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A method for portfolio choice with transaction costs[1]

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Abstract: This paper presents a simplex algorithm for solving portfolio selection problem with transaction costs. The transaction cost function is assumed to be a V-shaped function of the difference between an existing portfolio and a new one. Under some assumptions on the variance-covariance matrix of returns, we transform the non-differentiable and bi-objective programming problem to a mathematical programming problem which can be efficiently solved by a revised simplex algorithm. This method can be used by mutual funds managers in selecting their portfolios.

1 Introduction

Originally articulated by Markowitz (1952, 1959, 1987), the mean-variance methodology for portfolio selection has been central to research activities of this area and has served as a basis for the development of modern financial theory during the past five decades. Efforts in extending, improving and revising this approach have led to a wealth of research output in the forms of articles, monographs, and textbooks devoted to formulating risk and return of economic agents and to understanding diversification in investment strategies (Xia, Wang and Deng 2000).

Ballestero and Romero (1996) suggested that the application of Markowitz model to security portfolio selections normally consists of the following steps: (1) making probabilistic estimates of the future performance of securities, (2) determining the investment opportunity set as a constraint of the problem, by resorting to the mean-variance (E-V) efficient frontier, and (3) maximizing the expected utility of returns $Eu(R)$ of the investor on the frontier.

In the first phase, to determine the expected return of each security, one may take the arithmetical mean of historical data, or estimate the expected return by using a regression model. For the second phase, the E-V efficient frontier can be formulated as a typical convex quadratic program and can be solved efficiently. For the third phase to work, we need to make

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sure that there is an efficient portfolio which maximizes $Eu(R)$. As pointed out in [Levy and Markowitz (1979)], this is only possible if the returns from all securities are normally distributed or if the utility function $u(R)$ is concave and quadratic. However, as observed by Ballestero and Romero (1996). the normal distribution of returns is only a hypothesis which has not been empirically corroborated and quadratic utility functions present many logical flaws. This leaves open the possibility for alternative formulations to be applied in cases these assumptions are not met. Ballestero and Reomero (1996), for example, suggested a compromise programming solution . They considered different measures of distance and conclude that all those compromise programming problems derive solutions corresponding to "average" investors.

The main concern that leads to our study is about the transaction cost, an important factor considered by investors in financial markets. Most of the cases, investors usually start with an existing portfolio and the decisions are how to re-adjust to the changes in the security market. The problem is a revision of the existing portfolio to a new one that entails both purchases and sales of securities along with transaction costs. This is a V-shaped function of the difference between the old and the new portfolios. It has been noticed in empirical studies that ignoring transaction costs results in inefficient portfolios [Xia, Wang and Deng (2000), Yoshimoto (1996)].

In this paper, we consider portfolio selection with transaction costs. In Section 2, the model is formulated and a simplified form of the objective function is discussed. In Section 3, we transform the non-differentiable and bi-objective programming model to a mathematical programming problem which can be efficiently solved by a revised simplex algorithm. In Section 4, we extend the model to include a riskless asset which allows short sale.

2 Bi-objective Programming Model

As indicated in Markowitz (1952, 1959, 1987), a portfolio selection problem in a mean-variance context can be formulated as the following maximization problem $(P1)$:

$$\begin{array}{ll} \text{maximize} & \sum_{i=1}^{n} R_i x_i \\ \text{subject to} & \left(\sum_{i=1}^{n}\sum_{j=1}^{n} S_i S_j \rho_{i,j} x_i x_j\right)^{\frac{1}{2}} \leq \gamma \\ & \sum_{i=1}^{n} x_i = 1 \\ & x_i \geq 0,\ \ i = 1, \cdots, n \end{array} \tag{1}$$

where n is the number of risky securities, x_i is the proportion invested in security i, R_i is the expected rate of return on security i, S_i is the standard deviation of the rate of return on security i, $\rho_{i,j}$ is the correlation coefficient between the returns on securities i and j, $i, j = 1, \cdots, n$ and

γ is the maximum risk the investor can accept because the standard deviation of the return characterizes the risk of a portfolio.

