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LEARNING CURVES AS A TOOL OF ENTERPRISES DEVELOPMENT

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ABSTRACT

The aim of extensive research is to constitute the prediction and the control, of various technologies /enterprises, with various methods, in order to develop them in the modern and competitive environment. Learning curves constitute a precious tool in the hands of administration management. It has been scientifically proved that the repeated work on the same object has as a result the important reduction of production cost and the simultaneous increase of efficiency. Learning curves are resulted via the recording of these results of work; they represent the cost of production concerning the number of repetitions or the cost per unit of production concerning the production. The information that is exported from the learning percentage (factor of learning) constitutes an important tool for the development of enterprises in all the sectors.

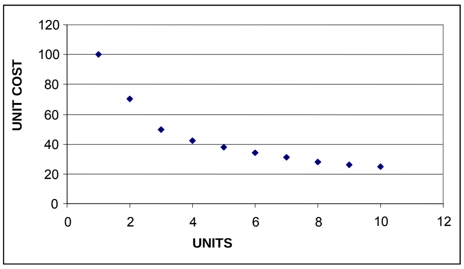
In this paper a comparison, of learning model of Wright with the polynomial Lagrange is attempted. In addition a report, on a relative research that has taken place in public and private companies of Northern Greece on the subject of the learning curves utilisation, is taken under consideration.

Key Words: Learning curves, Administration-Management of labor, Mathematic Models, Interjection and Prediction.

1. Introduction

Because of the continuous development of new technologies/companies in the developed economies, such as the USA and the EU, the research for methods of risk management is a necessity. These methods help in the comprehension and understanding of the optimisation of the efficiency of technologies and companies. Such methods, that provide important help in the decision making process, with regard to the planning and the control of investment, are the learning curves.

Learning curves have been studied analytically. From the experience of production of airplanes during the World War II, the constructors realised that the rate of improvement in the manufacturing process was stable and this could be attributed to a mathematic function; the labour hours that were required could be predicted with a function, with a high degree of precision. Thus, the significance of learning curves in the industry of airplanes in 1936 was established, when T. P. Wright in February of the same year published an article in the Journal of the Aeronautical Science. The learning curve is a line, which demonstrates the relation between the production time and cost of units and the total number of produced units or between the production time and cost of each unit and the number of repetitions (Fig. 1).



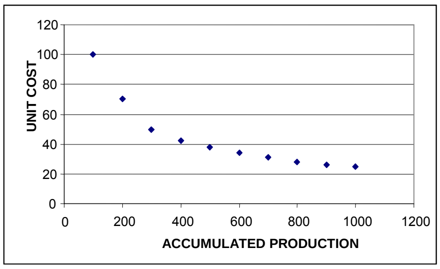


Fig. 1: *Typical learning curve*

The learning curves concern a wide variety of "products". Managers and researchers of many companies observed through the process of repetition or accumulation in different sectors, as aeronautics, the metallurgy, the manufacture and the writing of reports, that these processes are improved regularly after each repetition instead of remaining stable. Thus, the learning curves have been applied in all fields of activities, from the most common to the rarest. The learning curves are part of the enterprise strategy planning, as for example the pricing decision making, the main investment decision making and the functional expenses.

If we take the above for granted, when a process is applied from a technology / company for first time, it is natural to be deprived of experience for that particular procedure. However, while the process is repeated, the technology / company become more familiarized with this procedure and the result is the simultaneous increase of efficiency on the one hand and the reduction of labor cost per unit manufactured on the other. The rate of the labour cost per unit manufactured is steadily reduced, while with the help of the learning curves, it can be established from a point onwards. This rate of reduction can then be used for prediction of future expenses of labour as well as in the control of processes. The process of learning begins where the first unit from the production line is produced. From then on, every time the process is repeated, the accumulated production, the production time (cost) of each unit or the average time (average cost) that is received for the each unit production of accumulated production, will be a certain percentage of average time per unit of previous accumulative production (TIMOTHY L. SMUNT, 1999).

2. Basic models

The learning theory underlines that the repetition of a process leads to time reductions or to the reduction of the required effort (leading to costs reductions) for the particular process. Its significance is that direct labour time, for example, that is required for a unit to be completed, is decreased at a constant percentage each time the quantity of production is doubled. The learning curves are intended for the current production improvement and describe the plans of long-term improvement. The learning curves theory is based on the following conclusions:

- The time (the cost) that is required, for a given target or one unit of product to be completed will be less each time when the target is achieved or the accumulated production is increased.
- The time (the cost) will be decreased in a declining distribution.
- The reduction of time will follow a foreseeable distribution.

