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Two Heterogeneous Servers Queueing-Inventory System with Multiple Vacations and Server Interruptions

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Abstract

In this study, we shall scrutinize a continuous review inventory queuing system estimating with two heterogeneous servers named as first server and second server. We supposed that the first server is always available but it may subject to some interruptions and the second sever is perfectly reliable but it will undergo vacation during the inventory level is zero or the waiting hall is empty or both. Further, the restrictions are directed as the customer's arrival follows Poisson process, fixing the interrupted server is in exponential rate. In case the waiting hall is full, the arriving customers will enter the orbit. The first server alone allow to search and inviting the customer in the orbit with finite probability only when the waiting hall is empty with positive inventory level and the search time is imperceptible. The ordering quantity of the item is (s, Q) policy. The stationary distribution mode of the number of customers in the waiting hall, the number of customers in the orbit, status of the server and the inventory level is obtained by matrix methods. Some imperative system performance measures are derived in a steady state manner. Several numerical examples are presented to illustrate the optimality of this study.

Keywords: Inventory with service time, Markov process, Heterogeneous servers, (s, S) policy.

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1 Introduction

In the earlier models, the authors studied the inventory system without queuing policy. Nevertheless, in a real time condition, the service time holds significant effect on the queuing policy. The common people are not well familiar with the technical aspects of the particular item they wish to purchase; hence, the condition is not negligible. For an instance, if a customer be set to purchase an electronic item, at first handling and operation procedure of the item shall be known by the customer through demonstration / hands on exposure where the server requires some positive time that causes a time delay in purchasing the items. When the servers/ salesmen are engaged to attend the queries of the customers, the successive arrival of the customers need a hall for waiting. In such a case, this leads to formation of a queue. Hence, the total expected cost rate is influenced by one of the components of expected waiting time of a customer. In order to determine the expected number of units demanded per unit time, some measures of a queuing policy would also be needed. It is essential to study the queuing theory along with inventory control techniques.

Sigman and Simchi-Levi [20] have first measured the performance characteristics of the inventory system with positive service time under the steady state conditions. The random distribution of the service time and exponential distribution of lead and inter-arrival times are considered. The inventory with service policy is studied by Berman et al. [3] in which demand rate of the item and service rates are assumed to be deterministic and constant where the queue is considered only when the items are stocked out. The economic order quantity of the inventory is determined in such case the holding cost of the average inventory also includes the item in the service station. Berman and Kim [4] have investigated an inventory queuing model under the assumption that the inter-arrival time of customers and the service time of a customer are in exponential rate in which mean inter-arrival rate exceeds the mean service time. An inventory queuing system with Poisson arrival, arbitrarily distributed service times, zero lead times, demanding one item for each customer at a time and finite capacity of the waiting room is studied by Berman and Sapna [5]. The optimal ordering quantity is derived from the minimum long run expected total cost per unit time. Further, Elango [6] has discussed the Markovian inventory system along with service facility and instantaneous replenishment of orders. The service time is assumed to be exponentially distributed with its rate depending on the queue length. In the extension, Arivarignan et al. [1] have considered the inventory queuing system with exponential lead time. Arivarignan and Sivakumar [2] have worked the inventory problem with both exponential service and lead times but the demand of an item is considered to be arbitrarily distributed.

Practically, we find that the food, drugs and electronic items are perishable in nature.The perishability of the items are discussed by many authors in their studies because it is one of the significant component of total cost which contributes to total failure cost. For the details of research on perishability of an item, one can refer to [1, 7, 16, 17, 19]. Next, the arrival of customers may be allowed in an orbit when the waiting hall is full. Many authors discussed the retrial of customers in the orbit. Krishnamoorthy et. al [8], elaborately discussed the association of Inventory System with service, positive lead time, loss and retrial of customers. Krishnamoorthy and Islam [9] examined the behavior of retrial customers that demands a production inventory with positive service. In the model, the poison process of an arrival customer, exponential demand of the server from an orbital customer, (s, S) inventory production policy are all considered.

In a service process, not all servers can be reliable and occurrence of interruptions in service process is likely to be happened. In that sense, the occurrence of server interruption is studied by many authors. Krishnamoorthy et. al. [10] have designed a server interruption in an (s, S) inventory system with positive service time and zero lead time. In the extension, Krishnamoorthy et.al [11] have studied the server interruptions with inventory retrial queuing system. They assumed that the arrival customers are allowed in the orbit while the server is busy, the reattempt customers goes back to the orbit with fixed probability when the server has interruption, the arrival and retrial customers are lost at the stock-out period. Further, Poison course of customer arrival, exponential lead time, (s, S) ordering Policy, exponential service time, poison process of service interruption occurrence, exponential time of completion of service interruption, Poisson course of retrial customer for demanding the server are all considered.The various system characteristics are measured under the stability conditions which includes the waiting time of a customer in the orbit.

Introducing server vacation in a system will eliminate the idle time cost, which is one of the server related costs. In this vacation period, we can utilize that server for some other ancillary work, which will improvise the profitability of an organization. Narayanan et al. [14] designed a vacation to a server in an inventory system with correlated lead-time. Customers arriving in the service station according to a Markovian arrival process and service time for each customer follows the phase-type distribution. Sivakumar [21] framed the multiple server vacations in a (s, S) inventory system with infinite orbit. Inter-demand times, lead times, inter-retrial times and server vacation times that are considered to be exponentially distributed. Also, Padmavathi et.al.[15] investigated a single vacation on a retrial inventory system with modified multiple vacations. The last two models discussed above appear to be differing in such a way that server utility during vacation will break. Krishnamoorthy et.al. [12] Investigated the vacation of a server with the production inventory system and phase type service policy. Production and customer arrival are assumed based on the Markovian processes. The study revealed that, the system state distribution performed under the stability conditions. In particular, they envisaged the expected number of customers in the system during the server on vacation and also the server is busy. Also, Jeganathan et.al [13] discussed a perishable inventory system with server interruptions, multiple server vacations and N-Policy.

In this paper, we talk over a perishable inventory system with two heterogeneous servers including one server permits for vacation and another is unreliable server. The operating policy of ordering quantity is (s, Q) with exponential lead times. The paper is structured as follows. The assumptions, mathematical notations and description of the model are all elaborated in the section 2. Analysis and the steady state solutions of the model are discussed in section 3. Some quintessential system performance measures are derived in section 4. In the section 5, we calculate the total expected cost rate and present sensitivity analysis with numerical examples. Last section is given about the conclusion.

2 Description of the Model

Consider two servers at the service facility of the inventory retrial queuing system which are referred to as first server and second server. Each arriving customer demands a single item. Service discipline is under FCFS basis. After purchasing an item, before leaving the system, each customer must require to get some service on the item. It is supposed that the item shall not get perished during the time of servicing. The system holds a waiting space with finite capacity. Any arriving customers shall wait there if servers are busy with positive inventory level. If the waiting space is occupied, thereafter any arriving customer will enters the orbit with finite size. First server is available but sometimes interruptions may bind to occur during the servicing time and the second server leave for vacation each time when the server finds either the inventory level is zero or number of customers in the waiting space is zero or both. Any arriving customer is contemplating to be lost, whenever the customer found that the waiting space and orbit becomes engaged. All the activities discussed in the system are independent to each other. The following assumptions/ rules are also made:

- The arrival of a primary customer is subject to Poisson process with the rate λ.
- Both servers are heterogeneous and their service times are exponentially distributed with the rates μ_1 and μ_2 respectively.
- The continuous review selective technique is used for ordering the quantity Q and so the ordering policy is noted by (s, Q) where $s \geq 2$ is the reorder point and the maximum inventory of the system is S given by $S = s + Q$.
- The waiting space accommodates maximum R number of waiting customers including the customers attending their service. The maximum number of customers occupies the orbit is M.
- The lead-time is connoted to have a negative exponential distribution with parameter β.
- The lifetime of each item has negative exponential distribution with parameter $\gamma > 0$.
- The inter-retrial time is exponential distribution with linear rate $i\theta$ where i denotes the number of customers in the orbit.
- When the inventory level is positive and the waiting hall is empty, the first server searches an orbiting customer after completion of the service with a probability p, $0 \leq p \leq 1$. Otherwise, the server goes to free state with the probability q. The search time is considered as negligible.
- The vacation time of the second server is exponentially distributed with the parameter α.
- After completion of the vacation, there is at least two commodities and at least two customers in the waiting hall, then the second server starts the service immediately. In contrast, the server begins another vacation.
- The occurrence of the service interruption of the first server follows the Poisson process with the rate η_1. No further interruption can betide during the interruption.
- The duration of an interruption is exponentially distributed with the rate η_2.

