AMO - ADVANCED MODELING AND OPTIMIZATION, VOLUME 6, NUMBER 2, 2004

Customer migration, campaign budgeting, revenue estimation: the elasticity of Markov Decision Process on customer lifetime value

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KEYWORDS. Customer lifetime value, Higher-order Markov decision process.

Abstract

To predict the profitability of a customer, today's firms have to practice Customer Lifetime Value (CLV) computation. Different approaches are proposed in the last ten years to analyze the complex customer phenomenon. One of them is Markov Decision Process (MDP) model. The class of Markov Models is an effective and a flexibility decision model. Whereas the use of MDP model is limited by its assumption, in this paper, we attempt to introduce an extension model for MDP: Higher-order Markov Decision Model (HMDP). HMDP can perform excellently in CLV calculation and overcome the limitation of MDP. By using a real application, we will demonstrate how it can be used efficiently in a firm's daily operations.

1. Introduction

The prominent topic of today's marketers is Customer Relationship Marketing (CRM). Customers as assets become a theme of today's firm. Firms are seeking ways to maintain and develop long-term relationships with customers. More or less, CRM brings changes to our lives. For example, direct addressed advertisements are boomed into your mailbox or e-mail account everyday. And instead of being abandoned in a narrow space, you would try on clothes in a glamorous fitting room and be given advice from nice salesmen. In general, we have been served the best in these years. But, does CRM simply mean something that only involves the frontal staff of a firm? Actually, the revolution of CRM requires a boundaryless corporation across the firm. CRM is an integrated business strategy to manage and coordinate customer interactions, to understand and anticipate the existing and future need of its current and potential customers. Facilitating this strategy requires appropriate plan, management, process and technology [Zablah, 2004]. A better CRM can enforce a harmony mutual long-standing relationship between an organization and its customer. In a business terminology, CRM aims at building up business loyalty. Over years, relationship-marketing promoters claim enthusiastically that a loyal customer will repeat a business with the firm so that the business position of this firm will be strengthened and the profitability will be enhanced. As though, investors capitalize billion of dollars in the development of CRM. According to a study conducted by Aberdeen.com in 2003, it is estimated that US firms would spend $17.7 billion on CRM in 2006 compare to $10 billion in 2002. However, would it be that simple to claim loyal equals to profit? Certainly the answer of CRM software vendors is yes, whereas this is a question for the business executives should take a stop before investing expenditures on CRM. The gospel of loyalty becomes a common concept for the last ten years. In fact, there is a raising concern of how much or whether a loyal customer will generate a tremendous profit for a firm [Reinartz and Kumar, 2002]. The truth is, to grasp the metric of customer revenue, we have to exercise the Customer Lifetime Value (CLV) analysis.

The main aim of this paper is to give a tutorial implementation of higher-order Markov decision process for customer lifetime value based on the paper [Ching et al. 2003]. The rest of the paper is organized as follows. In Section 2, we introduce the concept of customer lifetime value. In Section 3, we give the notations and our models. In Section 4, we present some practical numerical examples. Finally a summary is given to conclude the paper in Section 5.

2. Customer Lifetime Value

In marketing research, quantitatively predict revenue generated by a customer's entire trading life with a company is called "Customer Lifetime Value" (CLV). The study of CLV is an art of forecast. Despite the importance of CLV, few companies have actually calculated it. Even so, common practices use traditional numerical illustrations, like graphical representation or tableau observation, to analyze the complex metric generated by its customers. One of the reasons for the hesitation of using mathematical models is the misunderstanding of the complexity of mathematical models [Gupta and Lehmann, 2003]. Besides, many marketers have an impression that mathematical models are hard to implement in a day-to-day operation. Indeed, to compute CLV efficiently and accurately, it is notable that company should move a step further to consider mathematical models [Berger and Nasr, 1998]. Moreover, with today's computer software, lots of mathematical models can be easily embedded into a personal computer.

The metric of CLV can be divided into two mainstreams: individual-approach or average-approach [Kumar et al, 2004]. Both approaches can estimate the customer value in either a discrete or continuous time planning horizon. The average-approach estimates the total value gained from all customers. These models incorporate a stochastic distribution function for the prediction of customer acquisition rate at different time horizon. Gupta and Lehmann [Gupta and Lehmann, 2004] present an average-approach that is easy to be modeled in an infinite horizon and to be computed by public available financial data. The root of their model is from a popular class of financial discount cash flow work where the time horizon discount value as well as customer retention rates are incorporated within. Their study successfully addressed the link between the customer value and the firm market value.

The use of individual-approach, on the other hand, address to the action level. The individual CLV approach aims at predicting the customer migration. The most well known method would be RFM [Recency, Frequency, and Monetary]. The cornerstone of RFM is that a customer's probability of consumption in the following horizon is governed by customers' recency of purchase, the frequency of purchase and the amount spend on purchase. Markov Decision Process (MDP) is one of the RFM models (Pfeifer and Carraway, 2000). MDP is so well-developed in the field of decision science that methodology for implementing are widely published. In CLV calculation, MDP can be used to model both customer retention and customer migration situations. However, the MDP assumption of static at the entire process is hard to achieve in a real setting [Jain and Singh, 2002]. The flexibility and simplicity of MDP build on an assumption: every probability incident solely relies on those in the previous state. That is, what happens today is the consequence of yesterday, only. And, the probability of incident is the same for everyday. However, in reality, most of the incidents are intertwined from more than one previous horizon. In addition, as supported by Reinartz and Kumar (2002), the general class of RFM approach, including MDP, will leave out non-frequent buyers. Reinartz and Kumar (2003) proposed a new approach that the profitable lifetime duration captures the dynamic nature of the customer-firm relationship through the time varying nature of the independent variable. That is no matter a customer is a frequent-loyaler or non-frequent-loyaler, the value of him/her would be studied. It is remarkable that their study deals with a single firm. Competitive information is out of the study scope.

