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ON BISECTOR RULED SURFACE OF GIVEN A SPECIAL CURVE IN MINKOWSKI 3- SPACE AND ITS APPLICATION

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Abstract. In this paper presents bisector surfaces of some special curves in Minkowski space. Firstly, we give the Bisector surface generated a point and a Bertrand curve of a given space curve Then, we show that the This Bisector surface is the ruled surface. Some applications is also given for these surfaces.

1. Introduction

The Bisector surface is a special surface because this surface is defined by any two objects in 2-dimensional or 3-dimensional space. These objects can be point-curve, curve-curve or surface-surface. Moreover the Bisector surface is the set of points which are equidistant from the two objects, [4]. Then, this surface is often used in scientific research from the past. For example, Horvath proved that all bisectors are topological images of a plane of the embedding Euclidean 3-space iff the shadow boundaries of the unit ball K are topological circles in [8], and Elber studyed a new computational model in $\mathbb{E}^3$ in [5]. The parallel curves are developed by Chrastinova in 2007. These curves are not easy to characterize in 3-dimensional space until this time. Additionally, he study parallel curves of a special curve as helices.

In this paper, we study Bisector surfaces of some special curves in Minkowski 3-space. The first, we give properties and the basic concepts of curves in $\mathbb{E}_1^3$. Then, we construct bisector surface generated by pedal and parallel curves of given timelike curve in $\mathbb{E}_1^3$. Moreover, we show how to generate the this surface. Finally, we give an example of these surfaces in $\mathbb{E}_1^3$.

2. **Preliminaires**

Given a spatial curve $\xi : s \to \xi(s)$, which is parameterized by arc-length parameter s. For each point of $\xi(s)$, the set $\{\mathbf{t}(s), \mathbf{n}(s), \mathbf{b}(s)\}$ is called the Frenet Frame along $\xi(s)$, where $\mathbf{t}(s)$, $\mathbf{n}(s)$, $\mathbf{b}(s)$ are the unit tangent, principal normal,

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and binormal vectors of the curve at the point $\xi(s)$, respectively. Derivative of the Frenet frame according to arc-length parameter is governed by the relations;

$$\begin{pmatrix} \mathbf{e}_1'(s) \\ \mathbf{e}_2'(s) \\ \mathbf{e}_3'(s) \end{pmatrix} = \begin{pmatrix} 0 & \kappa_1(s) & 0 \\ -\kappa_1(s) & 0 & \kappa_2(s) \\ 0 & -\kappa_2(s) & 0 \end{pmatrix} \begin{pmatrix} \mathbf{e}_1(s) \\ \mathbf{e}_2(s) \\ \mathbf{e}_3(s) \end{pmatrix},$$

where

$$\kappa_1(s) = \|\xi''(s)\|, \ \kappa_2(s) = \frac{(\xi'(s),\ \xi''(s),\ \xi'''(s))}{\|\xi''(s)\|^2}.$$

Assume that $\xi(s)$ is an arbitrary timelike curve in the space $\mathbb{E}_1^3$, then, the Frenet formulae of $\xi(s)$ are given by

$$\begin{pmatrix} \mathbf{e}_1'(s) \\ \mathbf{e}_2'(s) \\ \mathbf{e}_3'(s) \end{pmatrix} = \begin{pmatrix} 0 & \kappa_1(s) & 0 \\ \kappa_1(s) & 0 & \kappa_2(s) \\ 0 & -\kappa_2(s) & 0 \end{pmatrix} \begin{pmatrix} \mathbf{e}_1(s) \\ \mathbf{e}_2(s) \\ \mathbf{e}_3(s) \end{pmatrix},$$

where

$$\langle \mathbf{e}_1, \mathbf{e}_1 \rangle = -1, \ \langle \mathbf{e}_2, \mathbf{e}_2 \rangle = \langle \mathbf{e}_3, \mathbf{e}_3 \rangle = 1.$$

Assume that Minkowski 3-space is consider $\mathbb{E}_1^3 = \left[\mathbb{E}_1^3,\ (-,+,+)\right]$ and the Lorentzian inner product and vector product, respectively, are

$$\langle X, Y \rangle = -x_1y_1 + x_2y_2 + x_3y_3,$$
$$X \times Y = (x_2y_3 - x_3y_2, x_1y_3 - x_3y_1, x_2y_1 - x_1y_2).$$

where $X = (x_1, x_2, x_3)$ and $Y = (y_1, y_2, y_3) \in \mathbb{E}_1^3$, [25].

