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Optimal Analysis of Unemployment Model taking Policies to Control

Uzzwal Kumar Mallick[a] and Md. Haider Ali Biswas[b]

[a]Mathematics Discipline, Khulna University,Bangladesh
Email: mallickuzzwal@math.ku.ac.bd
[b]Mathematics Discipline, Khulna University,Bangladesh
Email: mhabiswas@yahoo.com

Abstract. Taking two policies, creating new vacancies and providing skilled manpower, a nonlinear mathematical model of unemployment is described in this paper with the cost function of policy making. This mathematical model is analysed with different constant control strategies and compared to the results with optimal control variables using Pontryagin's Maximum Principle. In this study, numerical simulations are investigated in absence of two policies or presence of one of two policies or presence of both policies. This study concludes a effective control strategy from different control strategies so that the number of unemployed persons at end of implemented policies of government is minimized while maximizing the value of objective function.

Keywords: Unemployment Model; Control of Policies; Numerical Analysis; Pontryagin's Maximum Principle; Optimal Control

AMS Classification: 97M40; 65L06; 49K30; 34H05; 49M37; 65K10

1. Introduction

Unemployment problem has become most immense concerns all over the world. Unemployment can be defined as a state of workless for a person fit and willing to work. We observe that the main reason for socio-economic damage, deletion of morality and social values is unemployment. With the increasing of unemployed population, other factors are significantly affected, such that the income per person, health costs, quality of health-care and poverty. But, according to World Employment Social Outlook of ILO in 2017, Global unemployment levels and rates are predicted to remain high in the short term, as the global labour force continues to grow. In particular, the global unemployment rate is predicted to rise modestly in 2017 to 5.8 per cent (from 5.7 per cent in 2016) representing 3.4 million more unemployed people globally (bringing total unemployment to just over 201 million in 2017). So unemployment is one of the most serious issues for every country. Several authors have contributed to different mathematical models to analyze and design optimal control strategies for unemployment problem. According to most of them, the population of developing countries has increased enormously but new opportunities for employment have not increased in the same proportion. So creating new opportunities is a priority for any vibrant economy. In 2003, Nikolopoulos and Tzanetis [1] developed a model considering housing allocation of homeless families due to a natural disaster. Using some concepts from this paper, Misra and Singh [2, 3] presented and analyzed nonlinear mathematical models for the control of unemployment. Where they assumed all entrants to category of the unemployed are fully qualified and competent to do any job. It is also considered that the number of unemployed persons increases continuously at a constant rate. For modeling process, the number of unemployed persons and employed persons are denoted by and respectively and number of vacancies is denoted by at any time . Inspired by [3], Pathan and Bhatahwala [4] proposed a model for better understanding of unemployment problem and its possible solution with self-employment. In that model, they considered no time delay by government and private sector in creating new vacancies. Motivated by [2], Monoli and Gani [5] proposed and discussed optimality of cost of new vacancies with nonlinear mathematical model for unemployment problem. In that study, retirement

[a]*CorrespondingAuthor.Email : mallickuzzwal@math.ku.ac.bd*

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and death of employed persons are considered as vacancies. They introduced two control variables and in the optimal control problem. Where, and are the implemented policies of government to provide employment from unemployed persons and to create new vacancies respectively. But, we observe the control profile in [5] that control is always zero. Now we take it positively, is always one. We think, there is no need of control . So, the control has to be changed for realistic of that model [6]. At present, the demand of skilled manpower has increased. So demands of time, policy of government provide skilled manpower with nurturing talent pool within the educational systems. So, we choose the control for the implemented policies of government to provide skilled manpower from unemployed population with nurturing talent pool within the educational systems or training or motivating program including effect rate $\alpha_3 = 0.05$. So, we assume that the following system of nonlinear differential equations for unemployment problem:

$$\dot{U}(t) = A - (kV(t) + \alpha_1 + \alpha_3 u_1(t))U(t) + \gamma E(t) \tag{1}$$

$$\dot{E}(t) = (kV(t) + \alpha_3 u_1(t))U(t) - (\alpha_2 + \gamma)E(t) \tag{2}$$

$$\dot{V}(t) = (\alpha_2 + \gamma)E(t) - \delta V(t) + \phi u_2(t)U(t) \tag{3}$$

with the initial conditions

$$U(0) = U_0, E(0) = E_0 \text{ and } V(0) = V_0 \tag{4}$$

We want to find the control strategies so that the number of employed at end of implemented policy of government is maximized while minimizing the cost of policy making. Hence we are maximizing the difference. Thus the objective functional is chosen to be

$$\text{Maximize } J(u_1(t), u_2(t)) = \int_{t_s}^{t_f} (A_1 E(t) - B_1 {u_1}^2(t) - B_2 {u_2}^2(t))dt \tag{5}$$