To fill the delinquency of these models, we intend to present a model that establishes base on MDP: Higher-order Markov Decision Process (HMDP). First we show how one can use HMDP to efficiently incorporate the estimation non-frequency customers' values. Also, although different individual CLV models are proposed, very few researchers have studied models which incorporate promotion budget allocation under competitive and stochastic situation. We will demonstrate how to model MDP or HMDP to achieve such situations. In addition, the solutions of our proposed MDP and HMDP models are computed by an optimization tool that solution formula can be embedded into an EXCEL worksheet to demonstrate the simplicity of incorporating these so called sophisticated models into daily operations. A real businesses application will be presented to show the efficiency of HMDP and MDP as well as the use of them in budgeting. Finally, we will pinpoint remarks for researchers and managers.

3. Definition and Modeling

To set an idea, we illustrate our work based on a real business problem encountered by Tel (we use a nickname here). A telecommunication company Tel is a long distance phone calling service provider. The call receiving quality of Tel is known to be the best in the market. Pros and cons, to pledge its service quality, Tel invests numerous for operation and maintenance. In recent year, the competition in the long distance phone calling service market has been set to intensify. Companies, specifically for middle-size and small enterprises, are shifting to use other long-distance call providers which provide cheap but only acceptable receiving quality. The cheap phone rates provided by competitors continue to impose pressure on this once dominant carrier.

Tel strives strategy to struggle out of this challenge. Improving and strengthen its relationship with customers is one of the thoughts. Tel will promote a special discount rate for existing customers as well as potential customers. Tel believes that a promotion is beneficial to a longer period as it stands. This is because customers establish a sense of business loyalty to Tel. Whereas money is the key ingredient of running business, this sense of business loyalty will flame out in a certain time. To launch promotion, additional operational costs are required for media advertisement, mailing category and hiring temporary staff for warm telephone call. Under this circumstance, Tel has to estimate four figures. The first one is the value of the campaign to each individual customer. Is it worth to give a discount rate to each customer? What is the return? The second is the duration of promotion under the budget control. The third is the promotion periodic pattern. The last one is the amount for capitalizing this campaign. None of these factors are necessarily mutually exclusive. Indeed, they are cross-related to each other.

We see that the class of Markov models is the most suitable for practicing CLV in this circumstance. The approach will be illustrated in this section. To simplify the discussion, reference for class of Markov models, i.e. both MDP and HMDP, will be called Markov. All Markov formulation is applicable to both MDP and HMDP. We will also identify the different setting or operation between MDP and HMDP.

Step 1: Specification of states and planning horizon

There are two core bases for Markov: planning horizon and state. The entire studying period is decomposed into an equal length of duration which is termed as the planning horizon. We fix “week” as the planning horizon for the Tel study and use whole planning horizon to describe the entire planning period. The planning horizon is indexed by t, where t = 1, 2, …, T. If Tel aims at estimating twenty weeks of its CLV, T would be equal to 20. Also, in Markov, the whole planning horizon would be either finite or infinite. A prefixed time line of the whole planning horizon is defined as a finite study (for example T=20). Infinite horizon refers to a plan that last forever.

The second Markov specification is state which is the mode of existence for the system. For example, states for daily weather forecast are sunny, cloudy and rainy. States defined as S_t where t= 1,2, …, T indicates the planning horizon under study. At each planning horizon, we classify all existing customers into three categories according to their long-distance call usage rate, starting from highest to lowest: 1-Gold, 2-Silver, and 3-Bronze. We add the fourth state, 4-Audit which represents an individual who is a customer of Tel ‘s competitors. In our following discussion, we use the word “outsider” to describe them. All four categories are states of the Markov, as illustrated by Figure 1. For example, if a customer migrates from Gold at week 1 to Bronze at week 2, the state for this customer is $S_1 = 1, S_2 = 3$.

Step 2: Calculation of transition probabilities

A customer will undergo a change of state at the end of every horizon. The change is in associate with a set of probabilities. A probability for a customer, who, at time t, in state $q_\bullet$ (falls into one of those categories) given that, at time t he or she was in states $q_\bullet$, is

$$P(S_t = q_k \mid S_1 = q_i, S_2 = q_k, \cdots, S_{t-1} = q_l).$$

For example, the probability for a customer who has the state pattern: Silver, Silver, Gold, Silver and Bronze from week 1 to week 5 is

$$P(S_5 = 3 \mid S_1 = 2 \ , S_2 = 2, S_3 = 1, S_4 = 2).$$

The formulation is indeed only a general representation of any stochastic process (a process characterized by probabilities).

Markov Decision Process

A key feature of MDP is built on the absence of all predecessor states, i.e. the future prospects of customers spend are a function only of the current state and not of the particular path for the customers took to reach this state. The process of MDP is assumed to specify only a recency, i.e. a state before. The transaction is a consequence of the previous state. The probability representation is thus truncated to

$$P(S_t = q_k \mid S_1 = q_i, S_2 = q_k, \cdots, S_{t-1} = q_l) = P(S_t = q_k \mid S_{t-1} = q_l).$$

Furthermore, in MDP, the transition is assumed to be independent of time (See Figure 2 for illustration), thereby leading to the form

$$p_{lk} = P(S_t = q_k \mid S_{t-1} = q_l) \text{ for all t.}$$

These are termed as transition probabilities, which are the probabilities that a state moving from one to another after a period of transition. Apart from states, there is one more factor in associate with p_{lk}, promotion or no-promotion. The transition patterns are correspondence to whether promotion has been launched or not. In 2003, Tel launched the same campaign for 8 weeks. Together with 12-week data for the no-promotion period, it is possible to compare the customer behavior between the period of promotion and no-promotion. Thus, one could compile two sets of transition probabilities where $p_{lk}^{(1)}$ stands for the probability at state k which transits from state l in which there is no promotion; similarly, $p_{lk}^{(2)}$ stands for the probability at state k which transits from state l in which there is promotion launched. It is assumed that all customers in the market are under study, thus the relationship of these transition probabilities are

$$\sum_{k=1}^{N} p_{lk}^{(1)} \text{ for } l = 1,2,\cdots,N \qquad \sum_{k=1}^{N} p_{lk}^{(2)} \text{ for } l = 1,2,\cdots,N$$

Empirical examples for the transition probabilities are listed in Table 1. The data is a promotion week's record. In this promotion week, the total number of customers in state 1 for the whole planning horizon is 5010 and the actual count of the number of customers from state 1 transit to 3 is 1013. Thus, the transition probability from state 1 to state 2 under promotion would be 1013/5010=0.2022.

