939	37992	.2454443278771E-12	.9952926525959E-06	.7166999699104E-04	.4941738082342E+00
8000	10492	60059	49362	.2473935753227E-12	.9992130474030E-06	.6273535872869E-04	.4913579815750E+00
9000	11503	65718	60857	.2443400799617E-12	.9930173357096E-06	.5581981500466E-04	.4936284741950E+00
10000	13247	75069	77572	.2475033018383E-12	.9994403202793E-06	.5019199707026E-04	.4896457224849E+00
TOTAL	72370	407980	2938.80(seconds)				

8 Barzilai-Borwein Algorithm: Raydan 1 Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	1134	5631	719	.5005000000000E+05	.9799764220712E-06	.9058109618420E-02	.9966768793218E+01
2000	1832	9419	2395	.2001000000000E+06	.2834005634720E-06	.4945794003181E-02	.9946909191285E+01
3000	2553	13298	5097	.4501500000000E+06	.2027337609428E-06	.3059750896996E-02	.9811017373802E+01
4000	2993	15958	8211	.8002000000000E+06	.1293583406582E-05	.2445662683258E-02	.9818149354912E+01
5000	3319	17971	11590	.1250250000000E+07	.1529467435878E-05	.1956603291895E-02	.9897027464942E+01
6000	4345	23262	18136	.1800300000000E+07	.1745287943513E-05	.1502963132624E-02	.9811412520151E+01
7000	4576	24565	22454	.2450350000000E+07	.3309740913059E-05	.1350454625335E-02	.9893270820240E+01
8000	4751	26063	27215	.3200400000000E+07	.2780428259946E-05	.1207835639047E-02	.9972276448823E+01
9000	5326	29278	34499	.4050450000000E+07	.2535212811847E-05	.1032125315464E-02	.9811253909930E+01
10000	6301	33159	43688	.5000500000000E+07	.2653059704327E-05	.9662872937886E-03	.9744228430034E+01
TOTAL	37130	198604	1740.04(seconds)				

9 Barzilai-Borwein Algorithm: Raydan 2 Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	7	16	5	.1000000000000E+04	.1427280152703E-07	.7702907530064E+00	.1672712031950E+01
2000	7	16	0	.2000000000000E+04	.1427280152714E-07	.7702907530065E+00	.1672712031949E+01
3000	7	16	11	.3000000000000E+04	.1427280152692E-07	.7702907530064E+00	.1672712031949E+01
4000	7	16	6	.4000000000000E+04	.1427280152703E-07	.7702907530064E+00	.1672712031950E+01
5000	7	16	11	.5000000000000E+04	.1427280152703E-07	.7702907530064E+00	.1672712031950E+01
6000	7	16	16	.6000000000000E+04	.1427280152692E-07	.7702907530064E+00	.1672712031950E+01
7000	7	16	17	.7000000000000E+04	.1427280152725E-07	.7702907530063E+00	.1672712031950E+01
8000	7	16	16	.8000000000000E+04	.1427280152801E-07	.7702907530062E+00	.1672712031949E+01
9000	7	16	22	.9000000000000E+04	.1427280152692E-07	.7702907530062E+00	.1672712031949E+01
10000	7	16	22	.1000000000000E+05	.1427280152736E-07	.7702907530062E+00	.1672712031949E+01
TOTAL	70	160	1.26(seconds)				

14 Barzilai-Borwein Algorithm: Generalized Tridiagonal 1 Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	29	89	28	.9972103074860E+03	.4378096525315E-06	.2147811685554E-01	.1597379953850E+00
2000	29	98	60	.1997210307486E+04	.4527448365099E-06	.2147763747589E-01	.1623921303940E+00
3000	29	89	83	.2997210307486E+04	.8093637007267E-06	.2134586354825E-01	.1597383553610E+00
4000	30	100	131	.3997210307486E+04	.2693968790268E-06	.2147739782913E-01	.1597384001488E+00
5000	29	93	149	.4997210307486E+04	.4411382725777E-07	.2134443882132E-01	.1637238522772E+00
6000	26	145	269	.5997210307486E+04	.1830476574316E-05	.2147731795340E-01	.1597384448911E+00
7000	29	89	203	.6997210307486E+04	.1039509069933E-06	.2147729513238E-01	.1617333037049E+00
8000	25	150	374	.7997210307486E+04	.4403905794259E-05	.2147727801678E-01	.1597384672453E+00
9000	28	155	433	.8997210307486E+04	.1731950212580E-05	.2147726470475E-01	.1597384746943E+00
10000	30	220	687	.9997210307486E+04	.1599457677859E-05	.2147725405517E-01	.1597384806526E+00
TOTAL	284	1228	24.17(seconds)				

