Dynamics of Curtailing Obesity

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Abstract

In recent years, the numbers of obese individuals in many countries are increased significantly. Many of the strategies that produce good weight loss and maintain health will help to reduce obesity by taking proper diet and regular exercise. Many people also prefer Bariatric surgery to become normal. In this paper, we are going to discuss two ways to prevent obesity one is by exercise and secondly by Bariatric surgery. In order to study the impact of obesity mathematically a system of non-linear ordinary differential equation is formulated. The stability analysis is carried out for obesity model. Numerical simulation is studied for validated data.

Keywords: Mathematical Model, Obesity, Bariatric Surgery, Basic Reproduction Number, Simulation

1. Introduction

"Health is wealth" the famous quote highlights the importance of good health in our life [6]. Life is not only to make it better in the present but also to prepare for future [5]. It is very difficult to stay healthy for some individual especially those who have passion of eating but there is always something which could keep us healthy like regular exercise, proper diet and vitamins [2]. Unhealthy person suffers from health problems like heart disease, obesity, diabetes, blood pressure, gastrointestinal problems etc.

Now-a-days, obesity has become a major problem in many countries. Obesity results when an individual intake more calories than it burn. There are more than 40 medical conditions that are associated with obesity like high blood pressure, high cholesterol, diabetes, heart disease, brain stroke and many more [4]. To reduce obesity individual prefers like regular exercise, takes proper diet and some go for surgery like Bariatric surgery. Bariatric surgery is an option if one has serious obesity and is not able to lose enough weight to improve one's health as compared to other method. There are three types of Bariatric operations most often: gastric band, gastric bypass and gastric sleeve [7]. Bariatric surgery costs around 2.5 to 5 lakhs, which is not affordable to everyone. Bariatric surgery may cause side effects like infection, diarrhoea, nutritional shortages and hernia. There are several ways to prevent obesity among which exercise is one of the most effective way. Exercise can help to reduce weight but it serves best when combined with a lower calorie food. Exercise has several other benefits such as lowering the risk of heart disease, stroke, high blood pressure, diabetes etc. Regular physical activity can also make an individual feel better because it helps to handle stress, control weight, to feel more energetic, reduces anxiety and many more [3].

In this paper, we will analyze how an individual recurs from obesity using an application of *SEIR* model. Section-2 consists of mathematical model including the description of notations, parametric value and basic reproduction number. Local and global stability of the system is studied in section 3.1 and 3.2 respectively. Section 4 and 5 consist of numerical simulation with their interpretation and conclusion respectively.

2. Mathematical Model

Here, we formulate a mathematical model for the analysis of how an obese individual gets recovered. The notations along with its parametric values are shown in Table1.

Table1: Notations and its Parametric Values

Notations		Parametric Values
$N(t)$	Sample size at any instant time t	2000
$O_B(t)$	Number of obese individuals at any instant time t	100
$B_S(t)$	Number of individuals opting for Bariatric surgery at any instant time t	10
$E_X(t)$	Number of obese individuals who prefers exercise at any instant time t	30
$R(t)$	Number of recovered individuals at any instant time t	15
B	New recruitment rate	0.05
β_1	Rate at which obese individual prefers Bariatric Surgery	0.30

β_2	Rate at which obese individuals prefers exercise	0.70
α	Rate at which obese individual moves exercise to Bariatric Surgery	0.004
η	Rate at which obese individual gets recovered through Bariatric Surgery	0.85
ε	Rate at which obese individual gets recovered through exercise	0.40
δ	Rate at which recover individuals become obese	0.06
μ	Natural escape rate	0.60

The transmission diagram of an individual gets recover from obesity is shown in figure 1

