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Probabilistic Continuous Review Inventory Model with Mixture Shortage and Varying Holding Cost under Constraint: Gamma Distribution

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Abstract: In this paper, we interested of studying a probabilistic continuous review inventory model with varying holding cost under holding cost constraint when the shortage is mixture. The model is derived under two different situations: Model (I_c) when the costs are crisp and Model (I_F) when the costs are fuzzy. The optimal values of order quantity, reorder point and the expected minimum total cost are obtained for both the two models, when the lead time demand follows gamma distribution. Finally numerical computations for optimum parameters of this model by using the mathematica program are presented.

1. Introduction

The continuous review inventory model $< Q, r >$ has been discussed over many years. A lot of conditions and assumptions represented in models in many papers and books, most of researchers discussed the case when the inventory is backordered or using the case of the lost sales as in [Fergany and Elwakeel, 2006, Gupta and Hira 1993]. Inventory model which present the case of backorder with lost sales case is known as the model with a mixture shortage, such as model analyzed by [Park, 1982]. In [Abuo-El-Ata, Fergany and Elwakeel, 2003] they introduced an inventory model with varying order cost and zero lead time under

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two restrictions. They derived the optimal maximum inventory level by using a geometric programming approach. Recently [Fergany, 2016] proposed a new general probabilistic multi-item, single-source inventory model with varying mixture shortage cost under two restrictions, one of them is on the expected varying backorder cost and the other is on the expected varying lost sales cost. Often in the inventory models, the cost components are considered as crisp values, but in the real life, because of various physical or chemical characteristics may be effect on the cost components, precise values of cost characteristics become difficult to measure the exact amount of order, holding and especially shortage cost. Thus, in controlling the inventory system it may allow some flexibility in the cost parameter values in order to treat the uncertainties which always fit the real situations. As a result, fuzzy set theory is presented to meet these requirements to certain extent. The Economic Order Quantity (EOQ) model from the fuzzy set theoretic by using trapezoidal fuzzy numbers for ordering and inventory holding costs have examined by [Park, 1987] cited in [Vijayan and Kumaran, 2007]. [Yao and Lee, 1999] has discussed a backorder inventory model which fuzzified the order quantity as triangular and trapezoidal fuzzy numbers and keeps the shortage cost as a crisp parameter. [Chang, 2003] performed the investigation of fuzzy lost sales on the periodic review inventory model with a mixture of backorder and lost sales under variable lead time. [Chiang, Yao and Lee, 2005] studied fuzzy inventory model with backorders where the parameters are represented by triangular fuzzy numbers. [Farithaasma and Henry, 2015] they have presented an inventory model with shortage together with the space constraint, where carrying cost, shortage cost, ordering cost and demand are assumed as fuzzy numbers in nature to make the inventory model more realistic. After that they transformed the minimization of the cost function subject to the constraint into a multi-objective inventory problem. Hence they used fuzzy optimization technique to find out the optimal results. [Elwakeel and Al-yazidi, 2016] discussed two different cases of the probabilistic continuous review mixture shortage inventory model with varying and constrained expected order cost, when the lead time demand follows some different continuous distributions. They presented two cases, the first case was when the total cost components are considered to be crisp values, and the other case was when the costs are considered as trapezoidal fuzzy numbers

[Vijayan and Kumaran, 2007] developed unconstraint continuous and periodic review inventory model with mixture shortage and constant units cost in case of all costs are fuzzy numbers and the cases when just one cost components is fuzzy and the remaining are crisp, with consideration that backorder is independent of time.

[Fergany, Ezzat and Gawdt, 2011] studied two different cases of continuous review inventory models with varying holding cost, under service level constraint with mixture shortage when lead time was reduction by the lead time crashing cost. In the first case we obtained the optimal lead time and the optimal order quantity in crisp values. The other case was when the average demand per year and the backorder fraction are considered triangular fuzzy numbers and the optimal policy was derived in fuzzy values.