Higher-order Markov Decision Process

The general first-order MDP is famous for its flexibility which can be implemented into many applications, and its objective optimality which best decision can be determined. However, in the real world, it is easy to violate MDP's "solely single recency" association. Although we seek for mathematical representations to simplify the real application problems, a dramatic truncation will bring delinquency to the results. On the other hand, a sophisticated mathematical model would narrow the scope of application. It is hard to find a two-way balanced solution.

Thanks to the fast-growing computational technology, there is a direct and simple improvement that can generate a better optimal strategy for MDP. This is the introduction of a higher number of recency, Higher-order Markov Decision Process (HMDP). To facilitate the discussion, in this paper we will consider a second-order Markov Decision Process (2-MDP) as a demonstration of this approach. The main difference between MDP and HMDP is the transition probability. The operation of a 2-MDP requires a specification of events happened in the previous two horizons. The probability of transition becomes

$$P(S_t = q_k \mid S_1 = q_i, S_2 = q_k, \cdots, S_{t-2} = q_i, S_{t-1} = q_l) = P(S_t = q_k \mid S_{t-2} = q_i, S_{t-1} = q_l)$$
$$= p_{ilk}$$

This is an extension of MDP. Given the above probabilities, the number of transition probabilities for the 2-MDP expand to N2×N compare with N×N of MDP, where N is the number of states. In Tel study, the number of transition probabilities is sixteen for MDP and sixty-four for HMDP. The convergence of infinite horizon to a stationary policy is nicely preserved by the 2-MDP. We termed this as a 2-transition probability. Similar to transition probability, sum of all 2-transition probabilities at every state are

$$\sum_{k=1}^{N} p_{ilk}^{(1)} \text{ for } i,l = 1,2,\cdots,N \;, \quad \sum_{k=1}^{N} p_{ilk}^{(2)} \text{ for } i,l = 1,2,\cdots,N \;,$$

where $p_{ilk}^{(1)}$ and $p_{ilk}^{(2)}$ account for the 2-transition probability for no-promotion and promotion respectively. For example, the 2-transition probability from state 4 to state 4 to state 3 under promotion would be 0.1815 (See Table 1).

Step 3: Define promotion cost, operation cost and profitability

As named, customer lifetime value, what are the values here? Values associated with states are the profit gained from a customer and the resources required for carrying out promotion. Tel estimates that the promotion budget at state i for a customer would be d_i for i = 1, 2, 3, 4. Also, the revenue obtained from a customer in State i with no-promotion would be $c_i^{(1)}$ and with promotion would be $c_i^{(2)}$, for i = 1, 2, 3, 4. As those for transition probabilities, these values are computed by 20 weeks data.

Step 4: The mathematical model

We can now start building the mathematical model. At each state, Tel has to decide whether launching promotion or not.

Markov Decision Process

At the beginning of every week, Tel aims at maximizing its expected net present value Vil(t):

$$V_i(t) = Max\left\{ c_i^{(1)} + \alpha \sum_{k=1}^{4} p_{ik}^{(1)} V_k(t-1), c_i^{(2)} - d_2 + \alpha \sum_{k=1}^{4} p_{ik}^{(2)} V_k(t-1) \right\} \quad (1)$$

The first choice in the parenthesis is the expected return of the ith type of customer when there is t planning horizon left if no-promotion is launched at this time horizon. Here $c_i^{(1)}$ is the net gained plus the transition probabilities multiply the gained at the t-1 following planning horizon, i.e. Vi(t-1) times the discount factor α. We have similar representation of the second choice for the expected return for launching promotion.

One of the advantages of MDP is that it can be proved mathematically that a stationary policy, vi for i =1,2,3,4, exists for an infinite planning horizon. A stationary policy is a policy that the choice of alternative depends only on the state of the system is in and is independent of time. This paper attempts not to get deep into this proof. But this nice property is extremely useful in practice. Interested readers may consult the book by Altman (Altman ,1999). The existence of a stationary policy implies at any time frame the decision depends only on the state of the customer is in and is independent of t. The stationary policy has the advantage of being computationally simpler. For the infinite horizon case, a MDP can be represented by the following linear programming problem see [Altman, 1999]:

$$\min x_0 = \sum_{i=1}^{4} v_i$$

Subject to $v_i \geq c_i^{(1)} + \alpha \sum_{k=1}^{4} p_{ik}^{(1)} v_k$, for i =1,2,3,4,5

$v_i \geq c_i^{(2)} - d_i + \alpha \sum_{k=1}^{4} p_{ik}^{(2)} v_k$, for i =1,2,3,4,5

$v_i \geq 0$, for i =1,2,3,4,5

Linear Programming model (LP) is a general decision tool (for more details please refer to Winston, 1994). In short, it aims to achieve the best value of the objective function ($\min x_0 = \sum_{i=1}^{4} v_i$ in this case) with a set of governed constraints. The set of constraints can be divided into two, one for the no-promotion period and the other for the promotion period. If it is a stationary policy, its expected reward is greater than all the right-hand-side value in (1). Also, the term Vi(t-1) in (1) is replaced by vi as this stationary policy achieves the best across the whole planning horizon. The above linear programming problem can be solved easily by using spreadsheet EXCEL.