The following notations are used throughtout this paper:

$\mathbf{e}$: a column vector of appropriate dimension containing all ones

$$\overset{k}{\underset{i=j}{\Omega}} c_i \quad : \quad \begin{cases} c_j c_{j-1} \cdots c_k & \text{if } j \geq k \\ 1 & \text{if } j < k \end{cases}$$

3 Analysis of the Model

It is noticed that the activities of the model under study, which is associated with the following regular irreducible homogeneous continuous time Markov chain (CTMC) $L(t) = \{(D(t), E(t), F(t), G(t)), t \geq 0\}$, where, during the period t, $t \geq 0$, $D(t)$ is the inventory level ($D(t) \in \{0, 1, \ldots, S\}$), $F(t)$ is the number of customers in the waiting hall ($F(t) \in \{0, 1, \ldots, R\}$), $G(t)$ is the number of customers in the orbit ($G(t) \in \{0, 1, \ldots, M\}$), $E(t)$ is the status of the server ($E(t) \in \{0, 1, 2, 3, 4, 5\}$) which is defined by

$$E(t) \quad : \quad \begin{cases} 0, & \text{if first server is free and second server is on vacation at time t,} \\ 1, & \text{if first server is free and second server is busy at time t,} \\ 2, & \text{if first server is busy and second server is on vacation at time t,} \\ 3, & \text{if first server is on interruption and second server is on vacation at time t,} \\ 4, & \text{if both servers are busy at time t,} \\ 5, & \text{if first server is on interruption and second server is busy at time t,} \end{cases}$$

The state space J of $L(t)$ is defined by $J : J_a \cup J_b \cup J_c \cup J_d \cup J_e \cup J_f$, where
$J_a : \{(0, 0, i_3, i_4) \mid i_3 \in V_0^R, i_4 \in V_0^M\}$,
$J_b : \{(i_1, 0, 0, i_4) \mid i_1 \in V_1^S, i_4 \in V_0^M\}$,
$J_c : \{(i_1, 1, 1, i_4) \mid i_1 \in V_1^S, i_4 \in V_0^M\}$,
$J_d : \{(1, 1, i_3, i_4) \mid i_3 \in V_2^R, i_4 \in V_0^M\}$,
$J_e : \{(i_1, i_2, i_3, i_4) \mid i_1 \in V_2^S, i_2 = 4, 5, i_3 \in V_2^R, i_4 \in V_0^M\}$,
$J_f : \{(i_1, i_2, i_3, i_4) \mid i_1 \in V_1^S, i_2 = 2, 3, i_3 \in V_1^R, i_4 \in V_0^M\}$.

Connote by level $(\mathbf{0})$ the collection of states given by $(\mathbf{0}) = \{(0, 0, i_3, i_4) : 0 \leq i_3 \leq R, 0 \leq i_4 \leq M\}$; Level $(\mathbf{1})$ the collection of states given by $(\mathbf{1}) = \{(1, 0, 0, i_4) : 0 \leq i_4 \leq M\} \cup \{(1, 1, 1, i_4) : 0 \leq i_4 \leq M\} \cup \{(1, i_2, i_3, i_4) : i_2 = 2, 3, 1 \leq i_3 \leq R, 0 \leq i_4 \leq M\}$; Level $(\mathbf{i_1}), i_1 \in V_2^S$ the collection of states given by $(\mathbf{i_1}) = \{(i_1, 0, 0, i_4) : 0 \leq i_4 \leq M\} \cup \{(i_1, 1, 1, i_4) : 0 \leq i_4 \leq M\} \cup \{(i_1, i_2, i_3, i_4) : i_2 = 2, 3, 1 \leq i_3 \leq R, 0 \leq i_4 \leq M\} \cup \{(i_1, i_2, i_3, i_4) : i_2 = 4, 5, 2 \leq i_3 \leq R, 0 \leq i_4 \leq M\}$. The levels $(\mathbf{0})$, $(\mathbf{1})$ and $(\mathbf{i_1}), i_1 \in V_2^S$ are of dimension $(R+1)(M+1)$, $(3R+1)(M+1)$ and $4R(M+1)$ respectively.

The rate matrix $\hat{\Theta}$ of $L(t) = \{(D(t), E(t), F(t), G(t)) : t \geq 0\}$ is given by

$$\hat{\Theta} \quad = \quad \begin{cases} \Gamma_i, & j_1 = i_1 - 1, \quad i_1 = 1, 2, \ldots, S, \\ \Delta_1, & j_1 = i + Q, \quad i_1 = 0, \\ \Delta_2, & j_1 = i + Q, \quad i_1 = 1, \\ \Delta_3, & j_1 = i + Q, \quad i_1 = 0, 1, \ldots, s, \\ \Psi_i, & j_1 = i_1 \qquad i_1 = 0, 1, \ldots, S, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

1. The sub-matrix $\Delta_i (i = 1, 2, 3)$ denotes the transitions from $(\mathbf{i})$ to $(\mathbf{i + Q})$;

2. The sub-matrix $\Gamma_i (i = 1, 2, \ldots, S)$ denotes the transitions from $(\mathbf{i})$ to $(\mathbf{i - 1})$;

3. The sub-matrix $\Psi_i (i = 0, 1, 2, \ldots, S)$ denotes the transitions from $(\mathbf{i})$ to $(\mathbf{i})$;

Since $\hat{\Theta}$ is the rate matrix of an irreducible, aperiodic, persistent non-null, finite state space Markov chain. Therefore the limiting probability distribution exists.

$$\mathbf{\Phi} \quad = \quad (\phi^{(\mathbf{0})}, \phi^{(\mathbf{1})}, \ldots, \phi^{(\mathbf{S})}).$$

Then the vector of limiting probabilities $\mathbf{\Phi}$ satisfies

$$\mathbf{\Phi}\hat{\Theta} = \mathbf{0} \quad \text{and} \quad \sum\sum\sum\sum_{(i_1,i_2,i_3,i_4)} \phi^{(i_1,i_2,i_3,i_4)} = 1 \tag{3.1}$$

The equation $\mathbf{\Phi}\hat{\Theta} = \mathbf{0}$ yields the following set of equations:

$$\begin{aligned} \boldsymbol{\phi^{i_1}}\Psi_{i_1} + \boldsymbol{\phi^{i_1+1}}\Gamma_{i_1+1} &= \mathbf{0}, && i_1 = 0,1,2,\dots,Q-1, \\ \boldsymbol{\phi^{i_1}}\Psi_{i_1} + \boldsymbol{\phi^{i_1+1}}\Gamma_{i_1+1} + \boldsymbol{\phi^{(i_1-Q)}}\Delta_{(i_1-(Q-1))}) &= \mathbf{0}, && i_1 = Q, Q+1, \qquad (*) \\ \boldsymbol{\phi^{i_1}}\Psi_{i_1} + \boldsymbol{\phi^{i_1+1}}\Gamma_{i_1+1} + \boldsymbol{\phi^{(i_1-Q)}}\Delta_3 &= \mathbf{0}, && i_1 = Q+2,\dots,S-1, \\ \boldsymbol{\phi^{i_1}}\Psi_{i_1} + \boldsymbol{\phi^{i_1-Q}}\Delta_3 &= \mathbf{0}, && i_1 = S. \end{aligned}$$