Our paper is divided into two models; Model (I_c): a probabilistic continuous review inventory model will be discussed with varying holding cost under holding cost constraint when the shortage is mixture by considering all costs are crisp values. Then we obtained the optimal value of order quantity Q^*, the optimal reorder point r^* and the expected minimum total cost $minE(TC(Q^*;\ r^*))$ when lead time demand follows gamma distribution. Model (I_F): the constraint continuous review inventory Model (I_c) will be recast with the same assumptions when all costs are trapezoidal fuzzy numbers rather than the crisp values. Again the optimal values of Q^*, r^* and $minE(TC(Q^*;\ r^*))$ are derived by using the sign distance method to deffuzzify the costs. Numerical computations for optimum parameters of both models by using the mathematica program are presented. Finally, we introduced a comparison between the results of the two models.

2. Notations and Assumptions:

2.1. List of notations:

$\boldsymbol{Q}$	The decision variable representing the order quantity per cycle,
$\boldsymbol{Q}^*$	The optimal value of the order quantity per cycle,
$\boldsymbol{r}$	The reorder point,
$\boldsymbol{r}^*$	The optimal value of the reorder point,
$<\boldsymbol{Q},\boldsymbol{r}>$	The continuous review inventory model, with Q,r are the decision variables,
$\boldsymbol{L}$	The lead time,
$\boldsymbol{X}$	The random variable represent the lead time demand,
$\overline{\boldsymbol{D}}$	The average annual demand,
$\overline{\boldsymbol{H}}$	The average on hand inventory,
$\boldsymbol{c_o}$	The inventory order cost per unit per cycle,
$\boldsymbol{c_h}$	The inventory holding cost per unit per cycle,
$\boldsymbol{c_h}\,(\boldsymbol{Q})$	The varying holding cost per cycle $=c_h Q^{\beta}$,
$\boldsymbol{\beta}$	A constant real number selected to provide the best fit of estimated cost function,
$\boldsymbol{c_b}$	The inventory backorder cost per unit per cycle,
$\boldsymbol{c_l}$	The inventory lost sales cost per unit per cycle,
$\tilde{\boldsymbol{c}}_{\boldsymbol{o}}$	The fuzzy order cost per unit per cycle,
$\tilde{\boldsymbol{c}}_{\boldsymbol{h}}$	The fuzzy holding cost per unit per cycle,
$\tilde{\boldsymbol{c}}_{\boldsymbol{b}}$	The fuzzy backorder cost per unit per cycle,
$\tilde{\boldsymbol{c}}_{\boldsymbol{l}}$	The fuzzy lost sales cost per unit per cycle,

$\boldsymbol{R(r)}$	The probability of the shortage $= \int_{r}^{\infty} f(x)dx$,
$\overline{\boldsymbol{S}}\,(\boldsymbol{r})$	The expected shortage quantity per cycle $= \int_{r}^{\infty} (x-r)f(x)dx$,
$\boldsymbol{K_h}$	The limitation on the expected annual holding cost,
$\boldsymbol{\lambda_h}$	Lagrange multiplier.
$\boldsymbol{G(Q, r, \lambda_h)}$	The Lagrange multiplier function of the expected annual total cost

2.2. Assumptions

1. Continuous review inventory model with varying holding cost.
2. Shortage cost is mixture and the backorder cost is dependent of time.
3. $\boldsymbol{\gamma}$ is a fraction of unsatisfied demand that will be backordered while the remaining fraction $(1-\gamma)$ is completely lost.
4. The model is under varying holding cost constraint.
5. Demand is a continuous random variable, the lead time is constant and the distribution of the lead time demand is known.

3. Model ($\mathbf{I}_c$) Mixture Probabilistic $< Q, r >$ with Varying Holding Cost under Holding Cost Constraint for crisp costs.