Higher-order Markov Decision Process

Similar to MDP, an optimal strategy exists for HMDP [Ching et al, 2003]. At every time step, the model aims at maximizing

$$V_{il}(t) = Max\left\{ c_l^{(1)} + \alpha \sum_{k=1}^{4} p_{ilk}^{(1)} V_{lk}(t-1), c_l^{(2)} - d_l + \alpha \sum_{k=1}^{4} p_{ilk}^{(2)} V_{lk}(t-1) \right\} \quad (2)$$

This is the total expected reward with state l and previous state i. The model seeks for the best move at the next time step. The probability of being in state k is $p_{ilk}^{(2)}$ if promotion is launched at the lth planning horizon. In state (i,l), if Tel launches promotion at t, then the expected reward would be the customer revenue generated by an lth type of customer, $c_l^{(2)}$, minus the promotion cost spends on this lth type customer, dl, plus the discounted expected sum of the next planning horizon $\alpha\sum_{k=1}^{4} p_{ilk}^{(2)} V_{lk}(t-1)$. Similar definition stands for the first choice of (2)'s parentheses. The stationary policy, vil where i,l=1,2,3,4, for HMDP exists. The LP that can find the optimal stationary policy for reference:

$$\min x_0 = \sum_{i=1}^{4}\sum_{l=1}^{4} v_{il}$$

Subject to $v_{il} \geq c_l^{(1)} + \alpha\sum_{k=1}^{4} p_{ilk}^{(1)} v_{lk}$, for i =1,2,3,4, l = 1,2,3,4,

$v_{il} \geq c_l^{(2)} - d_l + \alpha\sum_{k=1}^{4} p_{ilk}^{(2)} v_{lk}$, for i =1,2,3,4, l=1,2,3,4

$v_{il} \geq 0$, for i =1,2,3,4, l=1,2,3,4.

4. Numerical Examples

We will present a set of numerical results that can demonstrate the effectiveness of MDP and HMDP. The numerical data is a real data set provided by Tel.. Settings include the number of state, the state definition and the planning horizon, are stated as before. We have four states, 1,2,3,4, and we use week as our planning horizon. We seek for optimal stationary policy, i.e. optimal solution for infinite horizon.

In each period, we record the number of customers switching from state i to state j. Then, we divide it by the total number of customers in state i. Thus we get the estimations for the transition probabilities, one for the promotion period, $p_{lk}^{(1)}$, and the other one for the no-promotion period, $p_{lk}^{(2)}$. Similarly one can estimate the 2-transition probabilities. The calculated transition probabilities are shown in Table 2.

It leaves to compute the 2-transition probabilities for HMDP. Again for each period, we record the number of customers switching to state k given that the current state is i and the previous state is j. We then divide it by the total number of customers whose current state is i and previous state is j. Then we get the estimates for all 2-transition probabilities. The 2-transition probabilities under promotion $p_{ilk}^{(1)}$ and no-promotion $p_{ilk}^{(2)}$ period are presented in Table 3.

For simplicity, we fix same promotion cost for all types of customers and this standard promotion cost is termed as d. From the customer database, we can obtain the average revenue of a customer in different states in both the promotion period ($c_i^{(2)}$ for i = 1,2,3,4) and the no-promotion period ($c_i^{(1)}$ for i = 1,2 ,3 4). We remark that in the promotion period, a big discount was given to the customers and therefore the revenue was significantly less than the revenue in the no-promotion period. The revenue is shown in Table 4.

There are three aims for this real application. First is to estimate CLV for different types of customers under promotion and no-promotion period. Second, decide an appropriate promotion budget for launching promotion. Third, determine the best promotion period. To do so, we simply modify d in the worksheet. Then, check the optimal solution period with different d and see the periodic promotion desired by this optimum. Also, discount factor can be modified to tailor the economic situation.

To demonstrate the operation of optimizing MDP and 2-MDP, we present one set of the data in Table 3, $\alpha = 0.95$, the LP setting for generating MDP stationary policy is

$$\min x_0 = v_1 + v_2 + v_3 + v_4$$

Subject to

$$v_1 \geq 139.2 + 0.95(0.2380 v_1 + 0.2809 v_2 + 0.2069 v_3 + 0.2742 v_4),$$
$$v_2 \geq 51.72 + 0.95(0.3261 v_1 + 0.1158 v_2 + 0.1744 v_3 + 0.3837 v_4),$$
$$v_3 \geq 14.03 + 0.95(0.4964 v_1 + 0.0267 v_2 + 0.0623 v_3 + 0.4146 v_4),$$
$$v_4 \geq 0.95(0.8762 v_1 + 0.0053 v_2 + 0.0121 v_3 + 0.1064 v_4),$$
$$v_1 \geq 43.75 - 3 + 0.95(0.1372 v_1 + 0.4447 v_2 + 0.2034 v_3 + 0.2147 v_4),$$
$$v_2 \geq 18.09 - 3 + 0.95(0.2285 v_1 + 0.2148 v_2 + 0.2109 v_3 + 0.3458 v_4),$$
$$v_3 \geq 6.97 - 3 + 0.95(0.4163 v_1 + 0.0615 v_2 + 0.0992 v_3 + 0.4230 v_4),$$
$$v_4 \geq -3 + 0.95(0.8054 v_1 + 0.0191 v_2 + 0.0266 v_3 + 0.1489 v_4),$$
$$v_1, v_2, v_3, v_4 \geq 0,$$

For 2-MDP, the LP settings is

$$\min x_0 = v_{11} + v_{12} + v_{13} + v_{14} + v_{21} + v_{22} + v_{23} + v_{24} + v_{31} + v_{32} + v_{33} + v_{34} + v_{41} + v_{42} + v_{43} + v_{44}$$

