

# TEST FUNCTIONS FOR UNCONSTRAINED OPTIMIZATION

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1. Extended Freudenstein & Roth:

$$f(x) = \sum_{i=1}^{n/2} \left( -13 + x_{2i-1} + ((5 - x_{2i})x_{2i} - 2)x_{2i} \right)^2 \\ + \left( -29 + x_{2i-1} + ((x_{2i} + 1)x_{2i} - 14)x_{2i} \right)^2, \\ x_0 = [0.5, -2, 0.5, -2, \dots, 0.5, -2].$$

2. Extended Trigonometric:

$$f(x) = \sum_{i=1}^n \left( \left( n - \sum_{j=1}^n \cos x_j \right) + i(1 - \cos x_i) - \sin x_i \right)^2, \\ x_0 = [0.2, 0.2, \dots, 0.2].$$

3. Extended Rosenbrock:

$$f(x) = \sum_{i=1}^{n/2} c(x_{2i} - x_{2i-1}^2)^2 + (1 - x_{2i-1})^2, \\ x_0 = [-1.2, 1, \dots, -1.2, 1]. \\ c = 1000.$$

4. Extended White & Holst:

$$f(x) = \sum_{i=1}^{n/2} c(x_{2i} - x_{2i-1}^3)^2 + (1 - x_{2i-1})^2,$$

$$x_0 = [-1.2, 1, \dots, -1.2, 1],$$

$$c = 1.$$

5. Extended Beale:

$$f(x) = \sum_{i=1}^{n/2} \left( 1.5 - x_{2i-1}(1 - x_{2i}) \right)^2 + \left( 2.25 - x_{2i-1}(1 - x_{2i}^2) \right)^2$$

$$+ \left( 2.625 - x_{2i-1}(1 - x_{2i}^3) \right)^2,$$

$$x_0 = [1, 0.8, \dots, 1, 0.8].$$

6. Extended Penalty:

$$f(x) = \sum_{i=1}^{n-1} (x_i - 1)^2 + \left( \sum_{j=1}^n x_j^2 - 0.25 \right)^2,$$

$$x_0 = [1, 2, \dots, n].$$

7. Perturbed Quadratic:

$$f(x) = \sum_{i=1}^n ix_i^2 + \frac{1}{100} \left( \sum_{i=1}^n x_i \right)^2,$$

$$x_0 = [0.5, 0.5, \dots, 0.5].$$

8. Raydan 1:

$$f(x) = \sum_{i=1}^n \frac{i}{10} (\exp(x_i) - x_i),$$

$$x_0 = [1, 1, \dots, 1].$$

9. Raydan 2:

$$f(x) = \sum_{i=1}^n (\exp(x_i) - x_i),$$

$$x_0 = [1, 1, \dots, 1].$$

10. Summ of quadratics:

$$f(x) = \sum_{i=1}^{n-1} x_i^2 + c(x_{i+1} + x_i)^2,$$

$$x_0 = [1, 1, \dots, 1].$$

11. Diagonal 1:

$$f(x) = \sum_{i=1}^n \left( \exp(x_i) - \frac{x_i}{i} \right),$$

$$x_0 = [1/1, 1/2, \dots, 1/n].$$

12. Diagonal 2:

$$f(x) = \sum_{i=1}^n \left( \exp(x_i) - i \sin(x_i) \right),$$

$$x_0 = [1, 1, \dots, 1].$$

13. Hager:

$$f(x) = \sum_{i=1}^n \left( \exp(x_i) - \sqrt{i} x_i \right),$$

$$x_0 = [1, 1, \dots, 1].$$

14. Generalized Tridiagonal-1:

$$f(x) = \sum_{i=1}^{n-1} \left( x_i + x_{i+1} - 3 \right)^2 + \left( x_i - x_{i+1} + 1 \right)^4,$$

$$x_0 = [2, 2, \dots, 2].$$

15. Extended Tridiagonal-1:

$$f(x) = \sum_{i=1}^{n/2} \left( x_{2i-1} + x_{2i} - 3 \right)^2 + \left( x_{2i-1} - x_{2i} + 1 \right)^4,$$

$$x_0 = [2, 2, \dots, 2].$$

16. Extended Three Exponential:

$$f(x) = \sum_{i=1}^{n/2} \left( \exp(x_{2i-1} + 3x_{2i} - 0.1) + \exp(x_{2i-1} - 3x_{2i} - 0.1) + \exp(-x_{2i-1} - 0.1) \right),$$