15 Barzilai-Borwein Algorithm: Extended Tridiagonal 1 Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	45	215	33	.3502394439863E-06	.5501883210212E-06	.1255213942679E+00	.1064985959154E+04
2000	45	215	72	.7004769791538E-06	.5501896779332E-06	.1255213942678E+00	.1064986696491E+04
3000	45	215	104	.1050726785746E-05	.5501940420026E-06	.1255213942679E+00	.1064984307050E+04
4000	45	215	143	.1400958060630E-05	.5501859428883E-06	.1255213942678E+00	.1064985904061E+04
5000	45	215	176	.1751206115376E-05	.5501879188084E-06	.1255213942678E+00	.1064985104868E+04
6000	45	215	214	.2101436667442E-05	.5501883217062E-06	.1255213942679E+00	.1064985957968E+04
7000	45	215	247	.2451676049111E-05	.5501858518635E-06	.1255213942678E+00	.1064985963369E+04
8000	45	215	291	.2801937873559E-05	.5501890882843E-06	.1255213942678E+00	.1064984324871E+04
9000	45	215	324	.3152154847339E-05	.5501883026608E-06	.1255213942678E+00	.1064985971383E+04
10000	45	215	357	.3502394977517E-05	.5501883828384E-06	.1255213942678E+00	.1064985918139E+04
TOTAL	450	2150	19.61(seconds)				

16 Barzilai-Borwein Algorithm: Extended Three Expo Terms Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	12	43	5	.1279633348329E+04	.1510162931208E-07	.7178756380541E-01	.3906816509893E+00
2000	12	43	22	.2559266696658E+04	.1510162960514E-07	.7178756380541E-01	.3906816509893E+00
3000	12	43	33	.3838900044987E+04	.1510162931208E-07	.7178756380540E-01	.3906816509894E+00
4000	12	43	38	.5118533393317E+04	.1510162931208E-07	.7178756380541E-01	.3906816509893E+00
5000	12	43	50	.6398166741646E+04	.1510162924096E-07	.7178756380541E-01	.3906816509894E+00
6000	12	43	60	.7677800089975E+04	.1510162931208E-07	.7178756380540E-01	.3906816509893E+00
7000	12	43	72	.8957433438304E+04	.1510162931208E-07	.7178756380540E-01	.3906816509893E+00
8000	12	43	82	.1023706678663E+05	.1510162931208E-07	.7178756380541E-01	.3906816509892E+00
9000	12	43	93	.1151670013496E+05	.1510162924096E-07	.7178756380542E-01	.3906816509893E+00
10000	12	43	99	.1279633348329E+05	.1510162931208E-07	.7178756380542E-01	.3906816509891E+00
TOTAL	120	430	5.54(seconds)				

20 Barzilai-Borwein Algorithm: Extended Himmelblau Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	14	44	0	.1098215827633E-12	.1124138611012E-06	.1127015602006E-01	.3890793518509E-01
2000	14	44	11	.2196431998178E-12	.1124138646539E-06	.1127015602006E-01	.3890793518508E-01
3000	14	44	11	.3294647997267E-12	.1124138646539E-06	.1127015602006E-01	.3890793518508E-01
4000	14	44	16	.4392863996356E-12	.1124138646539E-06	.1127015602006E-01	.3890793518508E-01
5000	14	44	22	.5491079138163E-12	.1124138611012E-06	.1127015602006E-01	.3890793518509E-01
6000	14	44	28	.6589295994535E-12	.1124138646539E-06	.1127015602006E-01	.3890793518508E-01
7000	14	44	27	.7687510793428E-12	.1124138611012E-06	.1127015602006E-01	.3890793518509E-01
8000	14	44	33	.8785726621060E-12	.1124138611012E-06	.1127015602006E-01	.3890793518508E-01
9000	14	44	39	.9883942448693E-12	.1124138611012E-06	.1127015602006E-01	.3890793518508E-01
10000	14	44	44	.1098215697645E-11	.1124138539958E-06	.1127015602006E-01	.3890793518509E-01
TOTAL	140	440	2.31(seconds)				