3.1. Model Analysis

We know that when the number of units on hand and on order reaches to the reorder point r, we have to procure a replenishment quantity Q. In many situations, the customers of certain suppliers have high faith and loyalty when the system is out of stock, some customers are willing to wait for backorders. However, the remaining becomes impatient and turns to other suppliers, so lost sales result. The expected annual total cost can be expressed as follows:

$$E(Total\ Cost) = E(Order\ Cost) + E(Holding\ Cost) + E(Shortage\ Cost)$$

$$E(TC\ (Q,r)\ ) = E(OC) + E(HC) + \ E(SC),$$

where: $$E(SC) = E(BC) + E(LC).$$

The aim in this paper, is to minimize the expected annual total cost $E(TC(Q;\ r))$ under varying holding costs constraint. To solve this primal function, let us write it as follows:

$$E(TC) = c_o \frac{\overline{D}}{Q} + c_h Q^{\beta} \left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right]$$

$$+ \frac{c_b \gamma \overline{D}}{Q}\ \bar{S}(r) + \frac{c_l (1-\gamma)\overline{D}}{Q}\ \bar{S}(r), \qquad Q > 0 \qquad (3.1)$$

Subject to:

$$c_h Q^{\beta} \left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right] \leq \ K_h. \qquad (3.2)$$

To find the optimal values Q^* and r^* which minimize Equation (3.1) under the constraint (3.2) we use the Lagrange multiplier technique as follows:

$$G(Q,r,\lambda_h) = c_o \frac{\overline{D}}{Q} + c_h Q^{\beta} \left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right]$$

$$+ \frac{c_b \gamma \overline{D}}{Q}\ \bar{S}(r) + \frac{c_l (1-\gamma)\overline{D}}{Q}\ \bar{S}(r) \qquad (3.3)$$

$$+\lambda_h \left\{ c_h Q^{\beta} \left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right] - \ K_h \right\}$$

The optimal values Q^* and r^*can be found by setting each of the corresponding first partial derivatives of Equation (3.3) with respect to Q and r equal to zero, we get:

$$A(1+\beta)Q^{*\beta+2} + 2A\beta Q^{*\beta+1}[r^* - \mu + (1-\gamma)\bar{S}(r^*)] - B - 2M\bar{S}(r^*) = 0 \quad (3.4)$$

and the probability of the shortage is:

$$R(r^*) = \frac{AQ^{*\beta+2}}{M + (1-\gamma)AQ^{*\beta+1}} \qquad (3.5)$$

where: $A = (1+\lambda_h)c_h$, $B = 2c_o\overline{D}$ and $M = c_b\gamma\overline{D} + c_l(1-\gamma)\overline{D}$.

There is no closed form solution of Equations (3.4) and (3.5). If the lead time demand follows the Gamma distribution with parameter n, ρ then the probability of the shortage and the expected shortage quantity will be in the following form:

$$R(r) = e^{-\rho r} \sum_{k=0}^{n-1} \frac{(\rho r)^k}{k!}, \tag{3.6}$$

and

$$\bar{S}(r) = re^{-\rho r}\left(\frac{(\rho r)^{n-1}}{(n-1)!} - 1\right)\sum_{k=0}^{n-1}\frac{(\rho r)^k}{k!}, \tag{3.7}$$

To minimize the expected annual total cost we substitute Equations (3.6) and (3.7) into the Equations (3.4) and (3.5), but for solving these equations we have to use an iterative method, which is illustrated in the algorithm.

4. Model ($\mathbf{I_F}$): Mixture Probabilistic $< Q, r >$ with Varying Holding Cost under Holding Cost Constraint for fuzzy costs:

4.1. Model Analysis

Assume a continuous review inventory model with the same assumptions of the Model ($\mathbf{I_c}$). But consider all the costs c_o , c_h , c_b and c_l are fuzzy numbers. The new model is denoted by Model **($\mathbf{I_F}$)**, we express them by using trapezoidal fuzzy numbers, as the following form:

$$\widetilde{c_o} = (\widetilde{c_o} - \delta_1, \widetilde{c_o} - \delta_2, \widetilde{c_o} + \delta_3, \widetilde{c_o} + \delta_4),$$

$$\widetilde{c_h} = (\widetilde{c_h} - \delta_5, \widetilde{c_h} - \delta_6, \widetilde{c_h} + \delta_7, \widetilde{c_h} + \delta_8),$$

$$\widetilde{c_b} = (\widetilde{c_b} - \theta_1, \widetilde{c_b} - \theta_2, \widetilde{c_b} + \theta_3, \widetilde{c_b} + \theta_4),$$

