# RETAILER'S OPTIMAL ORDERING POLICY UNDER TWO STAGE TRADE CREDIT FINANCING 

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#### Abstract

Usually it is assumed that the supplier would offer a fixed credit period to the retailer but the retailer in turn would not offer any credit period to its customers, which is unrealistic, because in real practice retailer might offer a credit period to its customers in order to stimulate his own demand. Moreover, it is observed that credit period offered by the retailer to its customers has a positive impact on demand of an item but the impact of credit period on demand has received a very little attention by the researchers.

To incorporate this phenomenon, this paper develops an inventory model under two levels of trade credit policy by assuming the demand is a function of credit period offered by the retailer to the customers using discounted cash flow (DCF) approach. A DCF approach permits a proper recognition of the exact timing of cash flows associated with an inventory system under the trade credit. A theorem is then developed to determine the optimal replenishment policy for the retailer. Finally, numerical example is presented to illustrate the theoretical results followed by the sensitivity of parameters on the optimal solution.


Keywords: Inventory, credit-linked demand, two level credit policy, delay in payments, discounted cash flow

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## 1. INTRODUCTION

The traditional economic order quantity (EOQ) model assumes that the retailer must pay for the items as soon as he receives the items. However, this may not be true. In practice, usually the supplier offers the retailer a delay period for settling the account. Before the end of that period, the retailer can sell the goods and accumulate revenue and earn interest. An interest is charged if the retailer fails to settle the account by the end of the delay period. Owing to this fact, during the past few years, many articles dealing with various inventory models under trade credit have appeared in various research journals.

Haley and Higgins [6] introduced the first model to consider the economic order quantity under conditions of permissible delay in payments with deterministic demand, no shortages, and zero-lead time. Goyal [5] considered a model similar to that of Haley and Higgins with the exclusion of the penalty cost due to a late payment. Chung [3] presented the discounted cash flows (DCF) approach for the analysis of the optimal inventory policy in the presence of the trade credit. Shah [11] and Aggarwal and Jaggi [1] extended the Goyal's model to the case of deterioration. Jamal et al. [10] further generalized the model to allow for shortages. Jaggi and Aggarwal [9] extended Chung [3] to develop an inventory model for obtaining the optimal order quantity of deteriorating items in the presence of trade credit using the DCF approach. Hwang and Shinn [8] considered the problem of determining the retailer's optimal price and lot size simultaneously when the supplier permits delay in payments. Dye [4] in their paper considered the stock dependent demand for deteriorating items for partial backlogging and condition of permissible delay in payment. They assumed initial stock dependent demand function. Teng [12] provided an alternative conclusion from Goyal [5], and mathematically proved that it makes economic sense for a buyer to order less quantity and take benefits of the permissible delay more frequently. Chang, Hung and Dye [2] considered an inventory model for deteriorating items with instantaneous stockdependent demand and time-value of money when credit period is provided.

All the above articles implicitly assumed that the customer would pay for the items as soon as the items are received from the retailer. That is, they assumed that the supplier would offer the retailer a delay period but the retailer would not offer any delay
period to his/her customer. In most business transactions, this assumption is unrealistic. Huang [11] presented an inventory model assuming that the retailer also permits a credit period to its customer which is shorter than the credit period offered by the supplier, in order to stimulate the demand.

Moreover, in all the above articles, although the presence of credit period has been incorporated in the mathematical models but the impact of credit period on demand is unfortunately ignored. In reality, it is observed that demand of an item does depend upon the length of the credit period offered by the retailer. In order to incorporate the above phenomena, a demand function has been coined using which an inventory model has been formulated to determine the retailer's optimal replenishment policy when both the supplier as well as the retailer offers the credit period to stimulate customer demand using discounted cash flow (DCF) approach. A DCF approach permits a proper recognition of the financial implication of the opportunity cost and out-of-pocket costs in inventory analysis. It also permits an explicit recognition of the exact timing of cash flows associated with an inventory system under the trade credit.

