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MAXIMUM LIKELIHOOD ESTIMATION OF EXPONENTIAL PARAMETERS OF RELIABILITY SYSTEMS

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ABSTRACT

This paper presents a stable technique for obtaining the maximum likelihood estimator of parameters of exponential distribution of M components that form 1) series system 2) parallel system and 3) s out of M :G system. The data consists of lifetime of the system only, that is it is not known which component caused the system failure and can be applied not only for complete data but for randomly censored data also. The log likelihood function presented can be used for the estimation of two parameter Weibull distribution in all the three cases.

Key words : Maximum likelihood estimator-Exponential distribution-Reliability models-iterative method.

1. INTRODUCTION

In parameter estimation, the most interesting methods are MLE, graphical procedure Cran [2] moments method Falls [3] and Weighted

Abbreviated title : MLE of reliability systems least-square method Cheng and Fu [1]. It is widely known that maximum likelihood estimator is asymptotically unbiased and has minimum variance and it is the commonly used technique for parameter estimation. With the wide use of computers it is worthwhile to calculate the maximum likelihood estimator and it has become the major tool for parameter estimation of reliability models. Finite mixture distributions have been used widely in medicine, psychology, and botany as referred in Titterington, Smith and Markov [11]. The research on parameter estimation was done on the mixed normal, exponential, binomial distributions. For postmortem data Sinha [10] extended the approach of Mendenhall and Hader [6] for the MLE of mixed exponential to mixed-Weibull distributions. For non-postmortem data Kaylan and Harris [5] extended the approach of Hasselbald [4] for the MLE of the mixture from the exponential family to the mixed-Weibull distribution when the data are ungrouped and censored. Olsson [8] directly searched the maximum of the log likelihood function of the mixed Weibull distribution through the Nelder -Mead simplex procedure given by Olsson and Nelson [7] and that procedure applies only to the 2-Weibull mixture.

In this paper Maximum Likelihood Estimator of exponential parameters is presented without concomitant indicators. The estimation technique studied in this paper facilitates the estimation of the parameters of the life distribution of each component in a 1) series system 2) parallel system and 3) s out of M system. The algorithm is not new but the attempt to use it to estimate MLE in reliability models is very effective and it results in minimum variance of parameters. The Broyden–Fletcher– Goldfarb - Shanno (BFGS) method for multivariate optimization is used to provide absolute maximum of the likelihood function and this method is easy to understand and program.

ASSUMPTIONS

- $A_1, A_2, \ldots, A_M$ form a M- component system and

  (i) even if one component fails the system fails in a series system

  (ii) if all the M components fail the system fails in a parallel system

  (iii) if M-s+1 components fail the system fails in the more general s-out-of-M system.

- Component failures are statistically independent but not necessarily identical in all the above three cases and failed components are not replaced.
- The life distribution of each component is exponential
- Only the system–level life times are recorded. There are no concomitant indicators

Notation

t time

N sample size of life test

k kind of component k=1 or 2 or 3……or M

h_k exponential parameter for component k

t_i failure time i $0 < t_1 < t_2 < .... < t_M$

$f_k(.),\ F_k(.),\ R_k(.)$ pdf, cdf, sf of component k

$f\ (.),\ F\ (.),\ R\ (.)$ pdf, cdf, sf of system

2. SERIES SYSTEM

If the system has N components connected in series viz a N–out–of– N : G system then the system fails even if one component fails. Regardless of component distribution the pdf and reliability of the system are

$$f(t) = \sum_{i=1}^{M} \left(f_i(t) \prod_{\substack{j=1 \\ j \neq i}}^{M} R_j(t) \right) \qquad (1)$$

$$R(t) = \prod_{i=1}^{M} R_j(t) \qquad (2)$$

The likelihood function of randomly censored data is

$$L(q) = C \prod_{i=1}^{N} \left(f(t_i) \right)^{d_i} \left(R(t_i) \right)^{1-d_i} \qquad (3)$$

where C is a constant and

$$d_i = \begin{cases} 1; & \textit{if the system has failed} \\ 0; & \textit{otherwise} \end{cases}$$

