Multimodel Control of Discrete Systems with Uncertainties

S. Mezghani
LAIL UPRESA 8021 CNRS, Ecole Centrale de Lille, BP 48
F59651 Villeneuve d'Ascq Cedex FRANCE
Email: s_mezghani@hotmail.com

Abstract

The purpose of this paper is to introduce a multimodel approach to control uncertain systems. The basic problem is to minimize the number of models in the dynamic library in order to obtain the desired performance. A proposed solution is based on the Kharitonov principle. Each generic model and its corresponding controller have an unchanged structure. The multimodel control is obtained by fusion of the elementary controllers. A new strategy for multimodel control is proposed, which uses observers to improve the robustness of multimodel control in discrete systems. The principal idea of this control is to reinitialize the state of each model by the corresponding state computed from the process when the error between them exceeds a certain value. A good choice of observer, as well as a fusion applied to the coefficients of each controller, allows the multimodel control strategy to reach the desired objective for the multimodel control of the system. The desired performance is reached by the use of this new strategy of multimodel control. Thus, the significant aspect of this study is the introduction of observers into multimodel control as a technique to improve the validity of the generic models. The multimodel control obtained by this new strategy of control is presented. Computer simulation results show that a robust multimodel control design, based on an appropriate selection of models, plus reinitialization steps based on observers, is feasible and provides good performances.

Keywords : Multimodel approach, Multimodel control, Uncertain system, Residual generation, Validity, Observer, Fusion.

1 Introduction

Technological development is steadily increasing the complexity of many processes in various fields. So, the need for practical approaches which can be used in order to better understand and carry out such complex modelling and control tasks is more relevant than ever. The multimodel approach is used in the modelling of uncertain systems to derive a multimodel control law obtained by fusion of the different local controllers. This paper focuses on ill-defined systems where the coefficients of the plant are subject to variation within given intervals, and are hence referred to as interval plants. It is well known that multimodel approach is achieved by checking sub-system or models obtained by different methods. The main contribution of this paper concerns the determination of a sufficient number of linear models to be used in the control of a discrete time-interval plant system. In the literature, there exist two kind of models. In the first case, the models are local (see [10]). The validity domain for each local model (called an operating regime) is known a priori. In the second case, the models are generic and are introduced in the continuous case in [5]. Unlike local models, the validity domain of a generic model is unknown. We are interested in generic models obtained by means of the δ-operator and the Kharitonov principle. The extent to which each model in a dynamic library is a good representation of the process is defined by validity indexes ([4], [3], [7], [8]).

There are nine sections in this paper. Section 2 defines the process description to be studied. Section 3 determines the models used in the multimodel to control the uncertain system under consideration, thus leading to the algorithm for the model base. Section 4 introduces the strategy of determination for the multimodel control. Section 5 specifies the local controller design. Section 6 describes the multimodel control design. Section 7 improves the multimodel control leading to the reinitialization algorithm. Section 8 gives the simulation results, showing the interest of this method for the control of uncertain system. Finally, conclusions are drawn in Section 9.

2 Process Description

The plant to be controlled can be a non-linear or linear time-variant discrete system with uncertainties defined by a simplified Input/Output model of the form :

$$y(t) = -\sum_{i=1}^{n_A} a_i(.)y(t-i) + \sum_{i=0}^{n_B} b_i(.)u(t-d-i), \tag{1}$$

with $u(t)$ the input, $y(t)$ the output, d the integer value characterizing a pure time delay, a_i and b_i the model parameters (1) such that $b_i(.) \in [b_i^-, b_i^+]$ $(i = 1, ..., n_B)$ and $a_i(.) \in [a_i^-, a_i^+]$, $(i = 1, ..., n_A)$, where n_B (respectively n_A) is the number of coefficients in the numerator (respectively denominator) of the transfer function of the discrete system.

The corresponding transfer function of the model (1) is defined by :

$$H_p(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} = \frac{q^{-d}\sum_{i=0}^{n_B} b_i(.)q^{-i}}{1 + \sum_{i=1}^{n_A} a_i(.)q^{-i}} \tag{2}$$

where :

$$\begin{array}{rcl}
q^{-1} & \triangleq & \text{backward shift operator } (q^{-1}y(t) = y(t-1)), \\
B(q^{-1}) & = & b_1(.)q^{-1} + \cdots + b_{n_B}(.)q^{-n_B} = q^{-1}B^*(q^{-1}), \\
B^*(q^{-1}) & = & b_1(.) + b_2(.)q^{-1} + \cdots + b_{n_B}(.)q^{-n_B+1}, \\
A(q^{-1}) & = & 1 + a_1(.)q^{-1} + \cdots + a_{n_A}(.)q^{-n_A} = 1 + q^{-1}A^*(q^{-1}), \\
A^*(q^{-1}) & = & a_1(.) + a_2(.)q^{-1} + \cdots + a_{n_A}(.)q^{-n_A+1}, \\
b_i(.) & \in & [b_i^-, b_i^+] \quad i = 1, ..., n_B, \\
a_i(.) & \in & [a_i^-, a_i^+] \quad i = 1, ..., n_A.
\end{array}$$

The coefficients of the model (2) are taken to evolve in given intervals and are bounded.