After long simplifications, the above equations, except $(*)$, yields

$$\begin{aligned} \boldsymbol{\Omega_{i_1}} &= (-1)^{Q-i_1}\boldsymbol{\phi^{Q}} \underset{j=Q}{\overset{i_1+1}{\Omega}} \Gamma_j \Psi_{j-1}^{-1}, \qquad i_1 = Q-1, Q-2, \dots, 0 \\ &= (-1)^{Q}\boldsymbol{\phi^{Q}} \sum_{j=0}^{s-1}\left[\left\{\left(\underset{k=Q}{\overset{(s+1)-j}{\Omega}} \hat{\Gamma_k}\Psi_{k-1}^{-1}\right)\Delta_3\Psi_{S-j}^{-1}\right\}\left\{\left(\underset{l=S-j}{\overset{i_1+1}{\Omega}}\Gamma_l\Psi_{l-1}^{-1}\right)\Delta_2\Psi_{i_1}^{-1}\right\}\right],\ i_1 = Q+1 \\ &= (-1)^{2Q-i_1+1}\boldsymbol{\phi^{Q}} \sum_{j=0}^{S-i_1}\left[\left(\underset{k=Q}{\overset{s+1-j}{\Omega}}\Gamma_k\Psi_{k-1}^{-1}\right)\Delta_3\Psi_{S-j}^{-1}\left(\underset{l=S-j}{\overset{i_1+1}{\Omega}}\Gamma_l\Psi_{l-1}^{-1}\right)\right], \\ & \qquad i_1 = S, S-1, \dots, Q+2 \end{aligned}$$

where $\boldsymbol{\phi^{Q}}$ can be obtained by solving,

$$\boldsymbol{\phi^{Q+1}}\Gamma_{Q+1} + \boldsymbol{\phi^{Q}}\Psi_Q + \boldsymbol{\phi^{0}}\Delta_1 = \mathbf{0} \text{ and } \sum_{i_1=0}^{S}\boldsymbol{\phi^{i_1}}\mathbf{e} = 1,$$

4 System performance measures

In this section, we derive some measures of system performance in the steady state. Using this, we derive the total expected cost rate.

4.1 Expected Inventory Level

Let Ω_i denote the excepted inventory level in the steady state.

$$\Omega_i = \sum_{i_1=1}^{S} i_1 \Phi^{(i_1)}\mathbf{e}$$

4.2 Expected Reorder Rate

Let Ω_r denote the expected reorder rate in the steady state.

$$\begin{aligned}\Omega_r &= \sum_{i_4=0}^{M}(s+1)\gamma\phi^{(s+1,0,0,i_4)} + \sum_{i_4=0}^{M}(\mu_2+s\gamma)\phi^{(s+1,1,1,i_4)} \\ &\quad + \sum_{i_3=2}^{R}\sum_{i_4=0}^{M}\{(\mu_1+\mu_2+(s-1)\gamma)\phi^{(s+1,4,i_3,i_4)} + (\mu_2+(s-1)\gamma)\phi^{(s+1,5,i_3,i_4)}\} \\ &\quad + \sum_{i_3=1}^{R}\sum_{i_4=0}^{M}\{(\mu_1+s\gamma)\phi^{(s+1,2,i_3,i_4)} + s\gamma\phi^{(s+1,3,i_3,i_4)}\}.\end{aligned}$$

4.3 Expected Perishable Rate

Since $\Phi^{(i_1)}$ is the steady state probability vector for inventory level, the expected perishable rate Ω_p is given by

$$\begin{aligned}\Omega_p &= \sum_{i_1=1}^{S}\sum_{i_4=0}^{M}\{i_1\gamma\phi^{(i_1,0,0,i_4)} + (i_1-1)\gamma\phi^{(i_1,1,1,i_4)}\} \\ &\quad + \sum_{i_1=3}^{S}\sum_{i_3=2}^{R}\sum_{i_4=0}^{M}(i_1-2)\gamma\{\phi^{(i_1,4,i_3,i_4)} + \phi^{(i_1,5,i_3,i_4)}\} + \sum_{i_1=2}^{S}\sum_{i_3=1}^{R}\sum_{i_4=0}^{M}(i_1-1)\gamma\{\phi^{(i_1,2,i_3,i_4)} + \phi^{(i_1,3,i_3,i_4)}\}\end{aligned}$$

4.4 Expected Number of Customers in the Waiting Hall

Let Ω_{wh} denote the expected number of customers in the steady state. Then we have

$$\begin{aligned}\Omega_{wh} &= \sum_{i_3=1}^{R}\sum_{i_4=0}^{M} i_3\phi^{(0,0,i_3,i_4)} + \sum_{i_1=1}^{S}\sum_{i_4=0}^{M}\phi^{(i_1,1,1,i_4)} + \sum_{i_3=2}^{R}\sum_{i_4=0}^{M} i_3\phi^{(1,1,i_3,i_4)} \\ &\quad + \sum_{i_1=2}^{S}\sum_{i_3=2}^{R}\sum_{i_4=0}^{M} i_3\{\phi^{(i_1,4,i_3,i_4)} + \phi^{(i_1,5,i_3,i_4)}\} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{R}\sum_{i_4=0}^{M} i_3\{\phi^{(i_1,2,i_3,i_4)} + \phi^{(i_1,3,i_3,i_4)}\}\end{aligned}$$

4.5 Expected Waiting Time

Let Ω_{w1} denote the expected waiting time of the customers in the waiting hall. Then by Little's formula

$$\Omega_{w1} = \frac{\Omega_{wh}}{\Omega_a},$$

where Ω_{wh} is the expected number of customers in the waiting hall and the effective arrival rate of the customer (Ross [18]), Ω_a is given by

$$\begin{aligned}\Omega_a &= \sum_{i_3=0}^{R-1}\sum_{i_4=0}^{M}(\lambda+i_4\theta)\phi^{(0,0,i_3,i_4)}+\sum_{i_1=1}^{S}\sum_{i_4=0}^{M}(\lambda+i_4\theta)\phi^{(i_1,0,0,i_4)}+\sum_{i_1=1}^{S}\sum_{i_4=0}^{M}(\lambda+i_4\theta)\phi^{(i_1,1,1,i_4)}\\ &+\sum_{i_3=2}^{R-1}\sum_{i_4=0}^{M}(\lambda+i_4\theta)\phi^{(1,1,i_3,i_4)}+\sum_{i_1=2}^{S}\sum_{i_3=2}^{R-1}\sum_{i_4=0}^{M}(\lambda+i_4\theta)\{\phi^{(i_1,4,i_3,i_4)}+\phi^{(i_1,5,i_3,i_4)}\}\\ &+\sum_{i_1=1}^{S}\sum_{i_3=1}^{R-1}\sum_{i_4=0}^{M}(\lambda+i_4\theta)\{\phi^{(i_1,2,i_3,i_4)}+\phi^{(i_1,3,i_3,i_4)}\}\end{aligned}$$