Substituting (1) and (2) in (3), the likelihood function of the series system is

$$L(q) = C\prod_{k=1}^{N}\left[\sum_{i=1}^{M}\left(f_i(t_k)\prod_{\substack{j=1\\ j\neq i}}^{M} R_j(t_k)\right)^{d_k}\left(\prod_{j=1}^{M} R_j(t_k)\right)^{1-d_k}\right] \quad (4)$$

The log likelihood function of the above is

$$\ln L(q) = \ln C + \sum_{k=1}^{N}\left[d_k \ln\left(\sum_{i=1}^{M}\left(f_i(t_k)\prod_{\substack{j=1\\ j\neq i}}^{M} R_j(t_k)\right)\right) + (1-d_k)\sum_{j=1}^{M}\ln R_j(t_k)\right]$$

The 2-parameter Weibull distribution or 1-parameter exponential distribution can be examined as the failure distribution. Let us assume lifetimes follow exponential distribution with parameters $h_1, h_2, \ldots, h_M$. Differentiation of $\ln L(q)$ with respect to h_i (i = 1,2…,M) gives the following likelihood equations

$$\frac{\partial \ln L(q)}{\partial h_1} = \sum_{k=1}^{N}\left[d_k\left(\frac{\partial f_1(t_k)}{\partial h_1}\prod_{j=2}^{M} R_j(t_k)\right) + d_k\,\frac{\partial R_1(t_k)}{\partial h_1}\left(\sum_{i=2}^{M}\left(f_i(t_k)\prod_{\substack{j=2\\ j\neq i}}^{M} R_j(t_k)\right)\right)\Big/ f(t)\right]$$
$$+ \sum_{k=1}^{N}\left((1-d_k)\frac{\partial R_1(t_k)}{\partial h_1}\Big/ R(t)\right)$$

$$\frac{\partial \ln L(q)}{\partial h_2} = \sum_{k=1}^{N}\left[d_k\left(\frac{\partial f_2(t_k)}{\partial h_2}\prod_{\substack{j=1\\ j\neq 2}}^{M} R_j(t_k)\right) + d_k\,\frac{\partial R_2(t_k)}{\partial h_2}\left(\sum_{\substack{i=1\\ i\neq 2}}^{M}\left(f_i(t_k)\prod_{\substack{j=1\\ j\neq i,2}}^{M} R_j(t_k)\right)\right)\Big/ f(t)\right]$$
$$+ \sum_{k=1}^{N}\left((1-d_k)\frac{\partial R_2(t_k)}{\partial h_2}\Big/ R(t)\right)$$

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$$\frac{\partial \ln L(q)}{\partial h_M} = \sum_{k=1}^{N}\left[d_k \left(\frac{\partial f_M(t_k)}{\partial h_M} \prod_{\substack{j=1 \\ j \neq M}}^{M} R_j(t_k) \right) + \frac{\partial R_M(t_k)}{\partial h_M} \left(\sum_{\substack{i=1 \\ i \neq M}}^{M} \left(f_i(t_k) \prod_{\substack{j=1 \\ j \neq i, M}}^{M} R_j(t_k) \right) \right) \Big/ f(t) \right]$$
$$+ \sum_{k=1}^{N} \left((1-d_k) \frac{\partial R_M(t_k)}{\partial h_M} \Big/ R(t) \right)$$

Since a closed form solution does not exists for finding the roots of these likelihood equations the equations must be solved by iteration method given in section 5.