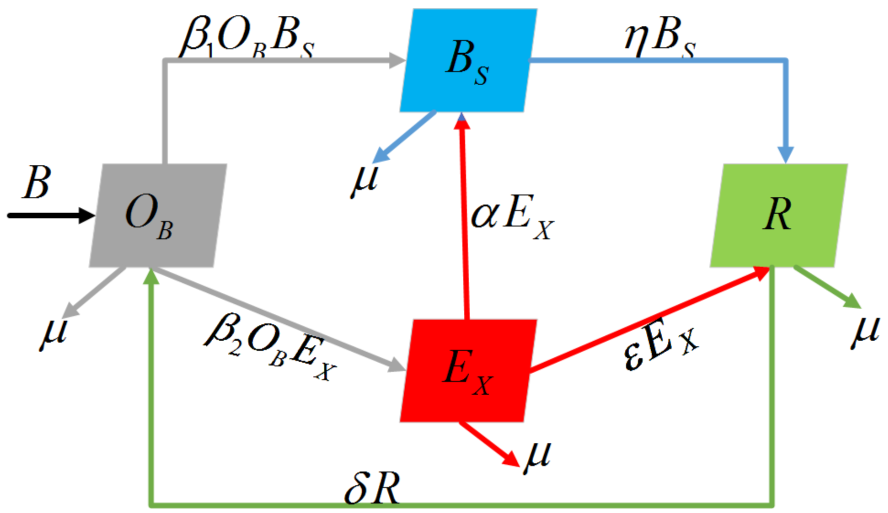

Figure 1: Transmission diagram from obesity to recovery

Obesity is the prime factor in this model. Obesity can be reduced by two ways. β_1% of obese individual opt for bariatric surgery and β_2% prefers exercise to recover from obesity. η and ε are the recovery rate from bariatric surgery and exercise respectively. This recovered individual may get obese at the rate δ. Here B and μ describes new recruitment rate and natural escape rate respectively.

The propose model can be described by the system of non-linear ordinary differential equations as

$$\frac{dO_B}{dt} = B - \beta_1 O_B B_S - \beta_2 O_B E_X + \delta R - \mu O_B$$

$$\frac{dB_S}{dt} = \beta_1 O_B B_S - \eta B_S + \alpha E_X - \mu B_S$$

$$\frac{dE_X}{dt} = \beta_2 O_B E_X - \alpha E_X - \varepsilon E_X - \mu E_X \tag{1}$$

$$\frac{dR}{dt} = \eta B_S + \varepsilon E_X - \delta R - \mu R$$

With $O_B + B_S + E_X + R \le N$ and $O_B > 0, B_S, E_X, R \ge 0$

Adding the above set of system of equations (1) we get,

$$\frac{d}{dt}\left(O_B + B_S + E_X + R\right) = B - \mu\left(O_B + B_S + E_X + R\right) \ge 0 \tag{2}$$

This gives $\lim_{t\to\infty}\sup\left(O_B + B_S + E_X + R\right) \le \frac{B}{\mu}$ (3)

Thus the feasible region for (1) is,

$$\Lambda = \left\{\left(O_B + B_S + E_X + R\right) / O_B + B_S + E_X + R \le \frac{B}{\mu}, O_B > 0, B_S, E_X, R \ge 0\right\}. \tag{4}$$

On solving set of equation (1) we get obese equilibrium point $E_0 = \left(\frac{B}{\mu}, 0, 0, 0\right)$.

Now, before find the other equilibrium point we need to calculate the basic reproduction number to know the motion of obese individual in the system using the next generation matrix method [1]. The next generation matrix method gives the spectral of matrix fv^{-1} where f and v are the Jacobian matrices of F and V evaluated with respect to each compartment at an obese equilibrium point.

Let $X = \left(B_S, E_X, R, O_B\right)$

$$\therefore \frac{dX}{dt} = F\left(X\right) - V\left(X\right) \tag{5}$$

where $F(X)$ and denotes the rate of new obese individuals in the compartment and $V(X)$ denote the transmission of obese individuals from one component to other which is follows:

$$F(X)=\begin{bmatrix}\beta_1 O_B B_S\\ \beta_2 O_B E_X\\ 0\\ 0\end{bmatrix} \text{ and } V(X)=\begin{bmatrix}\eta B_S-\alpha E_X+\mu B_S\\ \alpha E_X+\varepsilon E_X+\mu E_X\\ -\eta B_S-\varepsilon E_X+\delta R+\mu R\\ -B+\beta_1 O_B B_S+\beta_2 O_B E_X-\delta R+\mu O_B\end{bmatrix}$$

Now, the derivative of F and V of order 4×4 defined as

$$f=\left[\frac{\partial F_i(E_0)}{\partial X_j}\right] \text{ and } v=\left[\frac{\partial V_i(E_0)}{\partial X_j}\right] \text{for } i,j=1,2,3,4$$

$$\text{So, } f=\begin{bmatrix}\frac{\beta_1 B}{\mu} & 0 & 0 & 0\\ 0 & \frac{\beta_2 B}{\mu} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \text{ and } v=\begin{bmatrix}\eta+\mu & -\alpha & 0 & 0\\ 0 & \alpha+\varepsilon+\mu & 0 & 0\\ -\eta & -\varepsilon & \delta+\mu & 0\\ \frac{\beta_1 B}{\mu} & \frac{\beta_2 B}{\mu} & -\delta & \mu\end{bmatrix}$$