## 2. ASSUMPTIONS AND NOTATIONS

## Assumptions:

1. The supplier provides a fixed credit period $M$ to settle the account to the retailer and retailer, in turn, passes on a maximum credit period $N$ to its customers to settle the account. For simplicity, it is assumed that that the customer's credit period $N$ is less than or equal to the retailer's credit period $M$. It is also assumed that the customers would settle their accounts only on the last day of the credit period $N$.
2. The annual demand rate consists of (i) regular cash-demand and (ii) creditdemand. Hence, demand function at any time $t$ can be represented as

$$
D(t)= \begin{cases}\lambda+R(t) & 0 \leq t \leq N \\ \lambda & N \leq t \leq T\end{cases}
$$

where $\lambda$ is known, constant and uniform regular cash-demand rate during the cycle $(0, T)$ and $R(t)$ is the credit-demand rate during the customer's credit period
$N$; we assume $R(t)=\alpha(N-t)$, where $0 \leq t \leq N$ and $\alpha$ is the rate of change of credit-demand which can be estimated from the past data.
3. Replenishment rate is instantaneous.
4. Shortages are not allowed.
5. Lead-time is negligible.
6. The model is for only one item.
7. A discounted cash flow (DCF) approach is used to consider the various costs at various times.

## Notations:

$q(t)$ : the inventory level at any time $t$
$Q$ : the order quantity
$A \quad:$ the ordering cost per order at time zero
$C$ : the unit purchase cost of the item at time zero
$P \quad:$ the unit selling price of the item at time zero
$I$ : out-of-pocket inventory carrying charge per \$ per year
$r \quad$ : opportunity cost (discount rate) per year.
$I_{e} \quad:$ the interest that can be earned per \$ per year
$I_{p} \quad$ : the interest charges payable per \$ per year $\left(I_{p}>I_{e}\right)$
$M \quad$ : credit period offered to retailer by the supplier for settling the accounts
$N \quad$ : credit period granted by the retailer to his/her consumers, also $N \leq M$
$T$ : the inventory cycle length in years
$Z(T) \quad$ : annual net profit

## 3. MODEL FORMULATION

As the demand function is

$$
D(t)=\left\{\begin{array}{lr}
\lambda+\alpha(N-t) & 0 \leq t \leq N \\
\lambda & N \leq t \leq T
\end{array}\right.
$$

therefore, the order quantity is

$$
Q=\int_{0}^{T} D(t) d t
$$

$$
\begin{gather*}
=Q_{1}+Q_{2}=\lambda T+\frac{\alpha}{2} N^{2}  \tag{1}\\
\text { where } Q_{1}=\int_{0}^{N}\{\lambda+\alpha(N-t)\} d t=\lambda N+\frac{\alpha}{2} N^{2} \\
Q_{2}=\int_{N}^{T} \lambda d t=\lambda(T-N)
\end{gather*}
$$

and the inventory level at any time $t$ during the cycle is (figure 1 )

$$
\begin{align*}
q(t) & = \begin{cases}q_{1}(t)=Q-\int_{0}^{t}\{\lambda+\alpha(N-t)\} d t & 0 \leq t \leq N \\
q_{2}(t)=Q_{2}-\int_{N}^{t} \lambda d t & N \leq t \leq T\end{cases} \\
& = \begin{cases}q_{1}(t)=\lambda(T-t)+\alpha(N-t)^{2} / 2 & 0 \leq t \leq N \\
q_{2}(t)=\lambda(T-t) & N \leq t \leq T\end{cases} \tag{2}
\end{align*}
$$



Figure 1

The present worth of retailer's annual net profit is calculated as follows:
Net Profit, $Z(T)=$ sales revenue + interest earned - purchase cost - ordering cost inventory carrying cost (out-of-pocket) - interest payable.

By using the discounted cash flow approach, the present worth of various components of the retailer's net profit is calculated as follows:

1. The present worth of the sales revenue is

$$
\begin{align*}
& =\frac{P}{T}\left\{\int_{0}^{T} \lambda e^{-r t} d t+e^{-r N} \int_{0}^{N} R(t) d t\right\} \\
& =\frac{P}{T}\left\{\frac{\lambda}{r}\left(1-e^{-r T}\right)+\frac{\alpha N^{2}}{2} e^{-r N}\right\} \tag{3}
\end{align*}
$$

2. The present worth of cost of placing orders

$$
\begin{equation*}
=A / T \tag{4}
\end{equation*}
$$

3. The present worth of cost of purchasing is

$$
\begin{equation*}
=\frac{C Q}{T}=\frac{C}{T}\left(\lambda T+\frac{\alpha N^{2}}{2}\right) \tag{5}
\end{equation*}
$$

