AMO - Advanced Modeling and Optimization, Volume 5, Number 3, 2003

ON A VOLUME FLEXIBLE STOCK-DEPENDENT INVENTORY MODEL

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ABSTRACT. An inventory model of a *volume flexible manufacturing system* for a deteriorating item is developed, taking a stock -dependent demand rate. It is assumed that the demand rate remains stock-dependent for an initial period after which a uniform demand rate follows as the stock comes down to a certain level. The unit production cost is taken to be a function of the finite production rate which is treated to be a decision variable. The mathematical expression for the *average profit function* is derived and it is maximized subject to the different constraints of the system using **Zoutendijk's Method** of constrained optimization, the algorithm of which is given. The solution procedure is illustrated with the help of a numerical example. Using the result of the example, sensitivity of the near optimal solution to changes in the values of the parameters of the system is analyzed.

Key words and phrases : volume flexible manufacturing system, deteriorating item, stock-dependent demand rate, constrained optimization, sensitivity anlysis.

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Introduction

In the Classical Economic Production Lot Size(**EPLS**) model, the production rate of a machine is regarded to be pre-determinded and inflexible[1]. Alder and Nanda[2], Sule[3,4], Axsater and Elmaghraby[5], Muth and Spearmann[6] extended the **EPLS** model to situations where *learning effects* would induce an increase in the production rate. Proteus[7], Rosenblat and Lee[8] and Cheng[9] considered the **EPLS** model in an imperfect production process in which the demand would exceed the supply.

Schweitzer and Seidmann[10] adopted, for the first time, the concept of flexibility in the machine production rate and discussed optimization of processing rates for a **FMS** (*flexible manufacturing system*). Obviously, the machine production rate is a decision variable in the case of a **FMS** and then the unit production cost becomes a function of the production rate. Khouja and Mehrez[11] and Khouja[12] extended the **EPLS** model to an imperfect production process with a flexible production rate. Silver[13], Moon, Gallego and Simchi-Levi[14] discussed the effects of slowing down production in the context of a manufacturing equipment of a family of items, assuming a common cycle for all the items. Gallego[15] extended this model by removing the stipulation of a common cycle for all the items.

But the above studies did not consider the demand rate to be variable. It is a common belief that large piles of goods displayed in a supermarket lead the customers to buy more. Silver and Peterson[16] and Silver[17] have also noted that sales at the retail level tend to be proportional to the inventory displayed. Baker and Urban[18] and Urban[19] considered an inventory system in which the demand rate of the product is a function of the on-hand inventory. Goh[20] discussed the model of Baker and Urban[18] relaxing the assumption of a constant holding cost. Mandal and Phaujder[21] then extended this model to the case of deteriorating items with a constant production rate. Datta and Pal[22] presented an inventory model in which the demand rate of an item is dependent on the on-hand inventory level until a given inventory level is achieved, after which the demand rate becomes constant. Giri , Pal , Goswami and Chaudhuri[24] extended the model of Urban[19] to the case of items deteriorating overtime. Ray and Chaudhuri[25] discussed an **EOQ** (*economic order quantity*) model with stock-dependent demand, shortage, inflation and time discounting of different costs and prices associated with the system. Ray, Goswami and Chaudhuri[26] studied the inventory problem with a stock-dependent demand rate and two levels of storage, *rented warehouse* (**RW**) and *own warehouse* (**OW**). Giri and Chaudhuri[27] extended the model of Goh[20] to cover an inventory of a deteriorating item and discussed both the case of nonlinear time-dependent and stock-dependent holding costs.

Volume flexibility is a major component in a **FMS**. The manufacturing flexibility which is capable of adjusting the production rate with the variability in the market demand is known as volume flexibility[28]. In the present paper, we consider a volume flexible manufacturing system for a deteriorating item with an inventory-level-dependent demand rate. In reality, the demand rate remains stock-dependent for some time and then becomes a constant after the stock falls down to a certain level. Several factors like limited number of potential customers and their goodwill, price and quality of the goods, locality of shop, etc. can be accounted for the change in the demand pattern.