3 Model determination

3.1 Introduction

The goal of this model base is to allow the control of uncertain systems described by the equation (2). The model base is composed of linear models having the same order and the same structure as those of the uncertain system. A minimal number of models is necessary. This study relates to the determination of the adequate method to determine a library of models with the aim of controlling this type of system.

The question which arises is : which method should be used to obtain the models in the base ?

To answer this question, we must have some preliminary results. The two principal ideas used for obtaining the model base will be the δ-operator and the extreme models using the Kharitonov Theorem.

The discrete approach of Kharitonov was treated by [1]. This approach is strictly algorithmic and does not allow conditions to be expressed in algebraic form. Moreover, the complexity of the algorithm increases with the order of the system, and is therefore not appropriate to our requirements. We have sought a parallel with the continuous approach by using the δ-operator, which ensures good results in the modeling level of discrete uncertain systems.

3.2 Algorithm of determination of the model base

The δ-operator and the extreme models derived using the Kharitonov theorem are the two basic ideas developed in computing the model base.

It is assumed that the transfer function of the discrete system with interval plant is defined with the shift operator denoted by q. The following procedure (or algorithm) explains how to determine the model base. The algorithm has three steps :

Step 1. In the discrete case, the delta operator is introduced to reduce the number of possible models by means of an approach akin to that of the continuous case [1]. A new description of the transfer function (2) is obtained by replacing the operator q by $(1 + T_e\delta)$, as shown in equation (3) :

$$H_p(\delta) = [H_p(q)]_{q=1+T_e\delta} \tag{3}$$

To obtain the transfer function with interval in the δ-operator from the discrete transfer function of the system (2), there exist either $2^{n_A+n_B}$ combinations between the different extreme coefficients of numerator and denominator, or two combinations for which all the coefficients of the transfer function have the maximum value in the first case and the minimum in the second case.

Step 2. The transfer function of the discrete system with interval in the δ-operator obtained from Step 1 is denoted by :

$$H_p(\delta) = \frac{\sum_{i=0}^{r-d} \beta_i \delta^{r-d-i}}{\sum_{i=0}^{r} \alpha_i \delta^{r-i}} \tag{4}$$

where

$$\beta_i \in [\beta_i^-, \beta_i^+] \qquad\qquad \alpha_i \in [\alpha_i^-, \alpha_i^+]$$

are defined as :

$$\beta_i^- = \sum_{j=0}^{n_B} b_j^- C_{r'-j}^{r'-i} T_e^{r'-i} \ , \ \beta_i^+ = \sum_{j=0}^{n_B} b_j^+ C_{r'-j}^{r'-i} T_e^{r'-i} \ , \ \alpha_i^- = \sum_{j=0}^{n_A} a_j^- C_{r-j}^{r-i} T_e^{r-i} \ , \ \alpha_i^+ = \sum_{j=0}^{n_A} a_j^+ C_{r-j}^{r-i} T_e^{r-i}$$

with

$$r = \max(n_A, n_B + d)$$
$$r' = r - d$$

The highest degree α_0 of the transfer function in the delta operator depends only on the sampling period T_e ($a_0 = 1$), and is a strictly positive coefficient. Then, the transfer function (4) is rewritten as :

$$H_p(\delta) = \frac{\sum_{i=0}^{r-d} \beta'_i \delta^{r-d-i}}{\sum_{i=0}^{r} \alpha'_i \delta^{r-i}} \tag{5}$$

where

$$\beta'_i \ \in \ [\frac{\beta_i^-}{\alpha_0}, \frac{\beta_i^+}{\alpha_0}] = [\beta_i'^-, \beta_i'^+] \tag{6}$$

$$\alpha'_i \ \in \ [\frac{\alpha_i^-}{\alpha_0}, \frac{\alpha_i^+}{\alpha_0}] = [\alpha_i'^-, \alpha_i'^+] \tag{7}$$