4. The present worth of cost of out-of-pocket inventory carrying is

$$
\begin{align*}
& =\frac{I C}{T}\left(\int_{0}^{N} q_{1}(t) e^{-r t} d t+\int_{N}^{T} q_{2}(t) e^{-r t} d t\right) \\
& =\frac{I C \lambda}{T r^{2}}\left(e^{-r T}+r T-1\right)+\frac{I C \alpha}{T r^{3}}\left(1-e^{-r N}-r N+\frac{r^{2} N^{2}}{2}\right) \tag{6}
\end{align*}
$$

The computation for interest earned and payable will depend on the following two possible cases based on the values of $T$ and $M$ :

## Case1. $M \leq T$

In this case, the retailer deposits the accumulated revenue from cash sales during the period $(0, M)$ and also from credit sales during the time period $(N, M)$ into an account that earns an interest rate of $I_{e}$. At $M$ the accounts have to be settled, it is assumed that accounts will be settled by proceeds of sells generated up to $M$ and by taking a short term loan at an interest rate of $I_{p}$ for the duration of $(T-M)$ for financing the unsold stock.
5. Consequently, the present worth of interest earned is

$$
=\frac{I_{e} P}{T}\left\{\int_{0}^{M} \lambda t e^{-r t} d t+\int_{N}^{M} \frac{\alpha N^{2}}{2} e^{-r t} d t\right\}
$$

$$
\begin{equation*}
=\frac{I_{e} P}{T}\left\{\lambda\left(\frac{1-e^{-r M}-r M e^{-r M}}{r^{2}}\right)+\alpha N^{2}\left(\frac{e^{-r N}-e^{-r M}}{2 r}\right)\right\} \tag{7}
\end{equation*}
$$

6. And the present worth of the interest payable is

$$
\begin{equation*}
=\frac{I_{p} C}{T} \int_{M}^{T} q_{2}(t) e^{-r t} d t=\frac{I_{p} C \lambda}{T}\left(\frac{e^{-r M}(T-M)}{r}+\frac{e^{-r T}-e^{-r M}}{r^{2}}\right) \tag{8}
\end{equation*}
$$

Using the equations (3) to (8), the present worth of retailer's annual net profit, $Z_{l}(T)$ can be expressed as

$$
\begin{align*}
Z_{1}(T)=\frac{P}{T}\{ & \left.\left\{\frac{\lambda}{r}\left(1-e^{-r T}\right)+\frac{\alpha N^{2}}{2} e^{-r N}\right)+I_{e} \lambda\left(\frac{1-e^{-r M}-r M e^{-r M}}{r^{2}}\right)+I_{e} \alpha N^{2}\left(\frac{e^{-r N}-e^{-r M}}{2 r}\right)\right\} \\
- & \frac{1}{T}\left\{A+C\left(\lambda T+\frac{\alpha N^{2}}{2}\right)+\frac{I C \lambda}{r^{2}}\left(e^{-r T}+r T-1\right)+\frac{I C \alpha}{r^{3}}\left(1-e^{-r N}-r N+\frac{r^{2} N^{2}}{2}\right)\right.  \tag{9}\\
& \left.+I_{p} C \lambda\left(\frac{e^{-r M}(T-M)}{r}+\frac{e^{-r T}-e^{-r M}}{r^{2}}\right)\right\}
\end{align*}
$$

## Case 2. $M \geq T$

In this case the credit period $M$ is more or equal to the cycle $T$, so the retailer earns interest on cash sales during the period $(0, M)$ and also on credit sales during the time period $(N, M)$ and pays no interest for the items kept in stock.
5. The present worth of the interest earned is

$$
\begin{align*}
& =\frac{I_{e} P}{T}\left\{\int_{0}^{T} \lambda t e^{-r t} d t+\int_{T}^{M} \lambda T e^{-r t} d t+\int_{N}^{M} \frac{\alpha N^{2}}{2} e^{-r t} d t\right\} \\
& =\frac{I_{e} P}{T}\left\{\lambda\left(\frac{1-e^{-r T}-r T e^{-r M}}{r^{2}}\right)+\alpha N^{2}\left(\frac{e^{-r N}-e^{-r M}}{2 r}\right)\right\} \tag{10}
\end{align*}
$$