The expected return and the risk of a portfolio are respectively given by

$$R_p = \sum_{i=1}^{n} R_i x_i \tag{2}$$

and

$$S_p = (\sum_{i=1}^{n} \sum_{j=1}^{n} S_i S_j \rho_{i,j} x_i x_j)^{\frac{1}{2}}. \tag{3}$$

By varying the value of γ in Problem $(P1)$, one can find an efficient frontier which is part of a hyperbola in the $S_p - R_p$ plane. The investor should choose his/her portfolio based on this efficient frontier. But this frontier consists of infinite number of elements and does not tell the investor which one he/she should choose from them. When the investor's utility function is known, he/she can determine a portfolio via maximizing his/her utility function on the efficient frontier. In most situations, however, the investor's utility function is not explicitly known in a portfolio selection problem. In this paper, a goal programming method is used to support the investor to make decisions of portfolio selection.

Elton *et al* (1973, 1978, 1995,) conducted an empirical study about the correlation coefficient structure of the returns of securities in the portfolio. They demonstrated that the performance of equal pairwise correlation coefficients between all securities is better than the performance with the different correlation coefficients based on the historical returns. Based on the above result, we assume that $\rho_{i,j} = \rho$, for all $i, j = 1, \cdots, n$ and $i \neq j$, where $0 \leq \rho \leq 1$. Thus, the variance of the expected return is

$$\begin{aligned} S_p^2 &= \textstyle\sum_{i=1}^{n} \sum_{j=1}^{n} S_i S_j \rho_{i,j} x_i x_j \\ &= \rho(\textstyle\sum_{i=1}^{n} \sum_{j=1}^{n})_{i \neq j} S_i S_j x_i x_j + \sum_{i=1}^{n} S_i^2 x_i^2 \\ &= \rho(\textstyle\sum_{i=1}^{n} S_i x_i)^2 + (1-\rho) \sum_{i=1}^{n} S_i^2 x_i^2 \end{aligned} \tag{4}$$

Sharpe (1963,1967) testified that the nonsystematic risk of the portfolio was very little compared with the systematic risk, especially when the limit of investing to each security was no more than five percent of the money which is a most common law for managing the mutual funds in many countries. He suggested that the total portfolio variability S_p should be expressed as $\beta_p \cdot S_I$, *i.e.,* the product of the portfolio beta and the standard deviation of the market return while ignoring the nonsystematic risk $\sum_{i=1}^{n} S_i^2 x_i^2$. Because $(1-\rho) \sum_{i=1}^{n} S_i^2 x_i^2 \leq \sum_{i=1}^{n} S_i^2 x_i^2$, the standard deviation of the expected return of portfolio $x = (x_1, x_2, \cdots, x_n)$ could be approximately expressed as

$$S_p(x) = \sqrt{\rho} \sum_{i=1}^{n} S_i x_i \tag{5}$$

Transaction cost is another important factor for an investor to take into consideration in portfolio selection. Ignoring the transaction cost in a portfolio selection model often leads to an inefficient portfolio in practice. If the portfolio at time t is known, we consider its revision at time $t+1$. With the same consideration as in Markowitz [10], we assume in this paper that the transaction cost at time $t+1$ is a V-shaped function of the difference between the given portfolio at time t and a new portfolio at time $t+1$. Hence, the transaction cost $c_{i,t+1}$ of the $i-$th security at time $t+1$ can be expressed by

$$c_{i,t+1} = k_{i,t+1} \mid x_{i,t+1} - x_{i,t} \mid \tag{6}$$

where $k_{i,t+1}$ is a constant rate with respect to the $i-$th security at time $t+1$. So the total transaction cost and the expected return of portfolio $x_{t+1} = (x_{1,t+1}, x_{2,t+1}, \cdots, x_{n,t+1})$ at time $t+1$ are

$$C_{t+1}(x_{t+1}) = \sum_{i=1}^{n} k_{i,t+1} \mid x_{i,t+1} - x_{i,t} \mid \tag{7}$$