22 Barzilai-Borwein Algorithm: Extended PSC1 Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	14	45	22	.3865995282465E+03	.3374117657495E-06	.2135197487898E-01	.4853503910413E+00
2000	14	45	50	.7731990564930E+03	.3374117658606E-06	.2135197487898E-01	.4853503910412E+00
3000	14	45	66	.1159798584739E+04	.3374117658606E-06	.2135197487898E-01	.4853503910413E+00
4000	14	45	98	.1546398112986E+04	.3374117656940E-06	.2135197487898E-01	.4853503910417E+00
5000	14	45	116	.1932997641232E+04	.3374117658606E-06	.2135197487898E-01	.4853503910418E+00
6000	14	45	143	.2319597169479E+04	.3374117658606E-06	.2135197487898E-01	.4853503910416E+00
7000	14	45	164	.2706196697726E+04	.3374117656385E-06	.2135197487898E-01	.4853503910414E+00
8000	14	45	193	.3092796225972E+04	.3374117658606E-06	.2135197487897E-01	.4853503910410E+00
9000	14	45	214	.3479395754219E+04	.3374117656385E-06	.2135197487897E-01	.4853503910410E+00
10000	14	45	236	.3865995282465E+04	.3374117656385E-06	.2135197487897E-01	.4853503910410E+00
TOTAL	140	450	13.02(seconds)				

24 Barzilai-Borwein Algorithm: Extended Block-Diagonal BD1 Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	21	69	11	.1720848949325E-17	.8204285422175E-10	.5260512966433E-01	.8174311837572E+00
2000	21	69	16	.3441689488314E-17	.8204277664451E-10	.5260512966433E-01	.8174311837573E+00
3000	21	69	33	.5162543409269E-17	.8204243280192E-10	.5260512966433E-01	.8174311837576E+00
4000	21	69	38	.6883374415468E-17	.8204235533457E-10	.5260512966434E-01	.8174311837578E+00
5000	21	69	55	.8604202724756E-17	.8204269917715E-10	.5260512966434E-01	.8174311837575E+00
6000	21	69	61	.1032508681854E-16	.8204243280192E-10	.5260512966433E-01	.8174311837566E+00
7000	21	69	77	.1204585442026E-16	.8204262170979E-10	.5260512966432E-01	.8174311837560E+00
8000	21	69	82	.1376669076601E-16	.8204262170979E-10	.5260512966431E-01	.8174311837547E+00
9000	21	69	93	.1548755464185E-16	.8204227786721E-10	.5260512966430E-01	.8174311837551E+00
10000	21	69	105	.1720840544951E-16	.8204269917715E-10	.5260512966430E-01	.8174311837544E+00
TOTAL	210	690	5.71(seconds)				

25 Barzilai-Borwein Algorithm: Extended Maratos Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	18	129	11	-.5003121103484E+03	.1210338931888E-06	.1195139934664E-02	.1004904284245E+01
2000	18	129	22	-.1000624220697E+04	.1210338931926E-06	.1195139934664E-02	.1004904284245E+01
3000	18	129	33	-.1500936331045E+04	.1210338955455E-06	.1195139934664E-02	.1004904284245E+01
4000	18	129	44	-.2001248441393E+04	.1210338932046E-06	.1195139934663E-02	.1004904284245E+01
5000	18	127	55	-.2501560551742E+04	.1210338955261E-06	.1195139934664E-02	.1004904284244E+01
6000	18	127	66	-.3001872662090E+04	.1210338954995E-06	.1195139934664E-02	.1004904284246E+01
7000	18	127	82	-.3502184772439E+04	.1210338955317E-06	.1195139934663E-02	.1004904284246E+01
8000	18	127	88	-.4002496882787E+04	.1210338931460E-06	.1195139934663E-02	.1004904284245E+01
9000	18	127	105	-.4502808993135E+04	.1210338931144E-06	.1195139934664E-02	.1004904284246E+01
10000	18	127	115	-.5003121103484E+04	.1210338930525E-06	.1195139934664E-02	.1004904284245E+01
TOTAL	180	1278	6.21(seconds)				