Hence, using the equations (3), (4), (5), (6) and (10) the present worth of retailer's annual net profit, $Z_{2}(T)$ in this case is

$$
\begin{align*}
Z_{2}(T)= & \frac{P}{T}\left\{\left(\frac{\lambda}{r}\left(1-e^{-r T}\right)+\frac{\alpha N^{2}}{2} e^{-r N}\right)+I_{e} \lambda\left(\frac{1-e^{-r T}-r T e^{-r M}}{r^{2}}\right)+I_{e} \alpha N^{2}\left(\frac{e^{-r N}-e^{-r M}}{2 r}\right)\right\}  \tag{11}\\
& -\frac{1}{T}\left\{A+C\left(\lambda T+\frac{\alpha N^{2}}{2}\right)+\frac{I C \lambda}{r^{2}}\left(e^{-r T}+r T-1\right)+\frac{I C \alpha}{r^{3}}\left(1-e^{-r N}-r N+\frac{r^{2} N^{2}}{2}\right)\right\}
\end{align*}
$$

Combining the above two cases, the present worth of retailer's annual net profit, $Z(T)$ can be expressed as

$$
Z(T)=\left\{\begin{array}{lcc}
Z_{1}(T) & \text { if } & M \leq T  \tag{12}\\
Z_{2}(T) & \text { if } & M \geq T
\end{array}\right.
$$

Our problem is to determine the optimum value of $T$ which maximizes $Z(T)$. Since $Z_{1}(M)$ $=Z_{2}(M)$, by taking the first and second order derivatives of $Z_{1}(T)$ and $Z_{2}(T)$ with respect to $T$, we get

$$
\begin{align*}
Z_{1}^{\prime}(T)=\frac{1}{T^{2}} & {\left[-P\left(\frac{\lambda}{r}\left(1-e^{-r T}(1+r T)\right)\right)-P I_{e} \lambda\left(\frac{1-e^{-r M}-r M e^{-r M}}{r^{2}}\right)+\frac{I C \lambda}{r^{2}}\left(e^{-r T}(1+r T)-1\right)\right.} \\
& +A+\frac{I C \alpha}{r^{3}}\left(1-e^{-r N}-r N+\frac{r^{2} N^{2}}{2}\right)+I_{p} C \lambda\left(\frac{e^{-r T}-e^{-r M}}{r^{2}}-\frac{M e^{-r M}-T e^{-r T}}{r}\right)  \tag{13}\\
& \left.-\frac{\alpha N^{2}}{2}\left(P e^{-r N}+P I_{e}\left(\frac{e^{-r N}-e^{-r M}}{r}\right)-C\right)\right] \\
Z_{2}^{\prime}(T)= & \frac{1}{T^{2}}\left[-P\left(\frac{\lambda}{r}\left(1-e^{-r T}(1+r T)\right)\right)-P I_{e} \lambda\left(\frac{1-e^{-r T}-r T e^{-r T}}{r^{2}}\right)+\frac{I C \lambda}{r^{2}}\left(e^{-r T}(1+r T)-1\right)\right.  \tag{14}\\
& \left.+A+\frac{I C \alpha}{r^{3}}\left(1-e^{-r N}-r N+\frac{r^{2} N^{2}}{2}\right)-\frac{\alpha N^{2}}{2}\left(P e^{-r N}+P I_{e}\left(\frac{e^{-r N}-e^{-r M}}{r}\right)-C\right)\right] \\
Z_{1}^{\prime \prime}(T)= & \frac{2}{T^{3}}\left[\frac{P \lambda}{r}\left(1-e^{-r T}\left(1+r T+r^{2} T^{2} / 2\right)\right)+\frac{\alpha N^{2}}{2}\left(P e^{-r N}+P I_{e}\left(\frac{e^{-r N}-e^{-r M}}{r}\right)-C\right)\right. \\
& +P I_{e} \lambda\left(\frac{1-e^{-r M}-r M e^{-r M}}{r^{2}}\right)-A+\frac{I C \lambda}{r^{2}}\left(1-e^{-r T}\left(1+r T+r^{2} T^{2} / 2\right)\right)  \tag{15}\\
- & \left.\frac{I C \alpha}{r^{3}}\left(1-r N+r^{2} N^{2} / 2-e^{-r N}\right)-\frac{I_{p} C \lambda}{r^{2}}\left(e^{-r T}\left(1+r T+r^{2} T^{2} / 2\right)-e^{-r M}(1+r M)\right)\right]
\end{align*}
$$