A surface in $\mathbb{E}_1^3$ is called a timelike surface if the normal vector on the surface is spacelike vector. A surface is called spacelike surface if the normal vector on surface is the timelike vector.

3. **The Bisector Surface Obtained a Point and a Bertrand curve**

In this paper, our goal is construct Bisector surface generated a point and a Bertrand curve in $\mathbb{E}_1^3$. Let $\alpha : I \subset \mathbb{R} \to \mathbb{E}_1^3$ be a timelike curve in $\mathbb{E}_1^3$ and (α, β) be Bertrand curve couple in Minkowski 3-space. Then, the curve β can be written as

$$\beta(s) = \alpha(s) + \lambda(s)\,\mathbf{n}_a(s), \tag{3.1}$$

where $\lambda(s)$ is a smooth function on I.

Theorem 3.1 Suppose that $\mathfrak{W} = (\omega_1, \omega_2, \omega_3)$ is a fixed point and (α, β) is called Bertrand curve couple. Then, the rational ruled bisector surface $\mathfrak{T}(s,t)$ is

$$\mathfrak{T}(s,t) = \mathfrak{T}(s) + t\mathcal{N}(s); \text{ for } s,t \in \mathbb{R},$$

where

$$\begin{aligned} \mathcal{N}(s) &= \mathbf{t}_\beta(s) \times (\beta(s) - \mathfrak{W}) \\ &= (\mathbf{t}_\beta^3(\beta_2 - \omega_2) - \mathbf{t}_\beta^2(\beta_3 - \omega_3), \\ &\quad \mathbf{t}_\beta^3(\beta_1 - \omega_1) - \mathbf{t}_\beta^1(\beta_3 - \omega_3), \\ &\quad \mathbf{t}_\beta^1(\beta_2 - \omega_2) - \mathbf{t}_\beta^2(\beta_1 - \omega_1)), \end{aligned}$$

and

$$t_1 = \frac{1}{\mathfrak{J}} \begin{vmatrix} k_1 & \mathbf{t}_\beta^2 & \mathbf{t}_\beta^3 \\ k_2 & n_2 & n_3 \\ k_3 & (\beta_2 - \omega_2) & (\beta_3 - \omega_3) \end{vmatrix},$$

$$t_2 = \frac{1}{\mathfrak{J}} \begin{vmatrix} -\mathbf{t}_\beta^1 & k_1 & \mathbf{t}_\beta^3 \\ -n_1 & k_2 & n_3 \\ -(\beta_1 - \omega_1) & k_3 & (\beta_3 - \omega_3) \end{vmatrix},$$

$$t_3 = \frac{1}{\mathfrak{J}} \begin{vmatrix} -\mathbf{t}_\beta^1 & \mathbf{t}_\beta^2 & k_1 \\ -n_1 & n_2 & k_2 \\ -(\beta_1 - \omega_1) & (\beta_2 - \omega_2) & k_3 \end{vmatrix},$$

where $\mathfrak{J}$ is

$$\begin{vmatrix} -\mathbf{t}_\beta^1 & \mathbf{t}_\beta^2 & \mathbf{t}_\beta^3 \\ -n_1 & n_2 & n_3 \\ -(\beta_1 - \omega_1) & (\beta_2 - \omega_2) & (\beta_3 - \omega_3) \end{vmatrix}.$$

Proof. Taking the derivative of the eq (3.1), it is clear that

$$\beta' = \mathbf{t}_\beta = (1 + \lambda\kappa_1)\mathbf{t}_\alpha + \lambda'\mathbf{n}_\alpha + \lambda\kappa_2\mathbf{b}_\alpha.$$

On the other hand, let $\mathfrak{T}$ be a bisector point of $\beta(s)$ and $\mathfrak{W}$ with its foot points at $\beta(s)$ and $\mathfrak{W}$, respectively. Then, it is clear that the point $\mathfrak{T}$ is contained both the normal plane $\mathcal{H}(s_0)$ and the bisector plane $\mathcal{H}_b(s_0)$. In that case $\mathcal{H}(s_0)$ and $\mathcal{H}_b(s_0)$ intersect in a line $h(s_0)$.