Fundamental Assumptions and Notations

Assumptions:

1. The inventory system involves only one item and is a self-production system.
2. Lead time is zero.
3. No shortages are permitted.
4. The time-horizon is infinite.
5. The production cost per unit item is a function of the production rate.
6. The production rate is considered to be a decision variable.

Notations:

P - The production rate per unit time ;

$I(t)$ - On-hand inventory at time 't' $\geq$ 0 ;

$R(I)$ - Demand-rate function varying with I(t) ;

S_0 - The stock-level, beyond which the demand rate becomes constant;

θ - Constant deterioration rate of the On-hand inventory,$0 < \theta < 1$;

C_h - Holding cost per unit per unit time;

C_s - Setup cost per production run;

$\eta(P)$ - The production cost per unit item ;

S_p - Salvage cost per unit item;

T - The duration of the production cycle;

∇ - Gradient operator.

Formulation of the Model

We consider a self-manufacturing system in which the items are manufactured in a machine and the market demand is filled by these manufactured items. The demand rate is dependent on the on-hand inventory down to a level S_0, beyond which it is assumed to be a constant, i.e.,

$$\begin{aligned} R(I) &= D + \gamma I(t),\ I > S_0 \\ &= D + \gamma S_0,\ 0 \leq I \leq S_0, \end{aligned}$$

where D and γ are non-negative constants and $D < P$. The production cost per unit is

$$\eta(P) = r + \frac{g}{P} + \alpha P$$

where r, g, α are all positive constants. This cost is based on the following factors:

1. The material cost r per unit item is fixed.
2. As the production rate increases, some costs like labour and energy costs are equally distributed over a large number of units. Hence the production cost per unit(g/P) decreases as the production rate(P) increases.

3. The third term(αP), associated with tool/die costs, is proportional to the production rate.

The production cycle begins with zero stock. Production starts at $t = 0$ and the stock reaches a level S_0 at time $t = t_1$ after meeting demands. The demand rate in the interval (0, t_1) is (D+γS_0). In the interval (t_1,t_2), production continues uninterruptedly and the demand rate depends on the instantaneous stock level. Production is stopped at time $t = t_2$. The demand rate continues to depend on the instanteneous inventory level until $t = t_3$ when the stock falls down to the level S_0 again. The inventory falls to the zero level at the end $t = T$ of the production cycle. This cycle of production is repeated over and over again. Therefore, the governing equations of this model are

$$\frac{dI(t)}{dt} + \theta I(t) = P - (D + \gamma S_0),\ 0 \le t \le t_1,\ I \le S_0; \qquad (1)$$

with $I(0) = 0$ and $I(t_1) = S_0$;

$$\frac{dI(t)}{dt} + (\theta + \gamma)I(t) = P - D,\ t_1 \le t \le t_2,\ I > S_0; \qquad (2)$$

$$\frac{dI(t)}{dt} + (\theta + \gamma)\ I(t) = -D,\ t_2 \le t \le t_3\ ,\ I > S_0 \qquad (3)$$

with $I(t_3) = S_0$;

$$\frac{dI(t)}{dt} + \theta\ I(t) = -(D + \gamma\ S_0),\ t_3 \le t \le T,\ I \le S_0 \qquad (4)$$

with $I(T) = 0$.

From equation (1), we have

$$\textstyle\int_0^t d(I\ e^{\theta s}) = (P - D - \gamma\ S_0)\ \int_0^t e^{\theta s}\ ds,$$

i.e.,

$$I(t) = \left(\tfrac{P-D-\gamma\ S_0}{\theta} \right)(1 - e^{-\theta t}),\ 0 \le t \le t_1. \qquad (5)$$

Now the condition $I(t_1) = S_0$ gives us

$$\frac{P - D - \gamma S_0}{\theta} (1 - e^{-\theta t_1}) = S_0$$
$$\Rightarrow e^{-\theta t_1} = 1 - \frac{\theta S_0}{P - D - \gamma S_0},$$

i.e.,

$$t_1 = - \frac{1}{\theta} ln \left[1 - \frac{\theta S_0}{P - D - \gamma S_0} \right], \tag{6}$$

where $P > D + (\theta + \gamma)S_0$.

