Modeling of Modified Fourier Technique for Solving Separable Non Linear Programming Problem

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Abstract

In this paper, we propose a technique for solving a separable non linear programming problem (SNPP) by approximating each separable function by a piecewise linear function and then Modified Fourier Technique is used. Earlier Gauss Elimination, Fourier Technique and Modified Fourier Technique have already studied for linear programming problem. Here we use Modified Fourier Technique in which the numbers of additional constraints generated are reduced to a considerable extent by selecting the appropriate variable for elimination. To explain our technique, few numerical examples of different types are given at the end.

Key words:-Modified Fourier Technique, Inequalities, Separable Non Linear Programming, Grid points.

1. Introduction

In nonlinear programming problem (NLPP) either objective function or constraints or both may be non linear. Separable programming is a particular class of NLPP which is important because it allows a non linear program to be approximated with arbitrary accuracy with a linear programming model. In this technique each non linear function is reframed with a piecewise linear approximation. Once a non linear program is reduced to linear program then Modified Fourier Technique is applied to obtain the solution.

The problem of solving a system of linear inequalities dates back at least as far as Fourier who in 1827 published a method for solving them and after whom the method of Fourier-Motzkin elimination is named. A lot of methods are available to solve linear as well as non linear programming problems. In [3] Karmarkar gave a new polynomial time algorithm for linear Programming problem. In [2] Williams gave a method to solve linear programming problem by Fourier Technique. In [4] Kanniappan, Thangavel studied Modified Fourier Technique of solving linear programming problem. Jain and Mangal [6, 7, 8] studied various elimination techniques for Fractional Programming Problem. In [9] Jain studied Fourier elimination technique for multi-objective linear programming problem. In [10] Jain proposed Modified Gauss Elimination Technique for solving SNPP. Jain [11, 12] studied Fourier Elimination and Gauss Elimination Technique for Multiobjective Fractional Programming Problem. In [5] Bhargava and Sharma applied Extended Modified Fourier method to solve integer programming problem. In [1] Gaur and Arora studied Multi-level multi-objective integer linear programming problem.

The main difference between Gauss Elimination Technique and Modified Fourier Technique is that in Gauss Elimination Technique variable is arbitrary selected for elimination while in Modified Fourier Technique proper variable is selected for elimination using specific rule.

The purpose of this paper is to use modified Fourier technique for solving a separable non linear programming problem by selecting a variable for elimination, which will eliminate all variables step by step until one variable left and finally by back substitution we will get the value of variables and get optimal solution.

2. The Problem

Let f(x) and g(x) be a real-valued functions on X. Let f(x) is function to be optimized so non linear program can be represented as follows

Max./Min. z = f(x)

subject to, $g_j(x) \leq b_j$

$x \geq 0$

for j=1,2,………….,m and $x \in X$ (2.1)

Because we are considering separable non linear programming problems so f(x) and g(x) can be expressed as

$$f(x) = \sum_{i=1}^{n} f_i(x_i)$$

$$g_j(x) = \sum_{i=1}^{n} g_{ij}(x_i) \quad (2.2)$$

Using equation (2.2) in (2.1) the above problem is written as

$$\text{Max./Min. } z = \sum_{i=1}^{n} f_i(x_i)$$

subject to, $\sum_{i=1}^{n} g_{ij}(x_i) \leq b_j$ for j = 1, 2 ,..........., m

$x_i \geq 0$ for i = 1, 2,………...n (2.3)

3. Methodology

Let us consider the following nonlinear programming problem

$$\text{Max./Min. } z = \sum_{i=1}^{n} f_i(x_i)$$

subject to, $\sum_{i=1}^{n} g_{ij}(x_i) \leq b_j$ for j = 1, 2 ,..........., m

$x_I \geq 0$ for i = 1, 2,………...n (3.1)