Suppose that $\mathcal{N}(s)$ is the direction vector of $h(t)$, then it is clear that $\mathcal{N}(s)$ is contained in both $\mathcal{H}(s)$ and $\mathcal{H}_b(s)$ and it is orthogonal to the normal vectors of $\mathcal{H}(s)$ and $\mathcal{H}_b(s)$. Therefore, the following equation can be written easily

$$\begin{aligned} \mathcal{N}(s) &= \mathbf{t}_\beta(s) \times (\beta(s) - \mathfrak{W}) \\ &= (\mathbf{t}_\beta^3(\beta_2 - \omega_2) - \mathbf{t}_\beta^2(\beta_3 - \omega_3), \\ &\quad \mathbf{t}_\beta^3(\beta_1 - \omega_1) - \mathbf{t}_\beta^1(\beta_3 - \omega_3), \\ &\quad \mathbf{t}_\beta^1(\beta_2 - \omega_2) - \mathbf{t}_\beta^2(\beta_1 - \omega_1)), \end{aligned}$$

which is a rational vector field.

An auxiliary plane ($\mathcal{AP}$) $\mathcal{H}_n(s)$ is orthogonal to the intersection line $h(s)$ and passes through the fixed point $\mathfrak{W}$. So $\mathfrak{T}(s)$ is the closest point of $h(s)$ to $\mathfrak{W}$, $\mathcal{AP}$ can be written as:

$$\mathcal{H}_n(s): \quad \langle \mathfrak{T} - \mathfrak{W}, \mathcal{N}(s) \rangle = 0.$$

If the above equations are considered together, we obtain the following equations for intersection point $\mathfrak{T}$:

$$\begin{aligned} \langle \mathfrak{T}, \mathbf{t}_\beta(s) \rangle &= \langle \beta(s), \mathbf{t}_\beta(s) \rangle, \\ \langle \mathfrak{T}, \mathcal{N}(s) \rangle &= \langle \mathfrak{W}, \mathcal{N}(s) \rangle, \\ \langle \mathfrak{T}, \beta(s) - \mathfrak{W} \rangle &= \frac{1}{2}(\|\beta(s)\|^2 - \|\mathfrak{W}\|^2). \end{aligned}$$

Then, we have the following matrix equation,

$$\begin{bmatrix} -\mathbf{t}_\beta^1 & \mathbf{t}_\beta^2 & \mathbf{t}_\beta^3 \\ -n_1 & n_2 & n_3 \\ -(\beta_1 - \omega_1) & (\beta_2 - \omega_2) & (\beta_3 - \omega_3) \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}, \tag{3.2}$$

where

$$k_1 = -\beta_1 \mathbf{t}_\beta^1 + \beta_2 \mathbf{t}_\beta^2 + \beta_3 \mathbf{t}_\beta^3,$$
$$k_2 = -\omega_1 n_1 + \omega_2 n_2 + \omega_3 n_3,$$
$$k_3 = \frac{1}{2}(\|\beta(s)\|^2 - \|\mathfrak{W}\|^2).$$

If eq. (3.2) can be solved, then following equation is obtained by

$$t_1 = \frac{1}{\mathfrak{J}} \begin{vmatrix} k_1 & \mathbf{t}_\beta^2 & \mathbf{t}_\beta^3 \\ k_2 & n_2 & n_3 \\ k_3 & (\beta_2 - \omega_2) & (\beta_3 - \omega_3) \end{vmatrix},$$
$$t_2 = \frac{1}{\mathfrak{J}} \begin{vmatrix} -\mathbf{t}_\beta^1 & k_1 & \mathbf{t}_\beta^3 \\ -n_1 & k_2 & n_3 \\ -(\beta_1 - \omega_1) & k_3 & (\beta_3 - \omega_3) \end{vmatrix},$$
$$t_3 = \frac{1}{\mathfrak{J}} \begin{vmatrix} -\mathbf{t}_\beta^1 & \mathbf{t}_\beta^2 & k_1 \\ -n_1 & n_2 & k_2 \\ -(\beta_1 - \omega_1) & (\beta_2 - \omega_2) & k_3 \end{vmatrix},$$

