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# AN ORDER-LEVEL INVENTORY MODEL FOR A DETERIORATING ITEM WITH WEIBULL DISTRIBUTION DETERIORATION, TIME-QUADRATIC DEMAND AND SHORTAGES.

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**Abstract**

An inventory model is developed for a deteriorating item having an instantaneous supply, a quadratic time-varying demand and shortages in inventory. A two-parameter *Weibull distribution* is taken to represent the time to deterioration. The model is solved analytically to obtain the optimal solution of the problem. It is then illustrated with the help of a numerical example. The sensitivity of the optimal solution towards changes in the values of different system parameters is also studied. Special features of a time-quadratic demand are discussed.

Key Words: Inventory, economic order quantity, time-quadratic demand, deterioration.

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## 1. Introduction

The aim of the paper is to develop an EOQ(Economic Order Quantity) model for a single-item inventory having a time-varying *quadratic* demand. Inventory modellers have so far considered only two types of time-dependent demands, *linear* and *exponential*. Linear time-dependence of demand implies a uniform change in the demand rate of the product per unit time. This is rarely seen to occur in the real market. On the other hand, an exponentially time-varying demand also seems to be unrealistic because an exponential rate of change is very high and it is doubtful whether the market demand of any product may undergo such a high rate of change as exponential. In the opinion of the authors, an alternative (and perhaps more realistic) approach is to consider quadratic time-dependence of demand which may represent all types of time-dependence

depending on the signs of the parameters of the time-quadratic demand function.
A brief review of the literature dealing with time-varying demands is made in the following paragraphs.
In formulating inventory models, two factors of the problem have been of growing interest to the researchers, one being the deterioration of items and the other being the variation in the demand rate with time. Silver and Meal[1] developed an approximate solution procedure for the general case of a deterministic, time-varying demand pattern. The classical no-shortage inventory problem for a linear trend in demand over a finite time-horizon was analytically solved by Donaldson[2]. However, Donaldson's solution procedure was computationally complicated. Silver[3] derived a heuristic for the special case of a positive, linear trend in demand and applied it to the problem of Donaldson[2]. Ritchie([4],[5],[6]) obtained an exact solution, having the simplicity of the EOQ formula, for Donaldson's problem for a linear, increasing demand. Mitra et al[7] presented a simple procedure for adjusting the economic order quantity model for the case of increasing or decreasing linear trend in demand. The possibilities of shortage and deterioration in inventory were left out of consideration in all these models.
Dave and Patel[8] developed an inventory model for deteriorating items with time-proportional demand. This model was extended by Sachan[9] to cover the backlogging option. Bahari-Kashani[10] discussed a heuristic model for obtaining order quantities when demand is time-proportional and inventory deteriorates at a constant rate over time. Deb and Chaudhuri[11] studied the inventory replenishment policy for items having a deterministic demand pattern with a linear (positive) trend and shortages; they developed a heuristic to determine the decision rule for selecting the times and sizes of replenishment over a finite time-horizon so as to keep the total costs minimum. This work was extended by Murdeshwar[12]. Subsequent contributions in this direction came from researchers like Goyal([13],[14]), Dave[15], Hariga[16], Goswami and Chaudhuri[17], Xu and Wang[18], Chung and Ting([19],[20]), Kim[21], Hariga([22],[23]), Jalan, Giri and Chaudhuri[24], Jalan and Chaudhuri[25], Giri and Chaudhuri[26], Lin, Tan and Lee[27], etc.
The assumption of the constant deterioration rate was relaxed by Covert and Philip[28] who used a two-parameter Weibull distribution to represent the distribution of time to deterioration. This model was further generalized by Philip[29] by taking a three-parameter Weibull distribution. Misra[30] also adopted a two-parameter Weibull distribution deterioration to develop an inventory model with a finite rate of replenishment.

