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ON THE TUBE SURFACES WITH THE FOCAL CURVE

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Abstract. In this paper, we study a surface called the tube about focal curve. Finally, we give the Gauss curvature and the mean curvature.

1. Introduction

The analysis of space curves in differential geometry is a classical subject. They are defined in parametric represantation and their geometric properties are expressed with the aid of derivatives and integrals [14,16,17]. One of the most important topics of curve analysis is the Frenet-Serret formulas which describe the kinematic properties that provide a coordinate system at each point of the curve. These formulas are named after the two French mathematicians who Jean Frederic Frenet (1847) and Joseph Alfred Serret (1851). On the other hand, for any unit tangent vector of the curve , the focal curve is defined as the centers of the osculating spheres, [15,20]. The focal curves can also consider as evolute curves. Thus, we can examine many features of the curve with the help of focal curves. Also, some curves and surfaces with new frames have been studied papers [1,2,5].

The branch of differential geometry dealing with surfaces. In the theory of surfaces one examines the shape of a surface, its curvature, the properties of various types of curves on a surface, aspects of deformation, the existence of a surface with given internal or external features, etc [4]. The tubes with spine curve (or bore) the curve are the circled surfaces generated by a circle with constant radius centered on and the plane of which is always normal to this curve.

One of the principal purpose of the classical differential geometry is the study of some classes of surfaces with special properties in $\mathbb{R}^3$ such as developable surfaces, ruled surfaces or minimal surfaces etc. Recently there appeared several articles on the study of ruled surfaces [3,7,12]. Like as ruled surfaces, circular surfaces might be also important subjects in several area. A circular surface is a typical surface with such the property. One of the examples of circular surfaces is the canal surface (tube) of a space curve. A canal surface is the envelope of a moving sphere with varying radius, defined by the trajectory $A(t)$ of its centers and a radius function $r(t)$ and canal surface is parametrized through Frenet frame of the spine curve $A(t)$. Canal surface is useful to represent various objects e.g. pipe, hose, rope or intestine of a body. Moreover, canal surface is an important instrument in surface modelling for CAD/CAM such as tubular surfaces, torus and Dupin cyclides. If we choose

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the radius function $r(t)$ as constant, we obtain tubular surfaces [9]. Canal surfaces and tubular surfaces have been studied by many researchers [8,11,18].

The differential geometry of space curves is a classical subject which usually relates geometrical intuition with analysis and topology. While mechanisms and robots operate, they generate work paths as spatial curves and form workspace bounded by surfaces. It is hence natural to use differential geometry for investigating these spatial curves and surfaces [6,10,13,19,21].

2. Preliminaries

A γ curve in 3-space $\mathbb{E}^3$ is a continuous mapping of class C^3, $\gamma : I \longrightarrow \mathbb{E}^3$, where I is a open interval on the real line. γ is as follows of the Frenet formula:

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T(s) \\ N(s) \\ B(s) \end{bmatrix} \tag{1}$$

where s is the arc-length parameter, t, n, b, κ and τ are the tangent unit vector, the normal unit vector, the binormal unit vector, curvature and torsion respectively.

For any unit speed curve, the focal curve is defined as the centers of the osculating spheres. According to frenet frame $\{t(s), n(s), b(s)\}$ of unity speed curve γ, the focal curve is given as follows:

$$C_\gamma(s) = (\gamma + c_1 N + c_2 B)(s), \tag{2}$$

where the coefficents c_1, c_2 are smooth functions that are called focal curvature of γ [20].

A surface in $\mathbb{E}^3$ denotes by $X(s,t)$. The unit normal vector field of this surface is defined as follows:

$$n = \frac{X_s \times X_t}{\|X_s \times X_t\|}. \tag{3}$$

The coefficients of the first and second fundemental form of this surface, respectively, is found by

$$\begin{aligned} E &= \|X_s\|^2, F =< X_s, X_t >, G = \|X_t\|^2, \\ e &=< n, X_{ss} >, f =< n, X_{st} >, g =< n, X_{tt} > . \end{aligned} \tag{4}$$

The Gaussian curvature K and the mean curvature H of the surface $X(s,t)$ are given as follows:

$$K = \frac{eg - f^2}{EG - F^2}, H = \frac{eG - 2fF + gE}{2(EG - F^2)}. \tag{5}$$