Similarly to the approach defined for continuous systems [6], by using the extreme models technique applied to the last transfer function (4) we obtain four extreme transfer functions defined by :

$$H_1(\delta) = \frac{\beta_{r'}'^- + \beta_{r'-1}'^- \delta + \beta_{r'-2}'^+ \delta^2 + \beta_{r'-3}'^+ \delta^3 + \dots}{\alpha_r'^- + \alpha_{r-1}'^- \delta + \alpha_{r-2}'^+ \delta^2 + \alpha_{r-3}'^+ \delta^3 + \dots} \tag{8}$$

$$H_2(\delta) = \frac{\beta_{r'}'^- + \beta_{r'-1}'^+ \delta + \beta_{r'-2}'^+ \delta^2 + \beta_3'^- \delta^3 + \dots}{\alpha_r'^- + \alpha_{r-1}'^+ \delta + \alpha_{r-2}'^+ \delta^2 + \alpha_{r-3}'^- \delta^3 + \dots} \tag{9}$$

$$H_3(\delta) = \frac{\beta_{r'}'^+ + \beta_{r'-1}'^- \delta + \beta_{r'-2}'^- \delta^2 + \beta_3'^+ \delta^3 + \dots}{\alpha_r'^+ + \alpha_{r-1}'^- \delta + \alpha_{r-2}'^- \delta^2 + \alpha_{r-3}'^+ \delta^3 + \dots} \tag{10}$$

$$H_4(\delta) = \frac{\beta_{r'}'^+ + \beta_{r'-1}'^+ \delta + \beta_{r'-2}'^- \delta^2 + \beta_3'^- \delta^3 + \dots}{\alpha_r'^+ + \alpha_{r-1}'^+ \delta + \alpha_{r-2}'^- \delta^2 + \alpha_{r-3}'^- \delta^3 + \dots} \tag{11}$$

Step 3. To obtain the four extreme discrete transfer functions in the shift operator (denoted by q), equations (8) to (11) are replaced by those obtained by setting the δ-operator defined as $\delta = \frac{q-1}{T_e}$. We note the difficulty of using the results suggested in the literature to generalize the theorem of Kharitonov in the discrete case, since the proposed algorithms do not make it possible to give analytical representations systematically. The sampling period also has an influence on the coefficients of the different models.

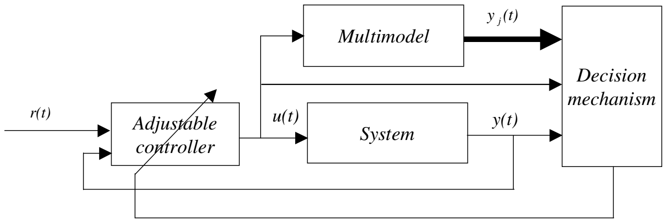


Figure 1: *Block diagram of the multimodel control strategy*

From Step 3 we obtain the four transfer functions denoted by H_j corresponding to the four discrete extreme models denoted by M_j. These models obtained (called generic models) have the same structure as the discrete system, and are defined by :

$$H_j(q^{-1}) = \frac{q^{-d}B_j(q^{-1})}{A_j(q^{-1})} \tag{12}$$

where
$j = 1, \ldots, 4$,
$B_j(q^{-1}) = b_{j1}q^{-1} + \cdots + b_{ji}q^{-i} + \cdots + b_{jn_B}q^{-n_B} = q^{-1}B_j^*(q^{-1})$,
$A_j(q^{-1}) = 1 + a_{j1}q^{-1} + \cdots + a_{ji}q^{-i} + \cdots + a_{jn_A}q^{-n_A} = 1 + q^{-1}A_j^*(q^{-1})$,
b_{ji} and a_{ji} are the coefficients of the polynomial B_j and A_j associated with the model M_j, defined as :

$$b_{ji} = \sum_{k=0}^{r'} \beta_{jk} T_e^{k-r'} C_{r'-k}^{r'-i} (-1)^{i-k} \tag{13}$$

$$a_{ji} = \sum_{k=0}^{r} \alpha_{jk} T_e^{k-r} C_{r-k}^{r-i} (-1)^{i-k} \tag{14}$$

3.2.1 Advantages

The main advantages of the chosen model base are :

* the small number of models used

* the systematic determination of these generic models

* the simplicity of implementation, especially if the control is based on fusion

3.2.2 Drawbacks

On the other hand, the model base is built up solely from models having the same structure as that of the initial system (2). Consequently, we may plan to add to the multimodel, if necessary, models having different structures. Thus, the global model can be obtained from the fusion of the models using the estimated validities.

4 Strategy of determination of the multimodel control

The multimodel approach can be used to control systems with uncertainty. To each model corresponds a controller. The process controller is an adjustable one. The multimodel control depends on the decision mechanism which determines the validity of each model at each sampling period of the discrete system. The multimodel control is obtained by fusion of the elementary coefficients of each controller, when the validity is computed as a linear combination of the coefficients of each controller. A block diagram illustrating this control strategy for such a complex system is given in figure (1), where $r(t)$ is the reference input, $u(t)$ is the plant input, $y(t)$ is the plant output, and $y_j(t)$ is the output of the model M_j.