To solve this problem, we first find range of x_i corresponding to non linear terms. We partition each x_i of non linear term into (r_i+1) grid points. As the number of grid points increases, the accuracy also increases. Suppose x_i ranges from l_i to u_i and also let

$$l_i = x_0 < x_1 < x_2 \ldots\ldots\ldots x_{r-1} < x_r = u_i$$

Suppose $(k+1)^{th}$ sub interval is $[x_k, x_{k+1}]$ so

$$x = \lambda x_k + (1-\lambda)\, x_{k+1} \quad (3.2)$$

where, $0 \leq \lambda \leq 1$

Now writing λ as $\lambda_k$and (1-λ) as λ_{k+1}.We get from equation (3.2)

$$x = \lambda_k x_k + \lambda_{k+1} x_{k+1} \lambda_k + \lambda_{k+1} = 1$$

where $0 \leq \lambda_k, \lambda_{k+1} \leq 1$ (3.3)

In general if $x_0 < x_i < x_r$ then

$$x_i = \sum_{k=0}^{r} \lambda_k x_k$$

and $\sum_{k=0}^{r} \lambda_k = 1$ (3.4)

Now at most two λ_k are positive and if two λ_k are positive then they must be consecutive.

Let $(x_k, f(x_k))$ and $(x_{k+1}, f(x_{k+1}))$ be two points. Then equation of this line is as follows

$$f(x) = f(x_k) + \frac{x - x_k}{x_{k+1} - x_k}(f(x_{k+1}) - f(x_k)) = \lambda f(x_k) + (1-\lambda) f(x_{k+1})$$

where $(1-\lambda) = \frac{x - x_k}{x_{k+1} - x_k}$, (3.5)

From equation (3.5), f(x) will reduce to

$$f(x) = \lambda_k f(x_k) + \lambda_{k+1} f(x_{k+1})$$

where, $\lambda_k + \lambda_{k+1} = 1$ (3.6)

In general $f_i(x_i)$ and $g_{ij}(x_i)$ is represented as

$$f_i(x_i) = \sum_{k=0}^{r_i} \lambda_{ik} f(x_k)$$

$$g_{ij}(x_i) = \sum_{k=0}^{r_i} \lambda_{ik} g_{ijk}(x_{ik}) \quad \text{with} \quad \sum_{k=0}^{r_i} \lambda_k = 1 \quad (3.7)$$

Here also atmost two λ_k are greater than zero and they must be consecutive. Using equation (3.1) and (3.7) the problem becomes as follows

$$\text{Max./Min. } z = \sum_{i=1}^{n} \sum_{k=0}^{r_i} \lambda_{ik} f(x_{ik})$$

subject to, $\sum_{i=1}^{n} \sum_{k=0}^{r_i} \lambda_{ik} g_{ijk}(x_{ik}) \leq b_j$ for j = 1,2…….m

$x_i \geq 0$ for i = 1, 2…….n (3.8)

where, $\sum_{k=0}^{r_i} \lambda_{ik} = 1$ and $0 \leq \lambda_{ik} \leq 1$

4. Modified Fourier Technique

In the problem defined by (3.8), we assume objective function as constraints so linear program becomes as follow

$$z - \sum_{i=1}^{n}\sum_{k=0}^{r_i} \lambda_{ik} f(x_{ik}) \leq 0$$

$$\sum_{i=1}^{n}\sum_{k=0}^{r_i} \lambda_{ik} g_{ijk}(x_{ik}) - b_j \leq 0$$

$$-x_i \leq 0 \qquad \text{for } i = 1, 2\ldots\ldots n \qquad (4.1)$$

Now above inequalities is represented as follows

$$Ax - b \leq 0$$

Now construct $I_j^+ I_j^-$ and I_j^0 for each variable j (except z) as follow

$$I_j^+ = \{i: A_{ij} > 0\}$$

$$I_j^- = \{i: A_{ij} < 0\}$$

$$I_j^0 = \{i: A_{ij} = 0\}$$

If any one of the sets I_j^+ or I_j^- is empty for a variable then the given linear problem is unbounded. Otherwise find minimum { $|I_j^+|$ * $|I_j^-|$ } where |C| denotes the number of constraints in the set C. Let j be the index corresponding to the minimum then eliminates this variable using Fourier variable elimination method. Repeat above steps until one programming variable is left and then by back substitution get optimum solution.

