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# LOT STREAMING IN A MULTISTAGE FLOW SHOP WITH A RANDOM PRODUCT LIFE CYCLE CONSIDERING SHORTAGES

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**Abstract**

For several decades, the lot streaming technique used to split a processing batch into several transfer batches has received much attention from researchers. All these papers have been made in infinite planing horizon without considering shortages. This paper extends the lot streaming technique by considering shortages and taking into account random life cycle of the product instead of deterministic life cycle . We have developed a simulation algorithm to find the optimal solution of our model. We have used this algorithm for normal as well as poisson distribution. Finally we have carried out sensitive analysis to check the effect of decision variable for changes in different parameters.

**INTRODUCTION** Today, in the age of time-based competition, reduction of manufacturing lead time plays a very important role of gaining competitive advantages in any kind of business. Lot streaming is an appealing concept in production management with almost universal application in multistage manufacturing systems. It is a procedure in which a large production lot is split into a smaller sub-lots and each sublot is processed serially by a given number of operations. In this way, several operations in different stages can be performed simultaneously, thereby accelerating production. The customer's demand is then satisfied by the output of the final production stage. A major benefit of this procedure for splitting a processing batch and overlapping operations in different stages is the reduction in manufacturing cycle time and inventories for an item, which requires several operations to be performed in a specific order. Due to this reason, lot streaming and its variations have received much attention from researchers.

The literature on inventory modeling is extensive and several studies have explored the concept related to the recognition of Work in Process(WIP)inventories, lot splitting, and overlapping of operations in multistage manufacturing system, under several different assumptions and in different contexts. In this section, we provide an overview of a small and representative sample of related studies to facilitate the positioning of our paper appropriately. We find it useful to recognize two main stream of studies: one dealing with time models and the other addressing the cost models. The studies of time models have been concerned with the time-related performance measures such as the mean flow time or the makespan time. In this area, Jacobs and Bragg [1] developed a concept of repetitive lots, in which a lot is divided into equal sublots by using simulation model. They show that the shop flow times could be significantly reduced when a simple form of lot streaming is used. Kropp and Smunt[2] investigated optimal lot splitting policies in a deterministic multi-process flow shop using a quadratic programming approach to determine the optimal sublot sizes that minimizes the manufacturing cycle time. Using predetermined production lot size and the number of sublots,they examined different lot splitting heuristic with respect to the scheduling measures, such as the total make-span and mean flow time. Baker and Pyke[3] suggested a computationally efficient algorithm for minimizing the manufacturing cycle time. They also developed several heuristic

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approaches to handle more than two sublots in flow shops using the technique of network analysis. Vickson and Alfredson[4] developed an exact scheduling algorithm for makespan minimization in two-machine flow shops and specially three machine flow shops with equal sized transfer batches. Trietch and Baker[5] presented an overview of basic time models and their solution procedure. Chen and Steiner[6] discussed a technique to minimize the makespan time with the detached and attached set up times in three-machine flow shops. Kalir and Sarin[7] suggested algorithms to find optimal number of sublot where the set up has an impact on the makespan time. Multiple-product lot streaming problems are more complex than the classical time models which do not consider splitting or overlapping. All the previous studies on optimizing lot streaming models for multiple-product problems were based on not more than three machine in flow shop.

Another part of the literature deals with cost models. The objective of solving a cost model is to determine the optimal processing lot size and the optimal number of sublots that minimize the total cost. In this field,Taha and Skeith[8] recognized the relationship between manufacturing cycle time and the cost of holding WIP inventories in developing a model for a single-product multistage production system; however they have not considered lot splitting. Szendrovits[9] first studied a cost model in which a constant lot size is produced through several operations with only one set up at each stage, but allowing transportation of sublots and overlapping of operations to reduce the manufacturing cycle time. In his model, the cost function is depended on both processing lot size and the number of sublot. Goyal[10] considered the effect of the number of sub-lots on the economic lot quantity by including the cost of moving sub-lots at different stages and introducing the cost of multiple set ups for the sub-lots at different stages. However, he assumed the time delay in transferring a production lot from one stage to the next to be zero. The resulting model is very similar to the one in Szedrovits[9]. Graves and Kostreva[11] adapted the Szendrovit's model to a Material Requirements Planning(MRP) framework to gain the efficiencies from overlapping operations. They examined a generic two-work station segment of a multistage manufacturing system and derived a cost function that considers setup cost, and the inventory holding costs. Assuming constant demand, identical production rates, and equal lot sizes, they determined the number of sublots that would minimize costs.

The above literature reveals that time models have attracted much more attention than cost models. Due to this reason, Chiu and Chang[12] developed two cost models for solving lot streaming problems in multistage flow shop. They first recognized the importance of reducing the makespan time and hence introduced the imputed cost associated with the makespan time. In their paper, they also proposed more complete and accurate method compared to those of Goyal[10] and Graves and Kostreva[11] to measure the cost of raw materials, WIP, and finished product inventories.

However none of the authers have given any emphasis on shortages. But generally in a flow shop supplier gets to start production after getting order and hence some accumulation of shortages will occur until it starts supplying the first finished product sublot. For this reason, we recognize the importance of shortage. Further, all the previous authors developed their respective model in the infinite planning horizon. But many researcher in EOQ model (see gurani[13], chang and kim[14], and Moon and Yun[15] ) have claimed that an infinite planning horizon does not exit in real life, and a finite horizon inventory model is theoretically superior and has greater practical utility. Moon and yun[15] also suggested that random planning horizon is even more realistic than fixed planning horizon.

