AMO - Advanced Modeling and Optimization, Volume 3, Number 1, 2001

Evolutionary Computation for Econometric Modeling

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ABSTRACT

This paper shows that, in case of high sensitivity to data econometric modeling, using evolutionary algorithms (Genetic Algorithms - GA and Simulated Annealing - SA) is better than using classical gradient techniques. The evaluation of the algorithms involved was performed on a short form of an economic macromodel. The optimization task is the model's solution, as function of the initial values (in the first stage) and of the objective functions (in the second stage). We proved that *a priori* information help "elitist" algorithms (like SA) to obtain best results; on the other hand, when one has equal believe concerning the choice among different objective functions, GA gives a straight answer. Analyzing the average related bias of the model's solution proved the efficiency of the stochastic optimization methods presented.

Introduction

This paper is a study on the initial values' influence on finding the optimal solution for some economic macromodel. Empirical trials showed the high sensitivity of the model's output to small variations of the input values. A second goal of our approach consists in rising some questions on the adequacy of different objective functions for the same econometric model.

The analysis of the suitable optimization techniques is depicted in *Section 1*. The numerical results of applying these techniques for finding the extreme global solution of the macromodel, including a comparative discussion on the several objective functions that one can associate to the macromodel, are presented in *Section 2*. We omit the concrete form of the econometric model, as irrelevant.

1 Evolutionary Computation for Function Optimization

When initiating a numerical analysis of some econometric model, one has to face a difficult problem: What optimization algorithm should he/she use? Usually, this problem is solved by a reduction mechanism: classical algorithms from Operation Research (SIMPLEX, e.g.) are excluded, as they are limited to convex regular functions. Gradient techniques (Gauss-Seidel, e.g.) are the first candidates, as their speed and precision on problems with a single local optima is highly appreciated. Probabilistic search algorithms (like GA – Michalewicz (1994) or SA – Laarhoven *et al.* (1987)) come next: they are easy to implement and they do not require supplementary information about the objective function; besides, they showed good performance on multi-modale problems, where gradient techniques usually fail.

How should an econometrician choose among these different methods (and we have mentioned just a few of them)? Are there some mathematical guidelines supporting this decision? Generally speaking, the answer to this question is negative. As Wolpert and Macready have recently proved, there is *no free lunch theorem for optimization* Wolpert *et al*. (1997). That is, for each optimization algorithm, the number of function classes on which it performs well is equal to the number of function classes on which it performs badly. In light of these results, the mathematician's advice would be the following: '*Concern about the particular function you want to optimize'*. Summing up, one has to take into account the existing experimental evidence (or convergence results, when available) for the particular class of problems he/she tackles. One must notice that evolutionary algorithms applications have some history in economy & financial engineering, e.g. Goonatilake (1995) or Przechlewski (1996).

2. Numerical Results

This chapter is divided into two sections. The first one deals with finding the global optimum of an economic macromodel (see, e.g., Gujarati (1997)) using different optimization techniques, namely:

- Gradient technique for non-linear systems (as included in EXCEL 7.0 – Solver, referred as *Gradient*, in the subsequent);
- Genetic Algorithm with real-valued chromosomes (implemented in Visual Basic, referred as *GA*);
- Simulated Annealing (implemented in C++ and referred as *SA*);

Section 2.2 presents an adequacy analysis of three different objective functions to the considered system.

2.1. Finding the Global Optimum of the Macromodel

We must mention that all the calculus below is restricted to the 1995-1996 annual data of the economic indicators, as they appear in Dobrescu (1996).

We considered some fixed intervals for each initial value of the endogenous variable (required at start, for each solving procedure). These intervals are large enough for ensuring a good exploration of potential solutions, yet appropriate to the specific economic data in study. We used 30 endogenous variables, but for readability reasons we selected the most representative nine variables. In figures 1-3 we present comparisons among the three optimization algorithms, on three different objective functions (each of them being supported by some economic reasoning and utility).

The comparison is w.r.t. the *averaged bias*, computed for each endogenous variable – as in Przechlewski *et al.* (1996):

$$\mathrm{BIAS}_i = \left| (1/m) * \Sigma_{k=1,m}(b_i^{(k)} - B_i) \right|$$

where B_i denotes the recorded statistical value for endogenous variable i, $b_i^{(k)}$ stands for the endogenous variable value i obtained in the k-th run, m is the number of runs (m=10). Additionally, some specific objective functions are computed, in order to give a global (economical) utility of the macromodel to be tested. They are usually non-linear functions of the most important economic indicators; in our case, we tested three such different objective functions, namely:

- Objective Function 1 - ***min [GDP-EXTDR]^2***;

- Objective Function 2 –

  min [GDP*EXGDPD/(GDPD*EXTDR)-1]^2;

- Objective Function 3 –

  min{[GDP*EXGDPD/(GDPD*EXTDR)-1]^2***+ [GDP/EXTDR -1]***^{2}***}.***

where: GDP stands for the *gross domestic product*, GDPD stands for the *gross domestic product deflator*, EXTDR is the *expected total disposable revenues* and EXGDPD is the *expected gross domestic product deflator*.