and

$$
\begin{align*}
Z_{2}^{\prime \prime}(T)= & \frac{2}{T^{3}}\left[\frac{P \lambda}{r}\left(1-e^{-r T}\left(1+r T+r^{2} T^{2} / 2\right)\right)+\frac{\alpha N^{2}}{2}\left(P e^{-r N}+P I_{e}\left(\frac{e^{-r N}-e^{-r M}}{r}\right)-C\right)\right. \\
& +P I_{e} \lambda\left(\frac{1-e^{-r T}+r T e^{-r M}}{r^{2}}\right)-A+\frac{I C \lambda}{r^{2}}\left(1-e^{-r T}\left(1+r T+r^{2} T^{2} / 2\right)\right)  \tag{16}\\
& \left.-\frac{I C \alpha}{r^{3}}\left(1-r N+r^{2} N^{2} / 2-e^{-r N}\right)\right]
\end{align*}
$$

Since, it is very difficult to handle above equations for finding the exact value of $T$, therefore, we make use of the second order approximations for the exponentials in equations (13) to (16), which follows as:

$$
\begin{aligned}
& e^{-r T}=1-r T+(r T)^{2} / 2 \\
& e^{-r N}=1-r N+(r N)^{2} / 2
\end{aligned}
$$

and

$$
e^{-r M}=1-r M+(r M)^{2} / 2
$$

Hence, equations (13) to (16) reduces to

$$
\begin{align*}
Z_{1}^{\prime}(T) & \approx \frac{2 A+\left(I_{p} C-I_{e} P\right) \lambda M^{2}-\alpha N^{2} S-\lambda T^{2}\left(r P+I C+I_{p} C\right)}{2 T^{2}}  \tag{17}\\
Z_{2}^{\prime}(T) & \approx \frac{2 A-\alpha N^{2} S-\lambda T^{2}\left(r P+I C+I_{e} P\right)}{2 T^{2}}  \tag{18}\\
Z_{1}^{\prime \prime}(T) & \approx \frac{-2 A-\left(I_{p} C-I_{e} P\right) \lambda M^{2}+\alpha N^{2} S}{T^{3}} \tag{19}
\end{align*}
$$

and $Z_{2}^{\prime \prime}(T) \approx \frac{-2 A+\alpha N^{2} S}{T^{3}}$
where $S=\left[P\left\{1-r N+\frac{(r N)^{2}}{2}+I_{e}\left(M-N-\frac{r}{2}\left(M^{2}-N^{2}\right)\right)\right\}-C\right]$
Consequently, $Z_{l}(T)$ is strictly concave on $T>0$ if

$$
\begin{equation*}
\alpha<\left\{2 A+\left(I_{p} C-I_{e} P\right) \lambda M^{2}\right\} / N^{2} S \tag{21}
\end{equation*}
$$

and $Z_{2}(T)$ is strictly concave on $T>0$ if

$$
\begin{equation*}
\alpha<2 A / N^{2} S \tag{22}
\end{equation*}
$$

Thus, there exists a unique value of $T_{1}$ which maximizes $Z_{1}(T)$ as

$$
\begin{equation*}
T_{1}^{*}=\sqrt{\frac{2 A+\left(I_{p} C-I_{e} P\right) \lambda M^{2}-\alpha N^{2} S}{\lambda\left[r P+\left(I+I_{p}\right) C\right]}} \tag{23}
\end{equation*}
$$

$T_{1}$ would satisfy the condition $M \leq T_{1}$ provided

$$
\begin{equation*}
2 A-\alpha N^{2} S \geq\left[I C+I_{e} P+r P\right] \lambda M^{2} \tag{24}
\end{equation*}
$$

or $\quad \alpha \leq \alpha_{1}^{U} \quad$ where $\alpha_{1}^{U}=\left\{2 A-\left(I C+I_{e} P+r P\right) \lambda M^{2}\right\} / N^{2} S$
Substituting (23) into (1) and (9), we can get the optimal values of $Q$ and $Z_{1}(T)$. Hence, the optimal order quantity for casel (i.e. $M \leq T$ ) is

$$
\begin{equation*}
Q^{*}\left(T_{1}^{*}\right)=\sqrt{\frac{2 A \lambda+\left(I_{p} C-I_{e} P\right) \lambda^{2} M^{2}-\alpha \lambda N^{2} S}{\left[r P+\left(I+I_{p}\right) C\right]}}+\frac{\alpha N^{2}}{2} \tag{25}
\end{equation*}
$$