The basic reproduction number R_0 calculated at an equilibrium point E_0 is

$$R_0=\frac{B\left(\beta_1(\alpha+\varepsilon+\mu)+\beta_2(\eta+\mu)\right)}{\mu(\eta+\mu)(\alpha+\varepsilon+\mu)} \tag{6}$$

On solving the set of equation (1), we get the other two points namely,

- $E_1=(O_B, B_S, 0, R)$ when person does not opt for exercise.

  where, $O_B=\dfrac{\eta+\mu}{\beta_1}$, $B_S=\dfrac{B\beta_1(\delta+\mu)-\mu\left(\mu(\eta-\delta+\mu)+\delta\eta\right)}{\mu\beta_1(\eta+\mu+\delta)}$, $E_X=0$,

  $R=\dfrac{\eta\left(-\mu(\eta+\mu)+B\beta_1\right)}{\beta_1\mu(\eta+\mu+\delta)}$

- $E^*=\left(O_B^*, B_S^*, E_X^*, R^*\right)$ when all exists.

  where, $O_B^*=\dfrac{\alpha+\varepsilon+\mu}{\beta_2}$, $B_S^*=\dfrac{\alpha\left[\mu\{-\mu(\alpha+\delta+\varepsilon+\mu)+\delta(\alpha-\varepsilon)\}+B\beta_2\right]}{\mu\begin{bmatrix}\beta_1\{-\varepsilon(2\mu+\varepsilon+\delta+\alpha)-\mu(\alpha+\delta+\mu)-\alpha\delta\}+\\ \beta_2\{\mu(\alpha+\delta+\eta+\varepsilon+\mu)+\alpha(\delta+\eta)-\eta(\varepsilon+\delta)\}\end{bmatrix}}$,

$$E_X^* = \frac{\begin{bmatrix}\{B\beta_2(\mu+\delta)\} - \mu\{\mu(\delta+\mu+\varepsilon+\alpha)+\delta(\varepsilon+\alpha)\} \\ \{-\beta_1(\mu+\varepsilon+\alpha)+\beta_2(\eta+\mu)\}\end{bmatrix}}{\beta_2\mu\begin{bmatrix}\beta_1\{-\varepsilon(2\mu+\varepsilon+\delta+\alpha)-\mu(\alpha+\delta+\mu)-\alpha\delta\}+ \\ \beta_2\{\mu(\alpha+\delta+\eta+\varepsilon+\mu)+\alpha(\delta+\eta)-\eta(\varepsilon+\delta)\}\end{bmatrix}},$$

$$R^* = \frac{\begin{matrix}\beta_1\varepsilon\left[-B\beta_2(\mu+\alpha+\varepsilon)+\mu\{\mu(2(\varepsilon+\mu)+\mu)+\varepsilon^2+\alpha(2\varepsilon+\mu)\}\right] \\ +\beta_2\begin{bmatrix}\eta\{\alpha(\beta_2 B-\mu(\varepsilon+\alpha))-\mu^2(\alpha+\varepsilon)+\varepsilon B\beta_2\} \\ -\mu\varepsilon\{\mu(\mu+\varepsilon+\alpha)+\eta\varepsilon-\beta_2 B\}\end{bmatrix}\end{matrix}}{\beta_2\mu\begin{bmatrix}\beta_1\{-\varepsilon(2\mu+\varepsilon+\delta+\alpha)-\mu(\alpha+\delta+\mu)-\alpha\delta\}+ \\ \beta_2\{\mu(\alpha+\delta+\eta+\varepsilon+\mu)+\alpha(\delta+\eta)-\eta(\varepsilon+\delta)\}\end{bmatrix}}$$

3. Stability Analysis

In this action local and global stability of an obese equilibrium point E_0 is discussed.