5. Numerical Examples

Here following three types of NLPP are considered in which

(a) Only objective function is non linear

(b) Only constraints is non linear

(c) Both objective function and constraints are non linear

(a) Let us consider NLPP in which objective function is non linear and constraints are linear

$$\text{Max } z = x_1^2 - 2x_1 + x_2$$

subject to, $x_1 + 2x_2 \le 5$

$2x_1 + x_2 \le 6$

$x_1, x_2 \ge 0$

From above constraints

$$0 \le x_1 \le 2$$

Let $f_1(x_1) = x_1^2 - 2x_1$

$f_2(x_2 = x_2$

$g_{11}(x_1) = x_1$

$g_{21}(x_2) = 2x_2$

$g_{12}(x_1) = 2x_1$

$g_{22}(x_2) = x_2$

Here only $f_1(x_1)$ is non linear so it is approximated as below.

Let there are 3 breaking points i.e. i=1 and r_i=2 and x_{ik} are

$$x_{10}=0 \quad x_{11}=1 \quad x_{12}=2$$

$\Rightarrow f_1(x_1) = \sum_{k=0}^{2} \lambda_{1k} f_1(x_{1k}) = 0\lambda_{10} + (-1)\lambda_{11} + 0\lambda_{12}$

$\Rightarrow f_1(x_1) = -\lambda_{11}$

Here $\lambda_{10}+\lambda_{11}+\lambda_{12} = 1$, So above problem reduces

$z+\lambda_{11}-x_2 \le 0$ (b1)

$x_1+2x_2 \le 5$ (b2)

$2x_1+x_2 \le 6$ (b3)

$-x_1 \le 0$ (b4)

$-x_2 \le 0$ (b5)

$-\lambda_{10} \le 0$ (b6)

$-\lambda_{11} \le 0$ (b7)

$-\lambda_{12} \le 0$ (b8)

$\lambda_{10} + \lambda_{11} + \lambda_{12} \le 1$ (b9)

Now the problem becomes linear. We select the variable for elimination using the Modified Fourier Techniqueby constructing I^+ , I^- and I^0 for each variable (except z) and selecting minimum of { | I_j^+ | * | I_j^- | }. We have the following sets

$I_{\lambda_{11}}^{+}$ = { b1, b9 }

$I_{\lambda_{11}}^{-}$ = { b7 }

$I_{\lambda_{11}}^{0}$ = { b2, b3, b4, b5, b6, b8 } and $I_{x2}^{+} = \{ b2, b3 \}$

$I_{x2}^{-} = \{ b1, b5 \}$

$I_{x2}^{0} = \{ b4, b6, b7, b8, b9 \}$

and $I_{x1}^{+} = \{ b2, b3 \}$

$I_{x1}^{-} = \{ b4 \}$

$I_{x1}^{0} = \{ b1, b5, b6, b7, b8, b9 \}$

and $I_{\lambda_{10}}^{+} = \{ b9 \}$

$I_{\lambda_{10}}^{-} = \{ b6 \}$

$I_{\lambda_{10}}^{0} = \{ b1, b2, b3, b4, b5, b7, b8 \}$

and $I_{\lambda_{12}}^{+} = \{ b9 \}$

$I_{\lambda_{12}}^{-} = \{ b8 \}$

$I_{\lambda_{12}}^{0} = \{ b1, b2, b3, b4, b5, b6, b7 \}$

Now

$\{ |I_j^{+}| * |I_j^{-}| \} = 2$ for $j=\lambda_{11}$

$\{ |I_j^{+}| * |I_j^{-}| \} = 4$ for $j=x_2$

$\{ |I_j^{+}| * |I_j^{-}| \} = 2$ for $j=x_1$

$\{ |I_j^{+}| * |I_j^{-}| \} = 1$ for $j=\lambda_{10}$

$\{ |I_j^{+}| * |I_j^{-}| \} = 1$ for $j=\lambda_{12}$

Minimum of {2, 4, 2, 1, 1} is 1 which is for λ_{10} and λ_{12} so selecting arbitrary λ_{12} and eliminating λ_{12} by (b8+b9) we get