Also from equation (2) , we have

$$\int_{t_1}^{t} d[I e^{(\theta+\gamma)s}] = (P - D) \int_{t_1}^{t} e^{(\theta+\gamma)s} ds ,$$

i.e.,

$$I(t) = \left(S_0 - \frac{P - D}{\theta + \gamma} \right) e^{(\theta+\gamma)(t_1 - t)} + \left(\frac{P - D}{\theta + \gamma} \right) , t_1 \leq t \leq t_2 . \tag{7}$$

Therefore,

$$I(t_2) = \left(S_0 - \frac{P - D}{\theta + \gamma} \right) e^{(\theta+\gamma)(t_1 - t_2)} + \left(\frac{P - D}{\theta + \gamma} \right). \tag{8}$$

Similarly, the equation (3) gives us

$$\int_{t_2}^{t} d[Ie^{(\theta+\gamma)s}] = -D \int_{t_2}^{t} e^{(\theta+\gamma)s} ds$$
$$\Rightarrow I(t)e^{(\theta+\gamma)t} - I(t_2)e^{(\theta+\gamma)t_2} = -\frac{D}{\theta+\gamma}[e^{(\theta+\gamma)t} - e^{(\theta+\gamma)t_2}]$$
$$\Rightarrow I(t) = I(t_2)e^{(\theta+\gamma)(t_2 - t)} - \frac{D}{\theta+\gamma}[1 - e^{(\theta+\gamma)(t_2 - t)}],$$

i.e.,

$$I(t) = \left(I(t_2) + \frac{D}{\theta+\gamma} \right) e^{(\theta+\gamma)(t_2 - t)} - \frac{D}{\theta + \gamma}, t_2 \leq t \leq t_3 \tag{9}$$

Since $I(t_3) = S_0$, we have

$$S_0 = [I(t_2) + \frac{D}{\theta+\gamma}] e^{(\theta+\gamma)(t_2 - t_3)} - \frac{D}{\theta+\gamma}$$
$$\Rightarrow e^{(\theta+\gamma)(t_2 - t_3)} = [\frac{S_0 + D/(\theta+\gamma)}{I(t_2) + D/(\theta+\gamma)}]$$
$$\Rightarrow t_2 - t_3 = \frac{1}{\theta+\gamma} ln [\frac{S_0 + D/(\theta+\gamma)}{I(t_2) + D/(\theta+\gamma)}] ,$$

i.e.,

$$t_3 = t_2 - \frac{1}{\theta + \gamma} ln \left[\frac{S_0 + D/(\theta+\gamma)}{I(t_2) + D/(\theta+\gamma)} \right] , \tag{10}$$

where $I(t_2) > S_0$.

Again the equation (4) becomes

$$\begin{aligned} &\int_{t_3}^{t} dI(Ie^{\theta s}) = -(D+\gamma S_0)\int_{t_3}^{t} e^{\theta s}\, ds \\ &\Rightarrow I(t)\, e^{\theta t} - S_0\, e^{\theta t_3} = -\left(\frac{D+\gamma\ S_0}{\theta}\right)\left(e^{\theta t} - e^{\theta t_3}\right) \\ &\Rightarrow I(t) = S_0\, e^{\theta(t_3-t)} - \frac{D+\gamma S_0}{\theta}\left\{ 1-e^{\theta(t_3-t)}\right\}, \end{aligned}$$

i.e.,

$$I(t) = \left(S_0 + \frac{D+\gamma S_0}{\theta}\right) e^{\theta(t_3-t)} - \left(\frac{D+\gamma S_0}{\theta}\right),\ t_3 \le t \le T. \qquad (11)$$

Now $I(T) = 0$ implies

$$\begin{aligned} &\left(S_0 + \frac{D+\gamma S_0}{\theta}\right) e^{\theta(t_3-T)} - \frac{D+\gamma S_0}{\theta} = 0 \\ &\Rightarrow e^{\theta(t_3-T)} = \frac{D+\gamma S_0}{D+(\gamma+\theta)S_0}, \end{aligned}$$

i.e.,

$$T = t_3 - \frac{1}{\theta}\, ln\left[\frac{D+\gamma S_0}{D+(\gamma+\theta)S_0}\right]. \qquad (12)$$