3.1 Local Stability

If all the eigenvalues of the Jacobian matrix of system (1) have negative real part then equilibrium point is locally stable [8][9]. For this, at $E_0 = \left(\frac{B}{\mu}, 0, 0, 0\right)$ the Jacobian matrix of system (1) is

$$J = \begin{bmatrix} -\mu & -\beta_1\frac{B}{\mu} & -\beta_2\frac{B}{\mu} & \delta \\ 0 & \beta_1\frac{B}{\mu}-\eta-\mu & \alpha & 0 \\ 0 & 0 & \beta_2\frac{B}{\mu}-\alpha-\varepsilon-\mu & 0 \\ 0 & \eta & \varepsilon & -\delta-\mu \end{bmatrix}$$

Thus, $trace(J) < 0$ provided $(\beta_1+\beta_2)\frac{B}{\mu} < 4\mu+\eta+\varepsilon+\alpha+\delta$ (7)

Hence, E_0 is globally stable.

3.2 Global Stability

The obesity model is globally stable if $\det\left(I - fv^{-1}\right) > 0$ [8][9]

$$\det\left(I - fv^{-1}\right) = 1 - R_0 = 1 - 0.07333 = 0.9267 > 0 \quad (8)$$

Hence, E_0 is globally stable.

4. Sensitivity Analysis

The sensitivity analysis for all model parameter is discussed in this section. The sensitivity index of the parameter value is computed by $\gamma_\theta^{R_0} = \frac{\partial R_0}{\partial \theta} * \frac{\theta}{R_0}$, where θ is the model parameter.[8]

Table 2: Sensitivity Analysis

Parameter	Value	Parameter	Value
B	+	α	-
β_1	+	η	-
β_2	+	ε	-
		μ	-

New recruitment rate, rate at which obese individuals prefers Bariatric surgery and the rate at which obese individuals prefers exercise all these model parameters helps to recover from obesity among individual in the society, whereas rate at which obese individuals moves from exercise to Bariatric surgery, rate at which obese individuals gets recovered through Bariatric surgery, rate at which obese individuals gets recovered through exercise and natural escape rate proves to be helpless. The rate at which recover individuals become obese have no impact on model.

5. Numerical Simulation

In this section, we will study the numerical results of all compartments.

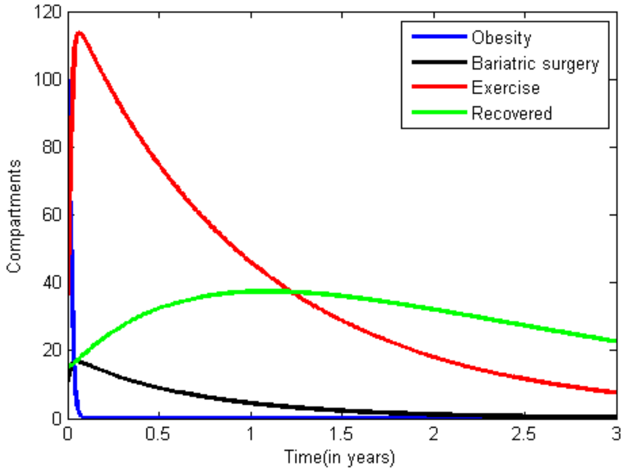

Figure 2: Movement of individuals in each compartment

Figure 2 indicates the numbers of obese individuals are decreasing which means that the individual have started up exercise or have made up their decision of going for Bariatric Surgery as these obesity increases the risk factors among them, so as to protect them from this disease or other problems, they adopt the path of surgery or exercise and gets recovered from this obesity.

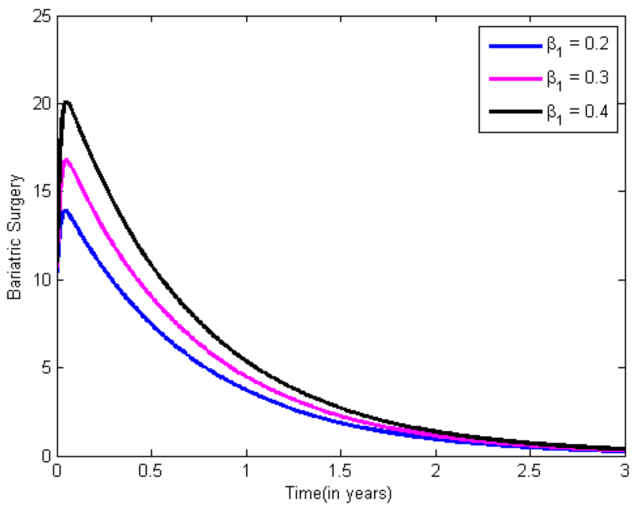

Figure 3: Effect of different values of β_1 on B_S

Figure 3 shows that if the demand of obese individuals for Bariatric Surgery (β_1) is increased from 20% to 40% the number of individuals preferring for Bariatric surgery increases from approximately 14 to 20 in the beginning but then after they decreases and become stable which means that they have understood the side effect of Bariatric surgery.