and

$$R_p(x_{t+1}) = \sum_{i=1}^{n} R_{i,t+1} x_{i,t+1} - \sum_{i=1}^{n} k_{i,t+1} \mid x_{i,t+1} - x_{i,t} \mid \tag{8}$$

respectively. Based on (5) and (8), the portfolio selection problem can be reformulated as the following non-differentiable and bi-objective programming problem $(P2)$:

$$\begin{array}{ll} \text{maximize} & R_p(x_{t+1}) = \sum_{i=1}^{n} R_{i,t+1} x_{i,t+1} - \sum_{i=1}^{n} k_{i,t+1} \mid x_{i,t+1} - x_{i,t} \mid \\ \text{minimize} & S_p(x_{t+1}) = \sqrt{\rho} \sum_{i=1}^{n} S_{i,t+1} x_{i,t+1} \\ \text{subject to} & \sum_{i=1}^{n} x_{i,t+1} = 1 \\ & x_{i,t+1} \leq 0.05, i = 1, \cdots, n \\ & x_{i,t+1} \geq 0, i = 1, \cdots, n. \end{array} \tag{9}$$

3 Goal Programming and Transformations

Denoting $R1_{i,t+1}, i = 1, \cdots, 10$ as the largest ten security returns and $S1_{i,t+1}, i = 1, \cdots, 10$ as the smallest ten security standard deviations in the stock market, then with the satisficing logic (Tamiz, Jones and Romero 1998) which considers trade-off between return and its risk, we derive an ideal point as follows:

$$
\begin{array}{l}
R_p^* = \frac{1}{10}\sum_i^{10} R1_{i,t+1} - \sum_{i=1}^{10} k_{i,t+1} \mid \frac{1}{10} - x_{i,t} \mid \\
S_p^* = \frac{\sqrt{\rho}}{10}\sum_i^{10} S1_{i,t+1}
\end{array} \tag{10}
$$

i.e., using all the money investing ten stocks with maximum returns as the ideal return of the investor while using all the money investing ten stocks with minimum risks as the ideal risk for the investor. Correspondingly, the investor can calculate the anti-ideal point (R_{p*}, S_{p*}), which is the worst solution for the investor with the most averse return and risk.

According to Ballestero and Romero (1998), we consider average risk aversion and use metric 2 to derive the following goal programming problem $(P3)$:

$$
\begin{array}{ll}
\text{minimize} & f = [(\frac{S_p - S_p^*}{S_{p*} - S_p^*})^2 + (\frac{R_p^* - R_p}{R_p^* - R_{p*}})^2]^{\frac{1}{2}} \\
\text{subject to} & \sum_{i=1}^n x_{i,t+1} = 1 \\
& S_p = \sqrt{\rho_t}\sum_{i=1}^n S_{i,t} x_{i,t+1} \\
& R_p = \sum_{i=1}^n R_{i,t+1} x_{i,t+1} - \sum_{i=1}^n k_{i,t+1} \mid x_{i,t+1} - x_{i,t} \mid \\
& x_{i,t+1} \leq 0.05, i = 1, \cdots, n \\
& x_{i,t+1} \geq 0, i = 1, \cdots, n
\end{array} \tag{11}
$$

Let $F = f^2$ and $x_{t+1} = (x_{1,t+1}, x_{2,t+1}, \cdots, x_{n,t+1})$. Denote the feasible solution set of Problem $(P3)$ by S. It is obvious that F is a strictly increasing function of f. Thus, x_{t+1} minimizes f over S if and only if it minimizes F over S. Hence, Problem $(P3)$ is equivalent to the following minimization problem $(P4)$:

$$
\begin{array}{ll}
\text{minimize} & F(x_{t+1}) = (\frac{\sqrt{\rho_{t+1}}\sum_{i=1}^n S_{i,t+1} x_{i,t+1} - S_p^*}{S_{p*} - S_p^*})^2 \\
& +(\frac{R_p^* - \sum_{i=1}^n R_{i,t+1} x_{i,t+1} + \sum_{i=1}^n k_{i,t+1} |x_{i,t+1} - x_{i,t}|}{R_p^* - R_{p*}})^2 \\
\text{subject to} & \sum_{i=1}^n x_{i,t+1} = 1 \\
& x_{i,t+1} \leq 0.05, i = 1, \cdots, n \\
& x_{i,t+1} \geq 0, i = 1, \cdots, n
\end{array} \tag{12}
$$