4.6 The Overall Rate of Retrials

Let Ω_{orr} denote the overall rate of retrials in the steady state. Then we have

$$\Omega_{orr}=\theta(\Omega_{or1}+\Omega_{or2})$$

where

$$\begin{aligned}\omega_{or1} &= \sum_{i_3=0}^{R-1}\sum_{i_4=1}^{M}i_4\phi^{(0,0,i_3,i_4)}+\sum_{i_1=1}^{S}\sum_{i_4=1}^{M}i_4\phi^{(i_1,0,0,i_4)}+\sum_{i_1=1}^{S}\sum_{i_4=1}^{M}i_4\phi^{(i_1,1,1,i_4)}+\sum_{i_3=2}^{R-1}\sum_{i_4=1}^{M}i_4\phi^{(1,1,i_3,i_4)}\\ &+\sum_{i_1=2}^{S}\sum_{i_3=2}^{R-1}\sum_{i_4=1}^{M}i_4\{\phi^{(i_1,4,i_3,i_4)}+\phi^{(i_1,5,i_3,i_4)}\}+\sum_{i_1=1}^{S}\sum_{i_3=1}^{R-1}\sum_{i_4=1}^{M}i_4\{\phi^{(i_1,2,i_3,i_4)}+\phi^{(i_1,3,i_3,i_4)}\}\end{aligned}$$

and Ω_{or2} is given by

$$\begin{aligned}\Omega_{or2} &= \sum_{i_4=1}^{M}i_4\phi^{(0,0,R,i_4)}+\sum_{i_4=1}^{M}i_4\phi^{(1,1,R,i_4)}+\sum_{i_1=2}^{S}\sum_{i_4=1}^{M}i_4\{\phi^{(i_1,4,R,i_4)}+\phi^{(i_1,5,R,i_4)}\}\\ &+\sum_{i_1=1}^{S}\sum_{i_4=1}^{M}i_4\{\phi^{(i_1,2,R,i_4)}+\phi^{(i_1,3,R,i_4)}\}\end{aligned}$$

4.7 The Successful Retrial Rate

Let Ω_{srr} denote the successful retrial rate in the steady state. Then

$$\Omega_{srr}=\theta(\Omega_{or1})$$

4.8 Expected Number of Customers in the Orbit

Let Ω_{w2} denote the expected number of customers in the steady state is given by

$$\Omega_{w2} = \Omega_{or1}+\Omega_{or2}$$

4.9 Expected Number of Customers in the System

Let Ω_{ws} denote the expected number of customers waiting in the system (including servicing customer) is given by

$$\Omega_{ws} = \Omega_{w2} + \Omega_{wh}$$

4.10 Expected Loss Rate of Customers

Let Ω_l denote the expected loss rate of customers in the steady state. Any arriving primary customer finds both the waiting hall and orbit full and leaves the system without getting service. These customers are considered to be lost. Thus we obtain

$$\begin{aligned}\Omega_l &= \lambda\{\phi^{(0,0,R,M)} + \phi^{(1,1,R,M)}\} + \sum_{i_1=2}^{S} \lambda\{\phi^{(i_1,4,R,M)} + \phi^{(i_1,5,R,M)}\} \\ &+ \sum_{i_1=1}^{S} \lambda\{\phi^{(i_1,2,R,M)} + \phi^{(i_1,3,R,M)}\}\end{aligned}$$

4.11 Effective Interruptions Rate

Let Ω_{ir} denote the effective interruption rate which is given by

$$\Omega_{ir} = \sum_{i_1=2}^{S}\sum_{i_3=2}^{R}\sum_{i_4=0}^{M} \eta_1 \phi^{(i_1,4,i_3,i_4)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{R}\sum_{i_4=0}^{M} \eta_1 \phi^{(i_1,2,i_3,i_4)}$$

4.12 Effective Repair Rate

Let Ω_{rep} denote the effective repair rate which is given by

$$\Omega_{rr} = \sum_{i_1=2}^{S}\sum_{i_3=2}^{R}\sum_{i_4=0}^{M} \eta_2 \phi^{(i_1,5,i_3,i_4)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{R}\sum_{i_4=0}^{M} \eta_2 \phi^{(i_1,3,i_3,i_4)}$$

4.13 Fraction of Successful Rate of Retrials

Let ω_{fsr} denote the fraction of successful rate of retrials which is given by

$$\Omega_{fsr} = \frac{\Omega_{srr}}{\Omega_{orr}},$$

4.14 Fraction of time the second server is on vacation

Let Ω_{fsv} denote the fraction of time the second server is on vacation is given by

$$\Omega_{fsv} = \sum_{i_3=0}^{R}\sum_{i_4=0}^{M} \phi^{(0,0,i_3,i_4)} + \sum_{i_1=1}^{S}\sum_{i_4=0}^{M} \phi^{(i_1,0,0,i_4)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{R}\sum_{i_4=0}^{M}\{\phi^{(i_1,2,i_3,i_4)} + \phi^{(i_1,3,i_3,i_4)}\}$$

4.15 Expected Number of Customers Waiting while second server is on vacation

Let Ω_{cw} denote the expected number of customers waiting while second server is on vacation which is given by

$$\Omega_{cw} = \sum_{i_3=1}^{R}\sum_{i_4=0}^{M} i_3\phi^{(0,0,i_3,i_4)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{R}\sum_{i_4=0}^{M} i_3\{\phi^{(i_1,2,i_3,i_4)} + \phi^{(i_1,3,i_3,i_4)}\}$$

5 Cost Analysis and Sensitivity Analysis

To compute the total expected cost per unit time (total expected cost rate), the following costs are considered.

C_h : The inventory carrying cost per unit item per unit time
C_s : Setup cost per order
C_p : Perishable cost per unit item per unit time
C_{wh} : Waiting cost of a customer in the waiting hall per unit time
C_{wo} : Waiting cost of a customer in the orbit per unit time
C_l : Cost due to loss of customers per unit per unit time.
C_i : Cost per interruption per unit time.
C_r : Cost per repair per unit time

The long run total expected cost rate is given by

$$TC(S,s,R,M) = C_h\Omega_i + C_s\Omega_r + C_p\Omega_p + C_{wh}\Omega_{w1} + C_{wo}\Omega_{w2} + C_l\Omega_l + C_i\Omega_{ir} + C_r\Omega_{rr}$$

where $\Omega's$ are as given in the measures of system performance.

5.1 Sensitivity Analysis

Here, S, s are the decision variables of the problem. Under the given sets of discrete values of the decision variables, we can determine the (local) optimal values of these variables which minimize the total cost function $TC(S, s)$ subject to all the costs and other parameters are kept constant values. From the table, considering the cost function $TC(S, s, 6, 3)$ and we can locate the local optima of S and s. Referring the figure 1, the sensitivity analysis is also carried out to see how the expected cost function behaves by changing values of the decision variables. By the procedure, first we determine the minimum values of each row and column. The minimum value of each row shown by bold form and the minimum value of each column showed by underlined.

Example 1.

Under the given cost structure and the fixed values of the controllable variables, we do investigated that the behavior of total expected cost rate which is the impact of the parameters. In this problem, we fix the cost values as $C_h = 0:001, C_s = 15, C_p = 2, C_w1 = 1, C_w2 = 1.6, C_l = 5, C_i = 0.1$ and $C_r = 10$, the controllable variables as S=115, s=4, R=6 and M=3, and the other fixed parameters values are shown under each figure (of 2-7). We observed the following from the figures.

1. Figures 2 to 7, illustrates the effect of the rate of lead time β on the rate of expected total cost TC. In each figure, we have also noticed that the expected curve is convex.

2. From figures 2 to 6, the long run expected cost rate attains a minimum value and slightly increases after exceeding the optimal value of β.

3. The optimal expected cost rate decreases when μ_1, μ_2, θ and α increase.

4. The optimal expected cost rate increases when λ increases.

5. When γ takes larger value, the total cost rate increases as β decreases and the total cost rate decreases as β increases. From the fig. 7, we also notice that the total cost rate attains minimum when the perishable rate γ of an item is the smallest value.

6. When we compare to μ_1, μ_2 is more sensitive on total cost rate. Similarly, comparing to η_1, η_2 is more sensitive on the total cost rate.

7. Comparing to all the figures, we attain the lowest cost rate for the largest value of θ and the highest cost rate for the smallest value of α.

Example 2.