Subject to

$$v_{11} \geq 139.2 + 0.95(0.4848 v_{11} + 0.1923 v_{12} + 0.1968 v_{13} + 0.1251 v_{14}),$$
$$v_{12} \geq 51.72 + 0.95(0.2302 v_{21} + 0.2434 v_{22} + 0.3224 v_{23} + 0.2040 v_{24}),$$
$$v_{13} \geq 14.03 + 0.95(0.1437 v_{31} + 0.1493 v_{32} + 0.4392 v_{33} + 0.2678 v_{34}),$$
$$v_{14} \geq 0.95(0.0818 v_{41} + 0.0575 v_{42} + 0.2114 v_{43} + 0.6493 v_{44}),$$
$$v_{21} \geq 139.2 + 0.95(0.2901 v_{11} + 0.2385 v_{12} + 0.2928 v_{13} + 0.1785 v_{14}),$$
$$v_{22} \geq 51.72 + 0.95(0.1445 v_{21} + 0.2531 v_{22} + 0.3992 v_{23} + 0.2032 v_{24}),$$
$$v_{23} \geq 14.03 + 0.95(0.0757 v_{31} + 0.1411 v_{32} + 0.4783 v_{33} + 0.3048 v_{34}),$$
$$v_{24} \geq 0.95(0.0349 v_{41} + 0.0847 v_{42} + 0.2253 v_{43} + 0.6551 v_{44}),$$

$$v_{31} \geq 139.2 + 0.95(0.1900v_{11} + 0.2624v_{12} + 0.3750v_{13} + 0.1727v_{14}),$$
$$v_{32} \geq 51.72 + 0.95(0.0872v_{21} + 0.1860v_{22} + 0.4763v_{23} + 0.2505v_{24}),$$
$$v_{33} \geq 14.03 + 0.95(0.0234v_{31} + 0.1064v_{32} + 0.5117v_{33} + 0.3584v_{34}),$$
$$v_{34} \geq 0.95(0.0086v_{41} + 0.0227v_{42} + 0.2400v_{43} + 0.7287v_{44}),$$
$$v_{41} \geq 139.2 + 0.95(0.1587v_{11} + 0.1343v_{12} + 0.2298v_{13} + 0.4771v_{14}),$$
$$v_{42} \geq 51.72 + 0.95(0.0753v_{21} + 0.0980v_{22} + 0.3069v_{23} + 0.5199v_{24}),$$
$$v_{43} \geq 14.03 + 0.95(0.0136v_{31} + 0.0329v_{32} + 0.3051v_{33} + 0.6484v_{34}),$$
$$v_{44} \geq 0.95(0.0041v_{41} + 0.0098v_{42} + 0.0904v_{43} + 0.8957v_{44}),$$
$$v_{11} \geq 43.75 - 3 + 0.95(0.6159v_{11} + 0.1855v_{12} + 0.1399v_{13} + 0.0587v_{14}),$$
$$v_{12} \geq 18.09 - 3 + 0.95(0.3671v_{21} + 0.2487v_{22} + 0.2562v_{23} + 0.1286v_{24}),$$
$$v_{13} \geq 6.97 - 3 + 0.95(0.2308v_{31} + 0.2086v_{32} + 0.3343v_{13} + 0.2263v_{14}),$$
$$v_{14} \geq -3 + 0.95(0.1494v_{41} + 0.1496v_{42} + 0.2189v_{43} + 0.4822v_{44}),$$
$$v_{21} \geq 43.75 - 3 + 0.95(0.3840v_{11} + 0.2630v_{12} + 0.2500v_{13} + 0.1030v_{14}),$$
$$v_{22} \geq 18.09 - 3 + 0.95(0.2304v_{21} + 0.2704v_{22} + 0.3757v_{23} + 0.1235v_{24}),$$
$$v_{23} \geq 6.97 - 3 + 0.95(0.1183v_{31} + 0.2043v_{32} + 0.4323v_{13} + 0.2451v_{14}),$$
$$v_{24} \geq -3 + 0.95(0.0834v_{41} + 0.1043v_{42} + 0.2371v_{43} + 0.5752v_{44}),$$
$$v_{31} \geq 43.75 - 3 + 0.95(0.3203v_{11} + 0.2271v_{12} + 0.3158v_{13} + 0.1368v_{14}),$$
$$v_{32} \geq 18.09 - 3 + 0.95(0.1563v_{21} + 0.2169v_{22} + 0.4353v_{23} + 0.1915v_{24}),$$
$$v_{33} \geq 6.97 - 3 + 0.95(0.0375v_{31} + 0.1661v_{32} + 0.4952v_{13} + 0.3012v_{14}),$$
$$v_{34} \geq -3 + 0.95(0.0205v_{41} + 0.0394v_{42} + 0.2662v_{43} + 0.6739v_{44}),$$
$$v_{41} \geq 43.75 - 3 + 0.95(0.2304v_{11} + 0.1482v_{12} + 0.2919v_{13} + 0.3295v_{14}),$$
$$v_{42} \geq 18.09 - 3 + 0.95(0.1172v_{21} + 0.1324v_{22} + 0.3033v_{23} + 0.4471v_{24}),$$
$$v_{43} \geq 6.97 - 3 + 0.95(0.0320v_{31} + 0.0549v_{32} + 0.3258v_{13} + 0.5873v_{14}),$$
$$v_{44} \geq -3 + 0.95(0.0088v_{41} + 0.0166v_{42} + 0.1225v_{43} + 0.8521v_{44}),$$
$$v_{11}, v_{12}, v_{13}, v_{14}, v_{21}, v_{22}, v_{23}, v_{24}, v_{31}, v_{32}, v_{33}, v_{34}, v_{41}, v_{42}, v_{43}, v_{44} \geq 0$$

Interested readers can download the EXCEL files for trial: http://hkumaths/~wkc/mdp.zip for MDP and http://hkumaths/~wkc/hmdp.zip for HMDP. All the transition probabilities, revenue, promotion cost and discount value are available in the sheet. To activate the LP solver, readers can click Tools->Solver->Solve. The generated expected reward and optimal policy are presented in the given worksheet. For further information concerning about using worksheet to build HMDP models, please refer to [Ching et al., 2004].

