AMO-Advanced Modeling and Optimization, Volume 7, Number 2, 2005

Determination of EOQ of multi-item inventory problems through nonlinear goal programming

S. Panda[1],* S. Senapati[2], K. Banerjee[2] and M. Basu[2†]

[1] Department of Mathematics, Bengal Institute of Technology, 1.no. Govt. Colony, Kolkata-700150, India

[2] Department of Mathematics, University of Kalyani, Kalyani-741235, West Bengal, India

Abstract

This paper describes how the proper priority structure of non-linear goal programming (NLGP) model can be selected for obtaining economic order quantity(EOQ) of multi-item inventory problems. In the solution process, sensitivity analysis of the priority under the given weight structure of goals has been performed. A set of solution is obtained. From the solutions, the ideal solution is identified. The D_1-distances of different solutions from the ideal solution are calculated. The solution corresponding to the minimum D_1-distance gives the best compromise solution. Finally, a case study demonstrates the applicability of the proposed technique.

Key words : Nonlinear goal programming, EOQ, D_1-distance.

1 Introduction:

After the discovery of EOQ formulae in 1918 by Wilson, a variety of EOQ models have been solved by many researchers under a variety of modeling assumptions and the literature reviewed by many authors (Raymond [13], Whitin [17], Hadley and Whitin [9], Clark [5], Silver [15] etc.). In most of the cases interest has been directed in studying the theory of single-product and single-installation system, i.e. one item stored in one location, with the successful utilization of several well-known optimization techniques to arrive at the optimal solution. Emphasis in research on implementing the theory of multiple items in multiple locations is relatively less. This is due to the lack of availability and hence proper utilization of adequate optimization techniques in multi-objective decision making area.

In real world problem situation, several conflicting objectives arise in multi-item inventory problems such as cost functions, budget etc. In some cases, the cost functions are non-linear in nature due to the associated set up cost. Now a days, goal programming (GP) technique, developed by Charnes and Cooper [2] and its methodology extended by Lee [12], Ignizio [10] etc., is used in the field of multi-item inventory control system (Romero[14]). In GP, the desired target levels are incorporated to each of the objectives. Then the objectives as well as the structural constraints are considered as goals by introducing under- and over-deviational variables to each of them to penalize in positive or negative way if they are not achieved. The goals are then rank-ordered according to their importance in the decision-making context. Once the priorities are selected, weights are given according to the relative importance of the goals at the same priority level. As a whole in the literature generally two types of GP are found: (1) the weighted deviations for all the goals are minimized using a single objective and (2) deviations are placed according to their importance in the decision making context in lexicographical ordering fashion and the optimality of higher priority level is obtained first then the achievement of optimality of next priority level. Using GP technique, Golanly et al. [8] have developed an inventory model which has been applied to a large chemical plant for the purpose of accomplishing several conflicting objectives in a

*Correspondence: tell: +91339433247197, fax: +913323458005, E.mail: shibaji_panda@yahoo.com

†Tell: +91339433186058, fax: +913325828282, E.mail: manjusri_basu@yahoo.com

quantitative manner. Using NLGP technique, Basu et.al.[1] have developed a solution procedure for solving multi-item inventory problems where the problem has the characteristic of dynamic programming. Charnes and Collomb[3] introduced the goal interval programming in which the decision maker has the freedom to choose an interval and penalize the deviation from either end of the interval. The popular U-penalty function was introduced by Charnes et.al[4] in which more deviation from the goal introduces more weight to penalize. In this method per unit penalty incurred between each pair of objectives by introducing upper bounds to the deviational variables. But for the introduction of boundaries to the deviational variables it is not definite that the deviational variables will form the starting basis[16]. Recently Jones and Tamiz[11] proposed an interesting preference modelling technique in which per unit penalty incurred at the more preferred objective by the elimination of the bounds of the deviational variables through the introduction of an extra objective to the next deviational target. And in this process the penalisation would be done according to the decision makers preference. However all the methods reported above require determination of proper priority structure but selection of priorities is a difficult task in a complex decision making environment. To overcome this difficulty the method discussed in this paper may be used in which instead of determining priority structure the best possible compromise solution is obtained.

In this paper, how the proper priority structure of GP model can be selected for obtaining the optimal solution is discussed. In the solution procedure sensitivity analysis with variation of priority under the given weight structure of the problem has been performed. Using the different priority structures, different possible solutions are obtained. The ideal solution of the model is then identified from the different solutions associated with different priority structures. Finally, the result shows that the minimum of the D_1-distances from the different solutions to the ideal solution identifies the best compromise solution.

