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AN EOQ MODEL WITH TIME-DEPENDENT DEMAND, INFLATION AND MONEY VALUE FOR A WARE-HOUSE ENTERPRISER

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ABSTRACT. A mathematical model for perishable items that follows Weibull distribution is developed , considering time dependent demand rate, inflation and money value. The concerned problem is solved analytically with the help of Simpson's 1/3 rd formula and Newton-Raphson method. An optimal production policy is derived with maximization of profit as the criterion of optimality. The result is illustrated with a numerical example. Sensitivity of the optimal solution to changes in the values of some key parameters is also studied.

1. Introduction

It is important to a supply manager in modern organization to control and maintain the inventories of deteriorating items. In general, deterioration is defined as damage, spoilage, decay, obsolescence, evaporation, pilferage etc., that result in decrease of usefulness of the original one. It is reasonable to note that a product may be understood to have a lifetime which ends when utility reaches zero. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. For items such as steel, hardware, glassware and toys, the rate of deterioration is so low that there is little need for considering deterioration in the determination of the economic lot size. But some items such as blood, fish, strawbery, alcohol, gasoline, radioactive chemical, medicine and food grains(i.e., paddy, wheat, pottato, onion etc.) deteriorate remarkably overtime.

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At first, Whitin[1] considered an inventory model for fashion goods deteriorating at the end of a prescribed storage period. Ghare and scharder[2] developed an **EOQ** model with an exponential decay and a deterministic demand. Thereafter, Covert and Philip[3] and Philip[4] extended **EOQ** (*Economic Order Quantity*) models for deterioration which follows *Weibull distribution* . Weiss[5] analysed decaying inventory with a fixed life perishability. Shah and Jaiswal[6] considered the rate of deterioration to be uniform. Misra[7] used a *two-parameter Weibull distribution* to fit the deterioration rate, and Deb and Chaudhuri[8] discussed a model with a variable rate of deterioration allowing shortages to occur. Mandal and Phaujdar[9] developed a model for deteriorating items (both constant and varying with time) with a stock-dependent consumption rate. Bahari-Kashani[10] discussed a heuristic for obtaining order quatities when demand is linear time-proportional and inventory deteriorates at a constant rate over time. His model was generalized by Goswami and Chaudhuri[11] to allow for shortages and a linear trend in demand. Also Dave and Patel[12], Sachan[13] developed **EOQ** models for a linear trend in demand. Other authors such as Dave[14], Elsayed and Teressi[15], Kang and Kim[16] , Mak[17], Raafat et al.[18] and Heng et al.[19] assumed either instantaneous or finite production with different assumptions on the patterns of deterioration. Mak[17], Aggarwal and Bahari-Kashani[20] and Wee[21] developed **EOQ** models to allow deterioration and an exponential demand pattern.

During the last 25 years, the economic situation of most of the countries have changed to such an extent due to large-scale inflation and consequent sharp decline in the purchasing power of money, that it has not been possible to ignore the effects of inflation and time value of money. Buzacott [22] was the first who developed the **EOQ** model taking inflation into account. Several other researchers like Jeya Chandra and Bahnar [23], Aggarwal [24], Misra [25,26], Bierman and Thomas[27], Sarkar and Pan[28] etc. have discussed the **EOQ** model in this direction , assuming constant demand rate. Thereafter, Datta and Pal[29], Bose et al.[30] and Ray and Chaudhuri[31] have developed the **EOQ** model incorporating the effects of inflation, time value of money, a linear time dependent demand rate.

In the present paper, we consider the money value and inflation of each cost and profit parameter ; deterioration rate follows two parameters *Weibull distribution*; salvage value is varying linearly with time; demand rate is constant up to a fixed time, after then it varies linearly with time. So our study of deteriorating inventory is somewhat different from those in the existing literature.

2. Fundamental Assumptions and Notations

Assumptions:

1. Demand rate varies with time.
2. Deterioration rate varies with time and follows a two-parameter *Weibull distribution* function.
3. Selling price per unit also varies with time.
4. Lead time is zero.
5. Shortages are not allowed.
6. Inflation and money value is considered.

Notations:

$Q(t)$- on-hand inventory at time $'t'(>0)$.

$D(t)$ - Demand rate at time $'t'(>0)$.

Q_0 - inventory at time $t=0$.

K- purchasing price per unit item.

$S_p(t)$- salvage price per unit of item.

$Z(t)$ - Weibull instantaneous rate function for the item stock.

i- inflation rate per unit time.

r - is the discount rate representing the time value of money.

K - purchasing cost per unit item.

C_h - inventory cost per unit per unit time.

C_s - set-up cost per order.

T - the duration of a cycle.