$$x_0 = [0.1, 0.1, \dots, 0.1].$$

17. Generalized Tridiagonal-2:

$$f(x) = \left( (5 - 3x_1 - x_1^2)x_1 - 3x_2 + 1 \right)^2 +$$

$$\sum_{i=1}^{n-1} \left( (5 - 3x_i - x_i^2)x_i - x_{i-1} - 3x_{i+1} + 1 \right)^2 + \left( (5 - 3x_n - x_n^2)x_n - x_{n-1} + 1 \right)^2,$$

$$x_0 = [-1, -1, \dots, -1].$$

18. Diagonal3 (1c1c):

$$f(x) = \sum_{i=1}^{n/2} \frac{1}{2} (x_{2i-1}^2 + cx_{2i}^2),$$

$$c = 10000,$$

$$x_0 = [1, 1, \dots, 1].$$

19. Diagonal full bordered:

$$f(x) = (x_1 - 1)^4 + \sum_{i=1}^{n-2} \left( \sin(x_{i+1} - x_n) - x_i^2 - x_{i+1}^2 \right)^2 + (x_n^2 - x_1^2)^2,$$

$$x_0 = [0.1, 0.1, \dots, 0.1].$$

20. Extended Himmelblau:

$$f(x) = \sum_{i=1}^{n/2} \left( x_{2i-1}^2 + x_{2i} - 11 \right)^2 + \left( x_{2i-1} + x_{2i}^2 - 7 \right)^2,$$

$$x_0 = [1, 1, \dots, 1].$$

21. Extended Powell:

$$f(x) = \sum_{i=1}^{n/4} \left( x_{4i-3} + 10x_{4i-2} \right)^2 + 5 \left( x_{4i-1} - x_{4i} \right)^2 +$$

$$\left( x_{4i-2} - 2x_{4i-1} \right)^4 + 10 \left( x_{4i-3} - x_{4i} \right)^4,$$

$$x_0 = [3, -1, 0, 1, \dots, 3, -1, 0, 1].$$

22. Tridiagonal Double Bordered:

$$f(x) = (x_1 - 1)^2 + \sum_{i=1}^{n-1} \left( x_i - \frac{1}{2}x_i^2 - \frac{1}{2}x_{i+1}^2 \right)^2,$$

$$x_0 = [-1, -1, \dots, -1].$$

23. Extended PSC1:

$$f(x) = \sum_{i=1}^{n/2} (x_{2i-1}^2 + x_{2i}^2 + x_{2i-1}x_{2i})^2 + \sin^2(x_{2i-1}) + \cos^2(x_{2i}),$$

$$x_0 = [3,0.1, \dots, 3,0.1].$$

24. Extended Block Diagonal BD1:

$$f(x) = \sum_{i=1}^{n/2} (x_{2i-1}^2 + x_{2i}^2 - 2)^2 + (\exp(x_{2i-1}) - x_{2i})^2,$$

$$x_0 = [0.1,0.1, \dots, 0.1].$$

25. Extended Maratos:

$$f(x) = \sum_{i=1}^{n/2} x_{2i-1} + c(x_{2i-1}^2 + x_{2i}^2 - 1)^2,$$

$$c = 1,$$

$$x_0 = [1.1,0.1, \dots, 1.1,0.1].$$

26. Full Hessian FH1 (Sum of quadratics, Quadratic inside):

$$f(x) = \sum_{i=1}^m \left( \sum_{j=1}^n ijx_j^2 - 1 \right)^2, \quad m = 500,$$

$$x_0 = [1/n, 1/n, \dots, 1/n].$$

27. Extended Cliff:

$$f(x) = \sum_{i=1}^{n/2} \left( \frac{x_{2i-1} - 3}{100} \right)^2 - (x_{2i-1} - x_{2i}) + \exp(20(x_{2i-1} - x_{2i})),$$

$$x_0 = [0, -1, \dots, 0, -1].$$

28. Quadratic Diagonal Perturbed:

$$f(x) = \left( \sum_{i=1}^n x_i \right)^2 + \sum_{i=1}^n \frac{i}{100} x_i^2,$$

$$x_0 = [0.5, 0.5, \dots, 0.5].$$

29. Full Hessian FH2 (Quadratic, perturbed with sin, cos):

$$f(x) = \left( \sum_{i=1}^n x_i \right)^2 + \sum_{i=1}^n \frac{i}{1000} (\sin x_i + \cos x_i),$$