Similarly, there exists a unique value of $T_{2}$ which maximizes $Z_{2}(T)$ as

$$
\begin{equation*}
T_{2}^{*}=\sqrt{\frac{2 A-\alpha N^{2} S}{\lambda\left[\left(r+I_{e}\right) P+I C\right]}} \tag{26}
\end{equation*}
$$

$T_{2}$ would satisfy the condition $N \leq T_{2} \leq M$ provided

$$
\begin{gathered}
{\left[I C+I_{e} P+r P\right] \lambda N^{2} \leq\left(2 A-\alpha N^{2} S\right) \leq\left[I C+I_{e} P+r P\right] \lambda M^{2}} \\
\text { or } \quad \alpha_{1}^{U} \leq \alpha \leq \alpha_{2}^{U} \quad \text { where } \quad \alpha_{2}^{U}=\left\{2 A-\left(I C+I_{e} P+r P\right) \lambda N^{2}\right\} / N^{2} S .
\end{gathered}
$$

Substituting equation (26) into (1) and (11), we can get the optimal values of $Q$ and $Z_{2}(T)$. Hence, the optimal order quantity for case 2 (i.e. $T \leq M$ ) is

$$
\begin{equation*}
Q^{*}\left(T_{2}^{*}\right)=\sqrt{\frac{2 A \lambda-\alpha \lambda N^{2} S}{\left[\left(r+I_{e}\right) P+I C\right]}}+\frac{\alpha N^{2}}{2} \tag{28}
\end{equation*}
$$

Combining the two possible cases, we obtain the following theorem.

## Theorem 1.

(a) If $\alpha<\alpha_{1}^{U}$ then $T^{*}=T_{1}^{*}$.
(b) If $\alpha_{1}^{U}<\alpha<\alpha_{2}^{U}$ then $T^{*}=T_{2}^{*}$.
(c) If $\alpha=\alpha_{1}^{U}$ then $T^{*}=M$.
(d) If $\alpha=\alpha_{2}^{U}$ then $T^{*}=N$.

Proof. It immediately follows from (21), (22), (24) and (27).

For the optimal ordering policy, calculate bounds on $\alpha$ i.e. $\alpha_{1}^{U}$ and $\alpha_{2}^{U}$, then determine the optimal cycle time for the retailer by using Theorem 1 .

## 4. NUMERICAL EXAMPLE AND OBSERVATIONS

Given $\lambda=1000$ units/year, $A=\$ 500 /$ order, $M=30$ days ( 0.0822 year), $N=10$ days (0.0274 year) $C=\$ 50 /$ unit, $P=\$ 60 /$ unit, $I_{p}=14 \%, I_{e}=9 \%, I=15 \%$ and $r=13 \%$.

First, we calculate bounds on $\alpha$ i.e. $\alpha_{1}^{U}=114595$ and $\alpha_{2}^{U}=131155$, then we calculate optimal cycle length $\left(T^{*}\right)$, order quantity $\left(Q^{*}\right)$ and annual profit ( $Z^{*}$ ) for different values of $\alpha$ using Theorem 1 and results are summarized in Table1.

| $\alpha$ | Theorem | $T^{*}$ (days) | $Q^{*}$ (units) | $Z\left(T^{*}\right)$ (\$) |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1(a) | $T_{1}^{*}=77.7$ | 212.9 | 5846 |
| 500 | 1(a) | $T_{1}^{*}=77.6$ | 212.7 | 5855 |
| 5000 | 1(a) | $T_{1}^{*}=76.2$ | 210.8 | 5935 |
| 10000 | 1(a) | $T_{1}^{*}=74.7$ | 208.5 | 6025 |
| 20000 | 1(a) | $T_{1}^{*}=71.7$ | 203.8 | 6210 |
| 30000 | 1(a) | $T_{1}^{*}=68.4$ | 198.7 | 6404 |
| 114595( $\alpha_{1}^{U}$ ) | 1(c) | $\mathrm{M}=30.0$ | 125.2 | 8749 |
| 120000 | 1(b) | $T_{2}^{*}=25.3$ | 114.3 | 9018 |
| $131155\left(\alpha_{2}^{U}\right)$ | 1(d) | $\mathrm{N}=10.0$ | 76.6 | 9896 |

The results clearly indicate that as $\alpha$ increases and approaches to $\alpha_{2}^{U}$, cycle length decreases while profit increases.