27 Barzilai-Borwein Algorithm: Quadratic Diagonal Perturbed Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	1490	13985	1450	.3514372529890E-10	.9967561245923E-06	.4975058375411E-03	.4093567951016E+02
2000	3631	36194	7558	.7507354765795E-10	.9692280805783E-06	.2487535375512E-03	.4036501007724E+02
3000	5502	55815	17527	.7321228199745E-10	.9570903018718E-06	.1658358292081E-03	.4009788645921E+02
4000	7137	73845	30989	.7092579335287E-10	.9899056398639E-06	.1243769234714E-03	.4015392723008E+02
5000	8893	92782	48900	.7435035502150E-10	.9584490499783E-06	.9950156352850E-04	.4003123821368E+02
6000	10183	106540	67871	.7628162663087E-10	.9602200231360E-06	.8291798335786E-04	.3998547571332E+02
7000	12380	132182	98839	.5619199948072E-10	.9033308401144E-06	.7107256558272E-04	.4088553063927E+02
8000	12881	137214	117535	.6880184560152E-10	.9229072407710E-06	.6218850040975E-04	.4030822454969E+02
9000	14039	150852	535595	.6448034050721E-10	.9151580699448E-06	.5527867085055E-04	.4022411728133E+02
10000	16625	181799	195035	.7064722151689E-10	.9277640757442E-06	.4975080651565E-04	.4011145369038E+02
TOTAL	92761	981208	11212.99(seconds)				

28 Barzilai-Borwein Algorithm: Extended Wood Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	1043	7083	1697	.4037673242032E-09	.9648453024914E-06	.2942612699834E-03	.1528495070879E+02
2000	930	6186	2960	.7419657435607E-09	.9479666676600E-06	.2942612699834E-03	.1056180012689E+02
3000	1126	7545	5416	.1234131324663E-08	.9767358496796E-06	.2942612699834E-03	.1472970173803E+02
4000	1096	7330	7042	.1675359971165E-08	.9824906638681E-06	.2942612699834E-03	.1709864607171E+02
5000	1156	7749	9321	.1948804260833E-08	.9779375458659E-06	.2942612699834E-03	.1056179947747E+02
6000	1040	7033	10161	.2274280993209E-08	.9654876220728E-06	-.2359035716004E+01	.1056179880342E+02
7000	1057	7095	12034	.2014007155735E-08	.8710436298794E-06	.2942612699834E-03	.1056179839631E+02
8000	1087	7465	14457	.3304406385658E-08	.9715346102683E-06	-.1160657209943E+01	.1056179814725E+02
9000	1094	7388	16153	.3393341931471E-08	.9368639226696E-06	-.6640136373147E+01	.1056179811101E+02
10000	1052	6978	16901	.4201669528555E-08	.9849183542420E-06	.2942612699834E-03	.1056179780499E+02
TOTAL	10681	71852	961.42(seconds)				

30 Barzilai-Borwein Algorithm: Quadratic QF1 Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	1364	7145	50	-.4999999995062E-03	.9908063103884E-06	.1004052347747E-02	.9978674596881E+00
2000	2566	13822	285	-.2499999995134E-03	.9865185247504E-06	.5016503507700E-03	.9995900811899E+00
3000	4012	22325	1011	-.1666666661774E-03	.9891957606534E-06	.3349523160179E-03	.9951985372656E+00
4000	5451	30313	1999	-.1249999995010E-03	.9986973612705E-06	.2512958194743E-03	.9924922529245E+00
5000	6673	37425	3142	-.9999999950129E-04	.9986937463454E-06	.2005035787007E-03	.9903830292281E+00
6000	7775	44272	4910	-.8333333284238E-04	.9906920449671E-06	.1673109714101E-03	.9948894626090E+00
7000	9023	50724	6509	-.7142857092961E-04	.9989431239685E-06	.1433651043223E-03	.9984929827784E+00
8000	10679	61275	8783	-.6249999950650E-04	.9934649710691E-06	.1254272211369E-03	.9958258416408E+00
9000	11852	67017	10946	-.5555555505642E-04	.9990984051018E-06	.1113420803190E-03	.9970881446523E+00
10000	12861	74610	14116	-.4999999950248E-04	.9974287517267E-06	.1004186197555E-03	.9942351606120E+00
TOTAL	72256	408928	517.51(seconds)				