Here the cost function is a nonlinear function of $u_1(t)$ and $u_2(t)$; we choose them quadratic cost function for concavity. Since the right hand sides of the state equations are linearly bounded with respect to $u_1(t)$ and $u_2(t)$. These bounds insure the compactness needed for the existence of the optimal control ([7],[8]). So we can apply the Pontryagins Maximum Principle [10] in our proposed model for optimal solution. Also, the parameters A_1, B_1, $B_2 \geq 0$ represents the desired weights on the achievement and systematic cost. Our aim is to find the control profile ${u_1}^*(t)$ and ${u_2}^*(t)$ of satisfying $\max\{J(u_1(t), u_2(t)) | 0 \leq u_1(t), u_2(t) \leq 1\} = J(u_1^*(t), u_2^*(t))$

2. Bounded State Variables of the Mathematical Model

Our analyzed model ((1)-(3)) without control variables is used for proof of bounded state variables. For this reason, we find the region of attraction [17] for the model in the form of the following lemma.

Lemma 1: The feasible set

$\Omega = \{(U(t), E(t), V(t)) : 0 \leq U(t) + E(t) \leq \frac{A}{\alpha};\ 0 \leq V(t) \leq \frac{A(\psi+\alpha-1)}{\alpha\delta}$

where $\alpha = \min(\alpha_1, \alpha_2)$ and $\psi = \min(\phi - \alpha_1, \gamma)$ is a region of attraction for the mathematical model and attracts all solutions initiating in the interior of positive octant.

Proof: Adding the first and second equation of the model ((1)-(3)), we get

$\dot{U}(t) + \dot{E}(t) = A - \alpha_1 U(t) - \alpha_2 E(t)$

$\Rightarrow \dot{U}(t) + \dot{E}(t) \leq A - \alpha\left(U(t) + E(t)\right)$, where $\alpha = \min(\alpha_1, \alpha_2)$

Taking the limit supremum, we get

$$\lim_{t\to\infty} Sup\left\{U(t) + E(t)\right\} \leq \frac{A}{\alpha}$$

Similarly, from third equation(3) of the model, we get

$$\lim_{t\to\infty} Sup\left\{V(t)\right\} \leq \frac{A(\psi + \alpha - 1)}{\alpha\delta}$$

3. Optimal Control Model for the Unemployment Problem

We can reformulate our proposed model $((1) - (3))$ with objective function (5) in optimal control problem as

maximize $\int_{t_s}^{t_f} L\left(t, x(t), u(t)\right) dt$

subject to

$\dot{x}(t) = f(x(t)) + g(x(t))u(t)$, $\forall t \in [t_s, t_f]$
$u(t) \in [0,1]$,$\forall t \in [t_s, t_f]$
$x(0) = x_0$
where,
$x(t) = (U(t), E(t), V(t))$, $L\,(t, x(t), u(t)) = A_1 E(t) - B_1 u_1^2\,(t) - B_2 u_2^2\,(t)$,

$$f(x) = \begin{bmatrix} A - kU(t)V(t) - \alpha_1 U(t) + \gamma E(t) \\ kU(t)V(t) - \alpha_2 E(t) - \gamma E(t) \\ \alpha_2 E(t) + \gamma E(t) - \delta V(t) \end{bmatrix},$$

$$g(x) = \begin{bmatrix} -\alpha_3 U(t) & 0 \\ \alpha_3 U(t) & 0 \\ 0 & \psi U(t) \end{bmatrix}$$

4. Evaluation of the Maximum Principle

We shall evaluate the necessary optimality condition of the Maximum Principle [10] for the above Optimal Control Problem. For the maximization of $J(u_1(t), u_2(t))$, the standard Hamiltonian function is given by $H[x(t), p(t), u(t)] = \lambda L(x(t), u(t)) + \langle p(t), f(x) + g(x)u(t) \rangle$, $\lambda \in R$
where, $p = (p_U, p_E, p_V)$ denotes the adjoint variables.
Let $(x^*(t), u^*(t))$ be an optimal solution. Then, the maximum principle asserts the existence of a scalar $\lambda \geq 0$, , an absolutely continuous function $p(t)$ such that the time argument $[t]$ denotes the evaluation along the optimal solution:
i. $\max\,\{|p(t)| : t \in [t_s, t_f]\} + \lambda > 0$
ii. $\dot{p}(t) = -H_x[x] = \lambda L_x[t] - \langle p(t), f_x[t] + g_x[t]u^*(t) \rangle$
iii. $p(t_f) = (0, 0, 0)$
iv. $H(x^*(t), p(t), u^*(t)) = \max\limits_{u} \{H(x^*(t), p(t), u(t))\,|0 \leq u \leq 1\}$
From the adjoint equations (ii) with adjoint variables $p = (p_U, p_E, p_V)$ in normal form $(i.e.\lambda = 1)$ are explicitly given by