But $(P4)$ is not easy to solve because of its non-differentiability. We consider the following minimization problem $(P5)$:

$$
\begin{array}{ll}
\text{minimize} & F(x_{t+1}) = (\frac{\sqrt{\rho_t}\sum_{i=1}^n S_{i,t+1} x_{i,t+1} - S_p^*}{S_{p*} - S_p^*})^2 \\
& +(\frac{R_p^* - \sum_{i=1}^n R_{i,t+1} x_{i,t+1} + x_{n+1,t+1}}{R*_p - R_{p*}})^2 \\
\text{subject to} & \sum_{i=1}^n k_{i,t+1} \mid x_{i,t+1} - x_{i,t} \mid \leq x_{n+1,t+1} \\
& \sum_{i=1}^n x_{i,t+1} = 1 \\
& x_{i,t+1} \leq 0.05, i = 1, \cdots, n \\
& x_{i,t+1} \geq 0, i = 1, \cdots, n
\end{array} \tag{13}
$$

We can prove the equivalence as given in the following theorem.

Theorem 1. $(x^*_{1,t+1},\cdots,x^*_{n,t+1})$ is an optimal solution to $(P4)$ if and only if there exists $x^*_{n+1,t+1}$ such that $(x^*_{1,t+1},\cdots,x^*_{n,t+1},x^*_{n+1,t+1})$ is an optimal solution to $(P5)$.

$(P5)$ is not differentiable. We consider the following transformation. Let

$$d^+_{i,t+1} = \begin{cases} x_{i,t+1}-x_{i,t} & \text{if } x_{i,t+1}-x_{i,t}\geq 0 \\ 0 & \text{otherwise,} \end{cases} \tag{14}$$

and

$$d^-_{i,t+1} = \begin{cases} x_{i,t}-x_{i,t+1} & \text{if } x_{i,t+1}-x_{i,t}\leq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

Obviously, for $i=1,\cdots,n$, we have

$$\begin{array}{l} x_{i,t+1}-x_{i,t}=d^+_{i,t+1}-d^-_{i,t+1}, \\ |x_{i,t+1}-x_{i,t}|=d^+_{i,t+1}+d^-_{i,t+1}, \\ d^+_{i,t+1}\cdot d^-_{i,t+1}=0, \\ d^+_{i,t+1}\geq 0, \\ d^-_{i,t+1}\geq 0. \end{array} \tag{16}$$

Hence, the transaction costs can be equivalently expressed as

$$c_{i,t}=k_{i,t+1}(d^+_{i,t+1}+d^-_{i,t+1}),\ i=1,\cdots,n \tag{17}$$

and $(P5)$ is equivalent to the following differentiable minimization problem $(P6)$:

$$\begin{array}{ll} \text{minimize} & F_1(x_{t+1},d^+_{t+1},d^-_{t+1})=\left(\frac{\sqrt{\rho_{t+1}}\sum_{i=1}^n S_{i,t+1}x_{i,t+1}-S^*_p}{S_{p*}-S^*_p}\right)^2 \\ & +\left(\frac{R^*_p-\sum_{i=1}^n R_{i,t+1}x_{i,t+1}+x_{n+1,t+1}}{R^*_p-R_{p*}}\right)^2 \\ \text{subject to} & \sum_{i=1}^n k_{i,t+1}(d^+_{i,t+1}+d^-_{i,t+1})\leq x_{n+1,t+1} \\ & \sum_{i=1}^n x_{i,t+1}=1 \\ & x_{i,t+1}-x_{i,t}=d^+_{i,t+1}-d^-_{i,t+1}, i=1,\cdots,n \\ & 0\leq x_{i,t+1}\leq 0.05, i=1,\cdots,n \\ & d^+_{i,t+1}\geq 0, d^-_{i,t+1}\geq 0, i=1,\cdots,n \\ & d^+_{i,t+1}\cdot d^-_{i,t+1}=0,\cdots,n \end{array} \tag{18}$$

where $d^+_{t+1}=(d^+_{1,t+1},d^+_{2,t+1},\cdots,d^+_{n,t+1})$, $d^-_{t+1}=(d^-_{1,t+1},d^-_{2,t+1},\cdots,d^-_{n,t+1})$.