$$\widetilde{c_l} = (\widetilde{c_l} - \theta_5, \widetilde{c_l} - \theta_6, \widetilde{c_l} + \theta_7, \widetilde{c_l} + \theta_8).$$

where δ_i and θ_i, $i = 1, 2, \dots 8$, are arbitrary positive numbers under the following restrictions:

$$\widetilde{c_o} > \delta_1 > \delta_2, \delta_3 < \delta_4,\ \widetilde{c_h} > \delta_5 > \delta_6 \text{ and } \delta_7 < \delta_8$$

simillarly, $\widetilde{c_b} > \theta_1 > \theta_2, \theta_3 < \theta_4,\ \widetilde{c_l} > \theta_5 > \theta_6$ and $\theta_7 < \theta_8$.

The lift and right $\alpha - \ cut$ of $\widetilde{c_o}, \widetilde{c_h}, \widetilde{c_b}$ and $\widetilde{c_l}$ are given as follows:

$$\tilde{c}_{ov}(\alpha) = \tilde{c}_o - \delta_1 + (\delta_1 - \delta_2)\alpha, \qquad \tilde{c}_{ou}(\alpha) = \tilde{c}_o + \delta_4 - (\delta_4 - \delta_3)\alpha,$$

$$\tilde{c}_{hv}(\alpha) = \tilde{c}_h - \delta_5 + (\delta_5 - \delta_6)\alpha, \qquad \tilde{c}_{hu}(\alpha) = \tilde{c}_h + \delta_8 - (\delta_8 - \delta_7)\alpha,$$

$$\tilde{c}_{bv}(\alpha) = \tilde{c}_b - \theta_1 + (\theta_1 - \theta_2)\alpha, \qquad \tilde{c}_{bu}(\alpha) = \tilde{c}_b + \theta_4 - (\theta_4 - \theta_3)\alpha,$$

$$\tilde{c}_{lv}(\alpha) = \tilde{c}_l - \theta_5 + (\theta_5 - \theta_6)\alpha, \qquad \tilde{c}_{lu}(\alpha) = \tilde{c}_l + \theta_8 - (\theta_8 - \theta_7)\alpha.$$

The expected annual total cost $E(TC\ (Q, r))$ with all cost components are fuzzy under the expected holding cost constraint is given by:

$$\tilde{E}(\tilde{c}_o, \tilde{c}_h\ , \tilde{c}_b, \tilde{c}_l) = \tilde{c}_o \frac{\bar{D}}{Q} + \tilde{c}_h Q^\beta \left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right]$$
$$+ \frac{\tilde{c}_b \gamma \bar{D}}{Q}\ \bar{S}(r) + \frac{\tilde{c}_l (1-\gamma)\bar{D}}{Q}\ \bar{S}(r), \qquad Q > 0 \tag{4.1}$$

Subject to:

$$\tilde{c}_h Q^\beta \left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right] \leq\ K_h. \tag{4.2}$$

To find the optimal values Q^* and r^* which minimize Equation (4.1) under the constraint (4.2) we use the Lagrange multiplier technique as follows:

$$\tilde{G}(\tilde{c}_o, \tilde{c}_h\ , \tilde{c}_b, \tilde{c}_l) = \tilde{c}_o \frac{\bar{D}}{Q} + \tilde{c}_h Q^\beta \left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right]$$
$$+ \frac{\tilde{c}_b \gamma \bar{D}}{Q}\ \bar{S}(r) + \frac{\tilde{c}_l (1-\gamma)\bar{D}}{Q}\ \bar{S}(r) \tag{4.3}$$
$$+\lambda_h \left\{\tilde{c}_h Q^\beta \left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right] - \ K_h\right\}.$$