5 Local controller design

Independently of the control of the different models, provided that their structure is identical, we adopt an ordering law of type R-S-T applied to each model of the multimodel. At each moment the equation of the R-S-T controller is given by

$$S_j(q^{-1})u_j(t) = T_j(q^{-1})y^*(t+d+1) - R_j(q^{-1})y_j(t), \tag{15}$$

where R_j, S_j and T_j are polynomials in q^{-1}, $u_j(t)$ is the controller law applied to the model M_j, $y_j(t)$ is the output of the model M_j, and $y^*(t)$ is the desired trajectory. The transfer function of the reference model is defined by

$$H_m(q^{-1}) = \frac{B_m(q^{-1})}{A_m(q^{-1})},$$

with

$$\begin{aligned} B_m(q^{-1}) &= b_{m0} + b_{m1}q^{-1} + \cdots + b_{mn_{B_m}}q^{-n_{B_m}}, \\ A_m(q^{-1}) &= 1 + a_{m1}q^{-1} + \cdots + a_{mn_{A_m}}q^{-n_{A_m}}, \end{aligned}$$

It is supposed that polynomials $A_j(q^{-1})$ and $B_j(q^{-1})$ do not have common factors.

The transfer function in the closed loop between the output of the reference model y^* and that of the model y_j is given by :

$$H_j^{CL}(q^{-1}) = \frac{T_j(q^{-1})q^{-d}B_j(q^{-1})}{P_j(q^{-1})}, \tag{16}$$

where $P_j(q^{-1}) = A_j(q^{-1})S_j(q^{-1}) + q^{-d}B_j(q^{-1})R_j(q^{-1})$ defines the poles in the closed loop.

Thus, for each model M_j, we have a controller C_j specified by the polynomials $R_j(q^{-1})$, $S_j(q^{-1})$ and $T_j(q^{-1})$.

The polynomials R_j, S_j and T_j have the following form :

$$R_j(q^{-1}) = r_{j0} + r_{j1}q^{-1} + ... + r_{jn_R}q^{-n_R}, \tag{17}$$

$$S_j(q^{-1}) = s_{j0} + s_{j1}q^{-1} + ... + s_{jn_S}q^{-n_S} = s_{j0} + q^{-1}S_j^*(q^{-1}), \tag{18}$$

$$T_j(q^{-1}) = GP_j(q^{-1}), \tag{19}$$

with

$$S_j^*(q^{-1}) = s_{j1} + s_{j2}q^{-1} + \ldots + s_{jn_s}q^{-n_s+1}, \tag{20}$$

$$G = \frac{1}{B_j(1)} \quad si \quad B_j(1) \neq 0.$$

Finally, the controller is written :

$$u_j(t) = \frac{1}{s_{j0}}[P_j(q^{-1})Gy^*(t+d+1) - S_j^*(q^{-1})u_j(t-1) - R_j(q^{-1})y_j(t)] \tag{21}$$

The poles in the closed loop of the multimodel are selected identical, like the system to be controlled.

6 Multimodel control design

The multimodel controller is designed to control complex systems. The multimodel control applied to the system is defined by the controller having as polynomials $R_g(q^{-1},t)$, $S_g(q^{-1},t)$ and $T_g(q^{-1},t)$, which are determined by the fusion of the polynomials $R_j(q^{-1})$, $S_j(q^{-1})$ and $T_j(q^{-1})$ for each local controller C_j by means of the validities $v_j^{\prime renf}(t)$, defined in ([7], [8]), computed at each sampling time t by :

$$R_g(q^{-1},t) = \sum_{j=1}^{m_d} v_j^{\prime renf}(t)R_j(q^{-1}) \tag{22}$$

$$S_g(q^{-1},t) = \sum_{j=1}^{m_d} v_j^{\prime renf}(t)S_j(q^{-1}) \tag{23}$$

$$T_g(q^{-1},t) = \sum_{j=1}^{m_d} v_j^{\prime renf}(t)T_j(q^{-1}) \tag{24}$$

satisfying the following condition :

$$\sum_{j=1}^{m_d} v_j'^{renf}(t) = 1 \tag{25}$$

with m_d the number of models in the base.

The multimodel control can be written as

$$u(t) = \frac{1}{s_{j0}}[T_g(q^{-1},t)y^*(t+d+1) - S_g^*(q^{-1},t)u(t-1) - R_g(q^{-1},t)y(t)] \tag{26}$$

7 Improvement of the multimodel control

7.1 Introduction

The aim of this section is to improve the multimodel control. The method we shall consider involves an observer for each model of the multimodel, in order to reinitialize regularly the state of the model to the value of the estimated state of the system for the corresponding model when there is a discrepancy.

We start this section with some preliminaries relating to the theory of observers in the case of linear models, which we shall subsequently apply to the the multimodel approach.

7.2 Preliminaries

The goal of this part is to reinitialize the state of the model(s) M_j, $j = 1, \cdots, m_d$, when the state of the process for the model M_j diverges from zero.