Taking all these in our mind, we have developed our model. The objectives of this study are fourfold. Firstly, we include the cost of raw material, WIP inventories, and the cost finished product inventories. Secondly, the shortage is being included. Thirdly, we consider the finite random planning horizon having a probabilistic density function. Fourthly, to derive the optimal solution of model, a simulation algorithm is developed which can be used for any probability distribution.

## 1. Assumptions and Notations

1.1. **Assumptions.** The mathematical model of the inventory problem is based on the following assumptions:

(1) Units of product are infinitely divisible and their production requires a fixed sequence of operation stages having only one machine with finite and constant production rates at each stage.
(2) The demand rate for the finished product is deterministic and constant over the random life cycle of the product.
(3) The life cycle of the product is probabilistic with a known probability density function.
(4) All sublots are of equal size in different stages. There are no production interruption times between any two adjacent sublots in the same stage.
(5) The number of transporters used to move sublots and the capacity of each transporter are unconstrained.
(6) The buffer area between two stages is sufficient to store sublots of any size.
(7) Shortages are allowed throughout the life cycle of the product.
(8) There is a constant set-up cost for each production stage but for model simplicity the set-up time for each stage is being neglected.
(9) Sublot movement cost and shipment cost of the finished product are fixed and constant(not dependent upon lotsize).
(10) After finishing the last stage the finished-product should be transferred to the customer immediately.
(11) Raw materials are replenished from some outside source at infinite rate. In the first stage, raw materials are replenished at the start time of each sublot's production. In addition, each replenishment quantity is equal to the sublot size.
(12) Transportation times are insignificant and hence ignored.
(13) The unit holding cost for each stage represents the cost of carrying one unit of physical inventory of a product on which the particular stage has been completed.

1.2. **Notations.** For convenience, the following natation is used throughout this paper:

(*i*) $n$ number of operation stages;
(*ii*) $b$ number of sublots(decision variable);
(*iii*) $j$ order of stage, $j$=1, 2, ...., $n$ where $n$th stage represents the last stage to complete production;
(*iv*) $D$ demand for the finished product per year(unit/ year);
(*v*) $Q$ processing lot size(units)(decision variable);
(*vi*) $t_j$ processing time per unit for stage $j$ (unit time/unit);
(*vii*) $S_j$ set up cost per cycle for stage $j$($/cycle),where a cycle is the time required to produce a processing lot;
(*viii*) $G_1$ sublot movement cost per movement($/movement);
(*ix*) $G_2$ finished product shipment cost per shipment($/shipment);
(*x*) $C_0$ value of raw materials per unit ($/unit);
(*xi*) $C_j$ value of work in process inventories per unit for stage $j$,$j$=1,2,....,n-1($/unit);
(*xii*) $C_n$ value of finished product inventories per unit($/unit)where $C_0 < C_1 < .... < C_n - 1 < C_n$;
(*xiii*) $h$ inventory holding cost rate per unit time for stage $j$;
(*xiv*) $r$ cost per unit time($/unit time);
(*xv*) $p$ the product life cycle(random variable);
(*xvi*) $f(p)$ the probability density function of $p$;
(*xvii*) $C_s$ shortages cost for unit item per unit time;

## 2. The model and the cost function

In the proposed system, there is no inventory held at the beginning and at the end of each cycle along the planning horizon(see fig1). The cycle starts with accumulating shortages from starting time of production. The total demand $Q$ will be produced in $b$ equal sublots each of size $\frac{Q}{b}$. The first sublot will be received at time $\frac{Q}{b}\sum_{j=1}^{n} t_j$ after the production of lot having size $\frac{Q}{b}$. After that