In order to compare the result generated by MDP and HMDP, we plotted the expected reward generated by these two models with different discount factors and promotion costs in Figure 3. In Figure 4, the top left chart corresponds to discount value 0.99. The expected reward for MDP and HMDP are similar. Only the expected reward of HMDP is slightly higher than that of MDP when promotion cost was increased to 6 units. However, for a higher discount value, the difference is significant. The top-right figure and bottom correspond to discount value 0.95 and 0.90. For promotion costs 1 to 6, the expected reward of HMDP is higher than those of MDP.

5. Summary

This paper illustrates how the class of Markov Processes is appropriate for CLV modeling. A major advantage is the flexibility in the way of incorporating budgeting, customer retention rates and expected revenue into a single model. Secondly, as shown, the advantage for both HMDP and MDP is the existence of an optimal stationary policy. Also, the generation of optimal solution is convenient and fast. Firms can easily incorporate it into a day-to-day by using a PC. Moreover, numerous of extensions can be incorporated into Markov models. For example, users can dedicate the promotion period into the solution LP.

Readers would wonder: what are the differences for these two models? What are the criteria for using them? The greatest assumption underlay MDP is the sole dependence of a state before. For most of the situations, this would prohibit the accuracy of MDP. If an application that forecasts to be an intertwined of several consequences, it is suggested to use HMDP. On the other hand, the main draw back for HMDP is abundance of data is required for the transition probabilities estimation. Thus, it is recommended that HMDP is used in the case where sufficient data are available. In any case, Markov type models are well-defined and convenient to use in real settings.

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FIGURES

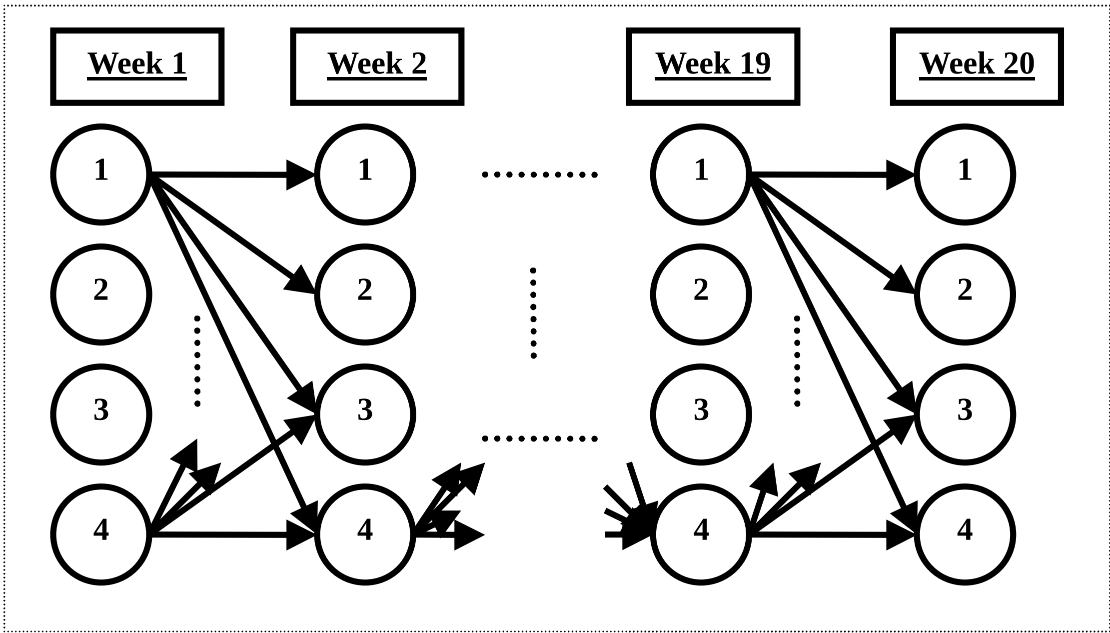

Figure 1. A full customer mobilizing network of *Tel*.

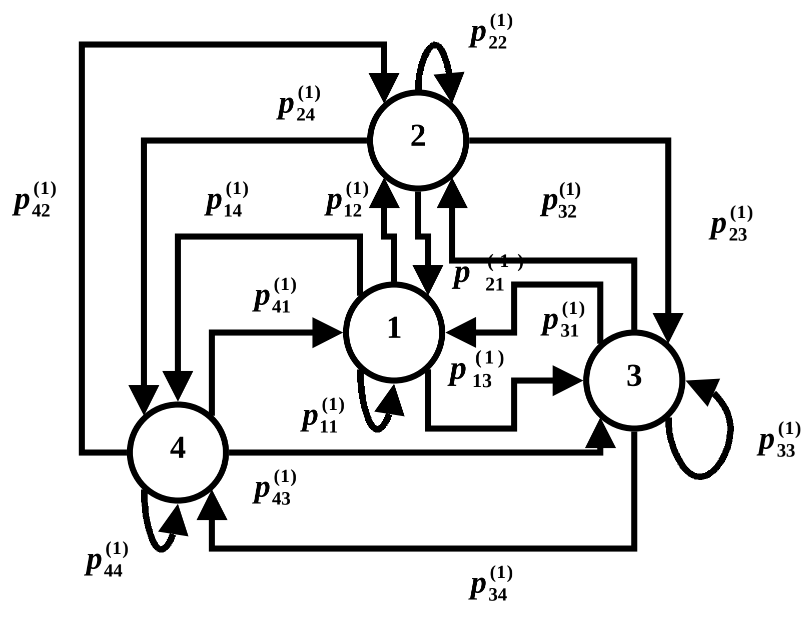

Figure 2. Truncated *Tel* problem into a Markov Decision Process (under promotion)