3. PARALLEL SYSTEM

If the system has N components connected in parallel, viz. a 1-out-of-N : G system and the system fails only if all the N components fail then the system pdf and reliability, regardless of component distribution is

$$f(t) = \sum_{i=1}^{N} \left(f_i(t) \prod_{j=1,\, j \neq i}^{N} F_j(t) \right) \quad \text{and} \quad (5)$$

$$R(t) = \sum_{i=1}^{N} \left(R_i(t) \prod_{j=1,\, j \neq i}^{i} F_j(t) \right) \qquad (6)$$

The likelihood function of parallel system with randomly censored data if results (5) and (6) are substituted in (3) are

$$L(q) = C\prod_{k=1}^{N}\left(\left[\sum_{j=1}^{M}\left(f_i(t_k)\prod_{\substack{j=1\\ j\neq i}}^{M}F_j(t_k)\right)\right]^{d_i}\left[\sum_{i=1}^{M}\left(R_i(t_k)\prod_{\substack{j=1\\ j\neq i}}^{i}F_j(t_k)\right)\right]^{1-d_i}\right)$$

The above can also be written as

$$L(q) = C\prod_{k=1}^{N}\left(\left[\sum_{j=1}^{M}\left(f_i(t_k)\prod_{\substack{j=1\\ j\neq i}}^{M}F_j(t_k)\right)\right]^{d_i}\left[1-\prod_{j=1}^{M}F_j(t_k)\right]^{1-d_i}\right) \qquad (7)$$

The log likelihood function of (7) is

$$\ln L(q) = \ln C + \sum_{.k=1}^{N}\left[d_k \ln\left[\sum_{j=1}^{M}\left(f_i(t_k)\prod_{\substack{j=1\\ j\neq i}}^{M}F_j(t_k)\right)\right] + (1-d_k)\ln\left[1-\prod_{j=1}^{M}F_j(t_k)\right]\right]$$

Differentiation of $\ln L(q)$ with respect to $h_1, h_2, \ldots\ldots, h_M$ gives the following likelihood equations

$$\frac{\partial \ln L(q)}{\partial h_1} = \sum_{k=1}^{N}\left[d_k\left(\frac{\partial f_1(t_k)}{\partial h_1}F_2(t_k)F_3(t_k)\ldots F_M(t_k)\right) + d_k\frac{\partial F_1(t_k)}{\partial h_1}\left(\sum_{i=2}^{M}\left(f_i(t)\prod_{\substack{j=2\\ j\neq i}}^{M}F_j(t_k)\right)\right)\middle/ f(t\right.$$

$$+\sum_{k=1}^{N}\left((1-d_k)\left(-\frac{\partial F_1(t_k)}{\partial h_1}\right)\prod_{j=2}^{M}F_j(t_k)\middle/ R(t)\right)$$

$$\frac{\partial \ln L(q)}{\partial h_2} = \sum_{k=1}^{N} \left[d_k \left(\frac{\partial f_2(t_k)}{\partial h_2} F_1(t_k) F_3(t_k) \ldots F_M(t_k) \right) + d_k \frac{\partial F_2(t_k)}{\partial h_2} \left(\sum_{\substack{i=1 \\ i \neq 2}}^{M} \left(f_i(t) \prod_{\substack{j=1 \\ j \neq i,2}}^{M} F_j(t_k) \right) \right) \Big/ f(t) \right]$$

$$+ \sum_{k=1}^{N} \left((1-d_k) \left(-\frac{\partial F_2(t_k)}{\partial h_2} \right) \prod_{\substack{j=1 \\ j \neq 2}}^{M} F_j(t_k) \Big/ R(t) \right)$$

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$$\frac{\partial \ln L(q)}{\partial h_M} = \sum_{k=1}^{N} \left[d_k \left(\frac{\partial f_M(t_k)}{\partial h_M} \prod_{\substack{j=1 \\ j \neq M}}^{M} F_j(t_k) \right) + d_k \frac{\partial F_M(t_k)}{\partial h_M} \left(\sum_{\substack{i=1 \\ i \neq M}}^{M} \left(f_i(t_k) \prod_{\substack{j=1 \\ j \neq i,M}}^{M} F_j(t_k) \right) \right) \Big/ f(t) \right]$$

$$+ \sum_{k=1}^{N} \left((1-d_k) \left(-\frac{\partial F_M(t_k)}{\partial h_M} \right) \prod_{\substack{j=1 \\ j \neq M}}^{M} F_j(t_k) \Big/ R(t) \right)$$

Since a closed form solution does not exists for finding the roots of these likelihood equations the equations must be solved by iteration methods. To solve these equations iteration method given in section 5 is used.