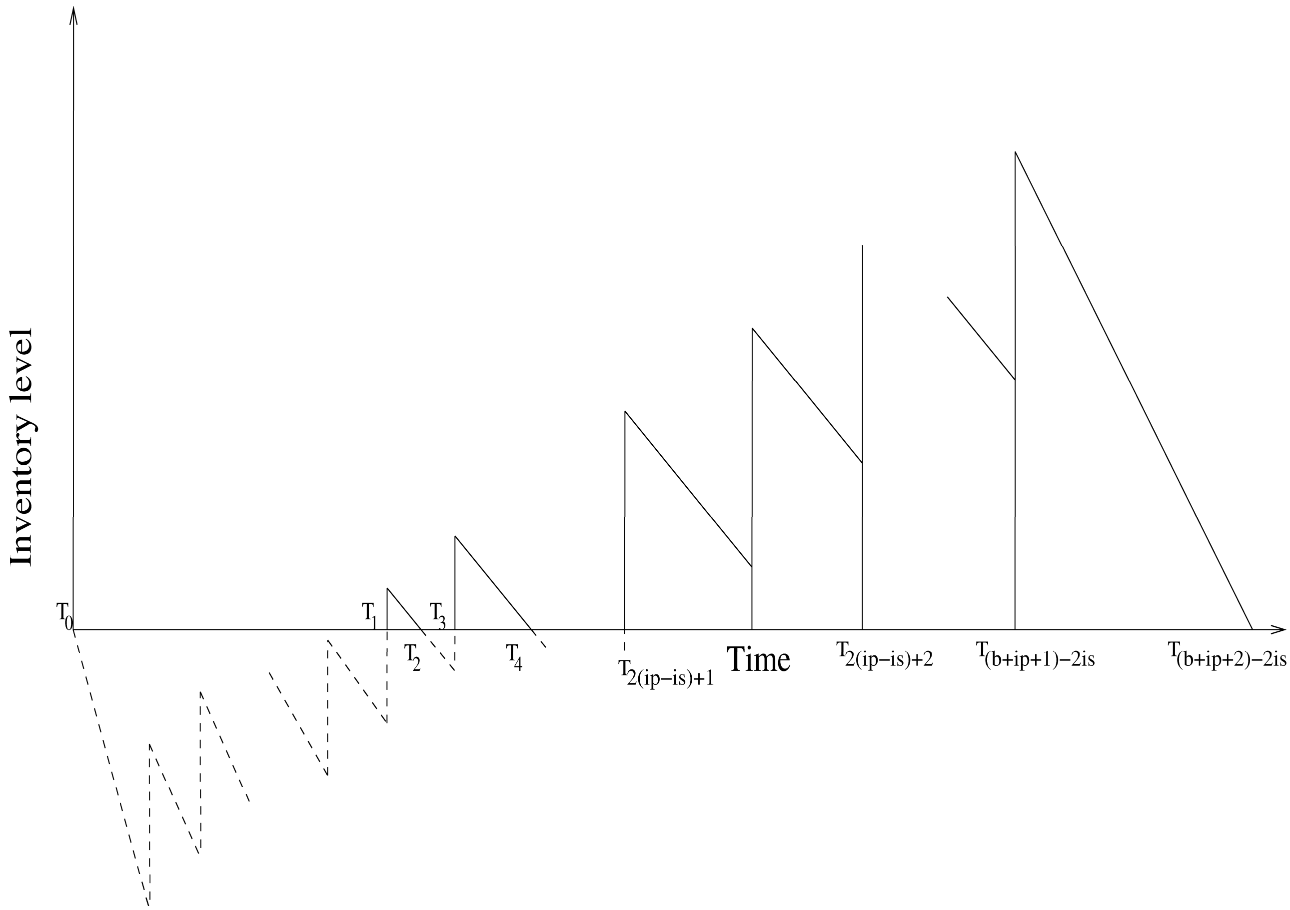


FIGURE 1. Pictorial representation of an inventory cycle

the rest of the sublot of size $\frac{Q}{b}$ will be received at an interval $\frac{Q}{b}\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j$ where

$$\delta_j = \begin{cases} 1, \text{ if } t_j \leq t_{j-1} \text{ where } t_0 = 0 \text{ and } j = 1, 2, ..., n \\ 0, \text{ otherwise} \end{cases}$$

It may so happen that after the reception of $i_s$th sublot for some $i_s$we have some inventory or stock at the beginning of the interval between $i_s$th and $(i_s + 1)$th replenishment and the initial time of this interval is denoted by $T_1$ and is given by $T_1 = \frac{Q}{b}\Big(\sum_{j=1}^{n} t_j + (i_s - 1)\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j\Big)$. After this stage, a situation may occur, say, after $i_p$th replenishment,where we have some stock at the end of the interval between $i_p$th and $(i_p + 1)$th replenishment. Finally after completing $b$th sublot we may have some stock and the time at which our stock meets the demand is the terminal time of each cycle. These situation are reflected in Fig1. The points $T_0$, $T_1$, $T_2$,..... given in Fig1 can be written as $T_i = p_i.\frac{Q}{b}$ for all $i$(decision variable), where $p_0 = 0$, $p_1 = \sum_{j=1}^{n} t_j + (i_s - 1)\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j$. All other $p_i$ and $i_s$, $i_p$ will be determined from our proposed algorithm given below.

2.1. **Algorithm 1.** Step 1. Input $t_j$ for $j = 1, 2, ...., n$ ; $D$ and $b$.
Step 2. Initialize $i = 1$.
Step 3. If $(i - D\sum_{j=1}^{n} t_j - D(i-1)\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j \geq 0)$ then $i_s = i$ go to step5.

Step 4. Increase $i$ by 1 and go to step3.

Step 5. If $i_s \geq b$ then this $b$ is not suitable for production. For this number of sublots the production will never meet demand throughout a cycle

Step 6. if $i_s = b$ then $p_1 = \sum_{j=1}^{n} t_j + (i_s - 1)\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j$, $p_2 = \Big((i_s - D\sum_{j=1}^{n} t_j - D(i-1)\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j)/D\Big) + p_1$ and $i_p = b$.

Step 7. If $i_s < b$ then $p_1 = \sum_{j=1}^{n} t_j + (i_s - 1)\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j$.

Step 8. If $\Big(1 - D\sum_{j=1}^{n} t_j - D(i-1)\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j\Big)/D \geq \sum_{j=1}^{n}(t_j - t_{j-1})\delta_j$ then $p_i = p_{i-1} + \sum_{j=1}^{n}(t_j - t_{j-1})\delta_j$ for $i = 2, .....(b - i_s + 1)$ and $p_{(b-i_s+2)} = \Bigg(\Big(i_s - D\sum_{j=1}^{n} t_j - D(i-1)\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j\Big)/D\Bigg) + p_{(b-i_s+1)}$ goto step14.

Step 9. If $\Big(i_s - D\sum_{j=1}^{n} t_j - D(i-1)\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j\Big)/D < \sum_{j=1}^{n}(t_j - t_{j-1})\delta_j$ then $p_2 = \Bigg(\Big(i_s - D\sum_{j=1}^{n} t_j - D(i-1)\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j\Big)/D\Bigg) + p_1$, $p_3 = p_1 + \sum_{j=1}^{n}(t_j - t_{j-1})\delta_j$ and initialize $k = 2$.