$$z+\lambda_{11}-x_2 \leq 0 \quad \text{(c1)}$$
$$X_1+2x_2 \leq 5 \quad \text{(c2)}$$
$$2x_1+x_2 \leq 6 \quad \text{(c3)}$$
$$-x_1 \leq 0 \quad \text{(c4)}$$
$$-x_2 \leq 0 \quad \text{(c5)}$$
$$-\lambda_{10} \leq 0 \quad \text{(c6)}$$
$$-\lambda_{11} \leq 0 \quad \text{(c7)}$$
$$\lambda_{10}+\lambda_{11} \leq 1 \quad \text{(c8)}$$

Now calculating $\{ |I_j^{+}| * |I_j^{-}| \}$ for other variables as earlier, we get minimum for λ_{10}. Therefore we get

$$z+\lambda_{11}-x_2 \leq 0 \quad \text{(d1)}$$
$$x_1+2x_2 \leq 5 \quad \text{(d2)}$$
$$2x_1+x_2 \leq 6 \quad \text{(d3)}$$
$$-x_1 \leq 0 \quad \text{(d4)}$$
$$-x_2 \leq 0 \quad \text{(d5)}$$

$$-\lambda_{11} \leq 0 \quad \text{(d6)}$$
$$\lambda_{11} \leq 1 \quad \text{(d7)}$$

Similarly calculating { $|I_j^+| * |I_j^-|$ } for other variables as earlier, we get minimum for x_1. Therefore we get

$$z+\lambda_{11}-x_2 \leq 0 \quad \text{(e1)}$$
$$2x_2 \leq 5 \quad \text{(e2)}$$
$$x_2 \leq 6 \quad \text{(e3)}$$
$$-x_2 \leq 0 \quad \text{(e4)}$$
$$-\lambda_{11} \leq 0 \quad \text{(e5)}$$
$$\lambda_{11} \leq 1 \quad \text{(e6)}$$

Similarly calculating { $|I_j^+| * |I_j^-|$ } for other variables as earlier, we get minimum for λ_{10}. Therefore we get

$$z-x_2 \leq 0 \quad \text{(f1)}$$
$$2x_2 \leq 5 \quad \text{(f2)}$$
$$x_2 \leq 6 \quad \text{(f3)}$$
$$-x_2 \leq 0 \quad \text{(f4)}$$
$$0 \leq 1 \quad \text{(f5)}$$

Similarly calculating { $|I_j^+| * |I_j^-|$ } for other variables as earlier, we get minimum for x_2. Therefore we get

$$2z \leq 5$$
$$Z \leq 6$$
$$0 \leq 5$$
$$0 \leq 6$$

So we get least upper bound for z which is equal to 2.5 and by solving above equations we get final solution $x_2 = 2.5$, $x_1 = 0$ and $z = 2.5$

(b) Let us consider NLPP in which objective function is linear and constraints are non linear.

$$\text{Max. } z = -3x_1 + 5x_2$$

subject to, $x_1^3 + 7x_2 \leq 8$

$$x_1 , x_2 \geq 0$$

Let $f_1(x_1) = -3x_1$

$f_2(x_2) = 5x_2$

$g_{11}(x_1) = x_1^3$

$g_{21}(x_2)= 7x_2$

Here only $g_{11}(x_1)$ is non linear so it is approximated as below

It is observed from the constraints set that $0 \leq x_1 \leq 2$.