$$\dot{p}_U(t) = (p_U - p_E)kV(t) - \alpha_1 p_U + u_1(t)\alpha_3(p_U - p_E) - \phi u_2(t) p_V \tag{6}$$

$$\dot{p}_E(t) = -\gamma p_U(\gamma + \alpha_2)\,(p_E - p_V)kV(t) - A_1 \tag{7}$$

$$\dot{p}_V(t) = (p_U - p_E)kV(t) + \delta p_V \tag{8}$$

We deduce from (iv) and get an explicit characterization of optimal control pair in normal form $i.e.(\lambda = 1)$ given in terms of the multipliers $p = (p_U, p_E, p_V)$ and we get,

$$\langle p, f(x^*(t)) + g(x^*(t))u^*(t) + L(x^*(t), u^*(t)) \geq \langle p, f(x^*(t)) + g(x^*(t))u(t) + L(x^*(t), u(t)) \tag{9}$$

Simplifying the above inequalities (9) and we get

$$u_1^*(t) = \max\left\{\min\left\{\tfrac{(p_E - p_U)\alpha_3 U(t)}{2B_1}, 1\right\}, 0\right\}$$

and

$$u_2^*(t) = \max\left\{\min\left\{\tfrac{\phi p_V U(t)}{2B_2}, 1\right\}, 0\right\}$$

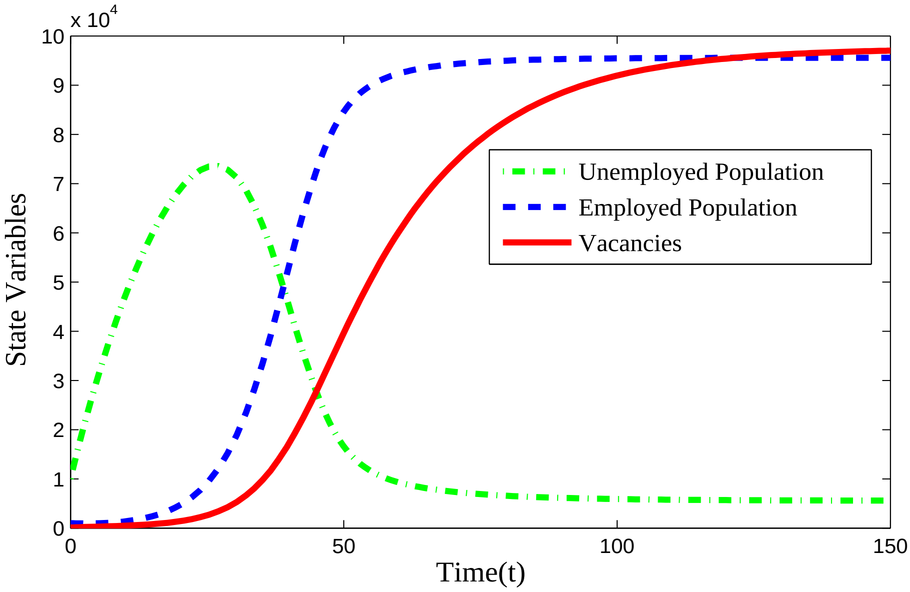

Figure 1. Asymtotical behaviour of state variables when taking no policy i.e. $u_1 = u_2 = 0$

Table 1. Explanation and Value of Parameters [5].

Parameters	Value	Explanation
A	5000	The constant rate of new faces in jobs market
k	0.00009	Employed rate for vacancies
α_1	0.04	Rate of migration and death of unemployed
α_2	0.05	Rate of retirement and death of employed
α_3	0.05	Rate of skilled person from training
γ	0.001	Rate of persons who fired from their jobs
ϕ	0.007	Rate of creating new vacancies
δ	0.05	Diminution rate of vacancies for lack of funds

5. Numerical Results and Discussions

Here, numerical simulations are investigated in absence of two policies or presence of one of two policies or presence of both policies. It is also compared with the results when both policies are optimized [Figures 9-11].

For solving the problem numerically, we use Forward-Backward Sweep Method [12]. The numerical optimal solution of the state equations and adjoint equations with objective function be found in MATLAB(R2014a) using the value of parameters from the Table 1 with weight parameters $A_1 = 20, B_1 = 4500, B_2 = 250$ and initial conditions $U(t_s) = 10^4, E(t_s) = 10^3, V(t_s) = 10^2$ are considered same as in [5].

For iterative process, we consider 1152 time-grid of 150 time units [5] and get increment of time $\Delta t = 0.13$. Since our optimal control problem is solved by indirect method, we accept convergence torerance of cost function at 10^{-8}.