2 Model formulation:

The general model of multi-item inventory problem is of the form

Find $\overline{q}(q_1, q_2, \cdots\cdots, q_n)$ so as to

$$minimize \sum_{i=1}^{n} \left(C_i q_i + \frac{C_{si}}{q_i} \right)$$

subject to

$$\sum_{i=1}^{n} a_{ij} q_i \leq=\geq b_j, \qquad 1 \leq j \leq m \tag{1}$$

and

$$0 < q_i \leq Q_i \qquad 1 \leq i \leq n$$

where $C_1, C_2, \cdots\cdots C_n$ are sum of ordering costs and $C_{s1}, C_{s2}, \cdots, \cdots, C_{sn}$ are the set-up costs and $Q_1, Q_2, \cdots\cdots Q_n$ are the level of inventory of $q_1, q_2, \cdots\cdots q_n$ respectively.

In (1) the cost related to the items $q_1, q_2, \cdots\cdots, q_n$ is the objective which is to be minimized subject to several restrictions. Since the cost related to each of the items are independent and they are interrelated by some constraints so the minimization of the entire objective function subject to a set of restrictions can be treated as the n objective functions subject to the same set of restrictions and due to this disaggregation the final solution will not be effected. Thus, the objective of (1) can be considered as n objective functions. In view of GP each objective $C_i q_i + \frac{Csi}{q_i}$ is considered as goal by introducing under- (d_{ik}^-) and over-(d_{ik}^+) deviational variables and a target level G_i, $(1 \leq i \leq n)$

The priority based NLGP model of multi-item inventory problem takes the form:

Find $\overline{q}(q_1, q_2, \cdots\cdots, q_n)$ so as to

$$minimize \quad P_1 \left[\sum_i \left(w_{i1}^- d_{i1}^- + w_{i1}^+ d_{i1}^+ \right) + \sum_j \left(w_{n+j,1}^- d_{n+j,1}^- + w_{n+j,1}^+ d_{n+j,1}^+ \right) \right]$$

$$minimize \quad P_2 \left[\sum_i \left(w_{i2}^- d_{i2}^- + w_{i2}^+ d_{i2}^+ \right) + \sum_j \left(w_{n+j,2}^- d_{n+j,2}^- + w_{n+j,2}^+ d_{n+j,2}^+ \right) \right]$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$minimize \quad P_k \left[\sum_i \left(w_{ik}^- d_{ik}^- + w_{ik}^+ d_{ik}^+ \right) + \sum_j \left(w_{n+j,k}^- d_{n+j,k}^- + w_{n+j,k}^+ d_{n+j,k}^+ \right) \right]$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$minimize \quad P_K \left[\sum_i \left(w_{ik}^- d_{ik}^- + w_{ik}^+ d_{ik}^+ \right) + \sum_j \left(w_{n+j,k}^- d_{n+j,k}^- + w_{n+j,k}^+ d_{n+j,k}^+ \right) \right]$$

subject to

$$C_i q_i + \frac{Csi}{q_i} + d_{ik}^- - d_{ik}^+ = G_i, \qquad 1 \le i \le n$$

$$\sum_{i=1}^{n} a_{ij} q_i + d_{n+j,k}^- - d_{n+j,k}^+ = b_j, \qquad 1 \le j \le m$$

and

$$0 < q_i \le Q_i \tag{2}$$

$$d_{ik}^- \; d_{ik}^+ = d_{n+j,k}^- \; d_{n+j,k}^+ = 0$$

and

$$d_{ik}^-, \; d_{n+j,k}^-, \; d_{ik}^+, \; d_{n+j,k}^+ \ge 0, \qquad 1 \le i \le n, \qquad 1 \le j \le m, \qquad 1 \le k \le K$$

where $P_k(1 \le k \le K : K \le n+m)$ is the k-th priority factor assigned to the set of goals that are grouped together in the problem formulation. $P_1, P_2, \cdots\cdots P_K$ stand for preemptive priority or priority weight determining the hierarchy of goals. Goals of the higher priority levels are satisfied first and only then may the lower priority goals be considered. Lower priority goals can not alter the goal attainment of higher priority level. Preemptive priority system is based on lexicographical ordering according to the importance of their components, Lee[12]. Thus, achievements of the goals at the priority level P_1 can never be satisfied by the goals at the priority level P_2 and so on. $w_{ik}^-, \; w_{n+j,k}^-(\ge 0)$ and $w_{ik}^+, \; w_{n+j,k}^+(\ge 0)$ are the numerical weights associated with the deviational variables $d_{ik}^-, \; d_{n+j,k}^-$ and $d_{ik}^+, \; d_{n+j,k}^+ \; (1 \le i \le n; \; 1 \le k \le K)$ respectively.