4. (s, M) RELIABILITY SYSTEM

The s - out - of - M : G systems are more general than purely series or parallel systems. Let M components be connected in such a way that the system fails if M-s+1 components fail. In this when an operating component fails, standby component becomes active and at least s out of M components must be good for the system to be good. It is equivalent to a (M-s+1) – out - of – M: F-system. There are two main advantages of using the system. It usually has much higher reliability than series system and is often less expensive than the parallel system.

The likelihood function using pdf and reliability function of (2,5) system in equation (3) is

$$L(q) = C\prod_{k=1}^{N}\begin{bmatrix} f_1(t_k)\begin{pmatrix} R_5(t_k)F_2(t_k)F_3(t_k)F_4(t_k)+R_4(t_k)F_2(t_k)F_3(t_k)F_5(t_k)+ \\ R_3(t_k)F_2(t_k)F_4(t_k)F_5(t_k)+R_2(t_k)F_3(t_k)F_4(t_k)F_5(t_k) \end{pmatrix}+ \\ f_2(t_k)\begin{pmatrix} R_5(t_k)F_1(t_k)F_3(t_k)F_4(t_k)+R_4(t_k)F_1(t_k)F_4(t_k)F_5(t_k)+ \\ R_3(t_k)F_1(t_k)F_4(t_k)F_5(t_k)+R_1(t_k)F_3(t_k)F_4(t_k)F_5(t_k) \end{pmatrix}+ \\ f_3(t_k)\begin{pmatrix} R_5(t_k)F_1(t_k)F_2(t_k)F_4(t_k)+R_4(t_k)F_1(t_k)F_2(t_k)F_5(t_k)+ \\ R_2(t_k)F_1(t_k)F_4(t_k)F_5(t_k)+R_1(t_k)F_2(t_k)F_4(t_k)F_5(t_k) \end{pmatrix}+ \\ f_4(t_k)\begin{pmatrix} R_5(t_k)F_1(t_k)F_2(t_k)F_3(t_k)+R_3(t_k)F_1(t_k)F_2(t_k)F_5(t_k)+ \\ R_2(t_k)F_1(t_k)F_3(t_k)F_5(t_k)+R_1(t_k)F_2(t_k)F_3(t_k)F_5(t_k) \end{pmatrix}+ \\ f_5(t_k)\begin{pmatrix} R_4(t_k)F_1(t_k)F_2(t_k)F_3(t_k)+R_3(t_k)F_1(t_k)F_2(t_k)F_4(t_k)+ \\ R_2(t_k)F_1(t_k)F_3(t_k)F_4(t_k)+R_1(t_k)F_2(t_k)F_3(t_k)F_4(t_k) \end{pmatrix}+ \end{bmatrix}^{d_{ki}} *$$

$$\begin{bmatrix} R_1(t_k)R_2(t_k)+R_2(t_k)R_3(t_k)F_1(t_k)+R_1(t_k)R_3(t_k)F_2(t_k)+ \\ R_3(t_k)R_4(t_k)F_1(t_k)F_2(t_k)+R_2(t_k)R_4(t_k)F_1(t_k)F_3(t_k) \\ +R_1(t_k)R_4(t_k)F_2(t_k)F_3(t_k)+R_4(t_k)R_5(t_k)F_1(t_k)F_2(t_k)F_3(t_k)+ \\ R_3(t_k)R_5(t_k)F_1(t_k)F_2(t_k)F_4(t_k)+R_2(t_k)R_5(t_k)F_1(t_k)F_3(t_k)F_4(t_k)+ \\ R_1(t_k)R_5(t_k)F_2(t_k)F_3(t_k)F_4(t_k) \end{bmatrix}^{1-d_k}$$