Firstly, we present and analyse the numerical simulations when taking no policy as shown in Figure 1. We observe that the peak of unemployed persons is 73608 at the time 27 units in Figure 1. Since the control variables $u_1(t)$ and $u_2(t)$ are absent i.e. zero. So, the value of objective function is 2.1293×10^8 units. Taking no policies, at the time 150 units, the number of unemployed persons and vacancies are

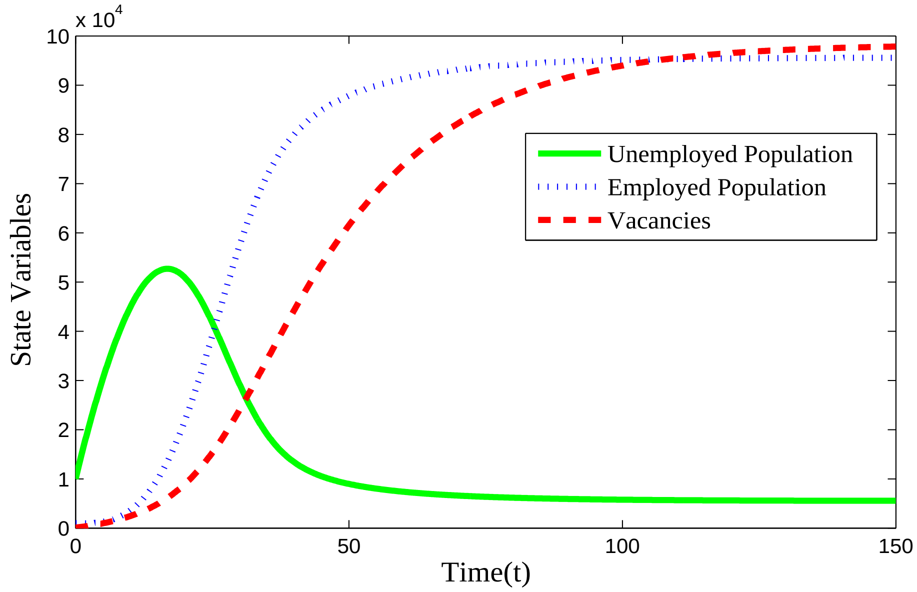

Figure 2. State trajectories when the second policy is active only i.e. $u_1 = 0$ & $u_2 = 1$

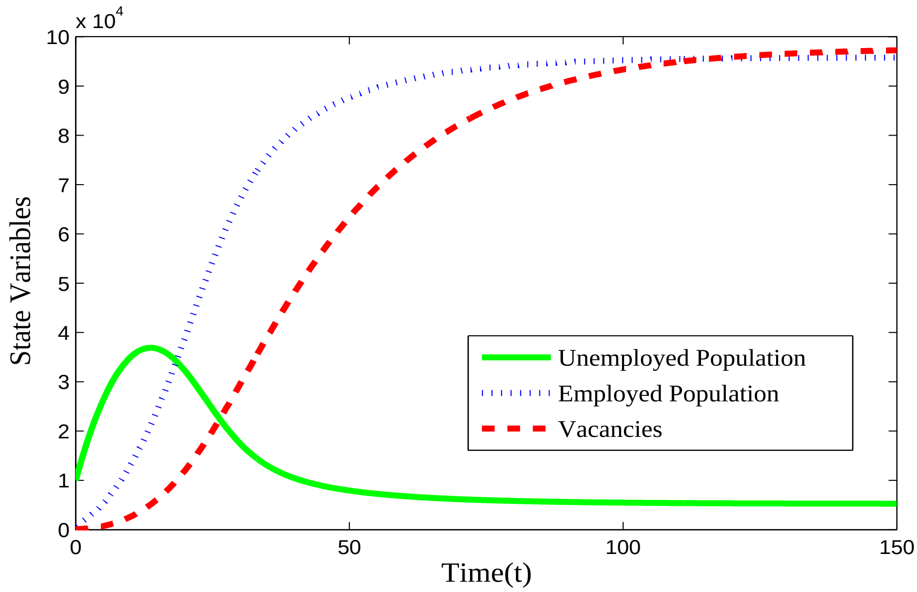

Figure 3. State trajectories when the first policy is active only i.e. $u_1 = 1$ & $u_2 = 0$

highest. We also observe that the state trajectories show the asymptotic behaviour of its. So, applying the second policy (creating new vacancies) only, it is observed that this policy indicates a significant decrease in the number of unemployed persons [Figure 2] compared with the results when taking no policy. Similarly, after the implementation of that policy (creating new vacancies), there are a eye-catching increase in the number of employed persons and vacancies. We observe a interesting matter that involving cost of policy making, the value of objective function is less than before. Determining the value of cost function, we found 2.2983×10^8 units. So we remind a well known proverb "Something is better than nothing" which is appropriate for the mathematical model. But when we are taking the first policy (providing skilled population) only, it is shown that this policy also indicates a significant decrease in the number of unemployed persons [Figure 3] compared with the results when taking the second policy (creating new vacancies) only. Taking the first policy, we find the value of cost function which is 2.3685×10^8 units. Similar to past, after the implementation of that policy (providing skilled population), there are great increase in the number of employed persons and vacancies. Besides 7 shows that the Comparison between that employed persons whose are appointed in job sectors for creating new vancancies and provided skilled population for government policy to control of policies.