Various models have been proposed, in order to describe the learning curves. These models examine the expiring period (Gruber, 1992), the number of units that are produced (Gruber, 1992) and the investment (Lieberman, 1984) as independent variables. The corresponding proposed dependent variables are the price (Chung, 2001), the cost (Waring, 1991), the production (Chung, 2001), the required work (Dompere, K. K. and Nti, K.O., 1991), or other certain variables. It is important to be acknowledged that different proposed models can yield different results in the attribution of learning in the various technologies or companies.

None of the various models for the learning curves, that have been proposed and used by the technologies or companies, is generally acceptable as superior. From many mathematical models of the learning curves, more important are the following:

Log-Linear (Wright): $y(X) = B_o X^n$ (1)

Stanford-B: $y(X) = B_o (X + c)^n$ (2)

DeJong: $y(X) = B_1 + B_o X^n$ (3)

S-Curve: $y(X) = B_1 + B_o (X + c)^n$ (4)

The Log-Linear (Wright) equation is the simplest and more common equation and is valid for a wide variety of processes. The Stanford - B equation is used for the configuration of processes, where the experience is carried from a product to another, in order for the workers to begin more effectively from where the asymptote of the learning curve predicts. The Stanford - B equation has been used for the modelling of airplanes and metallurgy production. The DeJong equation is used for the modelling of processes, where a part of the process cannot be improved. The DeJong equation is often used in factories, where the assembly line is limiting the improvement. The S - Curve equation combines the equations Stanford - B and DeJong, in order to shape the processes, in which the experience is carried by a product to the next and parts of the process can not be improved.

Wright's model of learning curves is presented below, which is a model of unit construction versus time. This model is based on the equation:

$Y_i = B_o X^n$ (5)

Where:

X = the accumulated production

Y_i = the total amount of the labour time which is required for the H units

B_o = the number of direct labour time which is required for the production of the first unit

n = logb/log2 where b = rate of learning. The exhibitor of improvement (n) can take any value between -1 and the zero.

This equation describes that the developing time of units is decreased with a constant percentage each time the double quantity of units is produced. The lower the rates of learning the faster the individual units are produced and the faster the average time per unit is decreased.

Crawford (Yelle, 1979) developed a similar log-linear model, using the same process, as it is presented for the accumulative average model. Nevertheless, in this case Y represents the unit cost for a particular unit x. For this reason, Crawford's approach is often reported as the model of "unit cost", that is to say:

$Y_i(X) = B_o X^n$ (6)

Where

X = the unit number

Y_i = the number of direct labour time (cost) that is required to produce the H^{th} unit

B_o = the number of direct labour time that is required for the production of the first unit

n = percentage of learning.

Finally the DeJong model is presented.

$Y_i = B_o + B_1 X^{-n}$ (7)

Where:

Y_i = the number of direct labour time (cost) that is required to produce the H^{th} unit or the total number of labour time that is required for the H units

B_1 = it is the biggest likely reduction (the difference from the price of first unit B_o),

B_o = it is asymptote (the minimal price),

X = it is the total number of produced units, and

n = percentage of learning.

In practice, the learning curves are drawn using a graphic representation in the logarithm scale. The curve is linear and the accumulative curve becomes linear after the first units. It requires clarification that the accumulative average cost is substantially linear after the eighth unit of production.

3. The shift of the natural limit

The lower part of the learning curve, which represents the period in which enormous sums of effort correspond in the minimal or null improvement of efficiency, is named "valley of despair". Somewhere an ascendant region is presented (fig. 2). This region represents a situation, where small sums of effort lead to essential improvements. Finally, the technological orbit approaches a natural limit of the efficiency (an obstacle), that cannot be exceeded. This natural limit can be approached, but it can not be exceeded. Thus the learning curve is asymptotic at the natural limit, where no further improvement is feasible.

Comprehending the nature and the cause of natural limits offers the possibility of exceeding the existing limits to technologies/companies. This leads to the shift of learning curve, since a new natural limit replaces an old one. The new natural limit offers the higher efficiency in one or more dimensions and/or the use of fewer imports. The application of this meaning is clarified in fig. 2, where the existence of a number of learning curves, with different natural limits for every one, is shown.