Here, we study the impact of the parameters β, λ, α, μ_1 and μ_2 on the expected number of customers in the system Ω_{ws}. Towards this end, we first fix the cost values as $c_h = 0.001$, $c_s = 15$, $c_p = 2$, $c_{w1} = 1$, $c_{w2} = 1.6$, $c_l = 5$, $c_i = 0.1$ and $c_r = 10$. From Figures $8 - 10$, we observe the following:

1. If λ increases, then Ω_{ws} increases.

2. When β, α and μ_1 increase, then Ω_{ws} decreases.

3. If μ_2 increases, then Ω_{ws} initially decreases then it stabilizes or slightly increases.

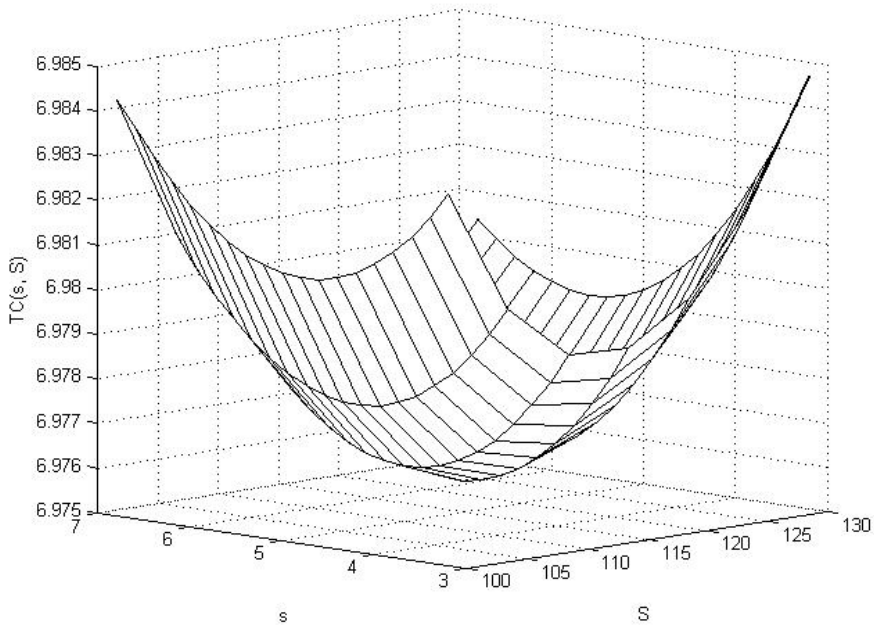
Example 3.

In this example, we investigate the impact of the parameters β, γ λ, α, μ_1, μ_2, η_1 and η_2 on the expected number of customers lost Ω_l. Towards this end, we first fix the cost values as $c_h = 0.001$, $c_s = 15$, $c_p = 2$, $c_{w1} = 1$, $c_{w2} = 1.6$, $c_l = 5$, $c_i = 0.1$ and $c_r = 10$. From Figures $11 - 14$, we observe the following:

1. If λ, γ and η_1 increase, then Ω_l increases.

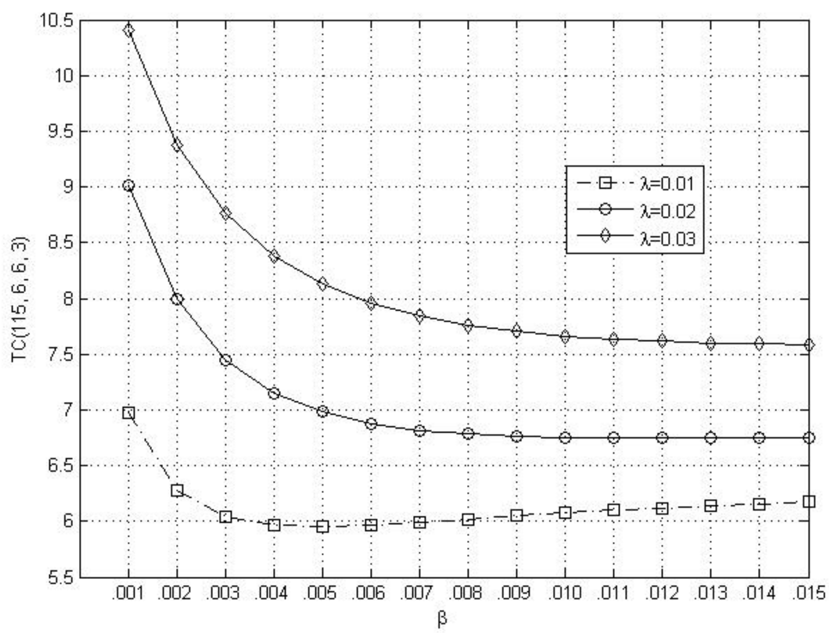
2. When β, α, η_2 and μ_1 increase, then Ω_l decreases.

3. If μ_2 increases, then Ω_l has looks like a convex curve.



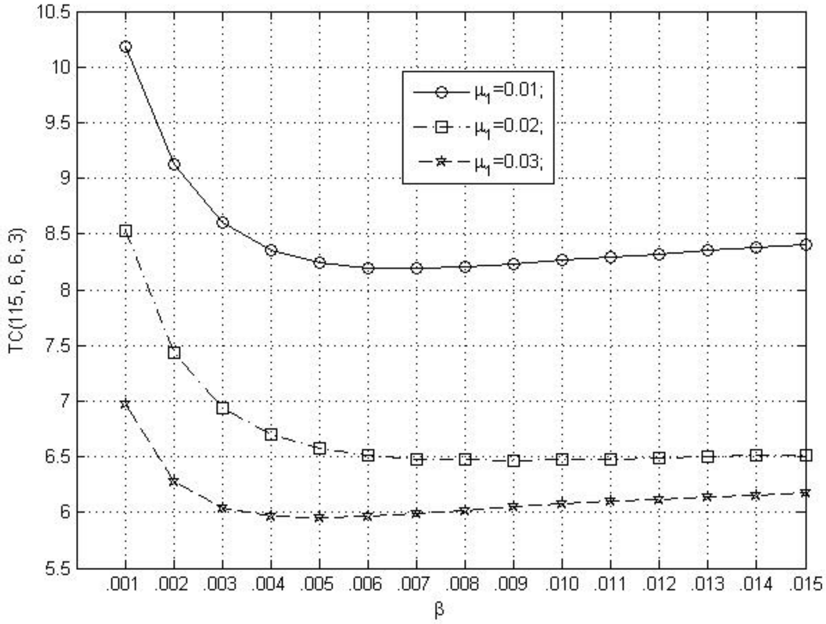
$\lambda = 1.4,\ \gamma = 0.02,\ \mu_2 = 1,\ \theta = 0.3,\ \eta = 3$

Figure 1: Convexity of the total cost for various combinations of S and s



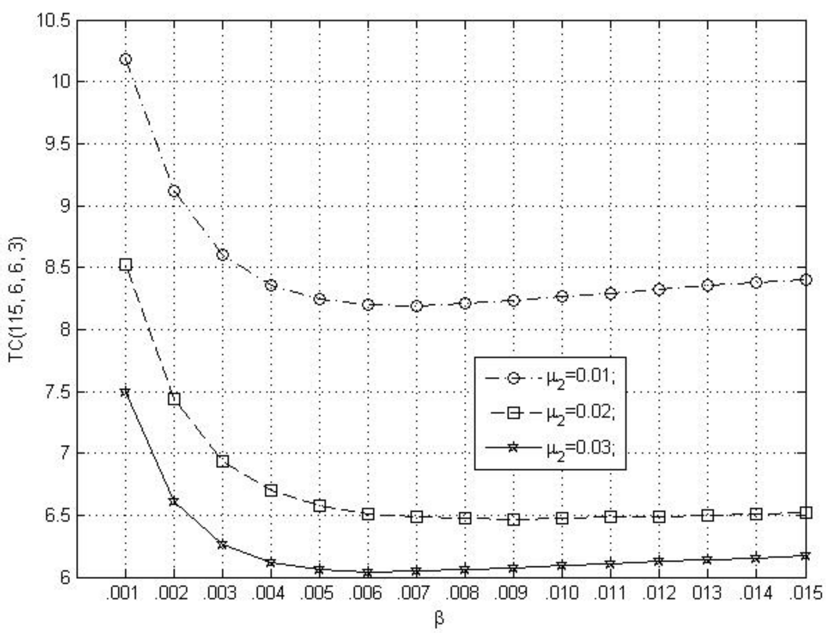
$\gamma = 0.01$, $\mu_1 = 0.001$, $\mu_2 = 0.04$, $\eta_1 = 0.01$, $\eta_2 = 0.2$, $\theta = 0.12$, $\alpha = 3$, $p = 0.4$,