$$x_0 = [1, 1, \dots, 1].$$

30. Full Hessian FH3 (Quartic, perturbed with sin, cos):

$$f(x) = \left( \sum_{i=1}^n x_i^2 \right)^2 + \sum_{i=1}^n \frac{i}{1000} (\sin x_i + \cos x_i),$$

$$x_0 = [1, 1, \dots, 1].$$

31. NONDQUAR (Tridiagonal Double Bounded Arrow-Down):

$$f(x) = (x_1 - x_2)^2 + \sum_{i=1}^{n-2} (x_i + x_{i+1} + x_n)^4 + (x_{n-1} + x_n)^2,$$

$$x_0 = [1, -1, \dots, 1, -1, \dots].$$

32. Tridiagonal White & Holst (c=4):

$$f(x) = \sum_{i=1}^{n-1} c(x_{i+1} - x_i)^2 + (1 - x_i)^2,$$

$$c = 4,$$

$$x_0 = [-1.2, 1, \dots, -1.2, 1].$$

33. Diagonal Double Bounded Arrow Up:

$$f(x) = \sum_{i=1}^n 4(x_i^2 - x_1)^2 + (x_i - 1)^2,$$

$$x_0 = [4, 0, \dots, 4, 0].$$

34. TRIDIA (Tridiagonal):

$$f(x) = \gamma(\delta x_1 - 1)^2 + \sum_{i=2}^n i(\alpha x_i - \beta x_{i-1})^2,$$

$$\alpha = 5 \quad \beta = 1, \quad \gamma = 1, \quad \delta = 1,$$

$$x_0 = [1, 1, \dots, 1].$$

35. ARWHEAD (Diagonal Double Bounded Arrow Down):

$$f(x) = \sum_{i=1}^{n-1} (-4x_i + 3) + \sum_{i=1}^{n-1} (x_i^2 + x_n^2)^2,$$

$$x_0 = [1., 1., \dots, 1.].$$

36. NONDIA (Diagonal Double Bounded Arrow up):

$$f(x) = (x_1 - 1)^2 + c(x_1 - x_1^2)^2 + \sum_{i=2}^n c(x_1 - x_i^2)^2, \quad c = 100,$$

$$x_0 = [-1., -1., \dots, -1.].$$

37. Extended Woods:

$$f(x) = \sum_{i=1}^{n/4} 100(x_{4i-3}^2 - x_{4i-2})^2 + (x_{4i-3} - 1)^2 + 90(x_{4i-1}^2 - x_{4i})^2 + (1 - x_{4i-1})^2 + 10.1\{(x_{4i-2} - 1)^2 + (x_{4i} - 1)^2\} + 19.8(x_{4i-2} - 1)(x_{4i} - 1),$$

$$x_0 = [-3., -1., -3., -1., \dots, -3., -1., -3., -1.].$$

38. Extended Hiebert:

$$f(x) = \sum_{i=1}^{n/2} (x_{2i-1} - c_1)^2 + (x_{2i-1}x_{2i} - c_2)^2,$$

$$c_1 = 10, \quad c_2 = 500,$$

$$x_0 = [0., 0., \dots, 0.].$$

39. BDQRTIC:

$$f(x) = \sum_{i=1}^{n-4} (-4x_i + 3)^2 + (x_i^2 + 2x_{i+1}^2 + 3x_{i+2}^2 + 4x_{i+3}^2 + 5x_n^2)^2,$$

$$x_0 = [1., 1., \dots, 1.].$$

40. DQDRTIC:

$$f(x) = \sum_{i=1}^{n-2} (x_i^2 + cx_{i+1}^2 + dx_{i+2}^2),$$

$$c = 1000, \quad d = 1000,$$

$$x_0 = [3., 3., \dots, 3.].$$

41. EG2:

$$f(x) = \sum_{i=1}^{n-1} \sin(x_1 + x_i^2 - 1) + \frac{1}{2} \sin(x_n^2),$$

$$x_0 = [1, 1, \dots, 1].$$

42. EDENSCH:

$$f(x) = 16 + \sum_{i=1}^{n-1} [(x_i - 2)^4 + (x_i x_{i+1} - 2x_{i+1})^2 + (x_{i+1} + 1)^2],$$