3 Solution procedure:

In conventional priority based NLGP, the solution under the decision makers imposed priority structure is considered as the optimal solution. But in different complex decision making situation, desired solution may not be acceptable under the composed weight structure. Thus a better solution is always expected for which alternative priority under the given weight structure may be considered.
To select the proper priority structure under the imposed weight structure, sensitivity analysis is performed. In the model, K priorities are considered. So involvement of K priorities indicates that $K!$ different solutions can be obtained from $K!$ problems arises for $K!$ number of different priority structures.
Let $\{q_1^{(r)}, q_2^{(r)}, \cdots\cdots q_n^{(r)}\}$, $1 \le r \le K!$ be the $K!$ number of solutions obtained by permuting the K priority levels.
Let for $i = n$

$$\min_{1 \le r \le K!} \left(C_n q_n^{(r)} + \frac{C_{sn}}{q_n^{(r)}} \right) \quad = \min_{1 \le r \le K!} \left(C_n q_n^* + \frac{C_{sn}}{q_n^*} \right)$$

where q_n^* is any value of $q_n^{(r)}$, $1 \leq r \leq K!$ then the ideal solution q^* is defined by $\{q_1^*, q_2^*, \cdots\cdots q_n^*\}$ Cohon[6].

But in practice ideal solution can never be achieved. The solution, which is closest to the ideal solution, is accepted as the best compromise solution, and the corresponding priority structure is identified as most appropriate priority structure in the planning context.

To obtain the best compromise solution, following GP problem is to be solved

$$\min_{1\leq r\leq K!} \sum_{i=1}^{n} \left(d_{ir}^{+} + d_{ir}^{-}\right)$$

subject to

$$q_i^* - q_i^{(r)} + d_{ir}^{-} - d_{ir}^{+} = 0, \quad 1 \leq r \leq n$$

and

$$d_{ir}^{+} \geq 0 \;\; d_{ir}^{-} \geq 0, \;\; d_{ir}^{+}.d_{ir}^{-} = 0, \quad 1 \leq r \leq K!, \quad 1 \leq i \leq n$$

where d_{ir}^{-} and $d_{ir}^{+}$are the under- and over- deviational variables respectively.

Now,

$$(D_1)^r = \sum_{i=1}^{n} \mid q_i^* - q_i^{(r)} \mid$$

is defined as the D_1-distance from the ideal solution $\{q_1^*, q_2^*, \cdots\cdots q_n^*\}$, to the r-th solution $\{q_1^{(r)}, q_2^{(r)}, \cdots\cdots q_n^{(r)}\}$, $1 \leq r \leq K!$

Therefore,

$$\begin{aligned} (D_1)_{opt} \;\; &= \;\; \min_{1\leq r\leq K!} (D_1)^r \quad = \min_{1\leq r\leq K!} \sum_{i=1}^{n} \mid q_i^* - q_i^{(r)} \mid \\ &= \;\; \min_{1\leq r\leq K!} \sum_{i=1}^{n} (d_{ir}^{+} + d_{ir}^{-}) \quad = \sum_{i=1}^{n} (d_{ir}^{+} + d_{ir}^{-}), \;\; say \\ &= (D_1)^p, \quad 1 \leq p \leq K! \end{aligned}$$

Hence, $\{q_1^{(p)}, q_2^{(p)}, \cdots\cdots, q_n^{(p)}\}$ is the best compromise solution according to the problem situation.

4 A case study:

To expound the model the Calcutta branch of the Private Warehousing Corporation (PWC) of India is considered. The capacity of the warehouse is 460 units. The Corporation stores different types of items for different depositors on monthly rent basis. Here, the three main depositors, viz. Food Corporation (FC), Farmers Fertilizers Co-operative (FFCO) and Oil Trading Corporation (OTC) have been considered. Some major items from each depositor are selected. Demand for each item is assumed to be uniform throughout the period. For best arrangement and protection of all types of items, different types of costs are involved. Total budget of the warehouse is 0.9 million dollar. To ensure the profit it has been realized that minimum capacity of the warehouse to be occupied is 80% that is 368 units. Among the selected items the warehouse management gives special attention to the four essential commodities rice, pulses, urea and mustard oil. The main objective of PWC is to minimize these costs. The data for all the items are given in Table-1.