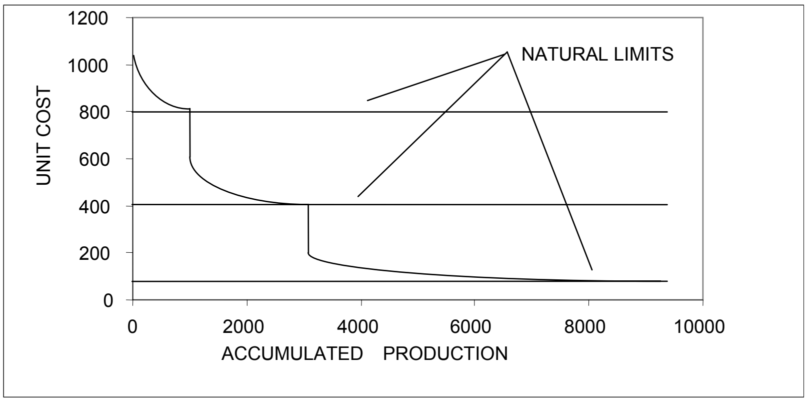


Fig. 2: *Shift of natural limit*

Nevertheless, with the emersion of new technological processes, a natural limit is exceeded periodically. Each time a natural limit is removed, a gradual reduction of learning curve is presented, representing new restrictive natural limit appearance. One abrupt learning curve shows that the target is easy to be achieved. Of course this leads faster to the valley of despair.

4. The approach of learning curves with polynomials.

A learning curve can be described by a numerical classification in logarithm table (Wright) or by other methods, depending on the form and the type of available variables. In the present paper the mathematic description is presented with the method of interjection and prediction (and for supervisory reasons we use the Lagrange polynomial). We can achieve also with other methods of interjection and prediction, the same or even better results, as the splines method (FRANCIS J. SCHEID, 1988).

The Lagrange polynomial is a polynomial of n -1 degree, which passes from n points and is described by the following function:

$$p(x) = \sum_{i=0}^{n} L_i(x) y_i \qquad (8)$$

Where L_i are the factors of Lagrange, that is to say:

$$L_i = \frac{(x - x_o)(x - x_1)...(x - x_{i-1})(x - x_{i+1})...(x - x_n)}{(x_i - x_o)(x_i - x_1)...(x_i - x_{i-1})(x_i - x_{i+1})...(x_i - x_n)} \qquad (9)$$

and they have the following attributes $L_i(x_k)=0$ for $k \neq i$, $L_i(x_i)=1$.

The fault that becomes in the calculation of f (x) price for some given x can be appreciated from the relation:

$$E(x) = (x - x_o)(x - x_1)...(x - x_n)\frac{f^{n+1}(\xi)}{(n+1)!} \tag{10}$$

Where n is the degree of symptotic polynomial and the ξ is the suitable point between the x_o, …, x_n.

5. Numerical example.

A company wants to construct 10 residences of type A. In the following table the required construction time of each residence is shown.

Table 1.

Id number(x)	*Construction time (y)*	*log(x)*	*log(y)*
1	100	0	2
2	70	0,30103	1,845098
3	50	0,477121	1,69897
4	42	0,60206	1,623249
5	38	0,69897	1,579784
6	34	0,778151	1,531479
7	31	0,845098	1,491362
8	28	0,90309	1,447158
9	26	0,954243	1,414973
10	25	1	1,39794

In order of some interjection or prediction to be done in the above data, the model of Wright and the polynomial Lagrange are applied. The prices are mentioned in the table 2.

Table 2.

(x)	Wright (y)	Lagrange(y)
2,3	60,8	62,46
4,5	40,25	39,92
6,7	31,51	31,76
8,3	27,63	26,71
1,2	90,68	94,14
11	23,24	418
12	22,03	1.772

From the above table becomes clear that the Lagrange polynomial can be applied with success in the control processes (interjection). On the contrary, in prediction processes it gives exceptionally wrong prices. In the same examples, lightly better results are given by the interjection with splines (FRANCIS J. SCHEID, 1988).

Wright's model provides the most satisfactory prices in the prediction as in the interjection too.

Finally the interjection – prediction models can be used very effectively for control processes. Despite the difficulty of their application, are rendered as an excellent control tool with the help of a PC. The models as the Wright, Stanford-B, DeJong and S-Curve are functional and effective enough in the prediction and in interjection too.

6. Factors that influence learning.

Main structural component of all technologies/companies is the human factor. Thus, all the learning theory is based on this. Based on the learning theory it is easier for somebody to learn things that are related to objects that he already knows. The probability of the new knowledge to be maintained is connected with the previous knowledge if this previous one contains "hooks" in which the new knowledge can be hung. In other words, new knowledge should ensure connectivity in the mental framework of the trainee. The change in the mental model happens more easily, when this change is based on previous well-known mental model.

The learning curves can be applied individually (personally), or in teams of individuals (enterprises). The personal learning results to an improvement when the individual repeats a process, after that efficiency is increased his gained experience. In other words practice makes it perfection.