$$x_0 = [0, 0, \dots, 0].$$

43. Broyden Pentadagonal:

$$f(x) = (3x_1 - 2x_1^2)^2 + \sum_{i=2}^{n-1} (3x_i - 2x_i^2 - x_{i-1} - 2x_{i+1} + 1)^2 + (3x_n - 2x_n^2 - x_{n-1} + 1)^2,$$

$$x_0 = [-1, -1, \dots, -1].$$

44. Almost Quadratic Perturbed:

$$f(x) = \sum_{i=1}^n ix_i^2 + \frac{1}{100} (x_1 + x_n)^2,$$

$$x_0 = [0.5, 0.5, \dots, 0.5].$$

45. Almost Quartic Perturbed:

$$f(x) = \sum_{i=1}^n ix_i^4 + \frac{1}{100} (x_1 + x_n)^2,$$

$$x_0 = [0.5, 0.5, \dots, 0.5].$$

46. FLETCHCR:

$$f(x) = \sum_{i=1}^{n-1} c (x_{i+1} - x_i + 1 - x_i^2)^2,$$

$$c = 100,$$

$$x_0 = [0.5, 0.5, \dots, 0.5].$$



47. ENGVAL1:

$$f(x) = \sum_{i=1}^{n-1} (x_i^2 + x_{i+1}^2)^2 + \sum_{i=1}^{n-1} (-4x_i + 3),$$

$$x_0 = [2, 2, \dots, 2].$$

48. DENSCHNA:

$$f(x) = \sum_{i=1}^{n/2} x_{2i-1}^4 + (x_{2i-1} + x_{2i})^2 + (-1 + \exp(x_{2i}))^2,$$

$$x_0 = [8, 8, \dots, 8].$$

49. DENSCHNB:

$$f(x) = \sum_{i=1}^{n/2} (x_{2i-1} - 2)^2 + (x_{2i-1} - 2)^2 x_{2i}^2 + (x_{2i} + 1)^2,$$

$$x_0 = [1000, 1000, \dots, 1000].$$

50. DENSCHNC:

$$f(x) = \sum_{i=1}^{n/2} (-2 + x_{2i-1}^2 + x_{2i}^2)^2 + (-2 + \exp(x_{2i-1} - 1) + x_{2i}^3)^2,$$

$$x_0 = [8, 8, \dots, 8].$$

51. DENSCHNF:

$$f(x) = \sum_{i=1}^{n/2} (2(x_{2i-1} + x_{2i})^2 + (x_{2i-1} - x_{2i})^2 - 8)^2 + (5x_{2i-1}^2 + (x_{2i} - 3)^2 - 9)^2,$$

$$x_0 = [2., 0., 2., 0., \dots, 2., 0.].$$

52. SINQUAD:

$$f(x) = (x_1 - 1)^4 + \sum_{i=1}^{n-2} (\sin(x_{i+1} - x_n) - x_1^2 + x_{i+1}^2)^2 + (x_n^2 - x_1^2)^2,$$