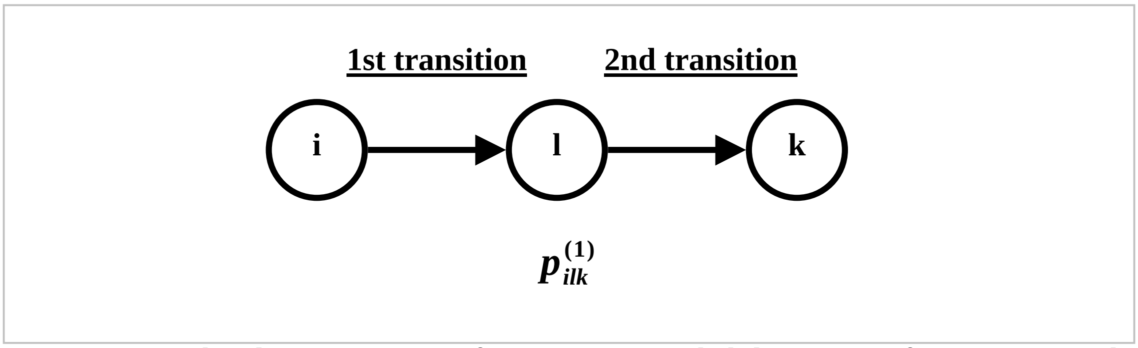

Figure 3: Graphical representation for transition probability transit from state *i* to *l* to *k* (with no-promotion)

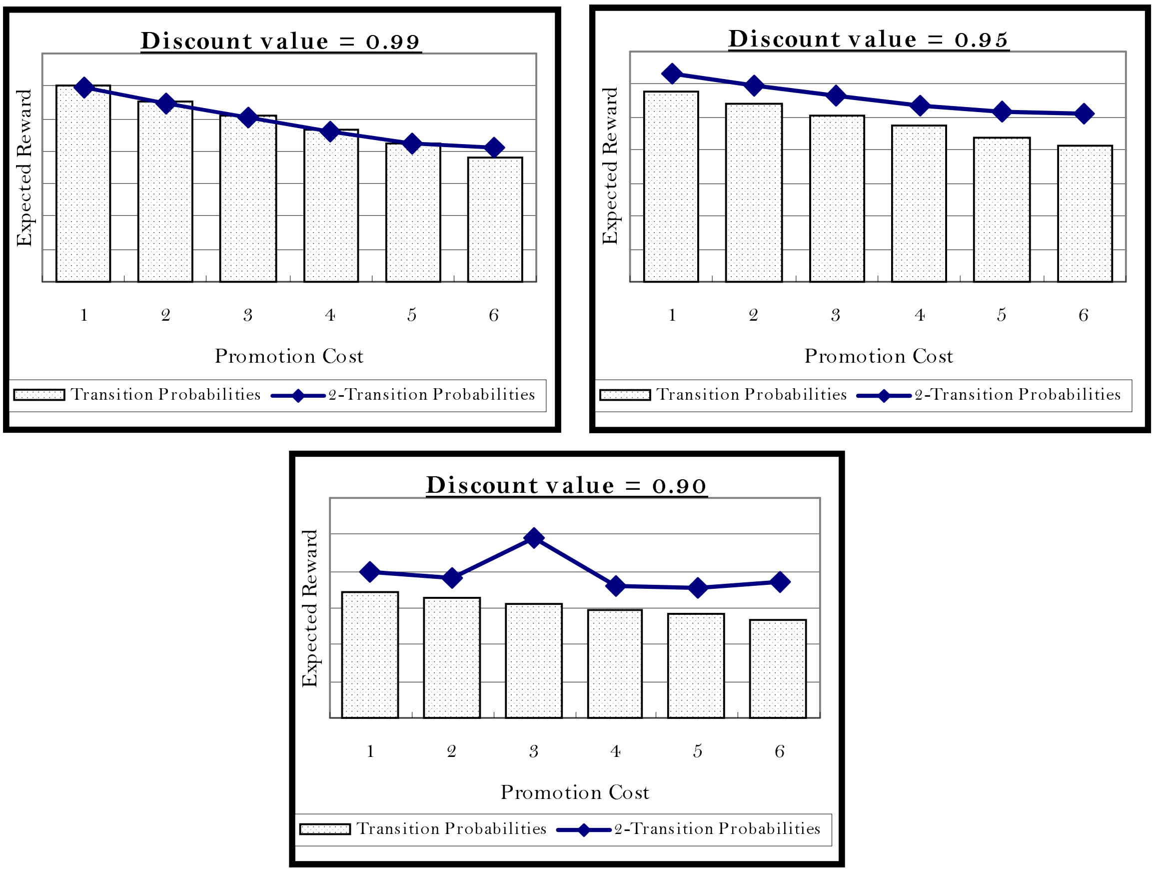

Figure 4. Comparison between MDP and HMDP

TABLES

Transition Probabilities for MDP at one of the promotion week																
States Transition	**(1,1)**	**(1,2)**	**(1,3)**	**(1,4)**	**(2,1)**	**(2,2)**	**(2,3)**	**(2,4)**	**(3,1)**	**(3,2)**	**(3,3)**	**(3,4)**	**(4,1)**	**(4,2)**	**(4,3)**	**(4,4)**
Actual Count	74	34	33	36	90	61	89	77	190	215	789	727	357	330	1257	4876
Transition Probabilities	0.4181	0.1921	0.1864	0.2034	0.2839	0.1924	0.2808	0.2429	0.0989	0.1119	0.4107	0.3785	0.0523	0.0484	0.1843	0.715