Hence, we have the following theorem.

Theorem 2. $(x^*_{1,t+1},\cdots,x^*_{n,t+1},x^*_{n+1,t+1})$ is an optimal solution to $(P5)$ if and only if $(x^*_{1,t+1},\cdots,$ $x^*_{n,t+1},x^*_{n+1,t+1},d^{+*}_{1,t+1},\cdots,d^{+*}_{n,t+1},d^{-*}_{1,t+1},\cdots,d^{-*}_{n,t+1})$ is an optimal solution to $(P6)$.

Since the feasible set X is linear and the objective function $F1$ is quadratic convex, Problem $(P6)$ is a convex quadratic programming problem. Based on the Kuhn-Tucker theorem and Theorems 1 and 2, we can equivalently transform $(P6)$ to the following minimization problem $(P7)$:

$$
\begin{array}{ll}
\text{minimize} & z_{t+1} + \sum_{i=1}^{4} \sum_{j=1}^{n} z_{i,j,t+1} \\
\text{subject to} & \\
& \sum_{i=1}^{n} x_{i,t+1} + z_{t+1} = 1 \\
& \frac{2}{a}\sqrt{\rho_{t+1}} \sum_{i=1}^{n} x_{i,t+1} S_{i,t+1} + \frac{2}{b} R_{j,t+1} \sum_{i=1}^{n} x_{i,t+1} R_{i,t+1} \\
& \quad -\frac{2}{b} k_{t+1} R_{j,t+1} d^{+}_{j,t+1} - \frac{2}{b} k_{t+1} R_{j,t+1} d^{-}_{j,t+1} \\
& -\lambda - h_j - y_j + sign(-c_j) z_{1,j} = -c_j \\
& \frac{2}{b} k_{t+1} \sum_{i=1}^{n} x_{i,t+1} R_{i,t+1} - \frac{2}{b} k^{2}_{t+1} d^{+}_{j,t+1} - \\
& \frac{2}{b} k^{2}_{t+1} d^{-}_{j,t+1} - h_j + g^{+}_{j} + z_{2,j} = \frac{2}{b} k_{t+1} R^{*}_{p} \\
& \frac{2}{b} k_{t+1} \sum_{i=1}^{n} x_{i,t+1} R_{i,t+1} - \frac{2}{b} k^{2}_{t+1} d^{+}_{j,t+1} - \\
& \frac{2}{b} k^{2}_{t+1} d^{-}_{j,t+1} + h_j + g^{-}_{j} + z_{3,j} = \frac{2}{b} k_{t+1} R^{*}_{p} \\
& x_{j,t+1} - d^{+}_{j,t+1} + d^{-}_{j,t+1} + z_{4,j} = x_{j,t+1-1} \\
& y_i \cdot x_{i,t+1} = 0, d^{+}_{i,t+1} \cdot d^{-}_{i,t+1} = 0, i = 1, \cdots, n \\
& g^{+}_{i} \cdot d^{+}_{i,t+1} = 0, g^{-}_{i} \cdot d^{-}_{i,t+1} = 0, i = 1, \cdots, n \\
& x_{i,t+1} \geq 0, d^{+}_{i,t+1} \geq 0, i = 1, \cdots, n \\
& d^{-}_{i,t+1} \geq 0, g^{+}_{i} \geq 0, i = 1, \cdots, n \\
& g^{-}_{i} \geq 0, y_i \geq 0, i = 1, \cdots, n
\end{array}
$$

where $a = (S_p * - S^{*}_{p})^2$, $b = (R^{*}_{p} - R_p *)^2$, $c_j = \frac{2}{a}\sqrt{\rho_{t+1}} S^{*}_{p} S_{j,t+1} - \frac{2}{b} R^{*}_{p} R_{j,t+1}$ and for each j,

$$
\text{sign}(-c_j) = \begin{cases} 1 & \text{if } c_j < 0 \\ -1 & \text{otherwise.} \end{cases}
$$