where $\mathfrak{J}$ is

$$\begin{vmatrix} -\mathbf{t}_\beta^1 & \mathbf{t}_\beta^2 & \mathbf{t}_\beta^3 \\ -n_1 & n_2 & n_3 \\ -(\beta_1 - \omega_1) & (\beta_2 - \omega_2) & (\beta_3 - \omega_3) \end{vmatrix}.$$

The rational ruled bisector surface $\mathfrak{T}(s,t)$ can be constructed as follows:

$$\mathfrak{B}(s,t) = \mathfrak{B}(s) + t\mathcal{N}(s)\,; \text{ for } s,t \in IR.$$

Now, we will examine the degenerate case.

Remark 3.2 Since the direction vector $N(s)$ is cross product of $\mathbf{t}_\beta(s)$ and $\beta(s) - \mathfrak{W}$, these vectors are linearly independent. On the contrary, we assume that these vectors $\beta(s)$ and $\beta(s) - \mathfrak{W}$ are linearly dependent. Since $\beta(s)$ is regular, $\mathbf{t}_\beta(s) \neq 0$ and so $\beta(s)$ and $\beta(s) - \mathfrak{W}$ parallel to each other. We can write

$$\mathfrak{W} - \beta(s) = \mathfrak{K}(s)\,\mathbf{t}_\beta, \tag{3.3}$$

where $\mathfrak{K}(s) \in \mathbb{R}$. From eq. (3.3),

$$\mathfrak{W} = \beta(s) + \mathfrak{K}(s)\,\mathbf{t}_\beta.$$

That is, the point $\mathfrak{W}$ is on tangent of curve $\beta(s)$ for all s.

4. **Application**

Let us consider a unit speed timelike curve in $\mathbb{E}_1^3$ by

$$\alpha = \alpha(s) = (\sqrt{2}s, \cos s, \sin s). \tag{3.3.1}$$

One can calculate its Frenet-Serret apparatus as the following, [9],

$$\mathbf{t}(s) = (\sqrt{2}, -\sin s, \cos s),$$
$$\mathbf{n}(s) = (0, -\cos s, -\sin s),$$
$$\mathbf{b}(s) = (-1, \sqrt{2}\sin s, -\sqrt{2}\cos s).$$

Then, the curvatures of α is given by

$$\kappa(s) = 1,$$
$$\tau(s) = \sqrt{2}.$$

On the other hand, $\mathcal{P}(s)$ parallel curve of a timelike $\alpha(s)$ curve with parametrized by arc-length in $\mathbb{E}_1^3$ obtained as follows

$$\mathcal{P}(s) = (\sqrt{2}s - 2, 2\cos s + 2\sqrt{2}\sin s, 2\sin s - 2\sqrt{2}\cos s), \tag{3.3.2}$$

where we choose $t = \sqrt{5}$. Taking the derivative of the eq. (3.3.2), we can computed by

$$\mathcal{P}(s) = \mathbf{t_P} = (\sqrt{2}, -2\sin s + 2\sqrt{2}\cos s, 2\cos s + 2\sqrt{2}\sin s).$$

Now, if we choose $Q = (-2, 2, 2)$ be a fixed point in $\mathbb{E}_1^3$, the direction vector $\mathcal{N}(s)$ of the intersection line $l(t)$ between two planes $\mathcal{L}(s)$ and $\mathcal{L}_b(s)$ obtained by

$$\begin{aligned}\mathcal{N}(s) = (&12 - (4 - 4\sqrt{2})\cos s - (4 + 4\sqrt{2})\sin s,\\ &2\sqrt{2} + (4 + 2\sqrt{2}s)\cos s - (2\sqrt{2} - 4s)\sin s,\\ &- 2\sqrt{2} + (2\sqrt{2} - 4s)\cos s + (4 + 2\sqrt{2}s)\sin s).\end{aligned}$$