3. Formulation of the Model

In this model, we consider the demand rate $D(t)$ to be a constant up to a certain time $t = \mu$ after which it varies linearly with time. Consequently as the demand rate increases the salvage value, $S_p(t)$, will be increased. If $Q(t)$ be the on-hand inventory at any time $t \geq 0$, the governing differential equation is as follows :

$$\frac{dQ(t)}{dt} + Z(t)Q(t) = -D(t), with \quad Q(0) = Q_0 \tag{1}$$

We define $D(t)$, $S_p(t)$ and $Z(t)$ as

$$D(t) = D_0 + b(t-\mu)H(t-\mu) \tag{2}$$

$$S_p(t) = P_i + a(t-\mu)H(t-\mu) \tag{3}$$

where

$$H(t-\mu) = \begin{cases} 1 & , \quad if \quad t \geq \mu \\ 0 & , \quad if \quad t < \mu \end{cases}$$

and

$$Z(t) = \alpha\beta t^{\beta-1} \tag{4}$$

where

D_0=initial demand rate ,
b=rate of change of demand rate with respect to t ,
P_i=initial salvage value per unit,
a=rate of change of salvage value per unit with respect to 't',

$\alpha =$ the scale parameter; $\alpha > 0$
$\beta =$ the shape parameter; $\beta > 0$
$t =$ time to deterioration; $t > 0$

is the instantaneous deterioration rate of the item stocked which may have a decreasing, constant or increasing rate of deterioration for $0 < \beta < 1$, $\beta = 1$and $\beta > 1$ respectively. But in reality β is greater than or equal to 1.

It is real fact that when the crops (like paddy, wheat, pottato, onion, ginger etc.) are harvested , then in the market, the salvage value of the items is remained constant up to a ceratin period(0 , μ). At the begining of harvesting , the business men stock the items in cold stores or ware houses. After that certain period$(0, \mu)$ the demand automatically increases in the market because all customers are bound to purchase their essential commodities due to decrease of the harvested item. Consequently, demand rate and salvage value gradually increase to the increase of time.

Substituting equ.(2) and equ.(4) in equ.(1) , we have

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1}Q(t) = -D_0, \quad 0 \le t \le \mu, \quad with \;\; Q(0) = Q_0 \tag{5}$$

and

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1}Q(t) = -D_0 - b(t-\mu), \quad \mu \le t \le T, \quad with \;\; Q(T) = 0 \tag{6}$$

Solutions of the equations (5) and (6) are given by

$$\begin{aligned} Q(t) &= \{Q_0 - D_0 \int_0^t e^{\alpha u^\beta} du\} e^{-\alpha t^\beta} \quad , \quad 0 \le t \le \mu \\ &= \{Q_0 - D_0 \int_0^t e^{\alpha u^\beta} du - b \int_\mu^t (u-\mu) e^{\alpha u^\beta} du\} e^{-\alpha t^\beta} \quad , \ \mu \le t \le T \end{aligned}$$

Now Q(T)=0 gives us

$$Q_0 = D_0 \int_0^T e^{\alpha t^\beta} dt + b \int_\mu^T (t-\mu) e^{\alpha t^\beta} dt$$

The total salvage value of the items in the market is

$$Sav(T) = D_0 \int_0^\mu r dt + b \int_\mu^T \{D_0 + b(t-\mu)\}\{r + a(t-\mu)\} dt$$

Then the total average profit during (0,T) be

$$TAP(T) = \frac{1}{T}\{Sav(T) - \frac{1}{2}C_h Q_0 - K Q_0 - C_s\}$$

$$= \frac{1}{T}[D_0 \int_0^{\mu} P_i dt + \int_{\mu}^{T} \{D_0 + b(t-\mu)\}\{P_i + a(t-\mu)\}dt$$
$$-(\frac{1}{2}C_h + K)\{D_0 \int_0^{T} e^{\alpha t^{\beta}} dt$$
$$+ b \int_{\mu}^{T} (t-\mu)e^{\alpha t^{\beta}} dt\} - C_s] \quad (7)$$

In this model , the implicit assumption is that the invested capital of the cycle is met from a loan on which the interest rate is r (constant) and the loan is repaid when the item is sold. There is no delay in settling accounts payable and no credit is granted when sales are made.

If we introduce the *inflation and money value* to the total average profit , then the equation(7) takes the form as

$$MTAP(T) = \frac{1}{T}[\int_0^{\mu} DP_i e^{(-Rt)}\, dt + \int_{\mu}^{T} \{D_0 + b(t-\mu)\}\{P_i + a(t-\mu)\}e^{(-Rt)}\, dt$$
$$-(\frac{1}{2}C_h + K)\{D_0 \int_0^{T} e^{(\alpha t^{\beta} - Rt)}\, dt$$
$$+b \int_{\mu}^{T} (t-\mu)e^{(\alpha t^{\beta} - Rt)}\, dt\} - C_s \int_0^{T} e^{(-Rt)}\, dt], \quad (8)$$
$$where \quad R = r - i$$