4 Extension to Case with a Riskless Asset

Riskless assets such as a short-term government bill or savings account can provide the investor certain return with its standard deviation zero. If the riskless asset is considered, we have the following goal programming problem $(P8)$:

$$
\begin{array}{ll}
\text{minimize} & F(x_{t+1}) = \left(\frac{\sqrt{\rho_{t+1}} \sum_{i=1}^{n} S_{i,t+1} x_{i,t+1} - S^{*}_{p}}{S_{p*} - S^{*}_{p}}\right)^2 \\
& +\left(\frac{R^{*}_{p} - \sum_{i=1}^{n} R_{i,t+1} x_{i,t+1} + R_{f,t+1} x_{f,t+1} + \sum_{i=1}^{n} k_{i,t+1} |x_{i,t+1} - x_{i,t}|}{R^{*}_{p} - R_{p*}}\right)^2 \\
\text{subject to} & \sum_{i=1}^{n} x_{i,t+1} + x_{f,t+1} = 1 \\
& 0 \leq x_{i,t+1} \leq 0.05, i = 1, \cdots, n
\end{array}
\tag{19}
$$

where $R_{f,t+1}$ and $x_{f,t+1}$ are denoted respectively as the return and the investment proportion of the riskless asset.

Based on a similar transformation, we can get the following minimization problem $(P9)$:

$$
\begin{array}{ll}
\text{minimize} & z_{t+1} + \sum_{i=1}^{4} \sum_{j=1}^{n} z_{i,j,t+1} \\
\text{subject to} & \\
& \sum_{i=1}^{n} x_{i,t+1} + x_{f,t+1} + z_{t+1} = 1 \\
& \frac{2}{a}\sqrt{\rho_{t+1}} \sum_{i=1}^{n} x_{i,t+1} S_{i,t+1} + \frac{2}{b} R_{j,t+1} \sum_{i=1}^{n} x_{i,t+1} R_{i,t+1} \\
& \quad + x_{f,t+1} R_{f,t+1} - \frac{2}{b} k_{t+1} R_{j,t+1} d^{+}_{j,t+1} - \frac{2}{b} k_{t+1} R_{j,t+1} d^{-}_{j,t+1} \\
& -\lambda - h_j - y_j + sign(-c_j) z_{1,j} = -c_j \\
& \frac{2}{b} k_{t+1} \sum_{i=1}^{n} x_{i,t+1} R_{i,t+1} + x_{f,t+1} R_{f,t+1} - \frac{2}{b} k^2_{t+1} d^{+}_{j,t+1} \\
& -\frac{2}{b} k^2_{t+1} d^{-}_{j,t+1} - h_j + g^{+}_j + z_{2,j} = \frac{2}{b} k_{t+1} R^{*}_p \\
& \frac{2}{b} k_{t+1} \sum_{i=1}^{n} x_{i,t+1} R_{i,t+1} + x_{f,t+1} R_{f,t+1} - \frac{2}{b} k^2_{t+1} d^{+}_{j,t+1} \\
& -\frac{2}{b} k^2_{t+1} d^{-}_{j,t+1} + h_j + g^{-}_j + z_{3,j} = \frac{2}{b} k_{t+1} R^{*}_p \\
& x_{j,t+1} - d^{+}_{j,t+1} + d^{-}_{j,t+1} + z_{4,j} = x_{j,t+1-1} \\
& y_i \cdot x_{i,t+1} = 0, d^{+}_{i,t+1} \cdot d^{-}_{i,t+1} = 0, i = 1, \cdots, n \\
& g^{+}_i \cdot d^{+}_{i,t+1} = 0, g^{-}_i \cdot d^{-}_{i,t+1} = 0, i = 1, \cdots, n \\
& x_{i,t+1}, d^{+}_{i,t+1}, d^{-}_{i,t+1}, g^{+}_i, g^{-}_i, y_i \geq 0, i = 1, 2, \cdots, n \\
& x_{f,t+1} \geq 0
\end{array}
$$