Differentiation of $\ln L(q)$ with respect to $h_1, h_2, ..., h_5$ gives the following likelihood equations

$$\frac{\partial \ln L}{\partial h_1} = \sum_{k=1}^{N} \frac{d_i}{f(t)} \left[\frac{\partial f_1(t_k)}{\partial h_1} \begin{pmatrix} R_5(t_k)F_2(t_k)F_3(t_k)F_4(t_k) + R_4(t_k)F_2(t_k)F_3(t_k)F_5(t_k) + \\ R_3(t_k)F_2(t_k)F_4(t_k)F_5(t_k) + R_2(t_k)F_3(t_k)F_4(t_k)F_5(t_k) \end{pmatrix} \right] +$$

$$\sum_{k=1}^{N} d_i \frac{\frac{\partial F_1(t_k)}{\partial h_1}}{f(t)} \begin{bmatrix} f_2\left(R_5(t_k)F_3(t_k)F_4(t_k) + R_4(t_k)F_4(t_k)F_5(t_k) + R_3(t_k)F_4(t_k)F_5(t_k)\right) + \\ f_3\left(R_5(t_k)F_2(t_k)F_4(t_k) + R_4(t_k)F_2(t_k)F_5(t_k) + R_2(t_k)F_4(t_k)F_5(t_k)\right) + \\ f_4\left(R_5(t_k)F_2(t_k)F_3(t_k) + R_3(t_k)F_2(t_k)F_5(t_k) + R_2(t_k)F_3(t_k)F_5(t_k)\right) + \\ f_5\left(R_4(t_k)F_2(t_k)F_3(t_k) + R_3(t_k)F_2(t_k)F_4(t_k) + R_2(t_k)F_3(t_k)F_4(t_k)\right) \end{bmatrix} +$$

$$\sum_{k=1}^{N} d_i \frac{\frac{\partial R_1(t_k)}{\partial h_1}}{f(t)} \begin{bmatrix} f_2(t_k)F_3(t_k)F_4(t_k)F_4(t_k) + f_3(t_k)F_2(t_k)F_4(t_k)F_5(t_k) + \\ f_4(t_k)F_2(t_k)F_3(t_k)F_5(t_k) + f_5(t_k)F_2(t_k)F_3(t_k)F_4(t_k) + \end{bmatrix} +$$

$$\sum_{k=1}^{N} (1-d_i) \frac{\frac{\partial F_1(t_k)}{\partial h_1}}{R(t)} \begin{bmatrix} R_2(t_k)R_3(t_k) + R_3(t_k)R_4(t_k)F_2(t_k) + R_2(t_k)R_4(t_k)F_3(t_k) \\ R_4(t_k)R_5(t_k)F_2(t_k)F_3(t_k) + R_3(t_k)R_5(t_k)F_2(t_k)F_4(t_k) + \\ R_2(t_k)R_5(t_k)F_3(t_k)F_4(t_k) \end{bmatrix} +$$

$$\sum_{k=1}^{N} (1-d_i) \frac{\frac{\partial R_1(t_k)}{\partial h_1}}{R(t)} \begin{bmatrix} R_2(t_k) + R_3(t_k)F_2(t_k) + R_4(t_k)F_2(t_k)F_3(t_k) + R_5(t_k)F_2(t_k)F_3(t_k)F_4(t_k) \\ \end{bmatrix}$$

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$$\frac{\partial \ln L}{\partial h_5} = \sum_{k=1}^{N} \frac{d_i}{f(t)} \left[\frac{\partial f_5(t_k)}{\partial h_5} \begin{pmatrix} R_4(t_k)F_1(t_k)F_2(t_k)F_3(t_k) + R_3(t_k)F_1(t_k)F_2(t_k)F_4(t_k) + \\ R_2(t_k)F_1(t_k)F_3(t_k)F_4(t_k) + R_1(t_k)F_2(t_k)F_3(t_k)F_4(t_k) \end{pmatrix} \right] +$$