These investigations were followed by several researchers like Shah and Jaiswal[31], Aggarwal[32], Roy-Chowdhury and Chaudhuri[33], etc. The models developed by Covert and Philip[28], Philip[29] and Misra[30] did not allow shortages in inventory and used a constant demand rate.
Recently Wee[34] and Jalan and Chaudhuri[35] worked with an exponentially time-varying demand.
In the present paper, we assume that time-dependence of the demand rate is quadratic. Deterioration rate is assumed to follow a two-parameter Weibull distribution and shortages in the inventory are allowed. An analytical solution of the model is discussed and it is illustrated with the help of a numerical example. Sensitivity of the optimal solution with respect to changes in different parameter values is also examined. The detailed justifications for the choice of a quadratic demand are given in the *Concluding Remarks* section.

## 2. Notations

The following notations are used in the model.
$C_1$ - inventory carrying cost per unit per unit time.
$C_2$ - shortage cost per unit per unit time.
$C_3$ - ordering cost per order.
$C_4$ - cost of a unit.
$q_0$ - size of the initial inventory.
$R(t)$ - demand rate at any time $t \geq 0$.
$T$ - cycle time.
$K$ - a constant value (0 ¡ $K$ ¡ 1).
$t_1$ - time during which there is no shortage (0 ¡ $t_1$ ¡$T$ ).
$Z(t)$ - instantaneous rate function for a two-parameter Weibull distribution $= \alpha\beta t^{(\beta-1)}$ where $\alpha$ is the scale parameter and $\beta$ is the shape parameter.
$T^*$ - optimal value of $T$.
${q_0}^*$ - optimal value of $q_0$.
${t_1}^*$ - optimal value of $t_1$.
$K^*$ - 0ptimal value of $K$.

## 3. Assumptions

The following assumptions are used in the model:
(i) The deterministic demand rate $R(t)$ varies quadratically with time, i.e, $R(t) = a + bt + ct^2$, where $a$, $b$ and $c$ are constants such that $a \geq 0, b \neq 0, c \neq 0$. Here $a$ stands for the initial demand rate and $b$ for the positive trend in demand.
(ii) Shortages in the inventory are allowed and completely backlogged.

(iii) The supply is instantaneous and the lead time is zero.
(iv) A deteriorated unit is not repaired or replaced during a given cycle.
(v) The holding cost, ordering cost, shortage cost and unit cost remain constant over time.
(vi) The distribution of the time to deterioration of the items follows the two-parameter Weibull distribution.

## 4. Formulation and solution

The instantaneous state of the inventory level $q(t)$ at any time $t$ is governed by the differential equations

$$\frac{dq(t)}{dt} + q(t)Z(t) = -(a + bt + ct^2), \qquad 0 \le t \le t_1 \tag{1}$$

$$\text{with} \quad q(0) = q_0 \quad \text{and} \quad q(t_1) = 0,$$

and

$$\frac{dq(t)}{dt} = -(a + bt + ct^2), \qquad t_1 \le t \le T \tag{2}$$

$$\text{with} \quad q(t_1) = 0.$$

The deterioration rate $Z(t)$ follows the two-parameter Weibull distribution given by

$$Z(t) = \alpha\beta t^{\beta-1}\ , \quad \alpha > 0, \quad \beta > 0, \quad t > 0. \tag{3}$$

By virtue of (3), (1) becomes

$$\frac{dq}{dt} + q\alpha\beta t^{\beta-1} = -(a + bt + ct^2), \qquad 0 \le t \le t_1. \tag{4}$$

This is a linear ordinary differential equation of first order and its integrating factor is

$$= exp\left\{\alpha\beta \int t^{\beta-1}dt\right\} = exp\{\alpha t^{\beta}\}.$$

Multiplying both sides of (4) by $\exp\{\alpha t^{\beta}\}$ and then integrating over [0,$t$], we have

$$q.exp\{\alpha t^{\beta}\} - q_0 = -\int_0^t (a + bt + ct^2)exp\{\alpha t^{\beta}\}dt, \quad 0 \le t \le t_1. \tag{5}$$

Using the condition $q(t_1)$=0, we have from (5)

$$q_0 = \int_0^{t_1} (a + bt + ct^2)exp\{\alpha t^{\beta}\}dt. \tag{6}$$

Substituting the value of $q_0$ from (6) in (5), we get

$$q(t) = \frac{\int_0^{t_1}(a+bt+ct^2)exp\{\alpha t^\beta\}dt - \int_0^{t}(a+bt+ct^2)exp\{\alpha t^\beta\}dt}{exp\{\alpha t^\beta\}}. \quad 0 \le t \le t_1 \quad (7)$$