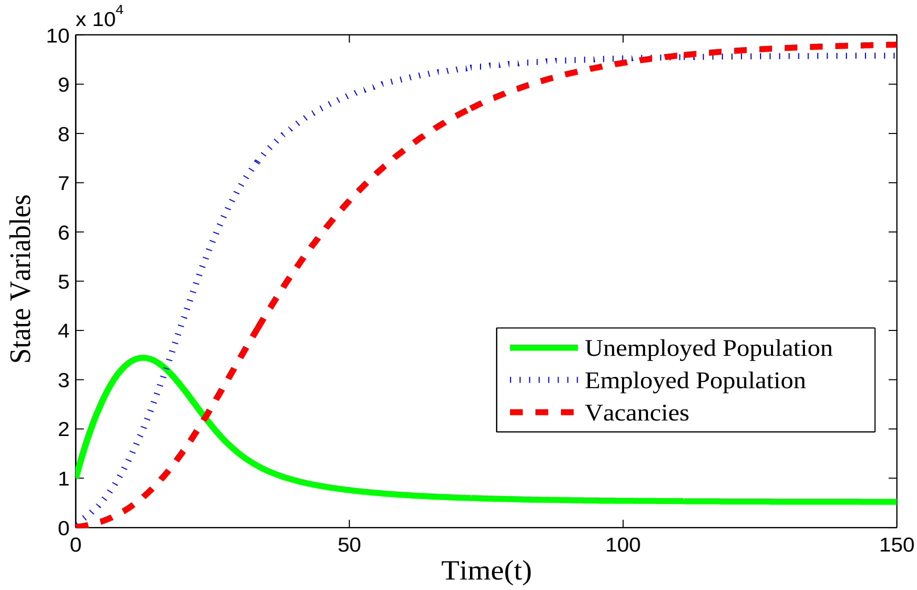

Figure 4. State Trajectories when both policies are active i.e. $u_1 = 1$ & $u_2 = 1$

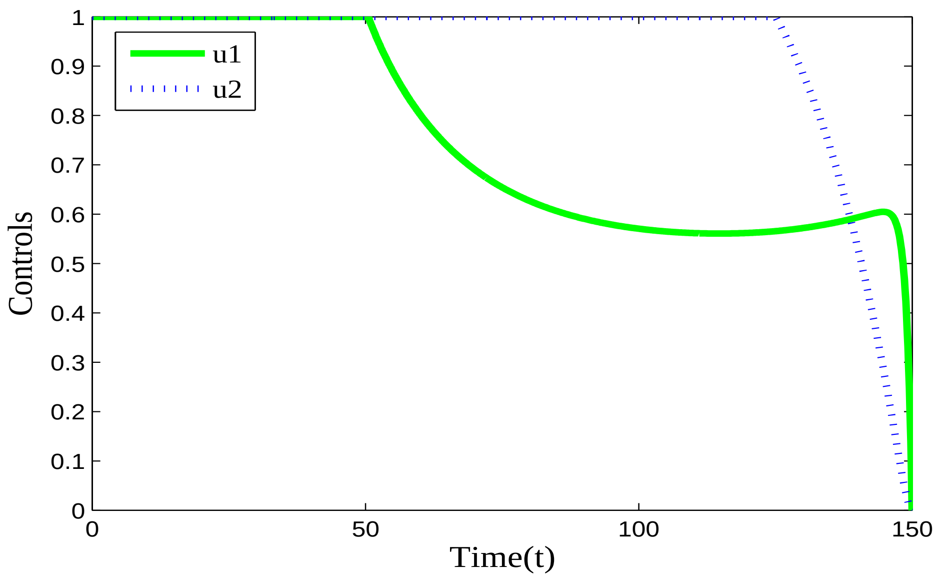

Figure 5. Optimal control profile to control of policies while minimizing the cost of policy making

Here it is clear that the most of unemployed are becomed as skilled population while minimizing the cost of policy of making. So, we declare that first policy is better than the second policy. But everyone know that the first policy is a useful policy for sustainable reduction of unemployed population. So we agree to this and decide the two policies to use for controlling of unemployed persons. Applying the both policies, we observe that the highest number of unemployed population is decreased 4 but slightly with compared to the results when the first policy (providing skilled population) is active only. Then, the value of objective function is 2.3846×10^8 units which is greater than before.