The intersection point $\mathfrak{B} = (b_1, b_2, b_3)$ of three planes: $\mathcal{L}(s)$, $\mathcal{L}_n(s)$, and $\mathcal{L}_b(s)$ can be computed by solving the following simultaneous linear equations in $\mathfrak{B}$:

$$\begin{aligned}\langle \mathfrak{B}, \mathbf{t_p}(s)\rangle &= r_1,\\ \langle \mathfrak{B}, \mathcal{N}(s)\rangle &= r_2,\\ \langle \mathfrak{B}, \mathcal{P}(s) - Q\rangle &= r_3,\end{aligned}$$

where

$$\begin{aligned}r_1 &= \langle \mathcal{P}(s), \mathbf{t_p}(s)\rangle\\ r_2 &= \langle Q, \mathcal{N}(s)\rangle,\\ r_3 &= \frac{1}{2}(\|\mathbf{P}(s)\|^2 - \|Q\|^2).\end{aligned}$$

Then, by Cramer's rule, eq. (3.3.3) can be solved as follows:

$$\begin{aligned}b_1 = \frac{1}{\mathfrak{J}}[&r_2(12 + (-4 + 4\sqrt{2})\cos s + (-4 + 4\sqrt{2})\sin s)\\ &+ n_2((-10 + 4\sqrt{2}s - 4\sqrt{2} + 2s)\cos s + (2\sqrt{2}\\ &+ 2\sqrt{2}s - 4s - 8)\sin s + 4s - 4\sqrt{2}) + n_3((4s + 8\\ &- 2\sqrt{2} - 2\sqrt{2}s)\cos s + (4\sqrt{2}s - 10 - 4\sqrt{2} + 2s)\sin s\\ &- 4s + 4\sqrt{2})],\end{aligned}$$

$$\begin{aligned}b_2 = \frac{1}{\mathfrak{J}}[&n_1((-10 - 4\sqrt{2} + 2s + 4\sqrt{2}s)\cos s - (8 - 2\sqrt{2}\\ &+ 4s - 2\sqrt{2}s)\sin s - 4\sqrt{2} + 4s) + r_2((4 + 2\sqrt{2}s)\cos s\\ &- (2\sqrt{2} - 4s)\sin s + 2\sqrt{2}) + n_3(4 + \sqrt{2} - 4s - \sqrt{2}s\\ &+ 2\sqrt{2}s^2)],\end{aligned}$$

$$b_3 = \frac{1}{\mathfrak{J}}[n_1((8-2\sqrt{2}+4s-2\sqrt{2}s)\cos s-(10+4\sqrt{2}-2s$$
$$-4\sqrt{2}s)\sin s-4s+4\sqrt{2})+n_2(-4-\sqrt{2}+4s+\sqrt{2}s$$
$$-2\sqrt{2}s^2)+r_2((2\sqrt{2}-4s)\cos s+(4+2\sqrt{2}s)\sin s-2\sqrt{2})],$$

where

$$\mathfrak{J}= -[-12+(4-4\sqrt{2})\cos s+(4+4\sqrt{2})\sin s]^2$$
$$+[2\sqrt{2}+(4+2\sqrt{2}s)\cos s-(2\sqrt{2}-4s)\sin s]^2$$
$$+[-2\sqrt{2}+(2\sqrt{2}-4s)\cos s+(4+2\sqrt{2}s)\sin s]^2,$$

$$r_2 = 24+(12\sqrt{2}-8s+4\sqrt{2}s)\cos s+(-12\sqrt{2}+8s+4\sqrt{2}s)\sin s,$$
$$n_1 = 12-(4-4\sqrt{2})\cos s-(4+4\sqrt{2})\sin s,$$
$$n_2 = 2\sqrt{2}+(4+2\sqrt{2}s)\cos s+(-2\sqrt{2}+4s)\sin s,$$
$$n_3 = -2\sqrt{2}+(2\sqrt{2}-4s)\cos s+(4+2\sqrt{2}s)\sin s.$$

Then, the rational ruled bisector surface $\mathfrak{B}(s,t)$ can be constructed as follows:

$$\mathfrak{B}(s,t) = \mathfrak{B}(s) + t\mathcal{N}(s)\,;\text{ for } s,t \in IR.$$

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