The solution of (2)is

$$q(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3), \qquad t_1 \le t \le T. \quad (8)$$

Hence the instantaneous level of inventory at any time t ∈ [0,T] is given by

$$\begin{aligned} q(t) &= \frac{\int_0^{t_1}(a+bt+ct^2)exp\{\alpha t^\beta\}dt - \int_0^{t}(a+bt+ct^2)exp\{\alpha t^\beta\}dt}{exp\{\alpha t^\beta\}}, \quad 0 \le t \le t_1 \\ &= a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3), \qquad t_1 \le t \le T. \quad (9) \end{aligned}$$

The inventory level at the beginning of the cycle must be sufficient enough to meet the total demand given by

$$\int_0^{t_1}(a+bt+ct^2)dt = at_1 + \frac{1}{2}bt_1^2 + \frac{1}{3}ct_1^3.$$

Also the total quantity of deteriorated items is given by

$$q_0 - \int_0^{t_1}(a+bt+ct^2)dt = q_0 - at_1 - \frac{1}{2}bt_1^2 - \frac{1}{3}ct_1^3.$$

Expressing the exponential term in (7) in infinite series and then integrating term by term, we have

$$q_0 = a\sum_{n=0}^{\infty}\frac{\alpha^n t_1^{n\beta+1}}{(n\beta+1)n!} + b\sum_{n=0}^{\infty}\frac{\alpha^n t_1^{n\beta+2}}{(n\beta+2)n!} + c\sum_{n=0}^{\infty}\frac{\alpha^n t_1^{n\beta+3}}{(n\beta+3)n!}. \quad (10)$$

The average inventory holding cost in $(0,t_1)$ is

$$\frac{1}{2}\frac{C_1}{T}q_0 t_1.$$

Although the inventory depletion curve is not a straight line here, the average inventory holding cost is taken here in the same form as it is used in deterministic EOQ models with no deterioration. Without this approximation, the model becomes too complex to be solved. Similar approaches have been adopted by Covert and Philip(1973) and Misra(1974). The average shortage cost in $[t_1, T]$ is

$$\frac{C_2}{T}\int_{t_1}^{T}(a+bt+ct^2)(T-t)dt$$
$$=\frac{C_2}{12T}[(T-t_1)^2\{6a+2b(T+2t_1)+c(T^2+2Tt_1+3t_1^2)\}].$$

Therefore, the total variable cost per unit time is

$$AVC=\frac{C_4}{T}(q_0-at_1-\frac{1}{2}bt_1^2-\frac{1}{3}ct_1^3)+\frac{1}{2}\frac{C_1}{T}q_0t_1$$
$$+\frac{C_2(T-t_1)^2}{12T}\{6a+2b(T+2t_1)+c(T^2+2Tt_1+3t_1^2)\}+\frac{C_3}{T}. \quad (11)$$

Since the length of the shortage interval is a part of the cycle time, we assume

$$t_1=KT, \qquad 0<K<1 \quad (12)$$

where $K$ is a constant to be determined in an optimal manner. Using (12), in (6) we have from (11),

$$\begin{aligned}AVC &= \left(\frac{C_4a}{T}+\frac{1}{2}C_1Ka\right)\int_0^{KT}exp\{\alpha t^\beta\}dt+\left(\frac{C_4b}{T}+\frac{1}{2}C_1Kb\right)\int_0^{KT}t\ exp\{\alpha t^\beta\}dt\\ &+ \left(\frac{C_4c}{T}+\frac{1}{2}C_1Kc\right)\int_0^{KT}t^2exp\{\alpha t^\beta\}dt-C_4aK-\frac{1}{2}C_4bK^2T\\ &- \frac{1}{3}C_4cK^3T^2+\frac{1}{2}C_2(1-K)^2aT+\frac{1}{6}C_2(1-K)^2(1+2K)bT^2\\ &+ \frac{1}{12}C_2(1-K)^2(1+2K+3K^2)cT^3+\frac{C_3}{T}.\end{aligned}$$