Transition Probabilities for HMDP at one of the promotion week																
States Transition	**(1,1,1)**	**(1,1,2)**	**(1,1,3)**	**(1,1,4)**	**(1,2,1)**	**(1,2,2)**	**(1,2,3)**	**(1,2,4)**	**(1,3,1)**	**(1,3,2)**	**(1,3,3)**	**(1,3,4)**	**(1,4,1)**	**(1,4,2)**	**(1,4,3)**	**(1,4,4)**
Actual Count	15	6	4	4	8	6	3	3	7	6	6	7	1	3	5	9
Transition Probabilities	0.5172	0.2069	0.1379	0.1379	0.4	0.3	0.15	0.15	0.2692	0.2308	0.2308	0.2692	0.0556	0.1667	0.2778	0.5
States Transition	**(2,1,1)**	**(2,1,2)**	**(2,1,3)**	**(2,1,4)**	**(2,2,1)**	**(2,2,2)**	**(2,2,3)**	**(2,2,4)**	**(2,3,1)**	**(2,3,2)**	**(2,3,3)**	**(2,3,4)**	**(2,4,1)**	**(2,4,2)**	**(2,4,3)**	**(2,4,4)**
Actual Count	9	8	2	3	8	7	11	5	4	15	35	12	5	5	12	37
Transition Probabilities	0.4091	0.3636	0.0909	0.1364	0.2581	0.2258	0.3548	0.1613	0.0606	0.2273	0.5303	0.1818	0.0847	0.0847	0.2034	0.6271
States Transition	**(3,1,1)**	**(3,1,2)**	**(3,1,3)**	**(3,1,4)**	**(3,2,1)**	**(3,2,2)**	**(3,2,3)**	**(3,2,4)**	**(3,3,1)**	**(3,3,2)**	**(3,3,3)**	**(3,3,4)**	**(3,4,1)**	**(3,4,2)**	**(3,4,3)**	**(3,4,4)**
Actual Count	15	7	8	2	16	15	35	17	38	77	299	172	13	23	156	405
Transition Probabilities	0.4688	0.2188	0.25	0.0625	0.1928	0.1807	0.4217	0.2048	0.0648	0.1314	0.5102	0.2935	0.0218	0.0385	0.2613	0.6784
States Transition	**(4,1,1)**	**(4,1,2)**	**(4,1,3)**	**(4,1,4)**	**(4,2,1)**	**(4,2,2)**	**(4,2,3)**	**(4,2,4)**	**(4,3,1)**	**(4,3,2)**	**(4,3,3)**	**(4,3,4)**	**(4,4,1)**	**(4,4,2)**	**(4,4,3)**	**(4,4,4)**
Actual Count	35	13	19	27	58	33	40	52	141	139	449	543	340	304	1110	4363
Transition Probabilities	0.3723	0.1383	0.2021	0.2872	0.3169	0.1803	0.2186	0.2842	0.1108	0.1093	0.353	0.4269	0.0556	0.0497	0.1815	0.7133

Table 1. Calculation of transition probabilities and 2-transition probabilities (with promotion)

	Promotion				No-Promotion			
States	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
1	0.1372	0.4447	0.2034	0.2147	0.2380	0.2809	0.2069	0.2742
2	0.2285	0.2148	0.2109	0.3458	0.2380	0.2809	0.2069	0.2742
3	0.4163	0.0615	0.0992	0.4230	0.4964	0.0267	0.0623	0.4146
4	0.8054	0.0191	0.0266	0.1489	0.8762	0.0053	0.0121	0.1064

Table 2. Transition Probabilities for MDP

	Promotion				No-Promotion			
States	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
(1,1)	0.6159	0.1855	0.1399	0.0587	0.4848	0.1933	0.1968	0.1251
(1,2)	0.3671	0.2481	0.2562	0.1286	0.2302	0.2434	0.3224	0.2040
(1,3)	0.2308	0.2086	0.3343	0.2263	0.1437	0.1493	0.4392	0.2678
(1,4)	0.1494	0.1496	0.2189	0.4822	0.0818	0.0575	0.2114	0.6493
(2,1)	0.3840	0.2630	0.2500	0.1030	0.2901	0.2385	0.2928	0.1785
(2,2)	0.2304	0.2704	0.3757	0.1235	0.1445	0.2531	0.3992	0.2032
(2,3)	0.1183	0.2043	0.4323	0.2451	0.0757	0.1411	0.4783	0.3048
(2,4)	0.0834	0.1043	0.2371	0.5752	0.0349	0.0847	0.2253	0.6551
(3,1)	0.3203	0.2271	0.3158	0.1368	0.1900	0.2624	0.3750	0.1727
(3,2)	0.1563	0.2167	0.4353	0.1915	0.0872	0.1860	0.4763	0.2505
(3,3)	0.0375	0.1661	0.4952	0.3012	0.0234	0.1064	0.5117	0.3584
(3,4)	0.0205	0.0394	0.2662	0.6739	0.0086	0.0227	0.2400	0.7287
(4,1)	0.2304	0.1482	0.2919	0.3295	0.1587	0.1343	0.2298	0.4771
(4,2)	0.1172	0.1324	0.3033	0.4471	0.0753	0.0980	0.3069	0.5199
(4,3)	0.0320	0.0549	0.3258	0.5873	0.0136	0.0329	0.3051	0.6484
(4,4)	0.0088	0.0166	0.1225	0.8521	0.0041	0.0098	0.0904	0.8957

Table 3. 2-Transition Probabilities for HMDP

State	**1**	**2**	**3**	**4**
Minutes	>40	21 to 40	1 to 20	0
Promotion	43.75	18	6.97	0
No-promotion	139.2	51.72	14.03	0

Table 4. Revenues