Let Inv_1 , Inv_2 , Inv_3 , Inv_4 be the total inventories in the intervals $0 \le t \le t_1,\ t_1 \le t \le t_2,\ t_2 \le t \le t_3,\ t_3 \le t \le T$ respectively. Then

$$\begin{aligned} Inv_1 &= \int_0^{t_1} I(t)\, dt \\ &= \int_0^{t_1}\left(\frac{P-D-\gamma S_0}{\theta}\right)\left(1-e^{-\theta t}\right) dt \\ &= \left(\frac{P-D-\gamma S_0}{\theta^2}\right)\left(\theta t_1 + e^{-\theta t_1} - 1\right); \end{aligned}$$

$$\begin{aligned} Inv_2 &= \int_{t_1}^{t_2} I(t)\, dt \\ &= \left\{ S_0 - \frac{P-D}{\theta+\gamma}\right\} e^{(\theta+\gamma)t_1} \int_{t_1}^{t_2} e^{-(\theta+\gamma)t}\, dt + \left(\frac{P-D}{\theta+\gamma}\right)\int_{t_1}^{t_2} dt \\ &= \frac{1}{(\theta+\gamma)}\left\{ S_0 - \frac{P-D}{\theta+\gamma}\right\}\left\{ 1-e^{-(\theta+\gamma)(t_2-t_1}\right\} + \left(\frac{P-D}{\theta+\gamma}\right)(t_2-t_1); \end{aligned}$$

$$\begin{aligned} Inv_3 &= \int_{t_2}^{t_3} I(t)\, dt \\ &= \left[I(t_2) + \frac{D}{\theta+\gamma}\right] e^{(\theta+\gamma)t_2}\int_{t_2}^{t_3} e^{-(\theta+\gamma)t} dt - \left(\frac{D}{\theta+\gamma}\right)\int_{t_2}^{t_3} dt \\ &= \frac{1}{(\theta+\gamma)}\left\{I(t_2) + \frac{D}{\theta+\gamma}\right\}\left\{1-e^{(\theta+\gamma)(t_2-t_3)}\right\} - \left(\frac{D}{\theta+\gamma}\right)(t_3-t_2); \end{aligned}$$

$$\begin{aligned} Inv_4 &= \int_{t_3}^{T} I(t) dt \\ &= \left(S_0 + \frac{D+\gamma S_0}{\theta}\right) e^{\theta t_3}\int_{t_3}^{T} e^{-\theta t}\, dt - \left(\frac{D+\gamma S_0}{\theta}\right)\int_{t_3}^{T} dt \\ &= \frac{1}{\theta^2}(\theta S_0 + D + \gamma S_0)\left(1-e^{\theta(t_3-T)}\right) - \left(\frac{D+\gamma S_0}{\theta}\right)(T-t_3) \end{aligned}$$

$$= \frac{S_0}{\theta} + \frac{D+\gamma S_0}{\theta^2}\ ln\ [\frac{D+\gamma S_0}{D+(\gamma+\theta)S_0}]\ , \text{ by (12).}$$

The values of θ, S_0 and $(D+\gamma S_0)$ must be such that $Inv_4 > 0$ is satisfied. Now the total deteriorated item (I_d) is

$$\begin{aligned} I_d &= \theta\{\textstyle\int_0^{t_1} I(t)\ dt + \int_{t_1}^{t_2} I(t)\ dt + \int_{t_2}^{t_3} I(t)\ dt + \int_{t_3}^{T} I(t)\ dt\} \\ &= \theta(Inv_1 + Inv_2 + Inv_3 + Inv_4). \end{aligned}$$

Therefore, the total demand in $(0, T)$ becomes $D_T = (Pt_2 - I_d)$. Then the average profit during time $(0, T)$ takes the form

$$\pi(P, t_2) = \frac{1}{T}[(Pt_2 - I_d)S_p - \{C_s + C_h(Inv_1 + Inv_2 + Inv_3 + Inv_4) + (r + \frac{g}{P} + \alpha P)Pt_2\}] \quad (13)$$

Therefore, we have to Maximize $\pi(P, t_2)$;

subject to the constraints :

$$\begin{aligned} D + (\theta+\gamma)S_0 - P &< 0\ , \\ -Inv_1 &< 0\ , \\ -Inv_2 &< 0\ , \\ -Inv_3 &< 0\ , \\ -I(t_2) + S_0 &< 0\ , \\ -t_2 + t_1 &< 0\ . \end{aligned}$$

The condition $D+(\theta+\gamma)S_0 - P < 0 \Rightarrow\ 0 < \frac{\theta S_0}{P-D-\gamma S_0} < 1$ which is necessary for the value of t_1 in eqn. (6) to be real.