Expressing the exponential terms in infinite series and then integrating term by term, we have

$$\begin{aligned}AVC &= \left(\frac{C_4a}{T}+\frac{1}{2}C_1Ka\right)\sum_{n=0}^{\infty}\frac{\alpha^n(KT)^{n\beta+1}}{(n\beta+1)n!}+\left(\frac{C_4b}{T}+\frac{1}{2}C_1Kb\right)\sum_{n=0}^{\infty}\frac{\alpha^n(KT)^{n\beta+2}}{(n\beta+2)n!}\\ &+ \left(\frac{C_4c}{T}+\frac{1}{2}C_1Kc\right)\sum_{n=0}^{\infty}\frac{\alpha^n(KT)^{n\beta+3}}{(n\beta+3)n!}-C_4aK-\frac{1}{2}C_4bK^2T-\frac{1}{3}C_4cK^3T^2\\ &+ \frac{1}{2}C_2(1-K)^2aT+\frac{1}{6}C_2(1-K)^2(1+2K)bT^2\\ &+ \frac{1}{12}C_2(1-K)^2(1+2K+3K^2)cT^3+\frac{C_3}{T}. \quad (13)\end{aligned}$$

Treating $K$ and $T$ as decision variables, the necessary conditions for the minimization of the average system cost are

$$\frac{\partial}{\partial T}(AVC) = 0 = \frac{\partial}{\partial K}(AVC).$$

After a little calculation, the condition $\frac{\partial(AVC)}{\partial T} = 0$ yields

$$\begin{aligned}
& C_4 a \sum_{n=0}^{\infty} \frac{n\beta}{n\beta+1} \frac{\alpha^n (KT)^{n\beta+1}}{n!} + C_4 b \sum_{n=0}^{\infty} \frac{n\beta+1}{n\beta+2} \frac{\alpha^n (KT)^{n\beta+2}}{n!} \\
+ \; & C_4 c \sum_{n=0}^{\infty} \frac{n\beta+2}{n\beta+3} \frac{\alpha^n (KT)^{n\beta+3}}{n!} + \frac{1}{2} C_1 a \sum_{n=0}^{\infty} \frac{\alpha^n (KT)^{n\beta+2}}{n!} \\
+ \; & \frac{1}{2} C_1 b \sum_{n=0}^{\infty} \frac{\alpha^n (KT)^{n\beta+3}}{n!} + \frac{1}{2} C_1 c \sum_{n=0}^{\infty} \frac{\alpha^n (KT)^{n\beta+4}}{n!} \\
- \; & \frac{1}{2} C_4 b K^{2} T^{2} - \frac{2}{3} C_4 c K^{3} T^{3} + \frac{1}{2} C_2 (1-K)^{2} a T^{2} \\
+ \; & \frac{1}{3} C_2 (1-K)^{2} (1+2K) b T^{3} + \frac{C_2}{4} (1-K)^{2} (1+2K+3K^{2}) c T^{4} \\
- \; & C_3 = 0. \qquad (14)
\end{aligned}$$

The other condition $\frac{\partial(AVC)}{\partial K} = 0$ leads to the result

$$\begin{aligned}
& C_4 a \sum_{n=1}^{\infty} \frac{\alpha^n (KT)^{n\beta}}{n!} + C_4 b \sum_{n=1}^{\infty} \frac{\alpha^n (KT)^{n\beta+1}}{n!} + C_4 c \sum_{n=1}^{\infty} \frac{\alpha^n (KT)^{n\beta+2}}{n!} \\
+ \; & \frac{1}{2} C_1 a \sum_{n=0}^{\infty} \frac{n\beta+2}{n\beta+1} \frac{\alpha^n (KT)^{n\beta+1}}{n!} + \frac{1}{2} C_1 b \sum_{n=0}^{\infty} \frac{n\beta+3}{n\beta+2} \frac{\alpha^n (KT)^{n\beta+2}}{n!} \\
+ \; & \frac{1}{2} C_1 c \sum_{n=0}^{\infty} \frac{n\beta+4}{n\beta+3} \frac{\alpha^n (KT)^{n\beta+3}}{n!} - C_2 a T (1-K) - C_2 b T^{2} K (1-K) \\
+ \; & \frac{1}{3} C_2 c T^{3} K (3K^{2} - 3K + 1) = 0. \qquad (15)
\end{aligned}$$