The model M_j of the multimodel, in the usual state-space form, is given by :

$$\begin{cases} x_j(t+1) &= A_j x_j(t) + B_j u_(t) \\ y_j(t) &= C_j x_j(t) \end{cases} \tag{27}$$

where $x_j(t)$, $u(t)$, $y_j(t)$ denote repectively state vector, input vector and output vector. A_j, B_j, C_j are known model matrices. The same model input-output form is described by the equations (12).

Let us consider an observer of the process for the model M_j given by

$$\begin{cases} \hat{x}_j(t+1) &= A_j\hat{x}_j(t) + B_j u(t) + K_j(y(t) - \hat{y}_j(t)) \\ \hat{y}_j(t) = &= C_j\hat{x}_j(t) \end{cases} \tag{28}$$

where $y(t)$ is the output of the process, $u(t)$ is the input of the process, $\hat{x}_j(t)$ is the estimated state of the process for the model M_j, and K_j is the observer feedback matrix. The state estimation error $e_j(t) = x_j(t) - \hat{x}_j(t)$ is introduced. Accordingly, the system error is

$$e_j(t+1) = (A_j - K_jC_j)e_j(t) \tag{29}$$

where K_j is obtained such that $(A_j - K_jC_j)$ is a asymptotically stable matrix.

7.3 Strategy for the improvement of the multimodel control

Contrary to the strategy of multimodel control studied in the previous section, an observer allowing the estimation of the state is associated with each model, as detailed in the previous paragraph. The new controller applied to the system is an adjustable one based on the reinitialization of the state of the model, if there is a discrepancy with the state estimated from the process starting from this model. Figure (2(a)) illustrates the general architecture of multimodel control using observers.

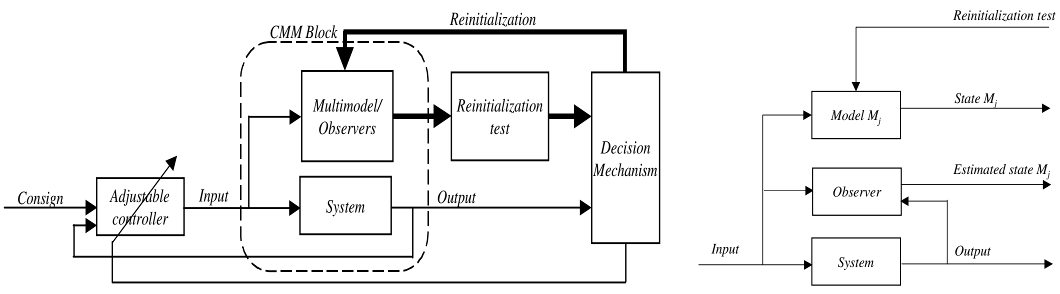
The *CMM block* contains the multimodel, the system and the corresponding observers. The figure (2(b)) shows the contents of this block in the case of a model of the multimodel.

7.4 Conditions and interest of reinitialization

Determine at each sampling instant t :

(i) The state $x_j(t)$ of each model M_j with $j = 1, \ldots, m_d$ from equation (27)

(ii) The estimated state $\hat{x}_j(t)$ of the process for the model M_j from equation (28),



(a) (b)

Figure 2: (a) General architecture of multimodel control using observers ; (b) *CMM block*

(iii) If $\|\hat{x}_j(t) - x_j(t)\| > \xi$ then the state $x_j(t)$ must be reinitialized to the estimated state $\hat{x}_j(t)$. In the converse case the state $x_j(t)$ of the model remains unchanged.

The value of ξ checking $\|\hat{x}_j(t) - x_j(t)\| > \xi$ is the precision defined by the operator. It can be either a constant, or depend on the estimated state at each sampling instant as $\frac{\|\hat{x}_i(t)\| \times 0.1}{100}$.

In general terms, reinitialization gives us an idea about the behavior of the state of the process by means of the model M_j. Moreover, it enables us to improve the validity of each model.

7.5 Improvement algorithm

The improvement flowchart for the multimodel control is presented in detail in figure (3).