The three conditions $-Inv_1 < 0$, $-Inv_2 < 0$ and $-Inv_3 < 0$ ensure that Inv_1, Inv_2 and Inv_3 must be positive.

The condition $-I(t_2)+S_0 < 0$ implies that $I(t_2)$, the inventory level at time t_2 , is higher than S_0.

The condition $-t_2 + t_1 < 0$ ensures that t_2 is greater than t_1 .

This problem can be solved by **Zoutendijk′s Method**[23] whose Algorithm is discussed below.

General Problem : Minimize $\{-\pi(\bar{X})\}$
subject to the constraints : $G_j(\bar{X}) < 0$, where $\bar{X} \in R^n$, $j = 1, 2..........m$.

Algorithm:

1. Start with an initial feasible point $\bar{X}_1$, evaluate $\pi(\bar{X}_1)$ and $G_j(\bar{X}_1)$, $j = 1, 2, ...m$. Set the iteration number as $i = 1$.

2. If $G_j(\bar{X}_i) < 0$, $j = 1, 2, ..., m$. (i.e., $\bar{X}_i$ is an interior feasible point), set the current search direction as $\bar{S}_i = -\nabla\pi(\bar{X}_i)$. Normalize $\bar{S}_i$ in a suitable manner.

3. Find a suitable step length λ_i along the direction $\bar{S}_i$ and obtain a new point $\bar{X}_{i+1}$ as $\bar{X}_{i+1} = \bar{X}_i + \lambda_i\bar{S}_i$.

4. Evaluate the objective function $\pi(\bar{X}_{i+1})$.

5. Test for the convergence of the method . If $\left| \frac{\pi(\bar{X}_i)-\pi(\bar{X}_{i+1})}{\pi(\bar{X}_i)} \right| \leq \epsilon$ where ϵ is a preassigned small positive quantity, terminate the iteration by taking $\bar{X}_{opt} \approx \bar{X}_{i+1}$. Otherwise, go to next step.

6. Set the new iteration number as $i = i + 1$, and repeat from step 2 onwards.

Numerical Example

We take the parameter values as $D = 50$, $\theta = 0.05$,$\gamma = 0.1$, $S_0 = 100$, $C_s = 300$, C_h= 0.1, $S_p = 6.0$, $r = 1.0$, $g = 250$, $\alpha = 0.01$ in appropriate units. We obtain the optimum results $t_1^* = 1.258883$, $t_2^* = 6.696204$, $t_3^* = 10.07596$, $T^* = 11.67682$, P^*= 141.9617, $\pi^* = 41.93613$.

Sensitivity Analysis

Using the same numerical example, the sensitivity of each variable t_1^* , t_2^*, t_3^*, T^*, P^* and π^* to changes in the values of each of the parameters θ, γ, S_0, C_s, C_h, S_p, r, g, α is examined which is shown in Table 1 . In the proposed model, t_1^* changes by 10.28 % , 03.27 % , 06.64 % , -12.67 % respectively with 50 % , 20 % , -20 % , -50 % changes in the value of θ. t_2^* changes by 1787.29 % , 44.52 % , -24.76 % , -37.03 % respectively with 50 % , 20 % , -20 % , -50 % changes in the value of γ. t_3^* changes by -14.14 % , -06.30 % , 00.46 % , 11.70 % respectively with 50 % , 20 % , -20 % , -50 % changes in the value of S_0. T^* changes by 39.74 % , 14.36 % , -14.63 % , -36.63 % respectively with 50 % , 20 % , -20 % , -50 % changes in the value of C_s . π^* changes by -92.74 % , -38.01 % , 39.89 % , 106.62 % , respectively with 50 % , 20 % , -20 % , -50 % changes in the value of $'r'$. Similarly P^* changes by (152.99 % , 11.64 % , -04.35 % , -04.46 %); (23.40 % , 09.38 % , -10.98 % , -04.01 %); (-22.42 % , -11.60 % , 13.83 % , 64.83 %) for 50 % , 20 % , -20 % , -50 % changes in the value of S_p , g and α respectively. In a similar manner, the changes in the solution variables for changes in other parameters can be computed. It is seen that t_1^*, t_2^* , t_3^* , T^*, P^* and π^* are moderately sensitive to changes in the parameters θ, γ, S_0, C_s, C_h, S_p, r, g, α. P^* is insensitive to changes in S_0; but P^* is fairly sensitive to the changes in the parameters S_p, g and α.