The optimal values $T^*$ of $T$ and $K^*$ of $K$ are obtained by solving (14) and (15). The sufficient conditions that these values minimize $AVC(T, K)$ are

$$\begin{aligned}
& \frac{\partial^2(AVC)}{\partial T^{*2}} \frac{\partial^2(AVC)}{\partial K^{*2}} - \left( \frac{\partial^2(AVC)}{\partial T^* \partial K^*} \right)^2 > 0 \\
& \frac{\partial^2(AVC)}{\partial T^{*2}} > 0, \quad \frac{\partial^2(AVC)}{\partial K^{*2}} > 0. \qquad (16)
\end{aligned}$$

Equations (14) and (15) can only be solved with the help of a computer oriented numerical technique for a given set of parameter values by truncating the infinite series. Once $T^*$ and $K^*$ are obtained, we get $t_1^*$ from

(12). We may then use (10) to determine the optimal EOQ $q_0^*$ and (13) to get the optimal average cost $AVC^*$.

**5. Numerical Example**

Equations (14) and (15) are now solved with the help of a computer using the following parameter values:
$C_1$= Rs. 0.001 per unit per day,
$C_2$= Rs. 10.00 per unit per day,
$C_3$= Rs. 20.00 per order,
$C_4$= Rs. 4.00 per unit,
$\alpha$= 0.002, $\beta$=1.5, $a$=10.0, $b$=2.0, $c$=1.0.
Based on these input data, the results are:
Optimum cycle time $T^*$=1.3532 days,
Optimum value $K^*$=0.630,
Economic order quantity $q_0{}^*$=9.46472 units,
Optimum stock-period $t_1{}^*$=0.8525 days and
Optimum average cost $AVC^*$=Rs. 36.4387 per day.
It is checked that this solution satisfies the sufficient conditions (16) for optimality.
To understand the benefits of choosing $K$ optimally rather than arbitrarily, we show the results for arbitrary choice of $K$ in Table 1 (for quadratic demand) and Table 2 (for linear demand). It is interesting to note that the results for arbitrary values of $K$ differ significantly from the optimal results. As $K$ increases from its lower values to its optimum value, each of $T^*$ and $q_0{}^*$ increases while $C^*$ decreases. The optimal results for a linear demand rate ($c$=0) are
Optimum cycle time $T^*$=7.30 days,
Optimum value $K^*$=0.9977,
Economic order quantity $q_0{}^*$=128.37 units,
Optimum stock-period $t_1{}^*$ =7.29 days and
Optimum average cost $AVC^*$=Rs. 29.99 per day.

Table 1. Results for arbitrary choice of $K$ in the case of quadratic demand

| $K$ | $T^*$ | $q_0^*$ | $TC^*$ |
|---|---|---|---|
| 0.500 | 1.06450 | 5.65786 | 36.9193 |
| 0.550 | 1.15658 | 6.85451 | 36.6591 |
| 0.560 | 1.17760 | 7.12820 | 36.6224 |
| 0.570 | 1.19930 | 7.41328 | 36.5818 |
| 0.580 | 1.22195 | 7.71210 | 36.5438 |
| 0.590 | 1.24570 | 8.02634 | 36.5109 |
| 0.600 | 1.27059 | 8.35705 | 36.4835 |
| 0.610 | 1.29670 | 8.70561 | 36.4607 |
| 0.620 | 1.32420 | 9.07420 | 36.4457 |
| 0.630* | 1.35320* | 9.46472* | 36.4387* |
| 0.640 | 1.38390 | 9.87993 | 36.4429 |
| 0.650 | 1.41645 | 10.3224 | 36.4591 |
| 0.660 | 1.45100 | 10.7950 | 36.4829 |