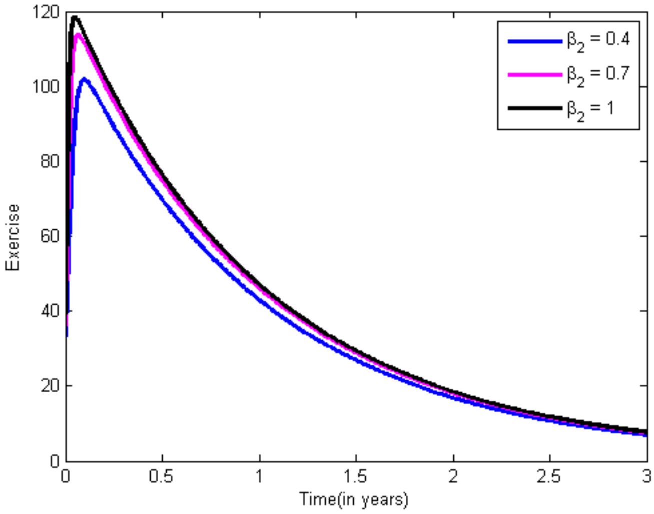

Figure 4: Effect of different values of β_2 on E_X

Figure 4 shows that initially the requirement of exercise for obese individuals prevails more but slowly it starts to cure the individuals from obesity which is observed from the figure.

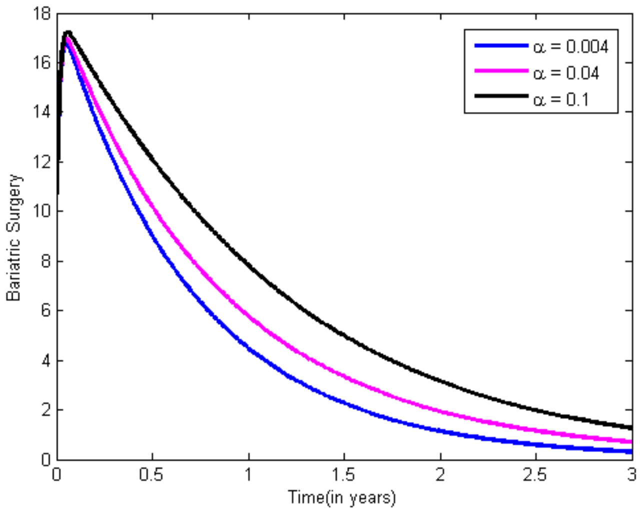

Figure 5: Effect of different values of α on B_S

Figure 5 indicates that if the rate of obese individual moving from exercise to Bariatric surgery(α) is increased from 0.4% to 10% the demand of opting for Bariatric surgery among obese individual increases in the first few years so as to protect them from harmful attack.

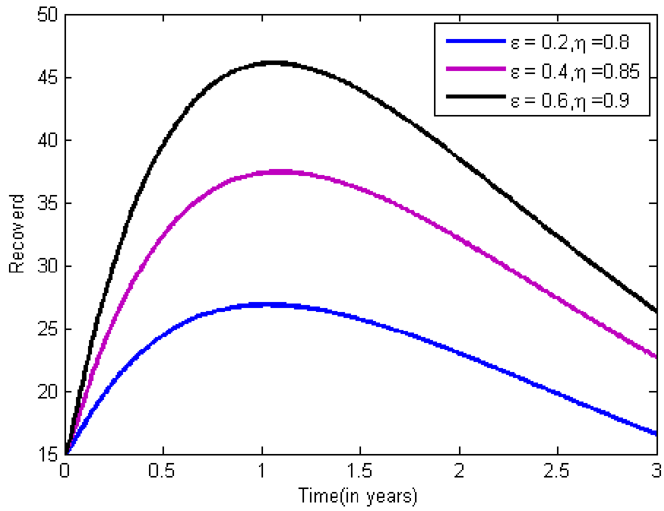

Figure 6: Effect of different values of ε and η on R

Figure 6 indicates that the collective impact of Bariatric surgery and exercise among obese individual helps in recover them from obesity faster as compared to the one opting for either only Bariatric Surgery or only exercise.