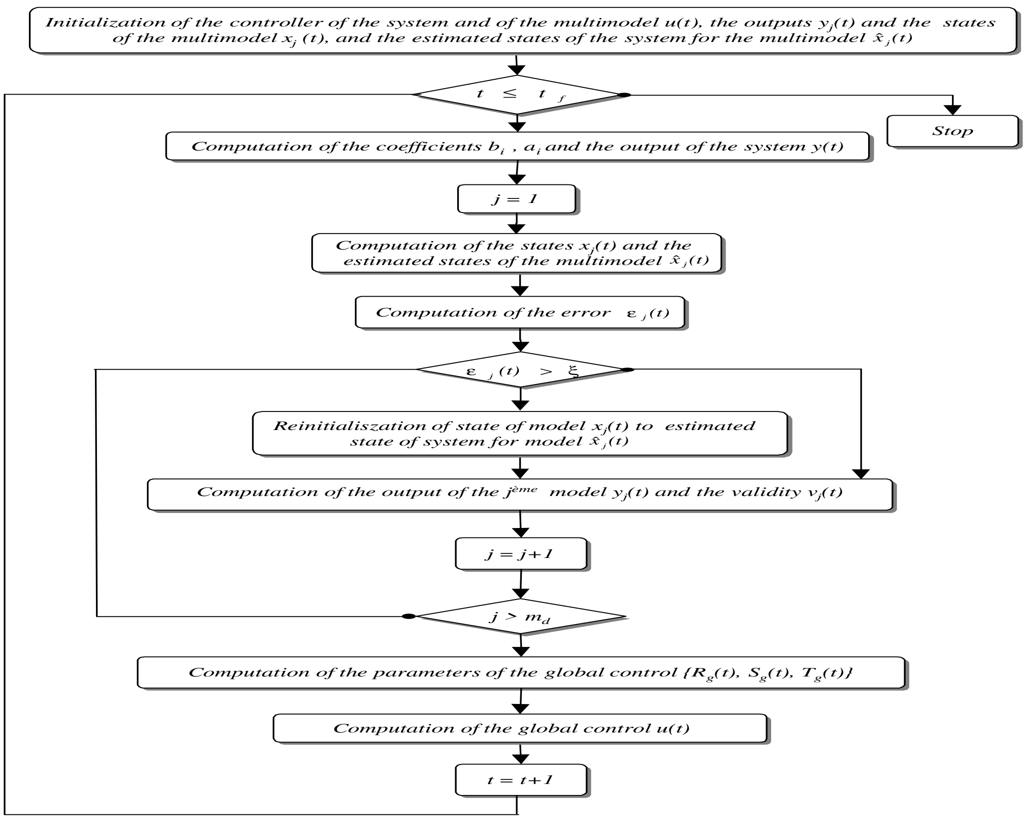


Figure 3: Flowchart for the improvement of the multimodel control

8 Illustrative examples

Let us consider the model of the discrete system with parameters varying with time, written as follows :

$$H_p(q^{-1}) = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}}. \quad (30)$$

The coefficients of the transfer function (30) vary as shown in the following equations :

$$b_i = b_i^{'} + \triangle b_i \ sin(w_{b_i} t + \varphi_{b_i}), \quad (31)$$
$$a_i = a_i^{'} + \triangle a_i \ sin(w_{a_i} t + \varphi_{a_i}). \quad (32)$$

The coefficients $b_i^{'}$, $a_i^{'}$, $\triangle b_i$ and $\triangle a_i$ are defined as :

$$b_i^{'} = \frac{b_i^+ + b_i^-}{2}, \qquad a_i^{'} = \frac{a_i^+ + a_i^-}{2}, \qquad \triangle b_i = \frac{b_i^+ - b_i^-}{2}, \qquad \triangle a_i = \frac{a_i^+ - a_i^-}{2}.$$

To illustrate our remarks, we present some numerical examples. We apply the two types of control previously described : first, control used without improvement, and, secondly, multimodel control with improvement. For each type of control we present a numerical example corresponding to an uncertain system with unstable reverse.

8.1 First control : Uncertain system (unstable reverse)

The coefficients of the system (30) are summarized in table (1).

b_1	$[0.0979; 0.1021]$	$\triangle b_1$	0.0021	$b_1^{'}$	0.1
b_2	$[0.1820; 0.2180]$	$\triangle b_2$	0.018	$b_2^{'}$	0.2
a_1	$[-1.3120; -1.2880]$	$\triangle a_1$	0.012	$a_1^{'}$	-1.3
a_2	$[0.4110; 0.4290]$	$\triangle a_2$	0.009	$a_2^{'}$	0.42

Table 1: Coefficients of the uncertain system with : $T_e = 0.3s$, $w_{b_i} T_e = w_{a_i} T_e = \frac{\pi}{50}$ and $\varphi_{b_i} = \varphi_{a_i} = 0$

Let us apply the different steps given by the construction algorithm of the model base. The transfer function limited in δ is given by :

$$H_p(\delta) = \frac{\beta_1' \delta + \beta_2'}{\delta^2 + \alpha_1' \delta + \alpha_2'}, \quad (33)$$

Table (2) summarizes the values of the different coefficients β_i' and α_i' of the transfer function (33).