Table 2. Results for arbitrary choice of $K$ in the case of a linear demand

| $K$ | $T^*$ | $q_0^*$ | $TC^*$ |
|---|---|---|---|
| 0.500 | 1.11064 | 5.86357 | 37.7119 |
| 0.600 | 1.33860 | 8.68182 | 37.1732 |
| 0.700 | 1.69340 | 13.2732 | 36.4657 |
| 0.800 | 2.33090 | 22.1726 | 35.4698 |
| 0.900 | 3.83980 | 46.7673 | 33.8048 |
| 0.950 | 5.60860 | 82.6019 | 32.1389 |
| 0.980 | 6.97980 | 117.149 | 30.5518 |
| 0.990 | 7.24700 | 125.481 | 30.1397 |
| 0.9977* | 7.30500* | 128.372* | 29.9925* |

Comparing the solution of the quadratic demand to that of the linear demand, we observe the following changes in the case of a quadratic demand:
(i) The cycle time decreases by 81.51 % nearly.
(ii) The stock period decreases by 88.32 %.
(iii) The economic lot size reduces by 92.33 %.
(iv) The average system cost increases by 21.51 % per day.
Reorders become more frequent in the case of a quadratic demand.

## 6. Sensitivity Analysis

We now study the effects of changes in the values of the system parameters $a, b, c$, $C_1$, $C_2$, $C_3$, $C_4$, $\alpha$ and $\beta$ on the optimal average cost, cycle time and EOQ derived by the proposed method. The sensitivity analysis is performed by changing each of the parameters by -50 %, -20

%, +20 %, and +50 %, taking one parameter at a time and keeping the remaining eight parameters unchanged.

Table 3. Sensitivity Analysis.

| Changing parameter | % change in the system parameter | % change in $T^*$ | % change in $q_0{}^*$ | % change in $TC^*$ |
|---|---|---|---|---|
| $a$ | -50 | 14.01 | -35.38 | -6.18 |
| | -20 | 5.16 | -13.13 | -2.02 |
| | +20 | -4.64 | 12.02 | 1.59 |
| | +50 | -10.74 | 28.33 | 3.37 |
| $b$ | -50 | 5.57 | 2.01 | 1.94 |
| | -20 | 2.08 | 0.74 | 0.71 |
| | +20 | -1.91 | -0.66 | -0.64 |
| | +50 | -4.52 | -1.53 | -1.51 |
| $c$ | -50 | 2.47 | 1.60 | 0.63 |
| | -20 | 0.93 | 0.59 | 0.23 |
| | +20 | -0.87 | -0.72 | -0.21 |
| | +50 | -2.07 | -1.29 | -0.47 |
| $C_1$ | -50 | 0.007 | 0.008 | -0.002 |
| | -20 | 0.002 | 0.002 | -0.001 |
| | +20 | -0.002 | -0.002 | 0.001 |
| | +50 | -0.004 | -0.006 | 0.002 |
| $C_2$ | -50 | 34.20 | 40.13 | 0.39 |
| | -20 | 9.84 | 11.18 | 0.15 |
| | +20 | -7.38 | -8.19 | -0.23 |
| | +50 | -15.69 | -17.25 | -0.66 |
| $C_3$ | -50 | -24.32 | -31.74 | -48.59 |
| | -20 | -8.39 | -9.32 | -19.22 |
| | +20 | 7.28 | 8.24 | 19.01 |
| | +50 | 16.72 | 19.17 | 47.19 |
| $C_4$ | -50 | -15.12 | -16.96 | -1.59 |
| | -20 | -6.15 | -6.89 | -0.65 |
| | +20 | 6.35 | 7.13 | 0.68 |
| | +50 | 16.22 | 18.23 | 1.75 |

| Changing parameter | % change in the system parameter | % change in $T^*$ | % change in $q_0{}^*$ | % change in $TC^*$ |
|---|---|---|---|---|
| $\alpha$ | -50 | 0.04 | 0.013 | 0.008 |
| | -20 | 0.02 | 0.008 | 0.006 |
| | +20 | -0.01 | -0.004 | -0.002 |
| | +50 | -0.04 | -0.013 | -0.008 |
| $\beta$ | -50 | 0.03 | 0.055 | 0.05 |
| | -20 | 0.01 | 0.007 | 0.02 |
| | +20 | -0.02 | -0.003 | -0.01 |
| | +50 | -0.04 | -0.023 | -0.02 |