The lift and right $\alpha - \ cut$ of the fuzzified cost function are respectively given by as follows:

$$\tilde{G}(\tilde{c}_o, \tilde{c}_h, \tilde{c}_b, \tilde{c}_l)_v(\alpha) = \tilde{c}_{ov}\frac{\bar{D}}{Q} + \tilde{c}_{hv}Q^{\beta}\left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right]$$

$$+\frac{\tilde{c}_{bv}\gamma\bar{D}}{Q}\,\bar{S}(r) + \frac{\tilde{c}_{lv}(1-\gamma)\bar{D}}{Q}\,\bar{S}(r) \qquad (4.4)$$
$$+\lambda_h\left\{\tilde{c}_{hv}Q^{\beta}\left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right] - K_h\right\},$$

and

$$\tilde{G}(\tilde{c}_o, \tilde{c}_h, \tilde{c}_b, \tilde{c}_l)_u(\alpha) = \tilde{c}_{ou}\frac{\bar{D}}{Q} + \tilde{c}_{hu}Q^{\beta}\left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right]$$

$$+\frac{\tilde{c}_{bu}\gamma\bar{D}}{Q}\,\bar{S}(r) + \frac{\tilde{c}_{lu}(1-\gamma)\bar{D}}{Q}\,\bar{S}(r) \qquad (4.5)$$
$$+\lambda_h\left\{\tilde{c}_{hu}Q^{\beta}\left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right] - K_h\right\}.$$

By using the sign distance method for Equations (4.4) and (4.5) we obtain the defuzzified value of $\tilde{G}(\tilde{c}_o, \tilde{c}_h, \tilde{c}_b, \tilde{c}_l)$ in the form:

$$d\left(\tilde{G}(\tilde{c}_o, \tilde{c}_h, \tilde{c}_b, \tilde{c}_l), \tilde{0}\right) = c_1\frac{\bar{D}}{Q} + c_2 Q^{\beta}\left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right]$$

$$+\frac{c_3\gamma\bar{D}}{Q}\,\bar{S}(r) + \frac{c_4(1-\gamma)\bar{D}}{Q}\,\bar{S}(r) \qquad (4.6)$$
$$+\lambda_h\left\{c_2 Q^{\beta}\left[\frac{Q}{2} + r - \mu + (1-\gamma)\bar{s}(r)\right] - K_h\right\}.$$

where:

$$c_1 = \frac{1}{4}(4c_o - \delta_1 - \delta_2 + \delta_3 + \delta_4), \qquad c_2 = \frac{1}{4}(4c_h - \delta_5 - \delta_6 + \delta_7 + \delta_8),$$
$$c_3 = \frac{1}{4}(4c_b - \theta_1 - \theta_2 + \theta_3 + \theta_4), \qquad c_4 = \frac{1}{4}(4c_l - \theta_5 - \theta_6 + \theta_7 + \theta_8).$$

The defuzzified value $d\left(\tilde{G}(\tilde{c}_o, \tilde{c}_h, \tilde{c}_b, \tilde{c}_l), \tilde{0}\right)$ considers the estimate of fuzzy cost function which is given in Equation (4.3), similarly as in the crisp case, to solve this primal function in Equation (4.6) and derived the optimal values $Q^*$and r^* equating to zero each of the corresponding first partial derivatives of Equation

(4.6) with respect to Q and r respectively, hence we get the following two equations:

$$(1+\lambda_h)(1+\beta)c_2Q^{*\beta+2}+2\beta(1+\lambda_h)c_2Q^{*\beta+1}[r^*-\mu+(1-\gamma)\bar{S}(r^*)]$$
$$-2c_1\overline{D}-2\bar{S}(r^*)[c_3\gamma\overline{D}+c_4(1-\gamma)\overline{D}]=0, \tag{4.7}$$

and the probability of the shortage is given by:

$$R(r^*)=\frac{(1+\lambda_h)c_2Q^{*\beta+1}}{(1+\lambda_h)(1-\gamma)c_2Q^{*\beta+1}+c_3\gamma\overline{D}+c_4(1-\gamma)\overline{D}}\,. \tag{4.8}$$

Also there is no closed form solution of Equations (4.7) and (4.8), so the optimal values Q^* and r^* for different values of β can be obtained by using the iterative procedure.

5. The algorithm:

Step 1: By assuming a value of β and a value of λ_h, input all the inventory model data. Put $r_0=\mu$ as an initial value so, $\bar{S}(r_0)=0$, hence compute the first order quantity Q_1.