Step 10. If $\Big(1 - (p_{2k-1} - p_{2k-2})D\Big)/D \geq \sum_{j=1}^{n}(t_j - t_{j-1})\delta_j$ then $p_{2k+i} = p_{2k+i-1} + \sum_{j=1}^{n}(t_j - t_{j-1})\delta_j$ for $i = 0, 1, ...., (b - k - i_s)$ and $p_{(2k+b-i_s+1-k)} = \Bigg(1 + \Big(1 - D(b-k)\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j\Big) - (p_{2k-1} - p_{2k-2})D\Bigg)/D + p_{(2k+b-i_s-k)}$ and $i_p = (k + i_s - 1)$ goto step14.

Step 11. If $\Big(1 - (p_{2k-1} - p_{2k-2})D\Big)/D < \sum_{j=1}^{n}(t_j - t_{j-1})\delta_j$ then $p_{2k} = \Big((1 - (p_{2k-1} - p_{2k-2})D\Big)/D + p_{2k-1}$, $p_{2k+1} = p_{2k-1} + \sum_{j=1}^{n}(t_j - t_{j-1})\delta_j$ and $k = k + 1$.

Step 12. If $k + i_s - 1 = b$ then $p_{2k} = \Big((1 - (p_{2k-1} - p_{2k-2})D\Big)/D + p_{2k-1}$ and $i_p = b$ goto step14.

Step 13. If $k + i_s - 1 \neq b$ then goto step 10.

Step 14. Stop.

2.2. **Related cost.** Following Chiu and chang's model[12], we have derived the following costs for an cycle having total demand $Q$ related to our proposed inventory model.
The value of the raw material in the first stage is

$$\frac{Q^2}{2b} t_1 C_0 \tag{1}$$

The value of the finished product inventories in the last stage is

$$\frac{Q^2}{2b} t_n C_n \tag{2}$$

Furthermore, the value of WIP inventories has two parts. In the first part, each sublot waiting to be produced in each stage expect for the first stage. Hence waiting time for producing a processing lot for $j$th stage is

$$\sum_{k=2}^{b} |t_{j-1} - t_j| \frac{Q}{b} = \frac{Q(b-1)}{2}|t_{j-1} - t_j| \quad \text{for } j = 2, ...., n \tag{3}$$

Then, the value of WIP inventories for $j$th stage can be obtained by multiplying $\frac{Q}{b}C_{j-1}$ with the result obtained from Equation(3, that is,

$$\frac{Q^2}{2}(1-\frac{1}{b})C_{j-1}|t_{j-1}-t_j| \quad \text{for } j=2,....,n \tag{4}$$

The total value of WIP inventories in the first part becomes

$$\frac{Q^2}{2}(1-\frac{1}{b})\sum_{k=2}^{b} C_{j-1}|t_{j-1}-t_j| \tag{5}$$

In the second part, the value of each sublot's WIP in the first stage is

$$\frac{Q^2}{2b}t_1C_1 \tag{6}$$

In this part, the value of WIP inventories in the last stage is

$$\frac{Q^2}{2b}t_nC_{n-1} \tag{7}$$

In this part, the value of each sublot's WIP in each stage expect for first and last stage is

$$\frac{Q^2}{2b}t_j(C_{j-1}+C_j) \quad \text{for } j=2,....,n-1 \tag{8}$$

Hence, the total value of WIP inventories in the second part is

$$\frac{Q^2}{2b}\left[t_1C_1+\sum_{j=2}^{n-1}t_j(C_{j-1}+C_j)+t_nC_{n-1}\right] \tag{9}$$

Now according the assumption(3) , the makespan time for a processing lot is

$$M(Q,b)=\frac{Q}{b}\left[\sum_{j=1}^{n}t_j+(b-1)\sum_{j=1}^{n}(t_j-t_{j-1})\delta_j\right] \tag{10}$$

The cost due to shortage for each cycle is

$$\frac{Q}{b}DC_s\left[\sum_{j=1}^{i_p-i_s+1}(p_{2j-1}-p_{2j-2})\right] \tag{11}$$

Sublot movement cost, finished product movement cost and set up cost for each cycle are $G_1b(n-1)$, $G_2b$ and $\sum_{j=1}^{n} S_j$ respectively. As a result, the total cost for a cycle, after little calculation, becomes

$$\frac{Q^2}{2}(1-\frac{1}{b})\sum_{k=2}^{b} C_{j-1}|t_{j-1}-t_j|+\frac{Q^2}{2b}\left[\sum_{j=1}^{n}t_j(C_{j-1}+C_j)\right]+\frac{Qr}{b}\left[\sum_{j=1}^{n}t_j+(b-1)\sum_{j=1}^{n}(t_j-t_{j-1})\delta_j\right]$$
$$+\quad G_1b(n-1)+G_2b+\sum_{j=1}^{n}S_j+\frac{Q}{b}DC_s\left[\sum_{j=1}^{i_p-i_s+1}(p_{2j-1}-p_{2j-2})\right] \tag{12}$$