SA's description: the maximal number of cycles was 350, with 50 uniform random draws on each cycle. The temperature (T) decreased from 10,000 down to 1,390 (on the longest run). The results were averaged on 10 different runs. One must notice that the values obtained by SA are closer to the reference values than those obtained by GA - see below. Referring, e.g., to the *domestic aggregate demand* (DAD) and GDP, the differences for the second objective function between GA and SA are obviously favorable to SA.

GA's description: real-valued chromosomes, uniform mutation, shuffle crossover, disruptive selection (i.e., the "worst" chromosome is maintained from a generation to another, in order to improve the information exchange in case of GA stagnation with the current population filled up with samples of the same individual), up to 100 chromosomes in a generation, and up to 1000 generations in one run. The initial population was randomly generated (but within some fixed intervals, provided by the econometrician), and the results were averaged on 10 different runs.

As for the results obtained by the Gradient technique, the starting points were the statistical values corresponding to year 1996. In case of the first objective function, as the model is proved to have a unique solution, the results can be obtained with different starting points, located "close" to the initial (statistical) values. It is worth noting that for extreme starting points, Gradient needs two successive runs (vs. one run, in case of "close" points) for reaching the desired solution. Going deeper in the Gradient procedure, one can explain this two-step behavior by successively missing and finding the correct search direction. Concerning the second objective function, the system solution is strongly depending on the initial values (due to the existence of multiple solutions, in this case). We stress that the choice of recorded statistical values as starting points is justified by our intention to give Gradient technique the best possible start.

2.2. Finding the Best Objective Function for the Macromodel

Choosing a certain objective function is usually a subjective matter. Hence, a new objective function (Obj_F. 1+2+3) can be introduced, as a linear convex combination of the three previous ones, with three weights, which are subject to optimization. Figure 4 depicts the weights (averaged on ten different runs) obtained for the aggregate objective function.

As one can notice in fig.4, Objective Function 2 (Obj_F.2) was preferred by both algorithms. Yet, the GA made the difference among objective functions more obvious.

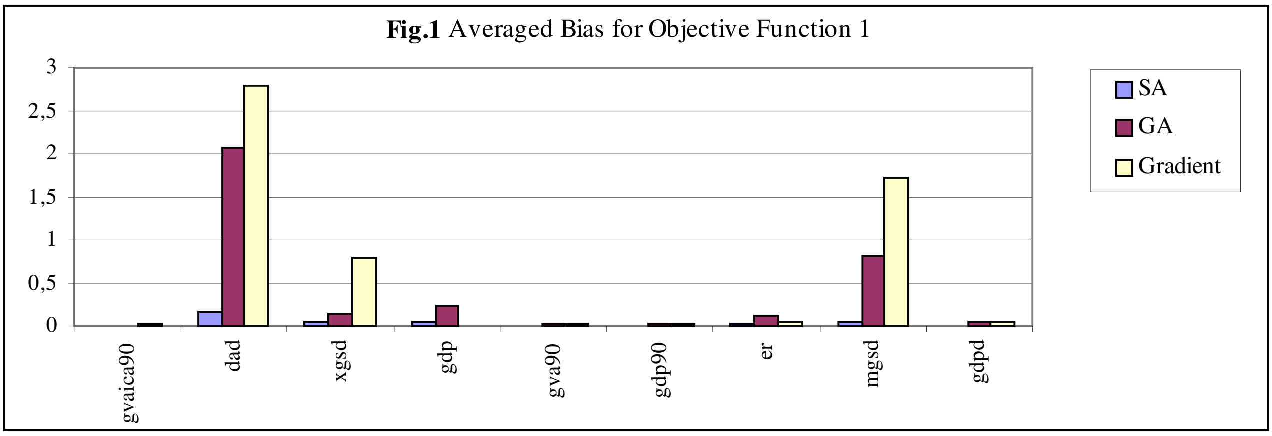


Fig.1 Averaged Bias for Objective Function 1
3
2,5
2
1,5
1
0,5
0
gvaica90
dad
xgsd
gdp
gva90
gdp90
er
mgsd
gdpd
SA
GA
Gradient