Now our object is to

$$Maximize \;\; MTAP(T)$$
$$s.t. \quad T > \mu$$

The above problem is solved by analytical method.*see Appendix-I*

3. Numerical Example

We have considered the values of parameters in appropriate units as follows :

$D_0 = 300$, $b = 3.50$, $P_i = 5.5$,$a = 0.25$, $r = 0.2$, $i = 0.08$, $K = 4.00$, $C_h = 0.750$, $C_s = 250$, $\alpha = 0.002$, $\beta = 1.50$, $\mu = 2.0$

Then the optimum solution of the proposed model be

Average Profit (P_{ro}^{max}) = 33.13042*, Optimum Time (T^*) = 29.47922*. Inventory^{optimum} = 2604.7170*

4.Sensitivity Analysis

Using the same numerical example, the sensitivity of each variable profit, inventory and time (T) is examined which is shown in Table-1. Profit is moderate sensitive with changes in μ, α , β r, i . T is less sensitive with changes in the parameter α. T has nofeasible solution with $(+50\%, +25\%)$ changes in β, r and -50% change in the parameter i and moderate sensitive with changes in μ, r, i. Total inventory is moderate sensitive with changes in β, r, i and is slightly sensitive with changes in μ, α.

5.Conclusion

From physical scenario , it is common belief that the demand rate of various crops is remained constant up to a time (μ) after which the demand rate varies with time. During the period $[0, \mu]$, some customers of a locality keep the crops by harvesting and some author customers, those don't have cultivation, are bound to purchase their essential crops. After the certain time μ, the amount of harvesting crops decreases continuously with time. Consequently, the demand rate increase with time t, $t \geq \mu$, and selling price increases with time t, $t \geq \mu$. Effects of *inflation and time value of money* can no longer be ignored as in the economy of many countries is in the grip of a large-scale inflation and a consequent sharp decline in the purchasing power of money.

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Appendix-I

Now,

$$\frac{dMTAP(T)}{dT.t} = -\frac{MTAP(T)}{T} + \frac{1}{T}[\{D_0 + b(T-\mu)\}\{P_i + a(T-\mu)\} \quad e^{-RT}$$
$$-(\frac{1}{2}C_h + K)\{D_0 + b(T-\mu)\}e^{(\alpha T^{\beta} - RT)} - C_s e^{-RT}]$$

and

$$\frac{d^2MTAP(T)}{dT} = \frac{MTAP(T)}{T^2} - \frac{1}{T}\frac{dMTAP(T)}{dT`}$$
$$-\frac{1}{T^2}[\{D_0 + b(T-\mu)\}\{P_i + a(T-\mu)\}e^{(-RT)}$$
$$-(\frac{1}{2}C_h + K)\{D_0 + b(T-\mu)\}e^{(\alpha T^{\beta} - RT)} - C_s e^{(-RT)}]$$
$$+\frac{1}{T}[\{-R\{D_0 + b(T-\mu)\}\{P_i + a(T-\mu)\} + \{b(P_i - a\mu)$$
$$+a(D_0 - b\mu) + 2abT\}\}e^{(-RT)}$$
$$-(\frac{1}{2}C_h + K)\{D_0(\alpha T^{\beta} - RT)$$
$$(\alpha\beta T^{\beta-1} - R) + b + b(T-\mu)(\alpha\beta T^{\beta-1} - R)\}$$
$$e^{(\alpha T^{\beta} - RT)} + C_s R e^{(-RT)}]$$

Since for maximum of $MTAP(T)$, $\frac{dMTAP(T)}{dT} |_{T^*} = 0$ and $\frac{d^2MTAP(T)}{dT}$ must be negative at $T = T^*$. In this problem some integrations are solved by Simpson's 1/3 rd formula, taking 70000 sub-interval of the intervals of integrations and $\frac{dMTAP(T)}{dT} = 0$ is solved by Newton-Raphson's method.

Table-1 *Sensitivity Analysis*

Parameters (% Change)	Time(T) (% Change)	Av. Profit (% Change)	Inventory (% Change)
+50%	+18.46	-60.23	+01.12
+25% μ	+09.97	-36.86	+00.71
-25%	-11.28	+59.17	-01.25
-50%	-23.85	+155.40	-03.44
+50%	+01.69	-35.52	+03.36
+25% α	+00.90	-17.37	+01.65
-25%	-01.04	+16.72	-01.59
-50%	-01.04	+16.72	-01.59
+50%	nofeasible	—	—
+25% β	nofeasible	—	—
-25%	-0.25	+39.83	-96.72
-50%	-02.06	+53.75	-96.70
+50%	nofeasible	—	—
+25% r	nofeasible	—	—
-25%	+54.68	+179.88	+88.88
-50%	+151.02	+1597.51	+776.65
+50%	+35.64	+166.69	+58.98
+25% i	+17.62	+40.38	+23.18
-25%	-16.17	-22.85	-16.10
-50%	nofeasible	—	—