Let there are three breaking points i.e. $x_{10} = 0$, $x_{11} = 1$ and $x_{12} = 2$ so $g_{11}(x_1)$ can be written as

$$g_{11}(x_1) = \sum_{k=0}^{2} \lambda_{1k} g_{11k}(x_{1k}) = \lambda_{10} \cdot g_{11}(x_{10}) + \lambda_{11} \cdot g_{11}(x_{11}) + \lambda_{12} \cdot g_{11}(x_{12})$$
$$= 0.\lambda_{10} + 2.\lambda_{11} + 16.\lambda_{12}$$

So the above problem reduces

$$z+3x_1 -5x_2 \leq 0 \qquad (g1)$$
$$2.\lambda_{11} + 16.\lambda_{12} + 7.x_2 \leq 8 \qquad (g2)$$
$$-x_1 \leq 0 \qquad (g3)$$
$$-x_2 \leq 0 \qquad (g4)$$
$$-\lambda_{10} \leq 0 \qquad (g5)$$
$$-\lambda_{11} \leq 0 \qquad (g6)$$
$$-\lambda_{12} \leq 0 \qquad (g7)$$
$$\lambda_{10} + \lambda_{11} + \lambda_{12} \leq 1 \qquad (g8)$$

Now the all constraints of problem become linear. We select the variable for elimination using the Modified Fourier Techniqueby constructing I^+ , I^- and I^0 for each variable (except z) and selecting minimum of $\{ | I_j^+ | * | I_j^- | \}$

Now calculating $\{ |I_j^+ | * | I_j^- | \}$ for other variables, we get minimum for x_1. Therefore we get

$$z -5x_2 \leq 0 \qquad (h1)$$
$$2.\lambda_{11} + 16.\lambda_{12} + 7.x_2 \leq 8 \qquad (h2)$$
$$-x_2 \leq 0 \qquad (h3)$$
$$-\lambda_{10} \leq 0 \qquad (h4)$$
$$-\lambda_{11} \leq 0 \qquad (h5)$$
$$-\lambda_{12} \leq 0 \qquad (h6)$$
$$\lambda_{10} + \lambda_{11} + \lambda_{12} \leq 1 \qquad (h7)$$

Now again calculating $\{ |I_j^+ | * | I_j^- | \}$ for other variables as earlier, we get minimum for λ_{10}. Therefore we get

$$z -5x_2 \leq 0 \qquad (i1)$$
$$2.\lambda_{11} + 16.\lambda_{12} + 7.x_2 \leq 8 \qquad (i2)$$
$$-x_2 \leq 0 \qquad (i3)$$
$$\lambda_{11} + \lambda_{12} \leq 1 \qquad (i4)$$
$$-\lambda_{11} \leq 0 \qquad (i5)$$

$$-\lambda_{12} \leq 0 \quad \text{(i6)}$$

Now again calculating { $|l_j^+| * |l_j^-|$ } for other variables as earlier, we get minimum for λ_{11}. Therefore we get

$$z - 5x_2 \leq 0 \quad \text{(j1)}$$
$$16.\lambda_{12} + 7x_2 \leq 8 \quad \text{(j2)}$$
$$\lambda_{12} \leq 1 \quad \text{(j3)}$$
$$-x_2 \leq 0 \quad \text{(j4)}$$
$$-\lambda_{12} \leq 0 \quad \text{(j5)}$$

Now again calculating { $|l_j^+| * |l_j^-|$ } for other variables as earlier, we get minimum for λ_{12}. Therefore we get

$$z - 5x_2 \leq 0 \quad \text{(k1)}$$
$$7x_2 \leq 8 \quad \text{(k2)}$$
$$0 \leq 1 \quad \text{(k3)}$$
$$-x_2 \leq 0 \quad \text{(k4)}$$

Now again calculating { $|l_j^+| * |l_j^-|$ } for other variables as earlier, we get minimum for x_2. Therefore we get

$$z \leq 40/7 \quad \text{(l1)}$$
$$0 \leq 8 \quad \text{(l2)}$$
$$0 \leq 1 \quad \text{(l3)}$$

So we get least upper bound for z which is equal to 40/7 and by solving above equations we get final solution $x_2=8/7$, $x_1 =0$ and $z=40/7$.