β_1'	$[0.3263; 0.3403]$	$\triangle\beta_1'$	0.007	${\beta_1'}^{moy}$	0.3333
β_2'	$[3.1100; 3.5567]$	$\triangle\beta_2'$	0.2234	${\beta_2'}^{moy}$	3.3334
α_1'	$[2.2933; 2.3733]$	$\triangle\alpha_1'$	0.04	${\alpha_1'}^{moy}$	2.3333
α_2'	$[1.1; 1.5667]$	$\triangle\alpha_2'$	0.2333	${\alpha_2'}^{moy}$	1.3334

Table 2: Values of different parameters of $H_p(\delta)$

The generic control law is written as :

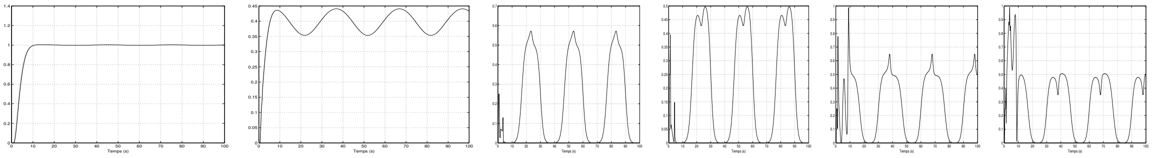
$$u_j(t) = \frac{1}{s_{j0}} \left[P_j(q^{-1}) y^*(t+d+1) - S_j^*(q^{-1}) u_j(t-1) - R_j(q^{-1}) y_j(t) \right] \quad (34)$$

The trajectory of desired reference is obtained by discretizing a continuous second order model of the form $\frac{w_0^2}{w_0^2 + 2w_0\xi s + s^2}$ for $w_0 = 0.6\ rad/s$, $\xi = 0.9$ and $T_e = 0.3\ s$. The transfer function of this reference model is written as :

$$\frac{B_m(q^{-1})}{A_m(q^{-1})} = \frac{0.0145 q^{-1} + 0.0131 q^{-2}}{1 - 1.6956 q^{-1} + 0.7233 q^{-2}} \quad (35)$$

The selected poles in the closed loop are the same in the model base as in the system. The polynomial $P_j(q^{-1})$ selected, equal to $(1 - 0.4463 q^{-1} + 0.0498 q^{-2})$, is obtained by discretizing a continuous second order model of the form $\frac{w_0^2}{w_0^2 + 2w_0\xi s + s^2}$ for $w_0 = 5\ rad/s$, $\xi = 1$ and $T_e = 0.3\ s$. All the initial conditions are equal to zero. The reference $r(t)$ is a level. The multimodel control is obtained by fusion of the elementary coefficients of each controller C_j.

The output of the system closely follows the reference trajectory as shows in figure (4(a)). The multimodel control is represented by figure (4(b)). The contribution of each model of the base is represented by the figures (4(c)), (4(d)), (4(e)) and (4(f)). The validities v_j with $(j = 1, \cdots, 4)$ of the four models overlap.



(a) (- - -) Reference trajectory y^*; (—) Output of the system y

(b) multimodel control u

(c) Validity of M_1

(d) Validity of M_2

(e) Validity of M_3

(f) Validity of M_4

Figure 4: Multimodel control applied to the uncertain system (unstable reverse).

8.2 Second control : Uncertain system (unstable reverse)

Let us consider the system defined previously (equation (30)). Table (3) illustrates the numerical values of the coefficients obtained by increasing the variation of the system parameters.

b_1	$[0.0998; 0.1002]$	$\triangle b_1$	0.0002	b_1'	0.1
b_2	$[0.1820; 0.2180]$	$\triangle b_2$	0.018	b_2'	0.2
a_1	$[-1.9; -0.7]$	$\triangle a_1$	0.6	a_1'	-1.3
a_2	$[-0.18; 1.02]$	$\triangle a_2$	0.6	a_2'	0.42

Table 3: Values of different parameters of the uncertain system (unstable reverse) with : $T_e = 0.3s$, $w_{b_i}T_e = w_{a_i}T_e = \frac{\pi}{50}$ and $\varphi_{b_i} = \varphi_{a_i} = 0$

Table (4) shows the values of the different coefficients β_i' and α_i' of the transfer function (33).

β_1'	$[0.3326; 0.3340]$	$\triangle\beta_1'$	0.0007	$\beta_1'^{moy}$	0.3333
β_2'	$[3.1310; 3.5357]$	$\triangle\beta_2'$	0.2024	$\beta_2'^{moy}$	3.3334
α_1'	$[0.3333; 4.3333]$	$\triangle\alpha_1'$	2	$\alpha_1'^{moy}$	2.3333
α_2'	$[-12; 14.6667]$	$\triangle\alpha_2'$	13.3334	$\alpha_2'^{moy}$	1.3334

Table 4: Values of different parameters of $H_p(\delta)$

We notice that the models of the base are unstable. The reference model and the poles selected in the closed loop are the same as in the previous example (equation (35)).