31 Barzilai-Borwein Algorithm: Extended Quadratic Penalty QP1 Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	17	73	5	.3990006250000E+04	.3122106247529E-07	.3277810617312E-03	.1251571637102E+00
2000	22	215	38	.7990003125000E+04	.3977215526731E-05	.1613255437472E-03	.1250781742047E+00
3000	8	909	248	.1199550365656E+05	.6724363033024E-01	-.1241938481306E+00	.8397200524533E-04
4000	25	233	88	.1599000156250E+05	.1437466982520E-04	.7912566101749E-04	.1248977486455E+00
5000	20	86	38	.1999000125000E+05	.3092903432741E-06	.6328957502694E-04	.1249800682805E+00
6000	21	89	55	.2399000104167E+05	.1006238188016E-06	.5429686473378E-04	.1249257020613E+00
7000	18	111	77	.2799000089285E+05	.8819634176593E-06	.3907733605601E-04	.1250137635865E+00
8000	22	175	132	.3199000078124E+05	.9633987679576E-05	.3866074608177E-04	.1249321060405E+00
9000	18	226	192	.3599000069444E+05	.5832832427048E-05	.2890047623688E-04	.1249811577676E+00
10000	22	106	104	.3999000062500E+05	.7183318325721E-06	.3092576970679E-04	.1249876694821E+00
TOTAL	193	2223	9.77(seconds)				

33 Barzilai-Borwein Algorithm: Quadratic QF2 Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	1646	8404	1796	-.1000124968765E+01	.9910145129773E-06	.2485818373266E-03	.3289436686795E+00
2000	3043	16380	6894	-.1000062492189E+01	.9792189975791E-06	.1254932007611E-03	.2510861703464E+00
3000	4528	24595	15456	-.1000041663195E+01	.9991933919977E-06	.8297135169677E-04	.3261439133345E+00
4000	5809	32397	27319	.9999687480468E+00	.9809306578003E-06	.6282388913734E-04	.3226306849978E+00
5000	6369	36008	37762	.9999749987499E+00	.7459159277666E-06	.5001679536536E-04	.3100161209538E+00
6000	9207	50610	63922	-.1000020832465E+01	.9999982169340E-06	.4183555299134E-04	.3067395006740E+00
7000	10125	56561	83564	-.1000017856505E+01	.9803916800011E-06	.3338995822878E-04	.3072240889832E+00
8000	12921	69525	116810	.9999843745118E+00	.9868599780117E-06	.3142548555095E-04	.3183962639035E+00
9000	11123	64698	122533	-.1000013888503E+01	.9920922585123E-06	.2781604743936E-04	.3142133099188E+00
10000	16378	87529	184605	.9999874996876E+00	.9998476024151E-06	.2259479221913E-04	.2902463521553E+00
TOTAL	81149	446707	6606.61(seconds)				

34 Barzilai-Borwein Algorithm: Extended EP1 Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	4	39	0	.7931762881473E+04	.3334041327741E-07	.2155464368951E-02	.2176993825951E-02
2000	4	39	5	.1586352576295E+05	.3334041327741E-07	.2155464368951E-02	.2176993825951E-02
3000	4	39	11	.2379528864442E+05	.3334041327741E-07	.2155464368951E-02	.2176993825950E-02
4000	4	39	6	.3172705152589E+05	.3334041327741E-07	.2155464368951E-02	.2176993825950E-02
5000	4	35	16	.3965881440736E+05	.3334041327741E-07	.2155464368951E-02	.2176993825950E-02
6000	4	35	11	.4759057728884E+05	.3334041327741E-07	.2155464368951E-02	.2176993825951E-02
7000	4	35	17	.5552234017031E+05	.3334041327741E-07	.2155464368951E-02	.2176993825951E-02
8000	4	35	22	.6345410305178E+05	.3334041327741E-07	.2155464368951E-02	.2176993825950E-02
9000	4	35	22	.7138586593325E+05	.3334041327741E-07	.2155464368951E-02	.2176993825950E-02
10000	4	35	22	.7931762881473E+05	.3334041327741E-07	.2155464368951E-02	.2176993825950E-02
TOTAL	40	366	1.32(seconds)				