## 3. Total cost

If we assume that the planning horizon $p$ having particular type of probabilistic distribution fully accumulates first $l$ cycle, and ends during $(l+1)$th cycle, then the total cost up to the beginning of $(l+1)$th cycle will be

$$l\left[\frac{Q^2}{2}(1-\frac{1}{b})\sum_{k=2}^{b} C_{j-1}|t_{j-1}-t_j|+\frac{Q^2}{2b}\left[\sum_{j=1}^{n}t_j(C_{j-1}+C_j)\right]+\frac{Qr}{b}\left[\sum_{j=1}^{n}t_j+(b-1)\sum_{j=1}^{n}(t_j-t_{j-1})\delta_j\right]\right.$$
$$\left.+\quad G_1b(n-1)+G_2b+\sum_{j=1}^{n}S_j+\frac{Q}{b}DC_s\left[\sum_{j=1}^{i_p-i_s+1}(p_{2j-1}-p_{2j-2})\right]\right] \tag{13}$$

The total cost during the last cycle, i.e.$(l+1)$th cycle will be

$$\frac{Q'^2}{2}(1-\frac{1}{b})\sum_{k=2}^{b} C_{j-1}|t_{j-1}-t_j| + \frac{Q'^2}{2b}\left[\sum_{j=1}^{n} t_j(C_{j-1}+C_j)\right] + \frac{Q'r}{b}\left[\sum_{j=1}^{n} t_j + (b-1)\sum_{j=1}^{n}(t_j - t_{j-1})\delta_j\right]$$
$$+ \quad G_1 b(n-1) + G_2 b + \sum_{j=1}^{n} S_j + \frac{Q'}{b} DC_s\left[\sum_{j=1}^{i_p-i_s+1}(p_{2j-1}-p_{2j-2})\right] \tag{14}$$

where

$$Q' = \frac{Q\Big(p - lp_{(b+i_p+2)-2i_s}\frac{Q}{b}\Big)}{\frac{Q}{b}\; p_{(b+i_p+2)-2i_s}} \tag{15}$$

(Here we assume that $Q^{'}$ will also be produced in $b$ sublots).
Since the planning horizon $p$ has a probability density funtion $f(p)$, the expected total cost in that whole horizon, say $C(Q)$, is

$$C(Q) = \sum_{l=0}^{\infty}\int_{\frac{Q}{b}lp_{(b+i_p+2)-2i_s}}^{\frac{Q}{b}(l+1)p_{(b+i_p+2)-2i_s}} [13+14]f(p)dp \tag{16}$$

## 4. Numerical computation method

We have observed that it is very difficult to solve Equation(16) analytically for any probabilistic distribution. To complement the analytical method in section 4, we develop a numerical algorithm which can be used for any distribution. The outline of the numerical algorithm is as follows:

4.1. **Algorithm 2.** Step 1. Input values all parameters .
Step 2. Start from $b = 1$.
Step 3. Using algorithm 1, find out all $p_i$, $i_s$ and $i_p$ .
Step 4. Take a sample of size, say 500, around the mean of the random product life cycle $p$ having particular probabilistic distribution.
Step 5. Initialize $Q = 0.001$.
Step 6. Compute $l = [p/p_{(b+i_p+2)-2i_s}Q]$ for each point of the sample space.This is the number of cycle for each point of the sample space using this Q.
Step 7. Compute $C(Q)$ for each point in that space using equations (13) and (14) and the $l$ values are computed in step 6.
Step 8. Compute the average $C(Q)$ for a given $Q$. Increase $Q$ by $\Delta$, say 0.001, that is, $Q = Q + \Delta$. Go to step 6 until $Q$ reaches a high value which is not appropriate for reorder quantity.
Step 9. Let $MC_b(Q)$ be the minimum of all these $C(Q)$.
Step 10. Increase $b$ by 1 and go to step 3 until $b$ becomes some high value, say 200.
Step 11. Let $OC(Q)$= Minimum$[MC_b(Q)]$, $b = 1, 2, 3, ...., 200$ and this minimum occurs for some $Q$ say $Q_{opt}$ and for some $b$ say $b_{opt}$.These $OC(Q)$, $Q_{opt}$ and $b_{opt}$ are optimal cost, optimal ordering quantity and optimal number of sublots respectively.
Step 12. Stop.

## 5. Numerical example

The following numerical example is considered for demonstration. Suppose that for a product the following data is available.
$D = 10000$ units per year.
$G_1 = \$2$ per movement.
$G_2 = \$10$ per shipment.
$C_0 = \$0.3$ per unit.
$h = \$0.002$ per year.
$r = \$100000$ per year.
$C_s = \$0.8$ per unit item per year.
We have also assumed that total working days in a year is 250 days and 8 hour is working hour in a day. In addition, processing time, set up cost, value cost have been given in Table-1.

Table-1

| Production stage $i$ | Unit cost after stage $i$ $C_i$ in \$ | Time required to process one unit in minutes | Set up cost in \$ |
|---|---|---|---|
| 1 | 0.4 | 3 | 200 |
| 2 | 0.5 | 5 | 210 |
| 3 | 0.6 | 4 | 220 |
| 4 | 0.7 | 6 | 230 |
| 5 | 0.8 | 2 | 240 |

As the life cycle of a product generally follows exponential distribution, in case1 we first give one illustration where we consider exponential distribution and later on in case2 we consider the normal distribution as an example of other distribution.

5.1. **Case1:** When the life cycle of the product follows exponential distribution with mean 2.