Particulars of data:

Using the data, the goal equations are appeared as:

4.1 Cost goals:

$$5q_1 + \frac{276}{q_1} + d_{1k}^{-} - d_{1k}^{+} = 380, \; (Rice); \qquad 6q_2 + \frac{150}{q_2} + d_{2k}^{-} - d_{2k}^{+} = 200 \; (Wheat)$$

$$7q_3 + \frac{300}{q_3} + d_{3k}^{-} - d_{3k}^{+} = 140\,(Sugar); \qquad 4q_4 + \frac{260}{q_4} + d_{4k}^{-} - d_{4k}^{+} = 100\,(Pulses)$$

$$14q_5 + \frac{680}{q_5} + d_{5k}^{-} - d_{5k}^{+} = 800\,(Ammonia); \qquad 17q_6 + \frac{690}{q_6} + d_{6k}^{-} - d_{6k}^{+} = 860\,(Urea)$$

$$12q_7 + \frac{450}{q_7} + d_{7k}^{-} - d_{7k}^{+} = 480\,(Potash); \qquad 40q_8 + \frac{620}{q_8} + d_{8k}^{-} - d_{8k}^{+} = 1600,\ (Mustard\ \ oil)$$

$$65q_9 + \frac{1030}{q_9} + d_{9k}^{-} - d_{9k}^{+} = 2000\,(Coconut\,oil); \qquad 127q_{10} + \frac{1380}{q_{10}} + d_{10k}^{-} - d_{10k}^{+} = 2600\,(Groundenut\,oil)$$

$$9q_{11} + \frac{400}{q_{11}} + d_{11k}^{-} - d_{11k}^{+} = 250\,(Rapeseed\,oil); \qquad 16q_{12} + \frac{610}{q_{12}} + d_{12k}^{-} - d_{12k}^{+} = 200\,(Vegetable\,oil)$$

Table – 1

Depositors	Items	Cost (C_i) (\$)/ unit	Cost (C_{si}) (\$)	Level of inventory (units)	Average investment \$/unit	Gn (\$)
FC	Rice	500	27600	80	550	38000
	Wheat	600	15000	40	650	20000
	Sugar	700	30000	20	750	14000
	Pulses	400	26000	30	460	10000
FFCO	Ammonia	1400	68000	60	1600	80000
	Urea	1700	69000	50	1950	86000
	Potash	1200	45000	40	1400	40888
OTC	Mustard	4000	62000	40	4500	160000
	Coconut	6500	103000	30	7500	200000
	Groundnut	12700	138000	20	14000	260000
	Rapeseed	900	40000	30	1200	25000
	Vegetable	1600	61000	20	1800	20000

4.2 Item restriction:

To ensure the profit, it has been realized that 80% capasity of the warehouse should be occupied. Therefore, the the goal equation appears as :

$$\sum_{i=1}^{12} q_i + +d_{13k}^{-} - d_{13k}^{+} = 368$$

4.3 Financial restriction:

To meet the total expenses incurred, a fixed amount per unit is provided each month for each item. So the goal equation is

$$55q_1 + 65q_2 + 75q_3 + 46q_4 + 160q_5 + 195q_6 + 140q_7 + 450q_8$$

$$+750q_9 + 1400q_{10} + 120q_{11} + 180q_{12} + d_{14k}^{-} - d_{14k}^{+} = 90000$$

4.4 The weight structure:

Three structures are assigned to include the goals in the model and the structures are designed in the decision making context according to the decision maker's choice. It should be noted that in general circumstance the financial restriction and the item restriction are treated as the hard constraints which are never be violated. But here our object is to determine the best compromise solution instead of assigning the priorities to the goals. Thus all the goals have the equal opportunity to acquire the highest priority in the decision making context once. The structure of the model is of the form:

$$S_K^1 \quad : \quad (3d_{1k}^+ + 2d_{4k}^+ + 2d_{6k}^+ + d_{8k}^+)$$
$$S_K^2 \quad : \quad (2d_{2k}^+ + d_{3k}^+ + 3d_{5k}^+ + 3d_{7k}^+ + d_{9k}^+ + d_{10,k}^+ + 2d_{11,k}^+ + d_{12,k}^+)$$
$$S_K^3 \quad : \quad (d_{13k}^- + d_{14k}^-)$$

The suffix K represents the level of priority $(K, = 1, 2, 3)$.
Now from the above three weight structures, $3! = 6$ priority structures under the imposed weight structure can be obtained. With these 6 set of priority structures, 6 problem can be solved. All the non-linear programming problems are solved by a computer program based on penalty function method algorithm[7]. The solutions of 6 problems are displayed in $Table-2$, along with the ideal solution.