Figure 2: TC vs β for different values of λ



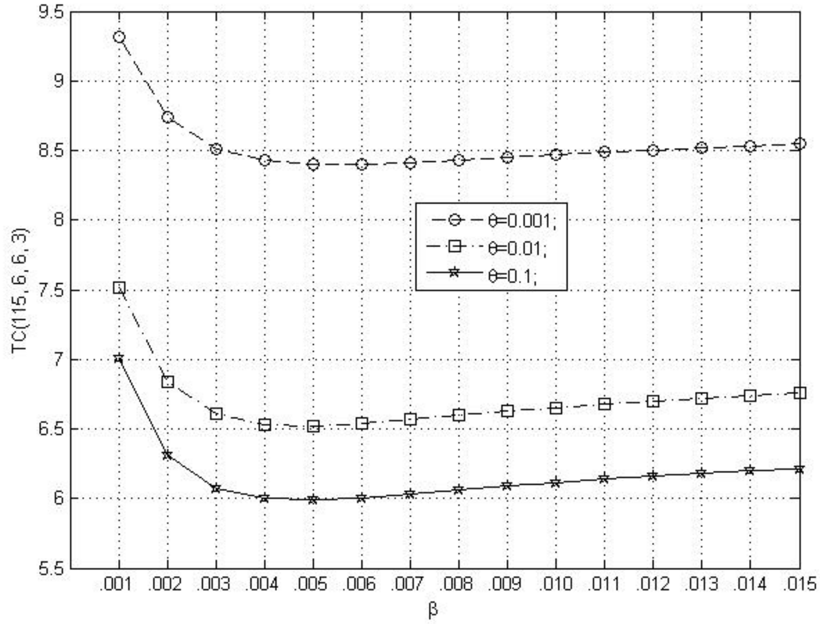
$\lambda = 0.01$, $\gamma = 0.01$, $\mu_2 = 0.04$, $\eta_1 = 0.01$, $\eta_2 = 0.2$, $\theta = 0.12$, $\alpha = 3$, $p = 0.4$,

Figure 3: TC vs β for different values of μ_1



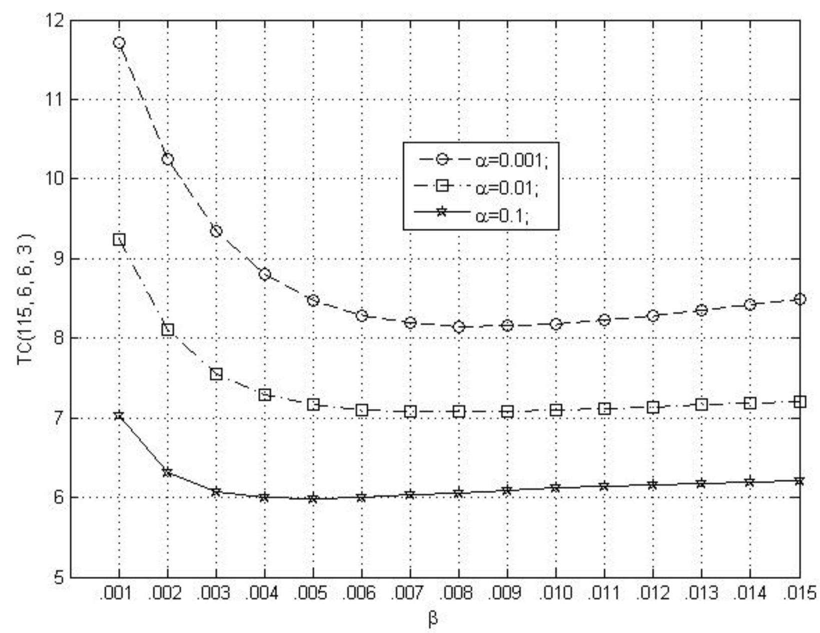
$\lambda = 0.01$, $\gamma = 0.01$, $\mu_1 = 0.001$, $\eta_1 = 0.01$, $\eta_2 = 0.2$, $\theta = 0.12$, $\alpha = 3$, $p = 0.4$,

Figure 4: TC vs β for different values of μ_2



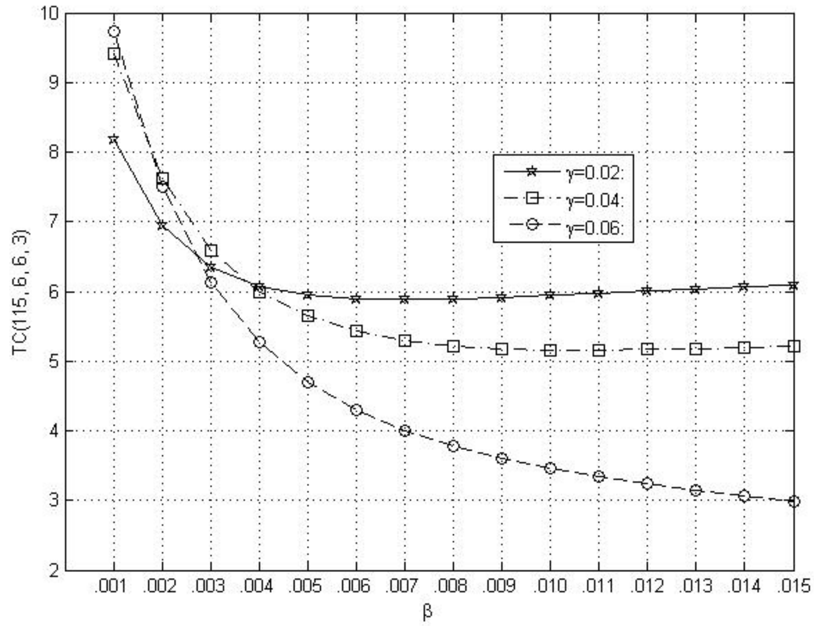
$\lambda = 0.01$, $\gamma = 0.01$, $\mu_1 = 0.001$, $\mu_2 = 0.04$, $\eta_1 = 0.01$, $\eta_2 = 0.2$, $\alpha = 3$, $p = 0.4$,

Figure 5: TC vs β for different values of θ



$\lambda = 0.01$, $\gamma = 0.01$, $\mu_1 = 0.001$, $\mu_2 = 0.04$, $\eta_1 = 0.01$, $\eta_2 = 0.2$, $\theta = 0.12$, $p = 0.4$,