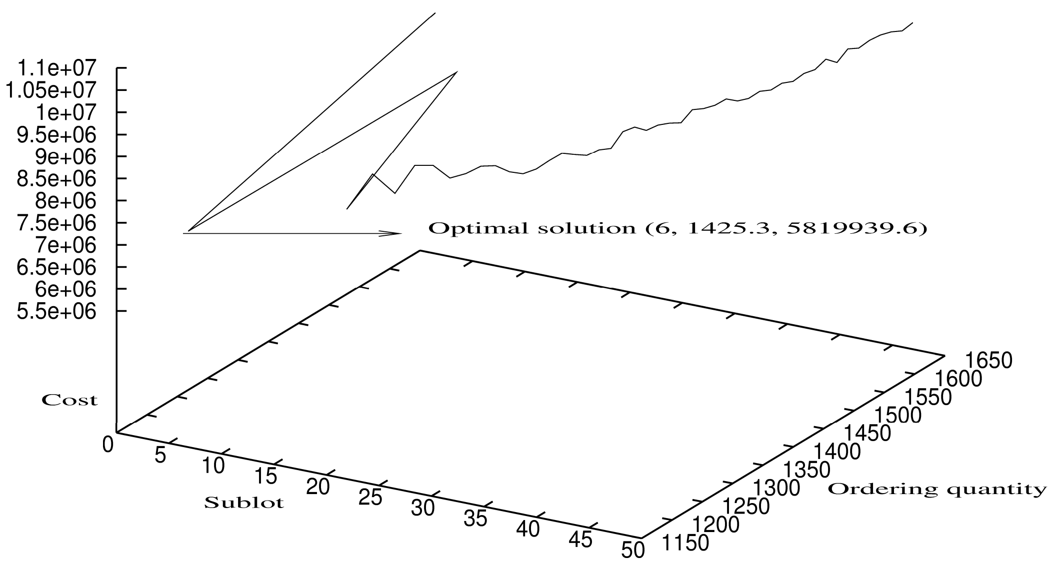


Figure 2. Graphical representation of optimal cost for exponential distribution

5.2. **Case 2:** When the life cycle of the product follows normal distribution with mean 2 and standard deviation 0.5.