Let $\gamma : (a,b) \longrightarrow \mathbb{E}^3$ be curve. Since the normal n and binormal b are perpendicular to γ, the circle $\theta \longrightarrow \cos\theta N(s) + \sin\theta B(s)$ is perpendicular to γ at $\gamma(s)$. As this circle moves along γ it traces out surface called the tube about γ. This surface is defined as follows [4]:

$$X(s,\theta) = \gamma(s) + r(\cos\theta N(s) + \sin\theta B(s)). \tag{6}$$

3. Tubes about Focal Curve

Let $C_\gamma(s) = (\gamma + c_1 n + c_2 b)(s)$ be focal curve of γ. The frenet vectors and Frenet formula of this curve are as follows.

$$T_F = C_\gamma'(s), N_F = \frac{T_F'}{\|T_F\|}, B_F = T_F \times N_F,$$
$$\begin{bmatrix} T_F'(s) \\ N_F'(s) \\ B_F'(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa_F & 0 \\ -\kappa_F & 0 & \tau_F \\ 0 & -\tau_F & 0 \end{bmatrix} \begin{bmatrix} T_F(s) \\ N_F(s) \\ B_F(s) \end{bmatrix}.$$

The tube surface about C_γ are defined by

$$X(s,\theta) = C_\gamma(s) + r(\cos\theta N_F(s) + \sin\theta B_F(s)). \tag{8}$$

The first and second fundemental forms of this surface, respectively, are obtained by

$$I = [(1 - r\kappa_F \cos\theta)^2 + r^2\tau_F^2]ds^2 + 2r^2\tau_F ds d\theta + r^2 d\theta^2. \tag{9}$$

and

$$II = [-r\tau_F^2 + \kappa_F(1 - r\kappa_F\cos\theta)\cos\theta]ds^2 - r\tau_F ds d\theta - r d\theta^2. \tag{10}$$

where κ_F and τ_F are denote the curvature and the torsion of the focal curve, respectively. The normal of the surface are as follows:

$$\tilde{n} = \sin\theta B_F + \cos\theta N_F \tag{11}$$

The Gauss curvature and mean curvature of the surface, respectevily, given by

$$K = \frac{\kappa_F \cos\theta}{r_F \kappa_F \cos\theta - 1}, \ H = \frac{r}{2}[1 - 2r\kappa_F\cos\theta]. \tag{12}$$

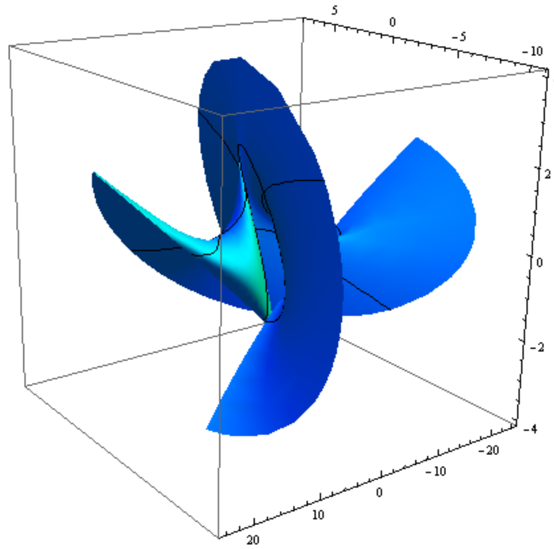
Example: Let $C_\gamma(s) = (-\frac{16}{9}\cos\frac{s}{5}, -\frac{16}{9}\sin\frac{s}{5}, \frac{4}{5}s)$ be a focal curve. The frenet vectors of this curve are as follows.

$$\begin{aligned} T_F &= (\frac{16}{45}\sin\frac{s}{5}, -\frac{16}{45}\cos\frac{s}{5}, \frac{4}{5}) \\ N_F &= (\cos\frac{s}{5}, \sin\frac{s}{5}, 0) \\ B_F &= (-\frac{4}{5}\sin\frac{s}{5}, \frac{4}{5}\cos\frac{s}{5}, \frac{16}{45}) \end{aligned} \tag{13}$$

The tube about $C_\gamma(s)$ is given by

$$X(s,\theta) = C_\gamma(s) + r(\cos\theta N_F + \sin\theta B_F) \tag{14}$$

The figure of the surface $X(s,\theta)$ is given as follows:



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