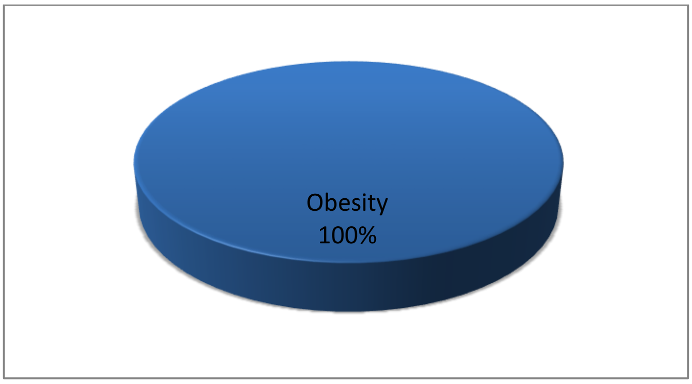

Figure 7: Percentage of Obesity at point E_0

Figure 7 shows that when individual do not opt for bariatric surgery or exercise, there is no chance for getting recovered due to which 100% obesity exists in the society.

The parametric values used for figure 8 and 9 are $B = 0.07, \beta_1 = 0.3, \beta_2 = 0.3, \alpha = 0.004, \eta = 0.05, \varepsilon = 0.04, \delta = 0.007, \mu = 0.06$ which defines the equilibrium point E_1 and E^* numerically.

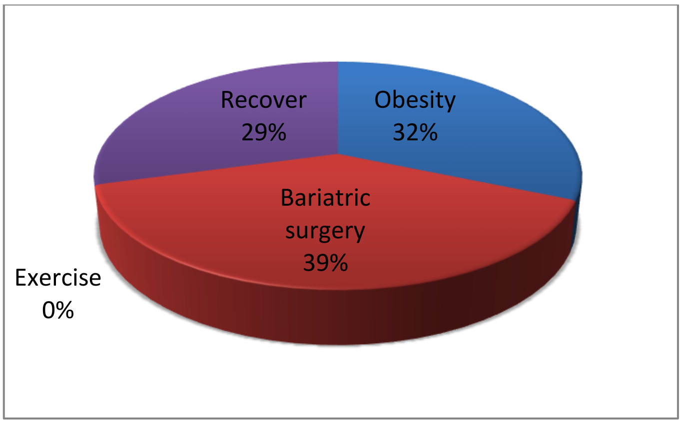

Figure 8: Individuals who do not opt for exercise

Figure 8 shows that out of 32% of obese individuals in society if 39% opts only for Bariatric surgery then 29% individuals recovered from obesity.

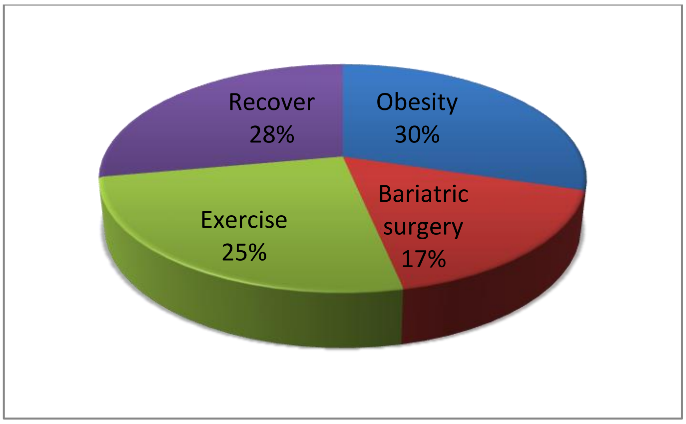

Figure 9: Individuals who do opt for exercise and bariatric surgery

Figure 9 shows that 28% of individuals get recovered, if 17% prefers bariatric surgery and 25% prefers exercise out of 30% of obese individuals.

5. Conclusion

Here, a non-linear mathematical model to curtail obesity is formulated. Also at the equilibrium point E_0 the given system is locally and globally stable. In today's fast life-style people suffers from many disease because of the excess weight. It is very important to maintain a healthy weight as it helps to lower the risk from developing disease, helps to feel good and brings more energy to enjoy one's life.

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