Step 2: Since we had known $R(r)$ of the Gamma distribution, then by using Q_1 compute r_1.

Step 3: Use r_1 and $\bar{S}\,(r_1)$ of Gamma distribution to compute a new order quantity Q_2. Use the value of Q_2 to find r_2 as in the step2. Repeat the steps until finding $Q_i=Q_{i+1}$ and $r_i=r_{i+1}$.

Step 4: Find the expected holding cost $E(HC)$ and the expected annual total cost $E(TC)$ by using the last values of Q_i and r_i.

Step 5: Check the constraint, if $E(HC)\le K_h$, then record the values Q_i and r_i as the optimal values Q_i^* and r_i^* which minimize the annual total cost under the constraint at this value of β, otherwise go to step 6.

Step 6: If $E(HC)>K_h$, go to step 1, and change the value of λ_h . Repeat all the steps until the constraint holds.

Step 7: Change the value of β, repeat all the procedures to compute the optimal values of Q_i^* and r_i^* which minimize the annual total cost under the constraint at another value of β, and so on.

6. The Numerical Study

For Model (I_c):

Consider a $< Q, r >$ model where ordering cost per inventory cycle (c_o) is 200 monetary unit per order, holding cost (c_h) per item per year 10 monetary unit, the shortage cost per unit backorder (c_b) and unit lost (c_l) respectively 20 and 30 monetary unit. Fraction of demand backordered during the stockout period is assumed to be 0.7. The annual demand $\overline{D}$ is chosen as 10, 000 units per year. There is a restriction that the average holding cost is either less than or equal to 3000 monetary unit. and the procurement lead time is constant. Determine Q^* and r^* when the lead time demand distributed as gamma with two parameters (n, ρ) which are chosen as 50 and 0.5 respectively. To establish the optimal decision variables Q^* and r^* substitute from equations (3.6) and (3.7) into (3.4) and (3.5) by iterative method at different values of β we can obtain the following results in Table 6.1, which illustrates the optimal values of λ_h^* for different values of β which give the optimum values of Q^* and r^* that minimize the expected total cost, when the lead time demand follows Gamma distribution:

Table 6.1: The optimal results of the model (I_c)

β	λ_h^*	Q^*	r^*	$E(TC)$
0.1	1.48	292.519	123.619	10143.6
0.2	3.08	173.479	120.029	15372.3
0.3	5.23	111.575	116.831	22903.8
0.4	7.893	76.873	113.971	33083
0.5	10.952	56. 185	111.38	46094.4

0.6	14.23	43.217	108.987	61938.9
0.7	17.49	34.788	106.711	80407.5
0.8	20.47	29.217	104.418	101172
0.9	22.76	25.649	101.925	123641

For Model (I_F):

Consider the same data as in the example of the Model (I_c). For the fuzzy costs components assume that, the values (based on arbitrary choices of δ_i and θ_i , $i = 1,2,\ldots 8$.) are given in Table 6.2 with their defuzziffied values and their percentage difference (P.D.) from the corresponding crisp values Pc_o, Pc_h, Pc_b and Pc_l respectively which denote the percentage decrease in c_o, c_h, c_b and c_l under fuzzy cases based on signed distance values from their corresponding crisp values.

Table 6.2: Fuzzy costs and their defuzzified values

Fuzzy costs	Their values	The sign distance method		P.D.	The value
$\tilde{c}_o$	(30,70,210,250)	$d\ (\tilde{c}_o, 0)$	140	Pc_o	30
$\tilde{c}_h$	(1,2,11,12)	$d\ (\tilde{c}_h, 0)$	6.50	Pc_h	35
$\tilde{c}_b$	(1,3,21,23)	$d\ (\tilde{c}_b, 0)$	12	Pc_b	40
$\tilde{c}_l$	(2,5,32,33)	$d\ (\tilde{c}_l, 0)$	18	Pc_l	40

By using the iterative method, the optimal values of Q^* and r^* that minimize the expected total cost, when the lead time demand follows Gamma distribution illustrated in Table 6.3.