$$\sum_{k=1}^{N} d_i \frac{\frac{\partial F_5(t_k)}{\partial h_5}}{f(t)} \begin{bmatrix} f_1\left(R_2(t_k)F_3(t_k)F_4(t_k) + R_3(t_k)F_2(t_k)F_4(t_k) + R_4(t_k)F_2(t_k)F_3(t_k)\right) + \\ f_2\left(R_1(t_k)F_3(t_k)F_4(t_k) + R_3(t_k)F_1(t_k)F_4(t_k) + R_4(t_k)F_1(t_k)F_4(t_k)\right) + \\ f_3\left(R_1(t_k)F_2(t_k)F_4(t_k) + R_2(t_k)F_1(t_k)F_4(t_k) + R_4(t_k)F_1(t_k)F_2(t_k)\right) + \\ f_4\left(R_1(t_k)F_2(t_k)F_3(t_k) + R_3(t_k)F_1(t_k)F_2(t_k) + R_2(t_k)F_1(t_k)F_3(t_k)\right) \end{bmatrix} +$$

$$\sum_{k=1}^{N} d_i \frac{\frac{\partial R_5(t_k)}{\partial h_5}}{f(t)} \begin{bmatrix} f_1(t_k)F_2(t_k)F_3(t_k)F_4(t_k) + f_2(t_k)F_1(t_k)F_3(t_k)F_4(t_k) + \\ f_3(t_k)F_1(t_k)F_2(t_k)F_4(t_k) + f_4(t_k)F_1(t_k)F_2(t_k)F_3(t_k) + \end{bmatrix}$$

$$\sum_{k=1}^{N}(1-d)_i \frac{\frac{\partial F_5(t_k)}{\partial h_5}}{R(t)}\begin{bmatrix} R_2(t_k)R_3(t_k)+R_3(t_k)R_4(t_k)F_2(t_k)+R_2(t_k)R_4(t_k)F_3(t_k) \\ R_4(t_k)R_5(t_k)F_2(t_k)F_3(t_k)+R_3(t_k)R_5(t_k)F_2(t_k)F_4(t_k)+ \\ R_2(t_k)R_5(t_k)F_3(t_k)F_4(t_k) \end{bmatrix}+$$

$$\sum_{k=1}^{N}(1-d)_i \frac{\frac{\partial R_5(t_k)}{\partial h_5}}{R(t)}\begin{bmatrix} R_4(t_k)F_1(t_k)F_2(t_k)F_3(t_k)+R_3(t_k)F_1(t_k)F_2(t_k)F_4(t_k)+ \\ R_2(t_k)F_1(t_k)F_3(t_k)F_4(t_k)+R_1(t_k)F_2(t_k)F_3(t_k)F_4(t_k) \end{bmatrix}$$

The estimation technique studied in this paper facilitates the estimation of the parameters of the life distribution of each component in a series, parallel and s-out-of-M systems. Here the likelihood estimation is done using the Broyden - Fletcher – Goldfarb - Shanno Method (BFGS) algorithm Rao [9]. The BFGS method can be considered as a quasi- Newton conjugate gradient, and variable metric method. In this method the inverse of the Hessian matrix is approximated and so can be called an indirect update method. Let us summarize the steps involved in the estimation of parameters $h_1, h_2, \ldots h_M$ in the next section.

5. Algorithm for MLE

Step 1

Assume the initial parameter vector be $h^{(1)} = \{h_1, h_2, h_3, \ldots\ldots, h_M\}$ and a MxM negative definite symmetric matrix $[B_1]$ as an initial estimate of the inverse of the Hessian matrix of L and let $[B_i]$ = - $[I]$ (unit matrix) for i=1 The log likelihood is calculated as

$L_i = \ln L(h^{(i)})$ and $\nabla L_i = \nabla \ln L\left(h^{(i)}\right) \quad i = 1$

Step 2

With the gradient of the function ∇L_i at the point $h^{(i)}$ set

$S_i = -[B_i]\nabla L_i \quad i = 1$

Step 3

Find the optimal step length l_i^{*} in the direction of S_i and set

$h^{(i+1)} = h^{(i)} + l_i^{*} * S_i$

l_i is the optimal step length which satisfies $Max \ln L(h^{(i)} + l_i S_i)$

Step 4

Test the point $h^{(i)}$ for optimality. If $\|\nabla \ln L_{i+1}\| \le e$ where e is a small pre defined quantity, take $h^{*} = h_{i+1}$ and stop the process. Otherwise, go to step 5.