35 Barzilai-Borwein Algorithm: Extended Tridiagonal 2 Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	37	125	11	.3893393944765E+03	.9589313399727E-06	.1431949664234E+00	.4842852195095E+01
2000	40	143	27	.7790685180764E+03	.4403551197019E-06	.1431723611831E+00	.4464944544051E+01
3000	41	143	39	.1168797641676E+04	.9568964724482E-06	.1431648352258E+00	.4672803525688E+01
4000	40	146	49	.1558526765277E+04	.8143984466524E-06	.1431610739553E+00	.4831359944981E+01
5000	40	141	66	.1948255888877E+04	.6193392062537E-06	.1431588177394E+00	.4477704142200E+01
6000	42	150	82	.2337985012477E+04	.7737495461013E-06	.1431573138230E+00	.4529492235348E+01
7000	43	162	105	.2727714136076E+04	.9860079527857E-06	.1431562397084E+00	.4813475510473E+01
8000	35	124	88	.3117443259676E+04	.8891391843069E-06	.1431554341834E+00	.4488489193721E+01
9000	40	146	120	.3507172383276E+04	.9327490767541E-06	.1431548077001E+00	.4621990390997E+01
10000	35	124	116	.3896901506876E+04	.9090059913675E-06	.1431543065362E+00	.4445474781728E+01
TOTAL	393	1404	7.03(seconds)				

36 Barzilai-Borwein Algorithm: BDQRTIC (CUTE) Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	704	5831	3114	.3983817950577E+04	.4386971498707E-04	.3592936307429E-05	.4459625650353E+00
2000	1690	15725	16824	.7989427682541E+04	.4850279523979E-03	.1772298719138E-05	.4459630406740E+00
3000	2423	22923	36981	.1199503741451E+05	.1791583604067E-03	.1194719808094E-05	.4459629732261E+00
4000	1438	13705	29451	.1600064714648E+05	.5088447662180E-03	.8752434248958E-06	.4459554612452E+00
5000	1216	11807	31829	.2000625687848E+05	.6351371034956E-03	.6997757159504E-06	.4459536578740E+00
6000	1655	16376	52916	.2401186661040E+05	.7932397246222E-03	.5920341464191E-06	.4459639420658E+00
7000	1604	15969	60286	.2801747634236E+05	.2433451868484E-03	.5161400776942E-06	.4459622946665E+00
8000	1718	16853	72974	.3202308607435E+05	.8619405224673E-03	.4321817299318E-06	.4459381876717E+00
9000	3482	35249	171411	.3602869580629E+05	.1703633156855E-02	.3990108128123E-06	.4459605056849E+00
10000	1744	17765	96197	.4003430553826E+05	.7628687680236E-03	.3456382458817E-06	.4459632867821E+00
TOTAL	17674	172203	5719.83(seconds)				

41 Barzilai-Borwein Algorithm: DQDRTIC (CUTE) Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	175	947	225	.9340199329322E-12	.9805660167331E-06	.2487576307259E-02	.4999341431248E+00
2000	162	850	407	.7231607021323E-12	.6487002215833E-06	.2487569237429E-02	.4999894888115E+00
3000	126	666	483	.2254693738195E-12	.9490106527877E-06	.2487566885592E-02	.4998218899488E+00
4000	125	669	648	.1624063036869E-12	.8056098556611E-06	.2487565710566E-02	.4999805306836E+00
5000	177	935	1131	.2174334132038E-12	.9320279581274E-06	.2487565005836E-02	.4998533481401E+00
6000	133	692	1006	.2441628720010E-12	.9872558939003E-06	.2487564536135E-02	.4999683582211E+00
7000	94	491	840	.3109897202440E-12	.9133790509662E-06	.2487564200692E-02	.4999889329114E+00
8000	102	527	1027	.2495248995202E-12	.9989861160787E-06	.2487563949142E-02	.4999262175800E+00
9000	165	827	1818	.2309196083838E-12	.9609876800702E-06	.2487563753511E-02	.4999771543752E+00
10000	134	724	1769	.1986289743905E-12	.5208253356941E-06	.2487563597018E-02	.4999438018160E+00
TOTAL	1393	7328	93.54(seconds)				