Finally, we determine the optimal solution for the policies of government as shown in Figure 5-8. When both control strategies are in use, the optimal value of objective function is 2.3854×10^8 units which is compared to objective functional (2.3846×10^8 units) with no control of the both policies. No control means both policies are active but they are constant and 1. We also observe that the highest number of unemployed persons, employed persons and vacancies are same in Figure 9-11 for the both cases of controls. But it is clearly seen in Table2 of numerical results. So, it is very interesting that we can not use our full ability but we are benefited. We also see that the peak of unemployed persons is 34463 at the time 13 units in Figure 6 when both control of policies are in use. But the peak

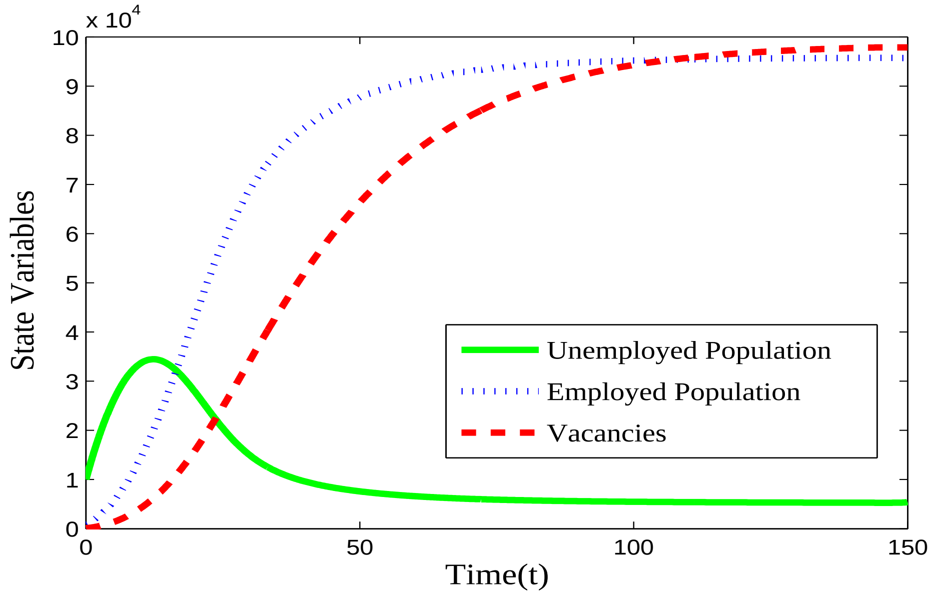

Figure 6. Optimal State Trajectories to control of policies while minimizing the cost of policy making

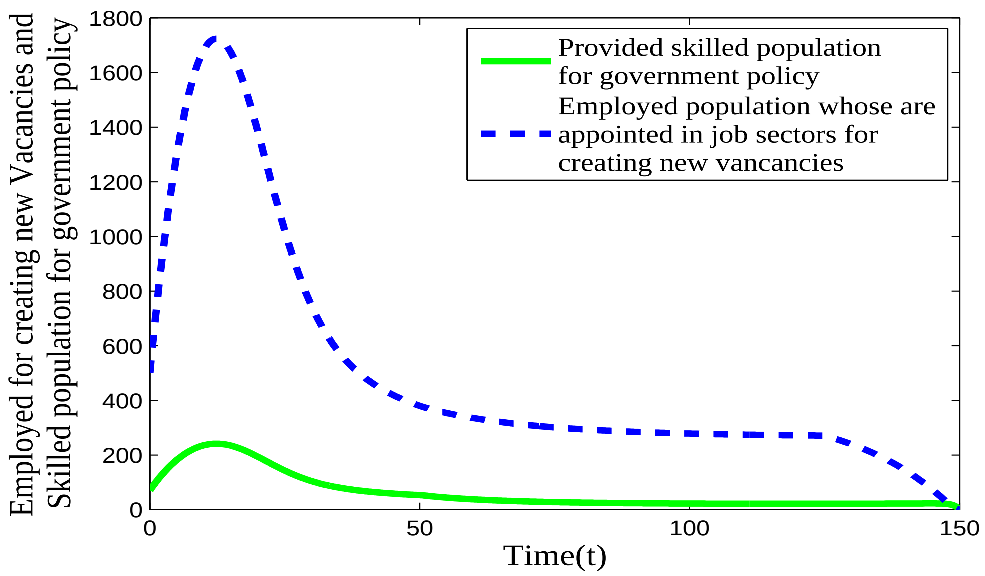

Figure 7. Comparison between that employed persons whose are appointed in job sectors for creating new vancancies and provided skilled population for government policy to control of policies

of unemployed persons is 73608 at the time 27 units when taking no policy. So, we decide that the condition of the unemployment problem is the best with two control strategies. It is clear that the number of employed and vacancies is excellent with two control strategies (see in Figure 10-11. We know that the transversality condition [figure 8] of the optimal control problem is zero at final time. So the figure 8 satisfies the completeness of optimal solution of the problem. It is clear in Table 2 that the objective functional is maximized for controlling of both policies of government to reduce the unemployed persons. So we decide that the control of both policies is better than no control of that policies. From this point, we believe that these control strategies can effectively reduce the unemployed persons in developing country after implementation of both policies of government. For different policy, the situation of unemployed population is presented in Figure 9. It is easy to say that the peak of unemployed population is smallest for the control of both policies. So, it is best strategies to reduce the unemployed population. Using those policies, the graph of employed population is presented in