On the basis of the results shown in the Table 3, the following observations can be made:

(1) $T^*$ decreases while $q_0{}^*$ and $AVC^*$ both increase with the increase in the value of the parameter $a$. However, $AVC^*$ and $T^*$ have low sensitivity to changes in $a$. On the other hand, $q_0{}^*$ has moderate sensitivity towards changes in $a$.

(2) $T^*$, $AVC^*$ and $q_0{}^*$ all decrease (increase) with the increase (decrease) of $b$. However, they are slightly sensitive to changes in $b$.

(3) Each of $T^*$, $q_0{}^*$ and $AVC^*$ decreases (increases) with the increase (decrease) of $c$ and they are slightly sensitive to changes in $c$.

(4) $T^*$, $q_0{}^*$ and $AVC^*$ are all insensitive to changes in the parameter $C_1$.

(5) $T^*$, $q_0{}^*$, $AVC^*$ all decrease (increase) with the increase (decrease) of $C_2$. $T^*$ and $q_0{}^*$ are moderately sensitive while $AVC^*$ is almost insensitive to changes in $C_2$.

(6) Each of $T^*$, $q_0{}^*$ and $AVC^*$ increases (decreases) with the increase (decrease) of $C_3$. The are all moderately sensitive to changes in $C_3$.

(7) Each of $T^*$, $q_0{}^*$ and $AVC^*$ increases (decreases) with the increase (decrease) of $C_4$. $q_0{}^*$ and $T^*$ have low sensitivity while $AVC^*$ is almost insensitive to changes in $C_4$.

(8) $T^*$, $q_0{}^*$, $AVC^*$ are insensitive to changes in $\alpha$.

(9) $T^*$, $q_0{}^*$, $AVC^*$ are insensitive to changes in $\beta$.

## 7. Concluding Remarks

While considering time-varying demands, inventory modellers usually take the demand to be either linearly dependent or exponentially dependent upon time. For the first case, the demand rate function is of the form $R(t) = a + bt$, $a \geq 0$, $b \neq 0$, which implies steady increase (or decrease) in demand, which may be rarely seen to occur in the real market. For the second case, the demand rate function is of the form

$R(t) = \alpha e^{\beta t}, \alpha > 0, \beta \neq 0$. In the real market situations, demand is unlikely to increase at a rate which is so high as exponential. Quadratic time-dependence of demand of the form $R(t) = a + bt + ct^2, a \geq 0, b \neq 0, c \neq 0$, seems to be a better representation of time-varying market demands. Here $a$ ($\geq 0$) stands for the initial demand rate. We have

$$\frac{dR(t)}{dt} = \dot{R}(t) = b + 2ct$$

and

$$\ddot{R}(t) = 2c.$$

Now $\dot{R}(t)$=0 gives $t = -\frac{b}{2c}$ which is positive if $b$ and $c$ are of opposite signs. For $b$¿0 and $c$¡0, $R(t)$ has a maximum at $t = -\frac{b}{2c}$. In this case, the demand rate gradually rises to a maximum $(a - \frac{b^2}{4c})$ and then gradually declines. This type of demand is quite appropriate for seasonal products like winter cosmetics. As the season progresses, the demand rate begins to rise, attains a peak in the mid-season and then wanes out towards the end of the season.
For $b$¡0 and $c$¿0, the demand rate gradually falls to a minimum and then increases. This type of situation is rare in the real market.
For $b$¿0, $c$¿0, the demand rate undergoes an *accelerated growth* which is found to occur in the case of spare parts of newly introduced state-of-the-art aircrafts, computers, etc. There is *accelerated decline* in demand for $b$¡0, $c$¡0; this happens in the case of spare parts of the obsolute aircrafts, computers, etc.

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