43 Barzilai-Borwein Algorithm: DIXMAANA (CUTE) Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	8	26	22	.1000000000016E+01	.4389236631501E-06	.1226524313423E+00	.5006185387641E+00
2000	8	26	44	.1000000000032E+01	.4389277123600E-06	.1226664847389E+00	.5005240710587E+00
3000	8	26	66	.1000000000048E+01	.4389582573880E-06	.1226738578097E+00	.5006259767265E+00
4000	8	26	88	.1000000000064E+01	.4389496570462E-06	.1226685008316E+00	.5006241189703E+00
5000	8	26	109	.1000000000080E+01	.4389471517657E-06	.1226709086250E+00	.5005850349003E+00
6000	8	26	132	.1000000000096E+01	.4389582573877E-06	.1226738578097E+00	.5006259767265E+00
7000	8	26	154	.1000000000112E+01	.4389533468463E-06	.1226707966499E+00	.5006249152929E+00
8000	8	26	176	.1000000000128E+01	.4389514917025E-06	.1226720145761E+00	.5006003598983E+00
9000	8	26	192	.1000000000145E+01	.4389582573878E-06	.1226738578097E+00	.5006259767266E+00
10000	8	26	220	.1000000000161E+01	.4389548211122E-06	.1226717149896E+00	.5006252337627E+00
TOTAL	80	260	12.03(seconds)				

44 Barzilai-Borwein Algorithm: DIXMAANB (CUTE) Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	8	28	22	.1000000000000E+01	.2198299308131E-07	.7391208271129E-01	.5146888964808E+00
2000	8	28	44	.1000000000000E+01	.3844169791540E-07	.7390230448043E-01	.5146102121798E+00
3000	8	28	66	.1000000000000E+01	.2221047289758E-07	.7389658145807E-01	.5146702129938E+00
4000	8	28	93	.1000000000000E+01	.2228623822050E-07	.7389325345363E-01	.5146625502075E+00
5000	8	28	115	.1000000000000E+01	.3831525921616E-07	.7389311044947E-01	.5146361940483E+00
6000	8	28	143	.1000000000000E+01	.2229889781155E-07	.7389178234743E-01	.5146618798340E+00
7000	8	28	165	.1000000000000E+01	.2233120175239E-07	.7389056665327E-01	.5146586504525E+00
8000	8	28	187	.1000000000000E+01	.3828329653097E-07	.7389081316785E-01	.5146426565224E+00
9000	8	28	214	.1000000000000E+01	.2232867577458E-07	.7389018313671E-01	.5146590687792E+00
10000	8	28	236	.1000000000000E+01	.2234930458080E-07	.7388949214940E-01	.5146570807162E+00
TOTAL	80	280	12.85(seconds)				

45 Barzilai-Borwein Algorithm: DIXMAANC (CUTE) Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	10	35	27	.1000000000000E+01	.1939323296229E-07	.4182099292620E-01	.5266257949421E+00
2000	10	35	61	.1000000000000E+01	.2906227509227E-07	.4181664044626E-01	.5234796804175E+00
3000	10	35	88	.1000000000000E+01	.3150872735504E-07	.4181388010036E-01	.5230084967313E+00
4000	10	35	121	.1000000000000E+01	.3357800340737E-07	.4181232400358E-01	.5223611398950E+00
5000	10	35	142	.1000000000000E+01	.3648441269072E-07	.4181231787598E-01	.5213770463454E+00
6000	10	35	182	.1000000000000E+01	.3708572148256E-07	.4181165863283E-01	.5213466878491E+00
7000	10	35	203	.1000000000000E+01	.3768211806182E-07	.4181108698380E-01	.5211325363514E+00
8000	10	35	236	.1000000000000E+01	.3920209485797E-07	.4181123779727E-01	.5206010694133E+00
9000	10	35	269	.1000000000000E+01	.3944331261495E-07	.4181091836628E-01	.5206446659608E+00
10000	10	35	297	.1000000000000E+01	.3962996112517E-07	.4181059227362E-01	.5205499188285E+00
TOTAL	100	350	16.26(seconds)				