Table 1: *Sensitivity Analysis*

Changing parameter in(%)		t_1^*	t_2^*	t_3^*	T^*	P^*	π^*
θ	+50	1.3883	4.8833	7.0585	8.6290	135.8456	24.6350
	+20	1.3001	5.6528	8.4405	10.0290	139.9558	34.3048
	-20	1.3424	9.5264	13.6156	15.2291	136.5070	50.7171
	-50	1.0993	123.4375	130.5588	132.1917	152.2191	74.5473
γ	+50	1.1375	126.3767	130.3652	131.8474	155.4316	73.7782
	+20	1.2217	9.6771	13.4629	15.0141	146.3745	50.7842
	-20	1.2478	5.0385	8.0524	9.7062	140.6663	34.5639
	-50	1.2790	4.2163	7.0012	8.7415	135.7072	25.1930
S_0	+50	2.0531	6.2040	8.6511	10.8351	141.8735	44.6257
	+20	1.5592	6.4515	9.4408	11.2882	141.9970	42.8400
	-20	0.9817	6.5000	10.1227	11.4565	141.5000	41.1946
	-50	0.5808	6.9485	11.2549	12.1440	142.3439	40.5015
C_s	+50	1.3704	10.7304	14.7161	16.3170	135.4992	30.9589
	+20	1.2917	8.0744	11.7522	13.3530	139.9434	37.1254
	-20	1.2130	5.3606	8.3671	9.9679	144.9651	47.4783
	-50	1.2120	3.6595	5.7987	7.3996	145.0331	57.8428
C_h	+50	1.2618	5.2080	8.0330	9.6339	141.9800	32.8058
	+20	1.2744	6.0152	9.1276	10.7284	140.9916	38.0860
	-20	1.2554	7.7621	11.4545	13.0554	142.1793	46.1549
	-50	1.2076	10.2054	14.5382	16.1390	145.3308	53.5354
S_p	+50	0.6465	291.8469	299.8896	301.4904	217.1918	394.8986
	+20	1.0419	270.1158	276.0579	277.6588	158.4912	147.9354
	-20	1.3651	4.5496	6.8570	8.4579	135.7813	-39.8138
	-50	1.3678	3.2075	4.7591	6.3600	135.6360	-156.0006
r	+50	1.3253	4.8981	7.4475	9.0483	137.9794	3.0430
	+20	1.2890	5.8232	8.8309	10.4318	140.1007	25.9964
	-20	1.2364	8.2314	12.0896	13.6905	143.4028	58.6627
	-50	1.1649	15.8558	20.9309	22.5317	148.3684	86.6486

Changing parameter in(%)		t_1^*	t_2^*	t_3^*	T^*	P^*	π^*
g	+50	0.8876	3.2432	5.96640	7.5672	175.1753	-18.9558
	+20	1.0781	4.6696	7.71267	9.3135	155.2737	15.3291
	-20	1.5664	13.6505	17.5322	19.1331	126.3721	73.6078
	-50	1.3561	81.9617	86.8966	88.4974	136.2690	137.1216
α	+50	2.1010	6.5581	8.5022	10.1030	110.1384	-5.0713
	+20	1.5884	6.4549	9.0780	10.6788	125.4862	20.7892
	-20	1.0093	8.1195	12.6550	14.2559	161.5913	68.1260
	-50	0.5831	292.7949	301.3344	302.9353	233.9999	153.9163

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