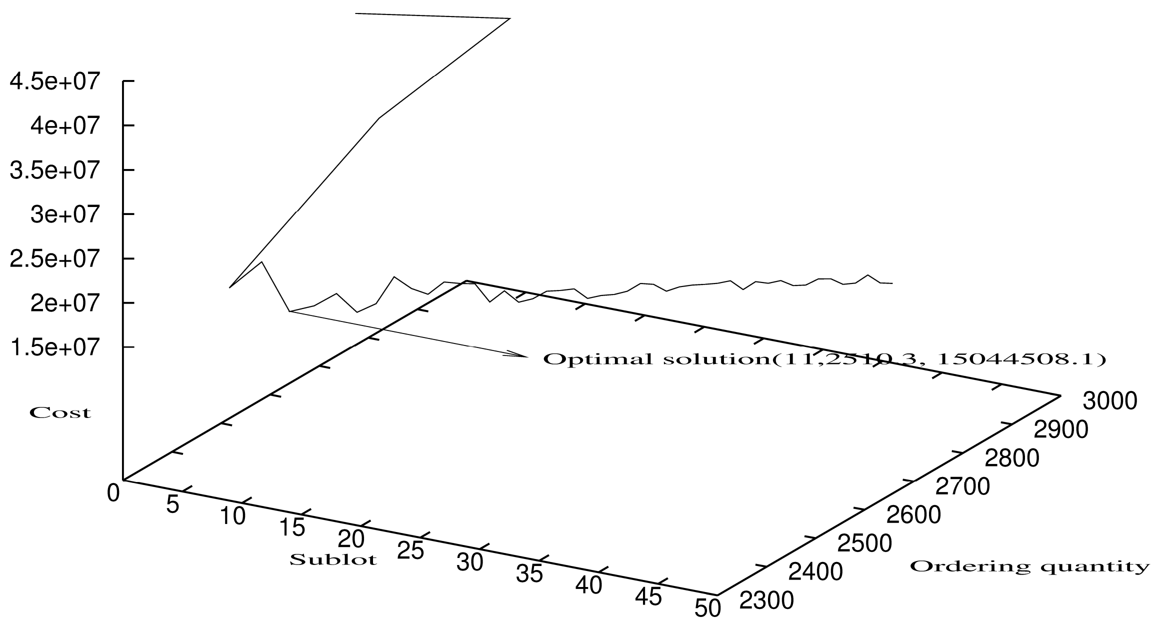


FIGURE 3. Graphical representation of optimal cost for normal distribution

Table-2

| | Exponential Distribution | | Normal Distribution | |
|---|---|---|---|---|
| sublot | optimal order | optimal cost | optimal order | optimal cost |
| 1 | b is not suitable | .... | .... | .... |
| 2 | b is not suitable | .... | .... | .... |
| 3 | 1623.19 | 8481692.86 | 2700.39 | 40542816.96 |
| 4 | 1198.70 | 7098856.14 | 2991.19 | 30882625.65 |
| 5 | 1623.19 | 7230706.71 | 2700.39 | 29252772.31 |
| 6* | 1425.29* | 5819939.62* | 2371.09 | 20958797.42 |
| 7 | 1450.49 | 6456690.84 | 2413.09 | 22797852.47 |
| 8 | 1470.09 | 5901385.71 | 2445.69 | 16414925.19 |
| 9 | 1485.59 | 6455544.86 | 2471.59 | 16494968.39 |
| 10 | 1498.29 | 6396929.08 | 2492.69 | 17461063.58 |
| 11* | 1508.89 | 6070283.81 | 2510.29* | 15044508.09* |
| 12 | 1517.79 | 6146094.57 | 2525.09 | 15785375.79 |
| 13 | 1525.39 | 6297539.31 | 2537.69 | 18673228.09 |
| 14 | 1531.99 | 6303491.02 | 2548.69 | 17263734.33 |
| 15 | 1537.79 | 6163174.76 | 2558.29 | 16571184.09 |
| 16 | 1542.89 | 6126028.26 | 2566.69 | 17939588.08 |
| 17 | 1547.39 | 6243265.82 | 2574.19 | 17791120.06 |
| 18 | 1551.39 | 6453750.55 | 2580.89 | 17801531.20 |
| 19 | 1554.99 | 6633563.46 | 2586.99 | 15808802.50 |
| 20 | 1558.29 | 6625018.37 | 2592.39 | 17145987.27 |
| 21 | 1561.29 | 6628868.25 | 2597.39 | 15965986.78 |
| 22 | 1563.99 | 6786124.98 | 2601.89 | 16478437.55 |
| 23 | 1566.49 | 6846210.88 | 2605.99 | 17457975.73 |
| 24 | 1568.69 | 7250305.77 | 2609.79 | 17667343.41 |
| 25 | 1570.79 | 7386857.93 | 2613.29 | 18004887.34 |
| 26 | 1572.79 | 7342479.88 | 2616.59 | 17081957.86 |
| 27 | 1574.59 | 7495076.93 | 2619.59 | 17563576.01 |
| 28 | 1576.29 | 7570822.80 | 2622.39 | 17863341.15 |
| 29 | 1577.89 | 7612782.19 | 2624.99 | 18392029.24 |
| 30 | 1579.29 | 7943162.46 | 2627.399 | 19467476.29 |
| 31 | 1580.69 | 8006176.63 | 2629.69 | 19547834.73 |
| 32 | 1581.99 | 8118325.45 | 2631.89 | 18950049.60 |
| 33 | 1583.19 | 8296641.39 | 2633.89 | 19637422.95 |
| 34 | 1584.39 | 8290026.72 | 2635.79 | 20039176.01 |
| 35 | 1585.49 | 8383944.43 | 2637.59 | 20332902.68 |
| 36 | 1586.49 | 8586080.82 | 2639.29 | 20664410.92 |
| 37 | 1587.49 | 8652019.77 | 2640.89 | 21153680.93 |
| 38 | 1588.39 | 8845273.00 | 2642.49 | 20356718.07 |
| 39 | 1589.29 | 8926543.86 | 2643.89 | 21468117.52 |
| 40 | 1590.09 | 9147675.45 | 2645.29 | 21494053.42 |
| 41 | 1590.89 | 9274753.85 | 2646.59 | 22029063.73 |
| 42 | 1591.59 | 9549793.55 | 2647.89 | 21673541.35 |
| 43 | 1592.39 | 9516208.94 | 2649.09 | 21948393.71 |
| 44 | 1592.99 | 9866794.66 | 2650.19 | 22878786.03 |
| 45 | 1593.69 | 9925004.84 | 2651.29 | 23106017.75 |
| 46 | 1594.29 | 10144708.08 | 2652.39 | 22703424.60 |
| 47 | 1594.89 | 10307289.92 | 2653.39 | 23077206.47 |
| 48 | 1595.49 | 10418362.90 | 2654.29 | 24232388.94 |
| 49 | 1596.09 | 10482929.24 | 2655.29 | 23556945.76 |
| 50 | 1596.59 | 10714302.51 | 2656.19 | 23731125.44 |

In both cases, optimal ordering quantity and optimal cost have been given for 1 to 50 sublots. Table-2 shows how the ordering quantity and total cost fluctuate with the number of sublots but still it has an optimal result. To have clearcut view, we have plotted two graph(Fig2 and Fig3) from this table for the above two distributions.

## 6. Sensitivity analysis

We have carried out sensitivity analysis in two stages. In the first stage, the aim is to investigate whether the unit processing time for stage $j$, $t_j$ will have a significant impact on the optimal solution. The values of $t_1, t_2, t_3, t_4, t_5$ have been taken in increasing, decreasing, completely balanced and other configurations. The result of the first stage are listed in Table3.

Table-3

| $(t_1, t_2, t_3, t_4, t_5)$ | Exponential Distribution | | | Normal Distribution | | |
|---|---|---|---|---|---|---|
| | sublot | optimal order | optimal cost | sublot | optimal order | optimal cost |
| (3, 5, 4, 6, 2) | 6 | 1579.3 | 5819939.6 | 11 | 2510.3 | 15044508.1 |
| (2, 3, 4, 5, 6) | 9 | 1460.9 | 5782425.2 | 16 | 2541.6 | 15374449.8 |
| (4, 4, 4, 4, 4) | 7 | 1482.1 | 5438351.1 | 14 | 2577.7 | 14727933.4 |
| (2, 6, 4, 5, 3) | 6 | 1425.3 | 5819942.2 | 11 | 2510.3 | 15044521.1 |
| (6, 5, 4, 3, 2) | 9 | 1460.9 | 5782404.6 | 16 | 2541.6 | 15374344.2 |
| (1, 0.5, 1, 2, 1) | 4 | 1623.2 | 4607315.7 | 7 | 2700.4 | 11524289.2 |
| (0.25, 0.25, 0.5, 0.5, 1) | 3 | 1623.2 | 4306583.6 | 5 | 2700.4 | 10455521.7 |
| (0.25, 0.25, 0.25, 0.25, 0.25) | 2 | 1623.2 | 4128052.7 | 3 | 2700.4 | 9820614.7 |