$$x_0 = [0.1, 0.1, \dots, 0.1].$$

53. HIMMELBG:

$$f(x) = \sum_{i=1}^{n/2} (2x_{2i-1}^2 + 3x_{2i}^2) \exp(-x_{2i-1} - x_{2i}),$$

$$x_0 = [1.5, 1.5, \dots, 1.5].$$

54. HIMMELBH:

$$f(x) = \sum_{i=1}^{n/2} (-3x_{2i-1} - 2x_{2i} + 2 + x_{2i-1}^3 + x_{2i}^2),$$

$$x_0 = [1.5, 1.5, \dots, 1.5].$$

55. DIXON3DQ:

$$f(x) = (x_1 - 2)^2 + \sum_{i=1}^{n-1} (x_i - x_{i+1})^2 + (x_n - 1)^2,$$

$$x_0 = [-1, -1, \dots, -1].$$

56. BIGGSB1:

$$f(x) = (x_1 - 1)^2 + \sum_{i=2}^n (x_i - x_{i-1})^2 + (1 - x_n)^2,$$

$$x_0 = [0, 0, \dots, 0].$$

57. Perturbed Quadratic:

$$f(x) = \left( \sum_{i=1}^n ix_i \right)^2 + \sum_{i=1}^n ix_i^2,$$

$$x_0 = [0.5, 0.5, \dots, 0.5].$$

58. Generalized Rosenbrock:

$$f(x) = (x_1 - 1)^2 + \sum_{i=2}^n c(x_i - x_{i-1}^2)^2,$$

$$c = 100,$$

$$x_0 = [-1.2, 1, \dots, -1.2, 1].$$

59. Quadratic Penalty QP1:

$$f(x) = \sum_{i=1}^{n-1} (x_i^2 - 2)^2 + \left( \sum_{i=1}^n x_i^2 - 0.5 \right)^2,$$

$$x_0 = [1, 1, \dots, 1].$$

60. Quadratic Penalty QP2:

$$f(x) = \sum_{i=1}^{n-1} (x_i^2 - \sin x_i)^2 + \left( \sum_{i=1}^n x_i^2 - 100 \right)^2,$$

$$x_0 = [1, 1, \dots, 1].$$

61. Tridiagonal TS1:

$$f(x) = \sum_{i=1}^{n-1} (x_i + x_{i+1} - i)^2,$$

$$x_0 = [1, 1, \dots, 1].$$

62. Tridiagonal TS2:

$$f(x) = \sum_{i=2}^n (x_{i-1} + x_i - i)^2,$$

$$x_0 = [1, 1, \dots, 1].$$

63. Tridiagonal TS3:

$$f(x) = \sum_{i=2}^n (x_{i-1} + x_i + i)^2,$$

$$x_0 = [2, 2, \dots, 2].$$

64. Trigonometric ET2:

$$f(x) = \sum_{i=1}^n \left( \left( n - \sum_{j=1}^n \sin(x_j) \right) + i(1 - \sin(x_i)) - \sin(x_i) \right)^2$$

$$x_0 = [0.2, 0.2, \dots, 0.2].$$

65. QP3:

$$f(x) = (x_1^2 + x_2^2 + \dots + x_n^2 - 0.25)^2 - \sum_{i=1}^{n-1} (x_i^2 - 1)^2,$$

$$x_0 = [1, 1, \dots, 1].$$

66. EG3:

$$f(x) = \sum_{i=1}^{n-1} \cos(x_1 + x_i^2 - 1) + \frac{1}{2} \cos(x_n^2),$$

$$x_0 = [2, 2, \dots, 2].$$

67. GENROSEN-2:

$$f(x) = (x_1 - 1)^2 + \sum_{i=2}^n c(x_{i-1}^2 - x_i)^2,$$

$$c = 100,$$

$$x_0 = [-1.2, 1, \dots, -1.2, 1].$$

68. PRODSin:

$$f(x) = (x_1^2 + x_2^2 + \dots + x_m^2)(\sin(x_1) + \sin(x_2) + \dots + \sin(x_n)),$$

$$m = n - 1,$$

$$x_0 = [5, 5, \dots, 5].$$

69. PROD1:

$$f(x) = (x_1 + x_2 + \dots + x_m)(x_1 + x_2 + \dots + x_n),$$

$$m = n,$$

$$x_0 = [1, 1, \dots, 1].$$

70. PRODCos:

$$f(x) = (x_1^2 + x_2^2 + \dots + x_m^2)(\cos(x_1) + \cos(x_2) + \dots + \cos(x_n)),$$

$$m = n - 1,$$

$$x_0 = [5, 5, \dots, 5].$$

71. PROD2 (m=1):

$$f(x) = (x_1^4 + x_2^4 + \dots + x_m^4)(1x_1 + 2x_2 + \dots + nx_n),$$

$$m = 1,$$

$$x_0 = [15, 15, \dots, 15].$$

72. ARGLINB (m=5):

$$f(x) = \sum_{i=1}^m \left( \sum_{j=1}^n ijx_j - 1 \right)^2,$$

$$m = 5,$$

$$x_0 = [0.01, 0.01, \dots, 0.01].$$

73. DIXMAANA:

$$f(x) = 1 + \sum_{i=1}^n \alpha x_i^2 \left( \frac{i}{n} \right)^{k1} + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 \left( \frac{i}{n} \right)^{k2} +$$

$$\sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 \left( \frac{i}{n} \right)^{k3} + \sum_{i=1}^m \delta x_i x_{i+2m} \left( \frac{i}{n} \right)^{k4},$$