(c) Let us consider NLPP in which both objective function and constraints are non linear and having separable functions

$$\text{Max. } z = x_1^2+x_2$$

subject to,
$$2x_1^2+4x_2 \leq 8$$
$$x_1, x_2 \geq 0$$

Let
$$f_1(x_1) = x_1^2$$
$$f_2(x_2) = x_2$$
$$g_{11}(x_1) = 2x_1^2$$
$$g_{21}(x_2) = 4x_2$$

Here two terms are non linear.

It is observed from the constraints set that $0 \leq x_1 \leq 2$.

Let there are 3 breaking points i.e. $x_{10} = 0$, $x_{11} = 1$ and $x_{12} = 2$ so $f_1(x_1)$ and $g_{11}(x_1)$ can be approximated as

$$f_1(x_1) = \sum_{k=0}^{2} \lambda_{1k} f_{1k} = 0\lambda_{10} + 1\lambda_{11} + 4\lambda_{12}$$

$$g_{11}(x_1) = 0\lambda_{10} + 2\lambda_{11} + 8\lambda_{12}$$

So above problem reduces to

$$z-\lambda_{11} -4\lambda_{12} - x_2 \leq 0 \quad (m1)$$
$$2\lambda_{11} + 8\lambda_{12} + 4x_2 \leq 8 \quad (m2)$$
$$\lambda_{10} + \lambda_{11} + \lambda_{12} \leq 1 \quad (m3)$$
$$-\lambda_{10} \leq 0 \quad (m4)$$
$$-\lambda_{11} \leq 0 \quad (m5)$$
$$-\lambda_{12} \leq 0 \quad (m6)$$
$$-x_2 \leq 0 \quad (m7)$$

Now calculating { $|I_j^+| * |I_j^-|$ } for other variables as earlier(in part a), we get minimum for λ_{10}. Therefore we get

$$z-\lambda_{11} -4\lambda_{12} - x_2 \leq 0 \quad (n1)$$
$$2\lambda_{11} + 8\lambda_{12} + 4x_2 \leq 8 \quad (n2)$$
$$\lambda_{11} + \lambda_{12} \leq 1 \quad (n3)$$
$$-\lambda_{11} \leq 0 \quad (n4)$$
$$-\lambda_{12} \leq 0 \quad (n5)$$
$$-x_2 \leq 0 \quad (n6)$$

Now again calculating { $|I_j^+| * |I_j^-|$ } for other variables as earlier, we get minimum for x_2. Therefore we get

$$4z-2\lambda_{11} -8\lambda_{12} \leq 8 \quad (o1)$$
$$2\lambda_{11} + 8\lambda_{12} \leq 8 \quad (o2)$$
$$\lambda_{11} + \lambda_{12} \leq 1 \quad (o3)$$
$$-\lambda_{11} \leq 0 \quad (o4)$$
$$-\lambda_{12} \leq 0 \quad (o5)$$

Now again calculating { $|I_j^+| * |I_j^-|$ } for other variables as earlier, we get minimum for λ_{12}. Therefore we get

$$4z \leq 16 \quad (p1)$$
$$4z + 6\lambda_{11} \leq 16 \quad (p2)$$
$$2\lambda_{11} \leq 8 \quad (p3)$$
$$\lambda_{11} \leq 1 \quad (p4)$$
$$-\lambda_{11} \leq 0 \quad (p5)$$

Finally calculating { $|I_j^+| * |I_j^-|$ } for other variables as earlier, we get minimum for λ_{11}. Therefore we get

$$4z \leq 16 \quad (q1)$$
$$4z \leq 16 \quad (q2)$$
$$0 \leq 8 \quad (q3)$$
$$0 \leq 1 \quad (q4)$$

So we get least upper bound for z which is equal to 4 and by solving above equations we get final solution $x_2 = 0$, $x_1 = 2$ and $z = 4$

6. Conclusion

The proposed technique for solving Separable Non Linear Programming Problem is much better than earlier existing techniques because in the proposed Modified Fourier Technique additional constraints generated are reduced to a considerable extent by selecting the appropriate variable for elimination and separable property is used to approximate non linear program into linear program. The procedure of solving NLPP by different numerical techniques are still updating in our research work like Relaxation Method etc.

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