In the second stage, the objective is to observe with the impact of changing the values of $C_s$, $G_1, G_2, h$ and $\sum_{j=1}^{n} S_j$ on the optimal solution or not. Table-4 shows the result of the second stage.It is seen from Table-3 that the sequence of processing time has a great impact on optimal solution. From Table-4, we see that the sublots, ordering quantity, and optimal cost are sensitive to the changes in the parameters $G_1$, $G_2$, $\sum_{j=1}^{n} S_j$, and $C_s$. It is also noted that sublots, ordering quantity, and optimal cost increases monotonically as $G_1$, $G_2$, and $C_s$ increases separately keeping all other parameters fixed. However in the case of $\sum_{j=1}^{n} S_j$ optimal cost is positively correlated to $\sum_{j=1}^{n} S_j$ but ordering quantity and sublots are negatively correlated to $\sum_{j=1}^{n} S_j$. Lastly we observe that parameter $h$ is positively correlated with only optimal cost but the parameter $r$ has no effect on optimal solution.

Table-4

| Parameter | Change | Exponential Distribution | | | Normal Distribution | | |
|---|---|---|---|---|---|---|---|
| | | sublot | optimal order | optimal cost | sublot | optimal order | optimal cost |
| $C_s$ | 0.2 | 6 | 1425.3 | 4705640.4 | 4 | 2991.2 | 11377424.9 |
| | 0.4 | 6 | 1425.3 | 5076032.7 | 11 | 2991.2 | 12973051.9 |
| | 0.8 | 6 | 1425.3 | 5819939.6 | 11 | 2510.3 | 15044508.1 |
| | 1.6 | 16 | 1558.3 | 7289537.8 | 21 | 2597.4 | 18473790.2 |
| | 3.2 | 16 | 1542.9 | 8071277.7 | 21 | 2597.4 | 23564350.2 |
| $G_1$ | 0.5 | 16 | 1542.9 | 5473899.5 | 21 | 2597.4 | 14007064.2 |
| | 1 | 6 | 1425.3 | 5661487.3 | 11 | 2510.3 | 14374724.1 |
| | 2 | 6 | 1425.3 | 5819939.6 | 11 | 2510.3 | 15044508.1 |
| | 4 | 6 | 1425.3 | 6143756.3 | 11 | 2510.3 | 16430540.1 |
| | 8 | 6 | 1425.3 | 6819037.5 | 11 | 2510.3 | 19388460.2 |
| $G_2$ | 2.5 | 16 | 1542.9 | 5316627.3 | 21 | 2597.4 | 13537178.6 |
| | 5 | 16 | 1542.9 | 5316627.3 | 11 | 2510.3 | 14209698.1 |
| | 10 | 6 | 1425.3 | 5819939.6 | 11 | 2510.3 | 15044508.1 |
| | 20 | 6 | 1425.3 | 6226150.4 | 11 | 2510.7 | 16786728.1 |
| | 40 | 6 | 1425.3 | 7081771.9 | 8 | 2545.7 | 20490329.4 |
| $\sum_{i=1}^{n} S_i$ | 275 | 8 | 1470.1 | 1676544.0 | 11 | 2510.3 | 5063357.7 |
| | 550 | 6 | 1425.3 | 2767340.7 | 11 | 2510.3 | 7785407.8 |
| | 1100 | 6 | 1425.3 | 5819939.6 | 11 | 2510.3 | 15044508.1 |
| | 2200 | 6 | 1425.3 | 1555137.4 | 11 | 2510.3 | 36822708.7 |
| | 4400 | 6 | 1425.3 | 49545532.9 | 11 | 2510.3 | 109419109.8 |
| $h$ | 0.0005 | 6 | 1425.3 | 5819831.9 | 11 | 2510.3 | 15043812.3 |
| | 0.001 | 6 | 1425.3 | 5819867.8 | 11 | 2510.3 | 15044151.3 |
| | 0.002 | 6 | 1425.3 | 5819939.6 | 11 | 2510.3 | 15044508.1 |
| | 0.004 | 6 | 1425.3 | 5820083.3 | 11 | 2510.3 | 15045221.7 |
| | 0.008 | 6 | 1425.3 | 5820370.6 | 11 | 2510.3 | 15046648.9 |
| $r$ | 25000 | 6 | 1425.3 | 5819939.6 | 11 | 2510.3 | 15044508.1 |
| | 50000 | 6 | 1425.3 | 5819939.6 | 11 | 2510.3 | 15044508.1 |
| | 100000 | 6 | 1425.3 | 5819939.6 | 11 | 2510.3 | 15044508.1 |
| | 200000 | 6 | 1425.3 | 5819939.6 | 11 | 2510.3 | 15044508.1 |
| | 400000 | 6 | 1425.3 | 5819939.6 | 11 | 2510.3 | 15044508.1 |

## 7. Conclusion

This paper deals with lot streaming problem in multistage flow shop. To reflect the realistic business situations, we have considered shortages in the planning horizon. We have also assumed that the planning horizon is a random variable having a probability density function. Since it is difficult to solve the problem analytically, to find optimal solution numerically we have developed two algorithms. Numerical examples(when life cycle of the product maintains exponential distribution or normal distribution) are provided for illustration purpose.Further, we have explained the nature of the optimal cost neatly with the help of graphs corresponding to exponential and normal distribution. Finally a sensitive analysis has been carried out, so as to check the effect of the decision variables for the corresponding changes in different parameter values. A future study should incorporate more realistic assumption in the proposed models such as unequal size of sublots, inclusion of mean flow time in the place of makespan time, and analytical solution method for any distribution.

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