Step 5

$d_i = h^{(i+1)} - h^{(i)} = l_i^{*} S_i$ and $g_i = \nabla \ln L(h^{(i+1)}) - \nabla \ln L(h^{(i)})$

$M_1 = d_i d_i^T \quad M_2 = d_i^T g_i \quad M_3 = d_i g_i^T \quad M_4 = g_i d_i^T$

$M_5 = g_i^T [B_i] g_i \quad M_6 = d_i g_i^T [B_i] \quad M_7 = [B_i] d_i g_i^T$

$$[B_{i+1}] = [B_i] + \left(1 + \frac{M_5}{M_2}\right)\frac{M_1}{M_2} - \frac{M_6}{M_2} - \frac{M_7}{M_2}$$

Step 6

Set the new iteration number as $i = i + 1$ and go to step 2

In step 3 to find the optimal step size cubic interpolation method is used. It finds the minimizing step length l^* using the cubic equation $f(l) = a + bl + cl^2 + dl^3$ as follows

(i) Use normalized S_i given in step 2 and minimize

$\ln L(h^{(i)} + l_i S_i)$

(ii) To establish lower and upper bound on the optimal step size l^*, assuming initial step size be t_0 and incrementing step size find two points A and B at which the slope $\frac{d \ln L}{dl}$ has different sign.

At $A = t_0$ find f_A and $f_A^{'}$ and

at $B = t_1$ find f_B and $f_B^{'}$ ($f_A^{'}$ *and* $f_B^{'}$ are of opposite sign)

(iii) To find optimal step length l_1^* we compute

$Z = \frac{3(f_A - f_B)}{B - A} + f_A^{'} + f_B^{'}$ and $Q = \sqrt{z^2 - f_A^{'} f_B^{'}}$

Using the results we get

$$l^* = A + \frac{\left(f_A^{'} + Z \pm Q\right)}{\left(f_A^{'} + f_B^{'} + 2Z\right)}(B - A)$$

(iv) Use the convergence criteria and

test if $\left\| \dfrac{f(l^*) - L(l^*)}{L(l^*)} \right\| \leq x_1$ and $\left| \dfrac{S^T \nabla L}{\|S\| \nabla L\|} \right|_{l^*} \leq x_2$ where x_1 *and* x_2 are pre-defined small numbers whose value depend on the accuracy desired. If we reached the optimal l^* value go to step 3 else go to (ii) and set $A = l^*$

If the step lengths l_i^* are found accurately, the matrix $[B_i]$, retains its positive definiteness as the value of i increases. However, in practical applications, the matrix $[B_i]$ might become indefinite or even singular if l_i^* are not found accurately. As such, periodical resetting of the matrix $[B_i]$ to the identity matrix is desirable. However, numerical experience indicates that BFGS method is less influenced by errors in l_i than other methods.

NUMERICAL RESULTS

A Simulation procedure is adopted to generate the life of series of 5 components that are from exponential distributions with

$h^{(0)} = \{1, 1.5, 2, 2.2, 3, 3.2\}$. The procedure is programmed in TURBO C and estimation in each case takes about 40 to 50 seconds with double precision computation on a personal computer with TURBO C OS. 1000 simulation runs are performed with N= 500. Table 1 summarizes the mean and standard deviation of h_k in all the three cases. The standard deviation of estimated values of the exponential parameters are greatly reduced. The estimates vary randomly around the input parameters used for data generation. The author could not make a comparative study of the results since no similar work is available.

Table 1

h / system	h_1^* $std(h_1)$	h_2 $std(h_2)$	h_3 $std(h_3)$	h_4 $std(h_4)$	h_5 $std(h_5)$
Series	1.9 0.01	2.4 0.02	3.6 0.001	3.4 0.012	4.0 0.002
Parallel	2.0 0.03	2.3 0.023	4.0 0.043	4.3 0.12	4.7 0.032
s out of M	1.1 0.012	2.8 0.12	3.7 0.014	2.8 0.043	3.1 0.105

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