46 Barzilai-Borwein Algorithm: DIXMAANE (CUTE) Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	288	1460	1296	.1000000000248E+01	.9939017014668E-06	.1361763808234E+00	.4564779476756E+03
2000	425	2335	4152	.1000000000492E+01	.9759981752216E-06	.1362042084316E+00	.8346499794994E+03
3000	557	2852	7619	.1000000000754E+01	.9774855034798E-06	.1361899785649E+00	.9667625847988E+03
4000	641	3299	11765	.1000000001071E+01	.9866844426260E-06	.1361914909896E+00	.1176566457436E+04
5000	639	3500	15582	.1000000001451E+01	.9975780211871E-06	.1361996050150E+00	.1661434023830E+04
6000	697	3594	19257	.1000000001849E+01	.9912052579467E-06	.1361932541177E+00	.1346576358545E+04
7000	627	3509	22009	.1000000002399E+01	.9983954097185E-06	.1361936502754E+00	.1972864556613E+04
8000	683	3604	25760	.1000000003064E+01	.9984319321939E-06	.1361984523389E+00	.1976312093716E+04
9000	556	2966	23870	.1000000003746E+01	.9845734921235E-06	.1361943459512E+00	.2053959022217E+04
10000	684	3575	31956	.1000000004306E+01	.9998110635956E-06	.1361945140381E+00	.1458265835236E+04
TOTAL	5797	30694	1632.66(seconds)				

49 Barzilai-Borwein Algorithm: Almost Perturbed Quadratic Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	1463	7534	61	.2403002032126E-12	.9852296724962E-06	.5037485383435E-03	.4931411093930E+00
2000	2851	15526	406	.2404453074725E-12	.9855736592356E-06	.2514934319056E-03	.4938334153198E+00
3000	4240	22764	1170	.2456560001349E-12	.9962090453775E-06	.1677372037543E-03	.4870008270747E+00
4000	5408	29703	2148	.2443335397880E-12	.9935191028626E-06	.1255189959129E-03	.4897117243606E+00
5000	6451	36437	3361	.2461664108071E-12	.9971927190704E-06	.1003144061171E-03	.4931143925082E+00
6000	7839	43925	5119	.2467342881198E-12	.9980429387803E-06	.8384235248528E-04	.4863881861711E+00
7000	9078	51773	6728	.2474324553625E-12	.9998097718619E-06	.7175479902322E-04	.4933808838119E+00
8000	10685	60868	9124	.2473481209723E-12	.9996381577023E-06	.6269438640655E-04	.4912908974119E+00
9000	11226	64071	10930	.2475698830609E-12	.9999924319577E-06	.5570687069613E-04	.4935759955902E+00
10000	13335	74844	14396	.2457566633313E-12	.9963970324218E-06	.5022927803209E-04	.4928729498487E+00
TOTAL	72576	407445	534.43(seconds)				

50 Barzilai-Borwein Algorithm: Tridiagonal Perturbed Quadratic Function

n	iter	fgcnt	time(c)	fxnew	ginf	stepmin	stepmax
1000	1442	7505	665	.2439313710014E-12	.9871685285403E-06	.4998464960674E-03	.4970418626804E+00
2000	2877	15595	3334	.2465305444490E-12	.9924406495315E-06	.2508236557424E-03	.4962940295818E+00
3000	4330	23251	9870	.2500185355886E-12	.9998319678873E-06	.1672774194999E-03	.4898430538833E+00
4000	5305	30159	18477	.2158499266019E-12	.9948363727272E-06	.1255284836410E-03	.4915172668642E+00
5000	7297	39851	31137	.2477300370630E-12	.9951736583747E-06	.1003993260351E-03	.4953225428596E+00
6000	8006	45554	41826	.2481458351834E-12	.9961934073396E-06	.8360247734854E-04	.4980988857436E+00
7000	9072	51488	54332	.2500229662572E-12	.9999043546163E-06	.7163914139718E-04	.4993705602530E+00
8000	10753	60362	71222	.2496114563014E-12	.9989560869645E-06	.6271072477460E-04	.4956777466304E+00
9000	11663	66566	88041	.2499902504010E-12	.9998654135821E-06	.5576462792020E-04	.4910849138953E+00
10000	13613	75456	107505	.2498120410905E-12	.9992275437975E-06	.5024335086511E-04	.4890979022345E+00
TOTAL	74358	415787	4264.09(seconds)				

Remark.
In contrast with the anticipative scalar approximation of Hessian algorithm, the Barzilai-Borwein algorithm was un able to solve a number of 9 problems considered in this numerical experiments. For all these problems this variant of the Barzilai-Borwein algorithm generated a negative or a very small initial stepsize.

March 21, 2005

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