Team learning also results from practice, but emanates also from the changes in administration, equipment and type of products. In team learning, we expect also to see the two types of learning simultaneously presented and to describe the combined effect in a single learning curve.

Certain general directives in order to improve the individual efficiency that is based on learning curves include the following:

- Suitable choice of workers. Not anyone learns at the same rate. Because of the differences between the individual persons and their innate ability, their age, their previous experience is rendered in each individual person a discreet learning curve. A trial period before the worker's engagement is required. These trials should be representative for the scheduled work.
- Suitable training. The more effective education, the bigger the rate of learning.
- Bonus. The profits of productivity, which are based on learning curves, are not achieved without remuneration. The remunerations can be money or not monetary (reward, employee of month, etc).
- Specialisation of work. As a general rule, the easier the target, the faster the learning.
- Implementation one or few tasks of each time. Learning is more effective in each individual task, when task is completed separately and not simultaneously.
- Utilisation of tools or equipment that helps or supports the efficiency.
- Fast and easy access for help.
- Possibility of target re-definition. The reception of more efficiency factors in the field of learning curve can, actually, shift the curve downwards.

A technology/company also learns. For the individual person, it is easy to conceive how knowledge is acquired and it is maintained in an individual process of learning. Of course, a main source of learning for technologies /companies is the individual education of employees. A technology/company acquires also the knowledge of its self development that emanate from the equipment, the methods, the product or the product line, the installations and so on. For example, while a technology/company advances, the knowledge is incorporated in the structure of its organisation, in the software and in the production planning.

7. Conclusions.

The learning curves resulted from the historical observation that the individual persons who execute repeated operations, present improvement in efficiency, while the operation is repeated in various time. With the suitable directives and repetition, the persons learn how to execute their work more effectively. Consequently, the direct work time per unit of product is decreased. This effect of learning would result from better methods of work, the tools, the plan of the product line or the supervision, as well as from the specialised human potential.

Learning curves is a tool which a technology/company can use to produce production programs, delivery plans, programs for personnel manning, as well as predictions and controls of budgets. The profits of the learning curves were underlined by professor Charles Bailey in the university of central Florida in his statement that "the likely applications of learning curves exceed by far their running use" (Anthes, 2001). The learning curves are influenced by various exterior powers; most of them are the human attributes.

Learning is influenced from factors that depend on the size of processes and on each process point of production. Briscoe and Roark (Briscoe, N.R. and Roark, S., 1991) determine these points as learning sources, and divide them in three basic categories the pre-treatment, the intermediate work and the exterior learning. These sources could easily be called before, at the duration, and after the production.

When the Lockheed had problems in the production of L -1011, he was accused for the fact that the enterprise engaged 2.000 inexperienced employees fast in the production. These employees were placed via a training program of four weeks in the manufacture of planes. The initial expenses were increased and were not decreased at the duration of initial production of planes because of the inexperience of the workers. In other words nor the more precise estimates of efficiency can ensure that our estimates will be precise. Because we examine human beings the approximate efficiency and the real efficiency can vary for many reasons. Consequently no new technology/company is supposed to support itself exclusively on the learning curves (Kondratief, 1984). Actually, the average proportion of investments in new technologies/companies is increased in the advanced industrial countries, because the products that are based on the mature technologies are transported continuously in the countries with low salaries. In other words, products that are provided by the new technologies/companies can be produced in the countries with high salaries. This happens, because the competitive advantage of new technologies /companies to decrease the cost of production is offered via the possibility of enterprise’s personnel to improve the production process. Accordingly, the competitive advantage of experienced technologies/ companies in the production of their products, that is based on the low expenses (usually the low salaries, decreased taxes etc), is contradictory to the factors as the natural resources and the expenses of transports. The new technologies/companies are important for the economic growth. In other words, the investment in the new technologies/companies is the form of competition that the USA and the EU can maintain in their borders.

After a research in Greece it was realised that learning curves are not applied in any sector by any public service or organisation. In the private sector in a sample of 36 companies from northern Greece the results were rather discouraging not only the use of learning curves but also for the effectiveness of their application.

Finally, the idea behind the learning curves is that people become better in the products production, while they become more experienced. While various methods exist, generally no model is acceptable, as the best one. It is very easy for the learning curves to be calculated with the computers utilisation and the information they provide can be used in the decision-making in all sectors, from the supply up to packing. They are rendered as a dynamic tool for crises management because of this the technologies/companies are in situation to appreciate their likely attitude in a potential crisis. While the learning curves are useful it is not a technique for cost reduction. Learning curves can be used in order to answer a lot of questions and their use will continue the increase because of the fact that the efficiency is a requirement and not a target.

8. References

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