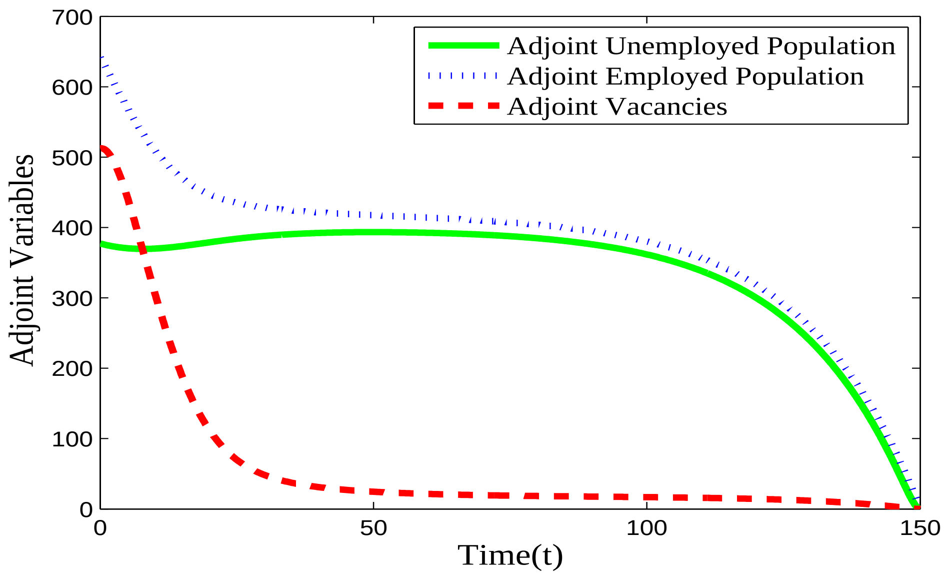

Figure 8. Adjoint trajectories for control of policies

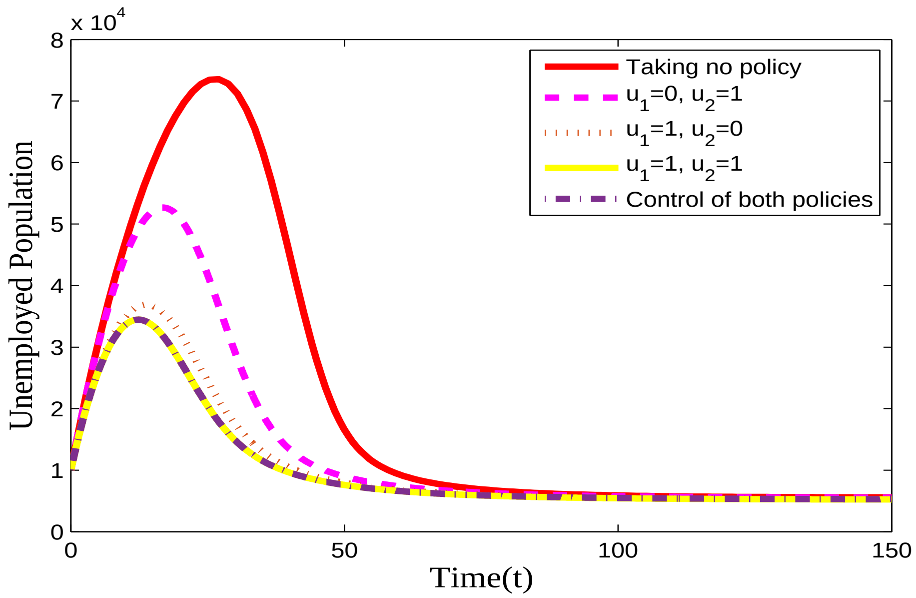

Figure 9. Unemployed population for different policy

Figure 10. We see that the upper curve of the graph is produced for the control of both policies. So, control of both polices are necessary for increasing of employed population. Similarly, Figure 11 shows the graph of vacancies for different policies. it is also observed that the number of vacancies is best for both control of polices. So, our declared optimal control of policies 5 is more useful than before.