Table – 2(Solutions)

Run	Priorities	*Solution*											
		q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{10}	q_{11}	q_{12}
1	$S_1^1S_2^2S_3^3$	76.04	33.29	**12.2**	21.79	55.97	44.84	34.26	34.29	**22.01**	**13.7**	**22.17**	**9.71**
2	$S_1^1S_2^3S_3^2$	76.12	33.4	12.22	21.88	56.06	44.94	34.36	34.34	24.73	14.22	22.96	9.74
3	$S_1^2S_2^1S_3^3$	77.68	36.44	15.27	25.73	56.24	47.42	36.86	36.72	25.95	15.68	26.17	11.62
4	$S_1^2S_2^3S_3^1$	77.69	36.24	15.07	25.53	56.04	47.52	36.66	36.52	25.75	15.47	25.97	11.42
5	$S_1^3S_2^1S_3^2$	**75.19**	**33.08**	12.88	**20.66**	**54.00**	**43.81**	**34.11**	**34.13**	23.66	13.72	22.95	9.83
6	$S_1^3S_2^2S_3^1$	75.74	33.98	13.31	21.29	54.66	44.48	34.90	34.92	24.36	14.02	23.71	10.08
Ideal Solution		**75.19**	**33.08**	**12.20**	**20.66**	**54.00**	**43.81**	**34.11**	**34.13**	**22.01**	**13.07**	**22.17**	**9.71**

Possible solutions of the problem

In $Table-3$ the D_1-distances of all possible solutions from the ideal solution are calculated. From Table-3 it is found that the minimum of the D_1-distances of possible solutions from the ideal solution is 3.88

Table – 3(D_1-distances of all the possible solutions from the Ideal Solution)

Run	$D_1 - Distances$
1	5.5
2	10.2
3	37.01
4	35.11
5	**3.88**
6	10.68

which corresponds to the priority structure $P_1 = S_1^3, P_2 = S_3^2, P_3 = S_3^2$. Therefore, the best compromise solution of the problem is:

$q_1 = 75.19$	$q_5 = 54$	$q_9 = 23.66$
$q_2 = 33.08$	$q_6 = 43.81$	$q_{10} = 13.72$
$q_3 = 12.88$	$q_7 = 34.11$	$q_{11} = 22.95$
$q_4 = 20.66$	$q_8 = 34.13$	$q_{12} = 9.83$

In $Table-4$ the goal description and its achievement are displayed. It is very interesting to note that though in the solution process each goal acquires highest priority once (since six problems are solved), the best compromise solution to the problem is found under the priority structure in which the financial goal and the item restriction goal have the highest priority. At the same time the goal for rice(q_1), pulses(q_4), urea(q_6) and mustered oil(q_8) are found in the final priority level though the decision maker wants to give a special attention to these four items. Since all the goals are achieved no problem arises form the compromise solution to the decision maker. If any one or more of the goals of the assumed final priority of the compromise solution is not achieved or the financial goal or the item restriction is not satisfied then the decision maker may choose the next best compromise solution to satisfy his desire and so on. Which is not found to happen if the priority level in the decision making context is assumed to be rigid. From this point of view this method is flexible in comparison to U-penalty function method[4] and penalty function method via preference modeling technique[11].