Further, the sensitivity analysis on $M$ and $N$ for $\alpha=10,000$ is shown in Table 2 for the following three cases:
casel. when $I_{e} P<I_{p} C$ (assuming $P=60, C=50, I_{p}=14 \%, I_{e}=9 \%$ ), case 2 . when $I_{e} P>I_{p} C$ (assuming $P=75, C=50, I_{p}=14 \%, I_{e}=12 \%$ ) and case3. when $I_{e} P=I_{p} C$ (assuming $P=70, C=50, I_{p}=14 \%, I_{e}=10 \%$ ).

| $M$ <br> (days) <br> (days) | $T^{*}$ (days) | $Q^{*}($ units $)$ | $Z\left(T^{*}\right)(\$)$ | $T^{*}$ (days) | $Q^{*}($ units $)$ | $Z\left(T^{*}\right)$ | $(\$)$ | $T^{*}($ days $)$ | $Q^{*}($ units $)$ | $Z\left(T^{*}\right)(\$)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 77.3 | 211.8 | 5299 | 74.1 | 203.1 | 20097 | 75.1 | 205.8 | 15164 |
| 30 | 0 | 77.7 | 212.9 | 5846 | 73.6 | 201.7 | 20702 | 75.1 | 205.8 | 15736 |
|  | 10 | 74.8 | 208.6 | 6025 | 66.2 | 185.1 | 21191 | 69.2 | 193.4 | 16113 |
|  | 20 | 65.5 | 194.6 | 6572 | 36.7 | 115.4 | 23122 | 48 | 146.6 | 17460 |
|  | 30 | 47.5 | 164.0 | 7623 | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ |
| 45 | 0 | 78.2 | 214.3 | 6099 | 73.0 | 200.0 | 21027 | 75.1 | 205.8 | 16020 |
|  | 10 | 75.2 | 209.8 | 6280 | 65.4 | 182.8 | 21528 | 69.1 | 193.2 | 16403 |
|  | 20 | 65.9 | 195.4 | 6838 | 35.6 | 112.5 | 23538 | 47.5 | 145.2 | 17776 |
|  | 30 | 47.5 | 163.8 | 7912 | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ |
|  | 40 | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ |
| 60 | 0 | 78.9 | 216.3 | 6339 | 72.1 | 197.5 | 21367 | 75.1 | 205.8 | 16302 |
|  | 10 | 76.1 | 212.1 | 6522 | 64.2 | 179.8 | 21883 | 69.1 | 192.9 | 16691 |
|  | 20 | 66.6 | 197.5 | 7087 | 35.6 | 112.5 | 23957 | 47.5 | 145.1 | 18090 |
|  | 30 | 45.7 | 159 | 8184 | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ |
|  | 40 | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ | $* *$ |

Table 2. Sensitivity on $N$ and $M(N \leq M)$
From the Table 2, it is quite clear that as the $M$ increases for any fixed $N$, annual net profit increases for all the three cases but both the cycle length ( T ) and order quantity (Q) increases for case1, decreases for case 2 and remains almost constant for the case 3 while as the $N$ increases for any fixed $M$, cycle length and order quantity decreases while annual net profit increases for all the three cases.

Numerical Results suggest that

- if $I_{e} P<I_{p} C$ and only the supplier offers credit period then retailer should order more quantity and if the retailer is also offering credit period, which is obviously less than the total available credit period from the supplier, then he should order less quantity than the usual order quantity.
- if $I_{e} P>I_{p} C$, then the retailer should order less quantity for availing the benefit of credit period more frequently.
- If $I_{e} P=I_{p} C$, then the retailer should order the usual order quantity i.e there is no effect of credit periods $M$ or $N$ on ordering policy.


## 5. CONCLUSION

This paper determines the retailer's optimal ordering policy under two-stage trade credit financing using discounted cash flow ( $D C F$ ) approach. The demand is also assumed to be dependent on the credit period offered by the retailer to its customers. A theorem is proposed which gives the decision rule for obtaining the optimal cycle length. Finally a numerical example is presented to illustrate the theoretical results followed by the sensitivity analysis on model parameters for the three different situations. Results suggest that retailer should order less quantity and take the benefits of the delay in payments more frequently which yields in more annual profit.

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