Table 2. Summary of objective functional and states at final time

Status of Controls	Objective functional	Unemployed Persons	Employed Persons	Vacancies
$u_1 = 0, u_2 = 0$	2.1293×10^8	73608	9.5561×10^4	9.0079×10^4
$u_1 = 0, u_2 = 1$	2.2983×10^8	52681	9.5559×10^4	9.7838×10^4
$u_1 = 1, u_2 = 0$	2.3685×10^8	36900	9.5753×10^4	9.7239×10^4
$u_1 = 1, u_2 = 1$	2.3846×10^8	34463	9.5781×10^4	9.8022×10^4
$0\leq u_1 \leq 1$ & $0\leq u_2 \leq 1$	2.3854×10^8	34463	9.5597×10^4	9.7653×10^4

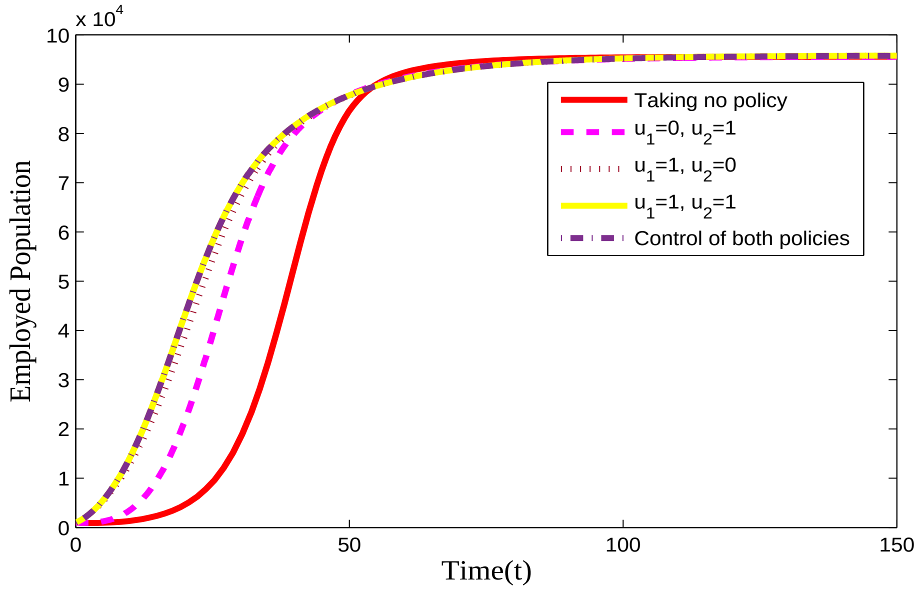

Figure 10. Employed population for different policy

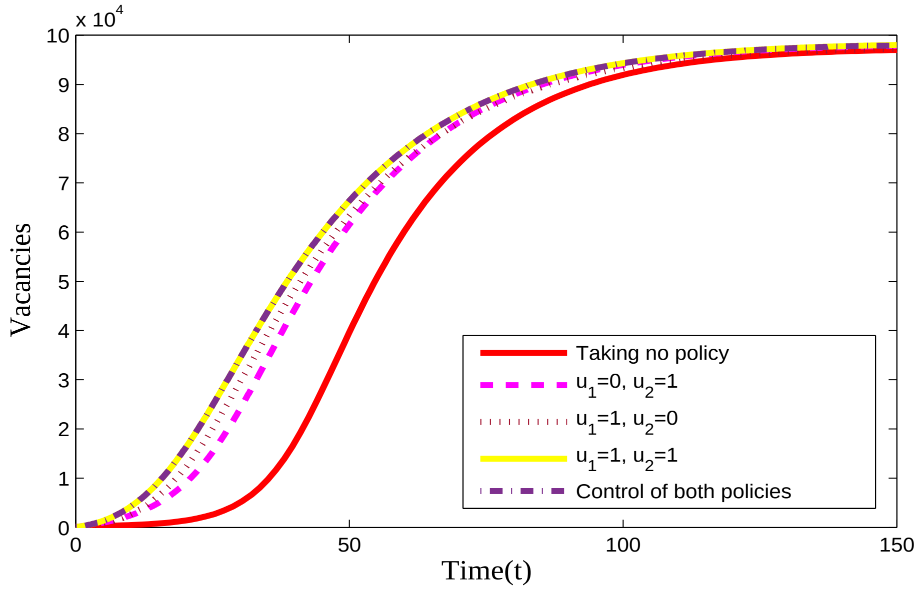

Figure 11. Vacancies for different policy

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6. Conclusion

This paper develop and analyse a model of unemployment to reduce unemployed population in the presence of two policies, one is creating new vacancies and other is providing skilled population. It is clear from above that the only one policy of government such as creating new vacancies is better than the taking no policy. Next time, we observe that the policy for providing skilled manpower is more effective for maximization of objective function than the policy of creating new vacancies. With taking both policies of government, we get some excellent results of the mathematical model where the value of objective function is maximized than before. But controlling the both policies, we have found the more minimized value of the objective function than the no control of the policies. We also seen that the unemployed persons are minimized for the controlling of both policies with the maximization of the value of objective function. we decide that the control of policies of government is more effective for maximization of objective function with the minimization of unemployed persons than the no control of policies.

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