Figure 6: TC vs β for different values of α

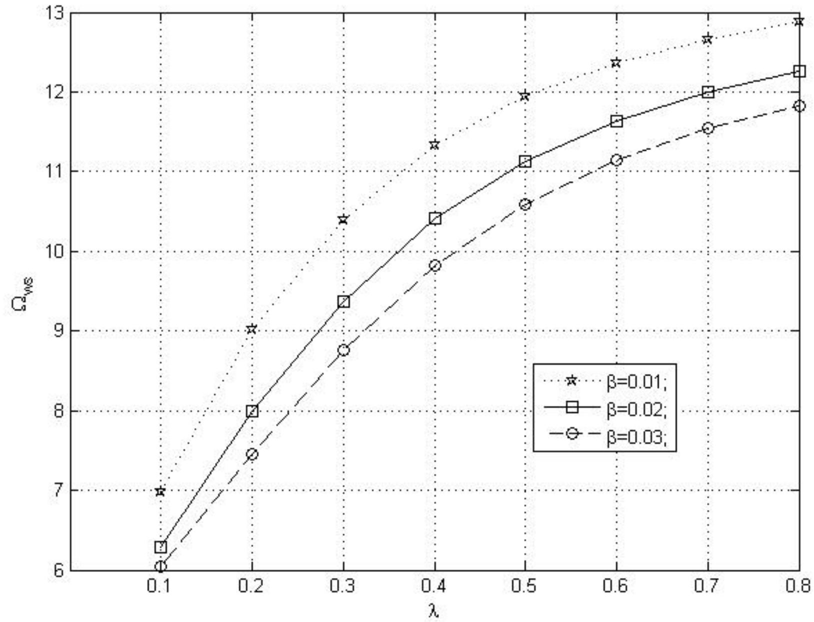


$\lambda = 0.01$, $\mu_1 = 0.001$, $\mu_2 = 0.04$, $\eta_1 = 0.01$, $\eta_2 = 0.2$, $\theta = 0.12$, $\alpha = 3$, $p = 0.4$,

Figure 7: TC vs β for different values of γ

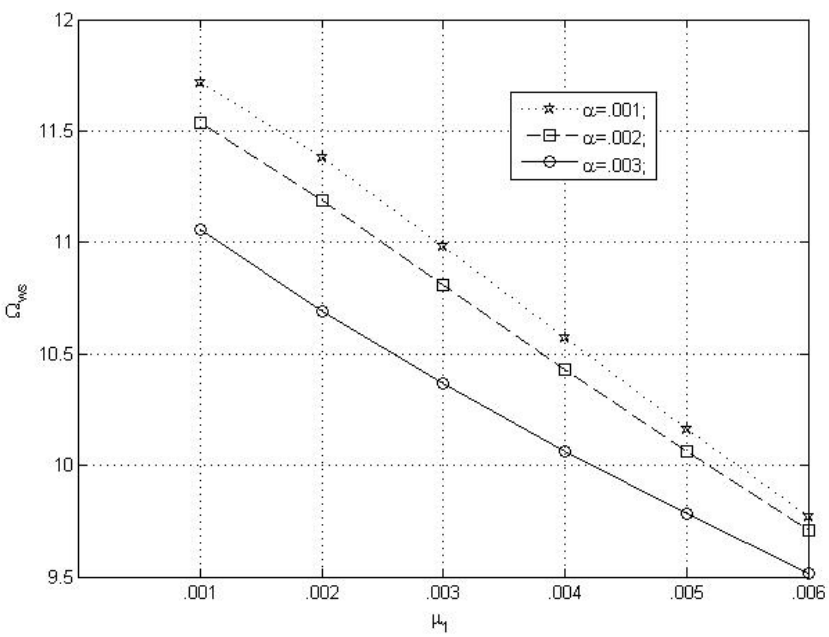
Table 1: Total expected cost rate as a function of S and s
$\lambda = 0.03$, $\beta = 0.001$, $\gamma = 0.01$, $\mu_1 = 0.001$, $\mu_2 = 0.01$, $\eta_1 = 0.01$, $\eta_2 = 2$, $\theta = 0.12$, $\alpha = 3$, $p = 0.4$,
$c_h = 0.001$, $c_s = 15$, $c_p = 2$, $c_{w1} = 2$, $c_{w2} = 1.6$, $c_l = 5$, $c_i = 0.1$, $c_r = 10$

S \ s	4	5	6	7	8
114	6.978272	6.976800	**6.976457**	6.977063	6.978535
115	6.978063	6.976548	**6.976154**	6.976700	6.978107
116	6.977930	6.976373	**6.975929**	6.976419	6.977761
117	6.977872	6.976274	**6.975782**	6.976216	6.977496
118	6.977885	6.976249	**6.975710**	6.976090	6.977309
119	6.977970	6.976296	**6.975712**	6.976040	6.977200
120	6.978124	6.976414	**6.975786**	6.976063	6.977165
121	6.978346	6.976601	**6.975930**	6.976157	6.977203
122	6.978634	6.976856	**6.976144**	6.976322	6.977314



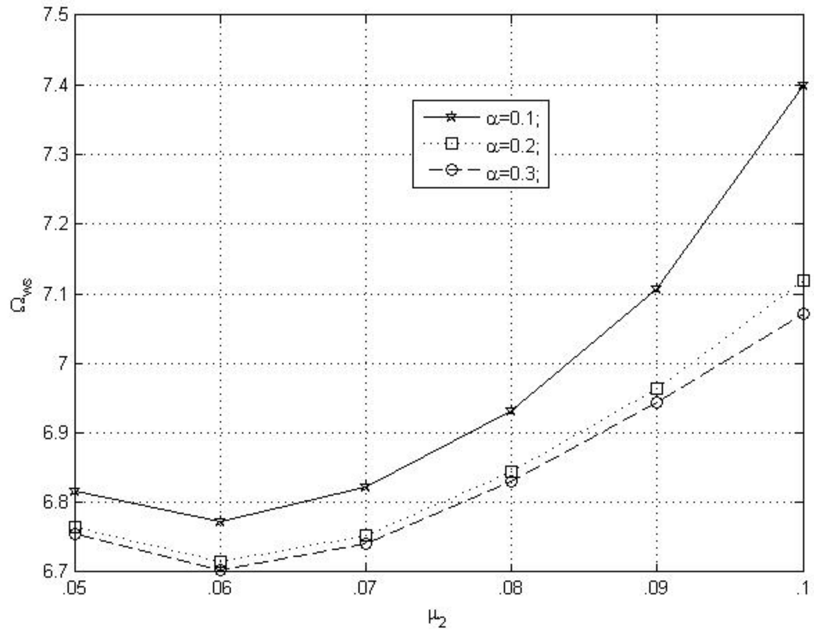
$\gamma = 0.01$, $\mu_1 = 0.001$, $\mu_2 = 0.04$, $\eta_1 = 0.01$, $\eta_2 = 0.2$, $\theta = 0.12$, $\alpha = 3$, $p = 0.4$,

Figure 8: Ω_{ws} vs λ for different values of β



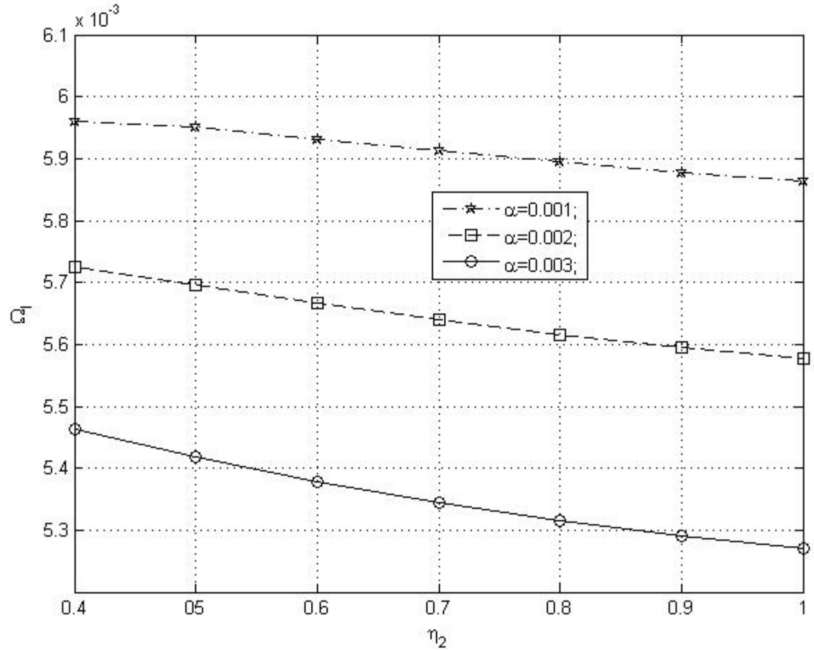
$\lambda = 0.01$, $\beta = 0.001$, $\gamma = 0.01$, $\mu_2 = 0.04$, $\eta_1 = 0.01$, $\eta_2 = 0.2$, $\theta = 0.12$, $p = 0.4$,