Table – 4(Goal description and achievement)

Priority	Goal description	Achievements	Results
P_1	Item and financial restriction goal	All the goals are achieved	To achieve all the goals 88.2% of the total capacity of the warehouse is occupied by all the items
P_2	Minimization of the costs for rice, pulses, mustered oil and urea	All the goals are achieved	rice 93.99%, pulses 68.87%, mustard oil 85.33% and urea 87.62%of level of inventory are met
P_3	Minimization of of the cost for Wheat, Sugar Ammonia,Potash, Coconut oil,Gro-undunt oil,Rape-seed oil and Vegatable oil	The goals of Sugar, Ammonia, Potash,Coconut oil, Groundnut oil,Rapeseed oil are achieved but wheat and vegetable oil are not achieved	Wheat 82.7%, Sugar 64.4% Ammonia 90%, Potash 85.28% Coconut oil 78.87%,Ground nut oil 68.6%, Rapeseed oil 76.5%, and Vegetable oil 49.15% of level of inventory are met

5 Conclusion:

The most widely used technique for the multi-item inventory problems subject to certain restrictions is the Lagrangian Multiplier method. Where the value of Lagrangian Multiplier is selected by trial method. So, the exact value may not be detected. Hence a trial optimal solution is obtained, which may not be desired to a decision maker. Also in practice, many difficulties may arise to control a large number of items under several restrictions because in that situation the value of Lagrangian Multiplier may not be determined. Thus to solve the GP model and to provide the best possible solution subject to the model constraints and priority structure of the goals this method may be used. The elegence of this method is that it is always possible get the best compromise solution without choosing the proper priority level rather than the U-penalty function method[4] or penalty function method via preference modeling technique[11], in both of which determination of proper priority level is essential for the successful determination of the solution. And the solution obtained by these two methods may not be acceptable to the decision maker. This situation may be overcomed by the reported technique. If the best compromise solution is not desired by the decision maker then the next best compromise solution can be choosen and so on. The process may be continued $K!$ times since K priority levels form $K!$ problems and thus the decision maker has $K!$ options from which he has to choose one. Which immediately implies that use of the reported technique to find EOQ of multi-item inventory problem in complex decision making context is most flexible among it's class. On the contrary, selection of goal structure in complex decision making context is very difficult and determining the best compromise solution, priority level to the goals which are to be achieved may be assigned.

Acknowledgement:

The first author expresses his sincerest thank to Techno India Group for supporting this research work.

6 References:

[1]Basu M, Ghosh D and Banerjee K, 1997. A Solution Procedure For Solving Multi-item Inventory Problem. International Journal of Management and Systems 15(1) 53-68.
[2]Charnes A and Cooper WW, 1961. Management Models and Industrial Application of Linear Programming. John Wiley & Sons. Inc. New York.
[3]Charnes A and Collomb B, 1972. Optimal economic stabilization policy: linear goal programming

models. Socio-Econ. Plann. Sci. 6, 431-435.

[4]Charnes A, Cooper WW, harrald J, Karwan K and Wallace W, 1976. A goal interval programming model for resource allocation in marine environment protection problem. J. of envir. Econ. Mgmt. 3, 347-362.

[5]Clark AJ, 1972. An Informal Survey of Multi-echelon Inventory Theory, Naval Research Logistics Quarterly, 40, 252-256.

[6]Cohon JL, 1978. Multiobjective Programming and Planning. Academic Press, New York.

[7]Deb Kalyanmoy, 2000. Optimization for Engineering Design. Prentice-Hall of India, New Delhi 153-171.

[8]Golanly B, Yadin N and Learner O, 1991. A Goal Programming Inventory Control Model Applied at a large Chemical Plant. Production and Inventory Management Journal, First Quarter, 16-23.

[9]Hadley G and Whitin TM, 1963. Analysis of Inventory Systems. Prentice-Hall, Inc., Englewood Chiffs, New Jersey.

[10]Ignizio JP, 1976. Goal Programming and Extensions. Lexington Books, Massachusettes .

[11]Jones DF and Tamiz M, 1995. Expanding the Flexibility of Goal Programming via Preference Modelling Techniques. Omega, 23(1) 41-48.

[12]Lee SM, 1972. Goal Programming for Decision Analysis. Auerbach, Philadelphia.

[13]Raymond FE, 1931. Quantity and Economy in Manufacture. McGraw-Hill Book Company, New York.

[14]Romero C, 1986. A Survey of Generalised Goal Programming(1970-1982). Eur, Jr. Opl. Res., pp183-191.

[15]Silver AE, 1981. Operations Research in Inventory Management. A Review and Critique. Oprs, res., pp. 628- 645.

[16]Tamiz M and Jones DF, 1993. GPSYS, specifications for an intelligent goal programming package. University of Portsmouth, U.K.

[17]Whitin TM, 1953. The Theory of Inventory Management. Prentice University Press, Princeton, New Jersy.