Table 6.3: The optimal results of the model (I_F)

β	λ_h^*	Q^*	r^*	$ETC(\tilde{c}_o, \tilde{c}_h, \tilde{c}_b, \tilde{c}_l)$	$P.D.TC$
0.1	0.071	449.592	125.64	6203.24	38.846

0.2	0.793	258.944	122.199	8658.11	43.677
0.3	1.78	162.182	118.953	12255.4	46.492
0.4	3.058	108.717	116.039	17220.6	47.947
0.5	4.583	77.2912	113.415	23703.5	48.576
0.6	6.31	57.755	111.018	31796.1	48.665
0.7	8.14	45.104	108.797	41465.9	48.430
0.8	9.978	36.6101	106.69	52621.3	47.988
0.9	11.67	30.8939	104.475	65088.0	47.357

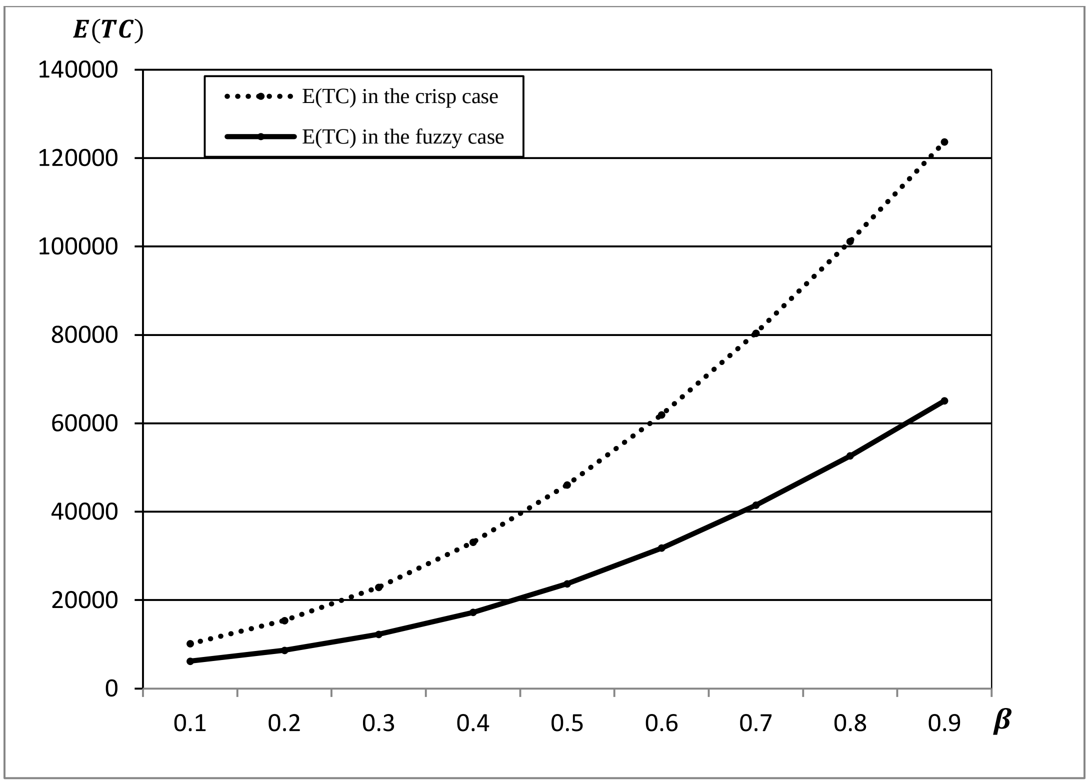

Figure 6.1: The comparing between the models (I_c) and (I_F)

7. The comparison and conclusion

By comparing the results between model (I_c) and the model (I_F) we have found that, at $\beta = 0.1$ the expected annual total cost in the crisp case is 10143.6 monetary unit (Table 6.1) while, it is 6203.24 monetary unit in the fuzzy case (Table 6.3) with percentage difference 38.8. This means that, we have been able to reduce the expected total cost by using the fuzzy costs by 38.8 % as shown in Figure 6.1, and so on for all values of β and this is exactly what we desired.

8. References

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