$$\alpha = 1, \quad \beta = 0, \quad \gamma = 0.125, \quad \delta = 0.125,$$

$$k1 = 0, \quad k2 = 0, \quad k3 = 0, \quad k4 = 0,$$

$$m = n/4,$$

$$x_0 = [2, 2, \dots, 2].$$

74. DIXMAANB:

$$f(x) = 1 + \sum_{i=1}^n \alpha x_i^2 \left( \frac{i}{n} \right)^{k1} + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 \left( \frac{i}{n} \right)^{k2} +$$

$$\sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 \left( \frac{i}{n} \right)^{k3} + \sum_{i=1}^m \delta x_i x_{i+2m} \left( \frac{i}{n} \right)^{k4},$$

$$\alpha = 1, \quad \beta = 0.0625, \quad \gamma = 0.0625, \quad \delta = 0.0625,$$

$$k1 = 0, \quad k2 = 0, \quad k3 = 0, \quad k4 = 1,$$

$$m = n/4,$$

$$x_0 = [2, 2, \dots, 2].$$

75. DIXMAANC:

$$f(x) = 1 + \sum_{i=1}^n \alpha x_i^2 \left( \frac{i}{n} \right)^{k1} + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 \left( \frac{i}{n} \right)^{k2} +$$

$$\sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 \left(\frac{i}{n}\right)^{k_3} + \sum_{i=1}^m \delta x_i x_{i+2m} \left(\frac{i}{n}\right)^{k_4},$$

$$\alpha = 1, \quad \beta = 0.125, \quad \gamma = 0.125, \quad \delta = 0.125,$$

$$k_1 = 0, \quad k_2 = 0, \quad k_3 = 0, \quad k_4 = 0,$$

$$m = n/4,$$

$$x_0 = [2, 2, \dots, 2].$$

76. DIXMAAND:

$$f(x) = 1 + \sum_{i=1}^n \alpha x_i^2 \left(\frac{i}{n}\right)^{k_1} + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 \left(\frac{i}{n}\right)^{k_2} +$$

$$\sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 \left(\frac{i}{n}\right)^{k_3} + \sum_{i=1}^m \delta x_i x_{i+2m} \left(\frac{i}{n}\right)^{k_4},$$

$$\alpha = 1, \quad \beta = 0.26, \quad \gamma = 0.26, \quad \delta = 0.26,$$

$$k_1 = 0, \quad k_2 = 0, \quad k_3 = 0, \quad k_4 = 0,$$

$$m = n/4,$$

$$x_0 = [2, 2, \dots, 2].$$

77. DIXMAANL:

$$f(x) = 1 + \sum_{i=1}^n \alpha x_i^2 \left(\frac{i}{n}\right)^{k_1} + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 \left(\frac{i}{n}\right)^{k_2} +$$

$$\sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 \left(\frac{i}{n}\right)^{k_3} + \sum_{i=1}^m \delta x_i x_{i+2m} \left(\frac{i}{n}\right)^{k_4},$$

$$\alpha = 1, \quad \beta = 0.26, \quad \gamma = 0.26, \quad \delta = 0.26,$$

$$k_1 = 2, \quad k_2 = 0, \quad k_3 = 0, \quad k_4 = 2,$$

$$m = n/4,$$

$$x_0 = [2, 2, \dots, 2].$$

78. VARDIM:

$$f(x) = \sum_{i=1}^n (x_i - 1)^2 + \left( \sum_{i=1}^n i x_i - \frac{n(n+1)}{2} \right)^2 + \left( \sum_{i=1}^n i x_i - \frac{n(n+1)}{2} \right)^4,$$

$$x_0 = \left[ 1 - \frac{1}{n}, 1 - \frac{2}{n}, \dots, 1 - \frac{n}{n} \right].$$

79. DIAG-AUP1:

$$f(x) = \sum_{i=1}^n 4(-x_1 + x_i^2)^2 + \sum_{i=1}^n (x_i^2 - 1)^2,$$

$$x_0 = [4, 4, \dots, 4].$$

80. ENGVAL8:

$$f(x) = \sum_{i=1}^{n-1} (x_i^2 + x_{i+1}^2)^2 - (7 - 8x_i),$$

$$x_0 = [2, 2, \dots, 2].$$