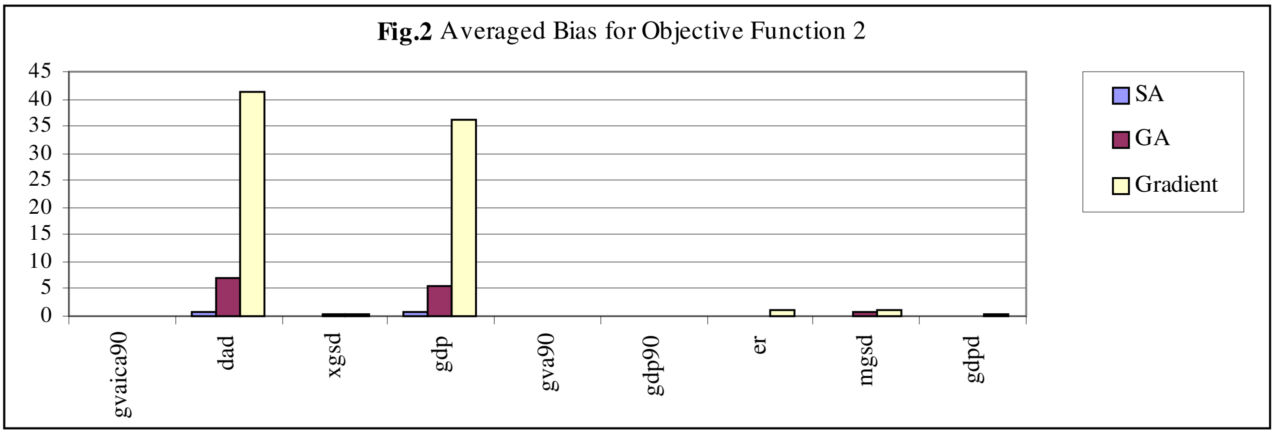


Fig.2 Averaged Bias for Objective Function 2
45
40
35
30
25
20
15
10
5
0
gvaica90
dad
xgsd
gdp
gva90
gdp90
er
mgsd
gdpd
SA
GA
Gradient

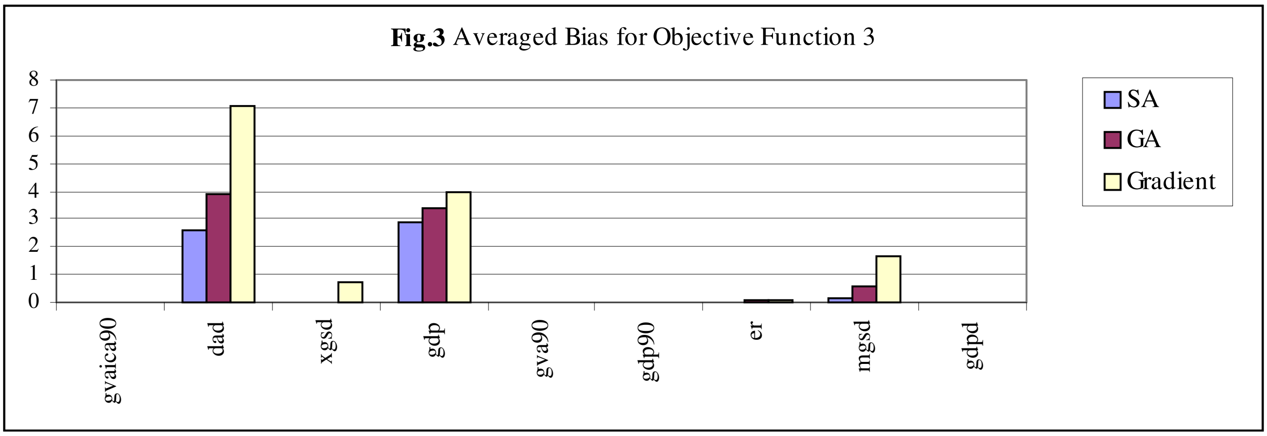
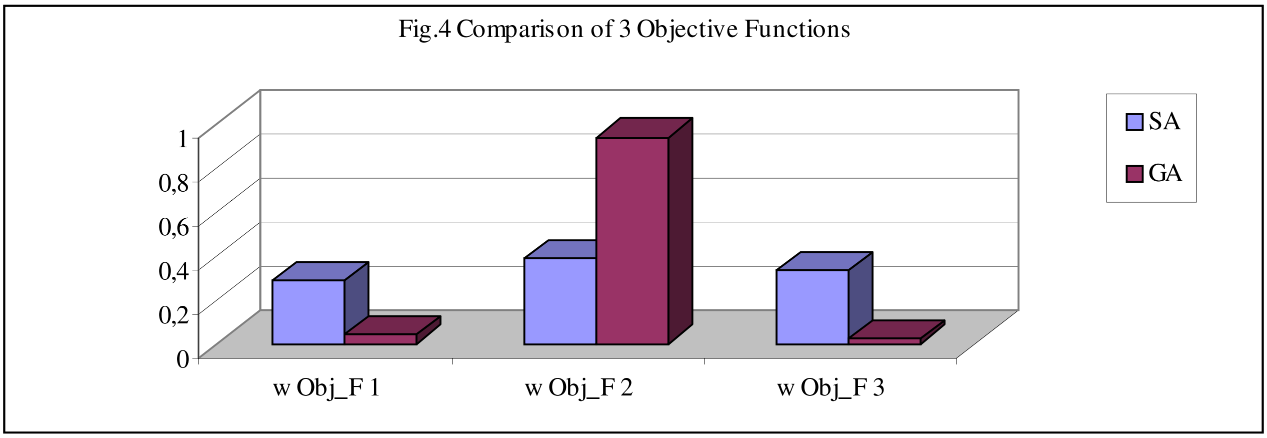


Fig.3 Averaged Bias for Objective Function 3
8
7
6
5
4
3
2
1
0
gvaica90
dad
xgsd
gdp
gva90
gdp90
er
mgsd
gdpd
SA
GA
Gradient



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