Although $(P7)$ and $(P9)$ are not linear programming problems, we can design a revised simplex algorithm to solve them efficiently. This can be easily implemented in designing a simplex algorithm by not taking y_i and $x_{i,t+1}$,$d^{+}_{i,t+1}$ and $d^{-}_{i,t+1}$, g^{+}_i and $d^{+}_{i,t+1}$, g^{-}_i and $d^{-}_{i,t+1}$, $(i = 1, \cdots, n)$ as basic variables at the same time in a simplex tableau so that $y_i \cdot x_{i,t+1} = 0, d^{+}_{i,t+1} \cdot d^{-}_{i,t+1} = 0$, $g^{+}_i \cdot d^{+}_{i,t+1} = 0$, $g^{-}_i \cdot d^{-}_{i,t+1} = 0$, $i = 1, \cdots, n$. With such a simplex algorithm, we can find an efficient portfolio by solving the two linear programs obtained by deleting the nonlinear constraints in $(P7)$ and $(P9)$ respectively.

Preliminary numerical experiments showed that the method has a few advantages over some of the existing methods for portfolio selection with transaction costs. Those results will be given in a separate paper.

References

Ballestero, E. and Romero, C. (1996), Portfolio selection: a compromise programming solution, Journal of Operational Research Society, 47, 1377-1386.

Ballestero, E. and Romero, C. (1998), Multiple Criteria Decision Making and its Applications to Economic Problems, Kluwer, Boston.

Elton,E.J. and Gruber,M.J. (1973), Estimating the dependence structure of share prices, Journal of Finance, 28, 1203-1232.

Elton, E.J. and Gruber, M.J. (1995), Modern Portfolio Theory and Investment Analysis (5th edition), John Wiley & Sons, New York, New York.

Elton, E.J., Gruber, M.J. and Urich, T.J. (1978), Are Betas best? Journal of Finance, 33, 1357-1384.

Kallberg, J.C. and Zembia, W.T. (1983), Comparison of alternative utility functions in portfolio selections problems, Management Science, 29, 1257-1276.

Levy, H. and Markowitz, H. (1979), Approximating expected utility by an function of mean and variance, American Economics Review, 69, 308-317.

Li,Z.F., Wang,S.Y. and Deng,X.T. (2000), A linear programming algorithm for optimal portfolio selection, International Journal of Systems Science, 31, 107-117.

Markowitz, H. (1952), Portfolio selection, Journal of Finance, 7, 77-91.

Markowitz, H. (1959), Portfolio Selection: Efficient Diversification of Investments, Wiley, New York.

Markowitz, H. (1987), Mean-Variance Analysis in Portfolio Choice and Capital Markets, Basil Blackwell, New York.

Sharpe, W.F. (1967), A linear programming algorithm for mutual funds portfolio selection, Management Science, 13, 499-510.

Sharpe, W.F. (1963), A simplified model for portfolio analysis, Management Science, 9, 277-293.

Tamiz, M., Jones, D. and Romero, C.(1998), Goal programming for decision making: an overview of the current state-of-the-art, European Journal of Operational Research, 111, 569-581.

Xia, Y.S., Liu, B.D., Wang, S.Y. and Lai, K.K.(2000), A new model for portfolio selection with order of expected returns, Computers & Operations Research, 27, 409-422.

Xia,Y.S., Wang,S.Y. and Deng,X.T. (2000), Criteria, models and strategies in portfolio selection, Advanced Modeling and Optimization, 2, 79-103.

Xia, Y.S., Wang, S.Y. and Deng, X.T. (2001), A compromise solution to mutual funds portfolio selection with transaction costs, European Journal of Operational Research, 129, 1-18.

Yoshimoto, A. (1996), The mean-variance approach to portfolio optimization subject to transaction costs, Journal of Operations Research Society of Japan, 39, 99-117.