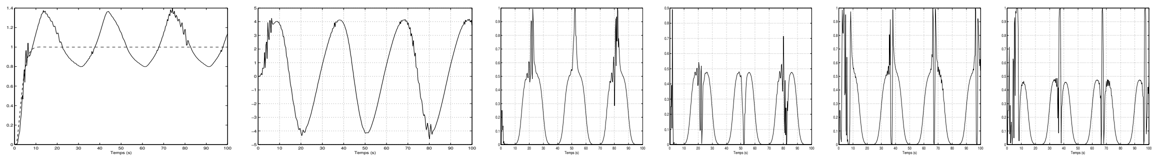
Let us consider, for each model M_j resulting from the multimodel, the matrices A_j, B_j, C_j and K_j (K_j are given such that the error $e_j(t+1) = (A_j - K_j C_j)e_j(t)$ converges asymptotically towards zero). The determination of the matrix K_j is carried out while placing the poles of the observer to a dynamics equalizes with $[0.01 \;\; 0.05]$ and by adjusting ξ with the value $0.01 \times \; \|\hat{x}_j(t)\|$.

The equation of the multimodel control, in the case of the R-S-T controller, is written as :

$$u(t) = T_g(q^{-1}, t)y^*(t + d + 1) - S_g^*(q^{-1}, t)u(t - 1) - R_g(q^{-1}, t)y(t) \qquad (36)$$

8.2.1 Improved multimodel control (with reinitialization)

Let us apply the multimodel control improvement algorithm to the system (30) with the new variations of the coefficients. According to simulations, the output of the system converges towards the reference trajectory as shows in figure (5(a)). The multimodel control is represented by the figure (5(b)). The contribution of each model of the base is represented by the figures (5(c)), (5(d)), (5(e)) and (5(f)). The figures (6(a)), (6(b)), (6(c)) and (6(d)) illustrate the reinitialization of the different models. For the model M_1, the figure (6(a)) shows commutation between two states. If $\|\hat{x}_j(t) - x_j(t)\| < \xi$, the reinitialization is not carried out. In the contrary case ($\|\hat{x}_j(t) - x_j(t)\| > \xi$), the reinitialization is then carried out. Similarly for models M_2, M_3 and M_4.



(a) (- - -) Reference trajectory y^*; (—–) Output of the system y

(b) Global control u

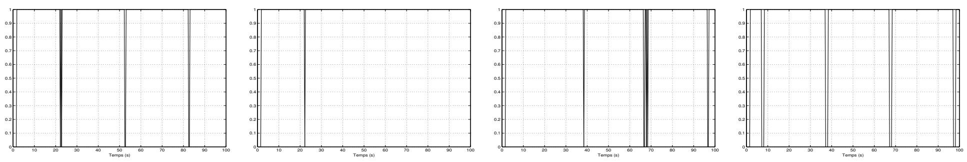
(c) Validity of M_1

(d) Validity of M_2

(e) Validity of M_3

(f) Validity of M_4

Figure 5: Improvement of the multimodel control applied to the system.



(a) Reinitialization of M_1

(b) Reinitialization of M_2

(c) Reinitialization of M_3

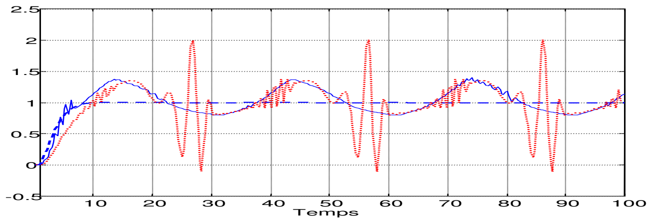
(d) Reinitialization of M_4

Figure 6: Reinitialization of the models of the multimodel applied to the system with $\xi = 0.01 \times \|\hat{x}_j(t)\|$.

8.2.2 Comparison of the different controls

Let us apply the multimodel control to the system defined above, without carrying out the test of reinitialization. According to simulations, the output of the system $y(t)$ diverges. To counter this the reinitialization test is introduced. Figure (5(a)) shows the output of the system which guaranteed a close correspondence to the reference trajectory.

The superposition of different curves (multimodel control with reinitialization and the control of the average model) are given in figure (7(a)). We note that the improved multimodel control reaches the desired objective, by guaranteeing the best performances.



(a) (- - -) Reference trajectory y^*; (—–) Output y using improved multimodel control; (....) Output y using the control of the average model

Figure 7: Comparison of the different controls.

9 Conclusions

The main idea introduced in this paper is the use of a multimodel approach with a limited number of models defined rigorously and in a systematic way. The assumption is that if the plant is sufficiently close to a model, in some sense, the validity of this latter has the greatest value. The objective, consequently, is to determine the most appropriate model at each time, using a suitable performance criterion based on the residual value, and to fuse the elementary controllers to obtain the global one. The theoretical basis for the design of such systems is presented in this paper in the context of linear time-variant models. The multimodel control presented satisfies such performance for a system where variations in parameters are small. Consequently, state space reinitialization can be of great value in the improvement of the multimodel control and in reaching the desired objective.

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