Figure 9: Ω_{ws} vs μ_1 for different values of α



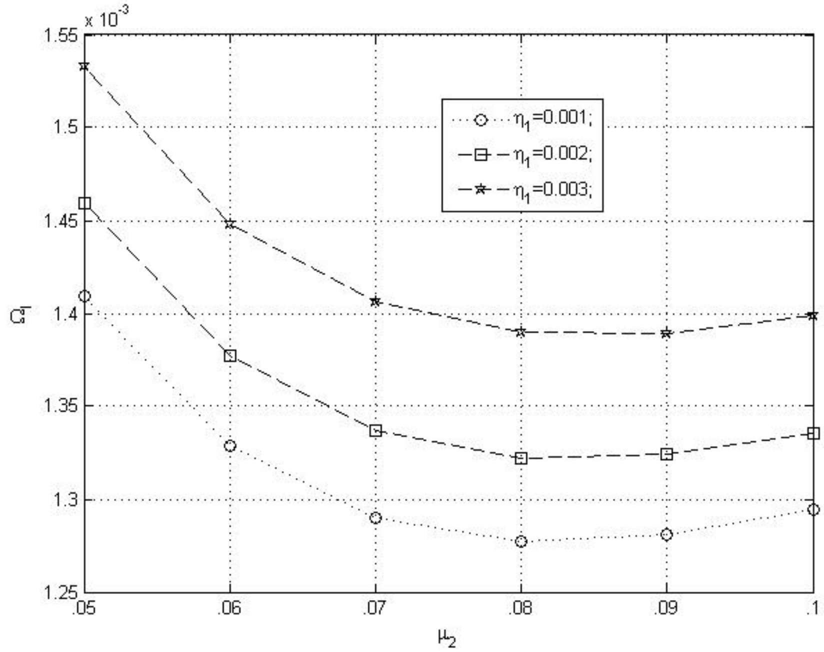
$\lambda = 0.01$, $\beta = 0.001$, $\gamma = 0.01$, $\mu_1 = 0.001$, $\eta_1 = 0.01$, $\eta_2 = 0.2$, $\theta = 0.12$, $p = 0.4$,

Figure 10: Ω_{ws} vs μ_2 for different values of α



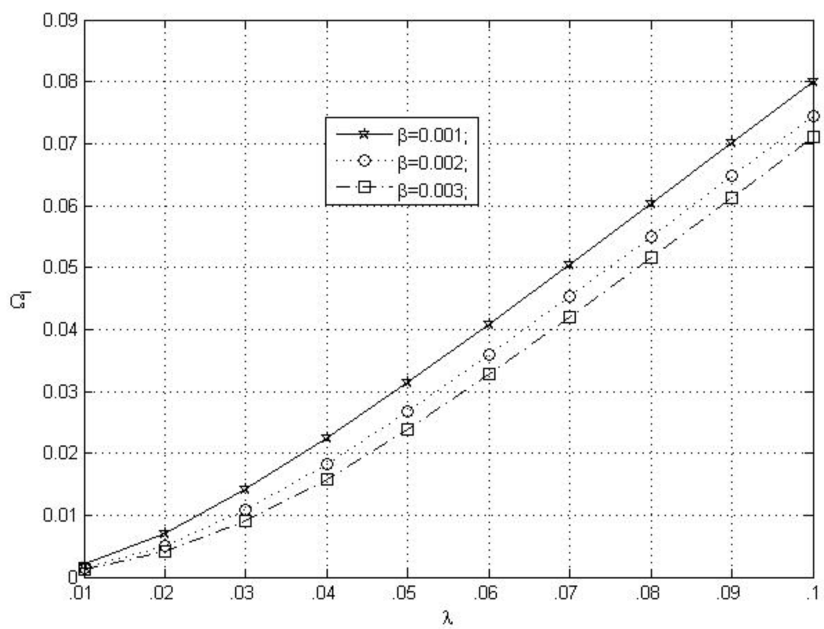
$\lambda = 0.01$, $\beta = 0.001$, $\gamma = 0.01$, $\mu_1 = 0.001$, $\mu_2 = 0.04$, $\theta = 0.12$, $\eta_1 = 0.01$, $p = 0.4$,

Figure 11: Ω_l vs η_2 for different values of α



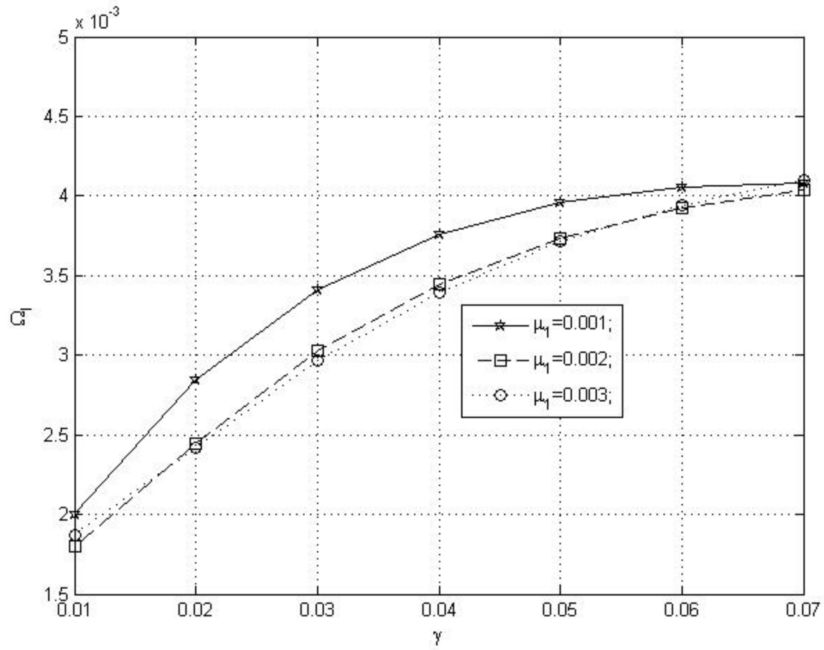
$\lambda = 0.01$, $\beta = 0.001$, $\gamma = 0.01$, $\mu_1 = 0.001$, $\eta_2 = 0.2$, $\theta = 0.12$, $\alpha = 3$, $p = 0.4$,

Figure 12: Ω_l vs μ_2 for different values of η_1



$\gamma = 0.01$, $\mu_1 = 0.001$ $\mu_2 = 0.04$, $\eta_1 = 0.01$, $\eta_2 = 0.2$, $\theta = 0.12$, $\alpha = 3$, $p = 0.4$,

Figure 13: Ω_l vs λ for different values of β



$\lambda = 0.01$, $\beta = 0.001$, $\mu_2 = 0.04$, $\eta_1 = 0.01$, $\eta_2 = 0.2$, $\theta = 0.12$, $\alpha = 3$, $p = 0.4$,

Figure 14: Ω_l vs γ for different values of μ_1

6 Conclusion

In this paper we have described a perishable inventory management at service facilities with two types of servers, one is reliable and another is non-reliable server. Under the steady state conditions, the joint distribution of the number of customers in the waiting hall, the number of customers in the orbit, status of the server and the inventory level is obtained using matrix techniques. The measures of important characteristics of the system are derived in the steady state. Under the numerical study, we have found the optimality of this model. Hence this model is suitable for cases for allotting vacation where the server is reliable and occurring interruption of service where the other server is not reliable. The reliable server vacations implies the advantage of elimination of ideal time cost of management, contributing the service of the server for allied tasks, utilizing server energy to other required job of the management.

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