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# Multimethod's technology for solving optimal control problems

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**Abstract**

Multimethod's technology for solving optimal control problems is implemented under the form of parallel optimization processes with the choice of a best approximation. Corresponding to this technology the solution is found by a multimethods algorithm consisting of a sequence of steps of different methods applied to the optimization process in order to accelerate it. Such a technology allow to take into account some particularities of problem of interest at all stages of its solving and improve the efficiency of optimal control search.

## Introduction

Technology of numerical solving of applied optimal control problems as usual is based on universal software which has well developed interface and rich arsenal of optimization methods. Such software allows to take into account a peculiarities of the problem under consideration by making of use diverse algorithmes of improvement at different stages of iteration process. Applcation of several numerical methods for solving single optimization problem was suggested in many works oriented on the development of software packages [Gurman,1996, Evtushenko,1982, Tyatyushkin, 1989, 1992].

Principal difficulty in applying of multimethod's technology consists of the fact that at each stage in the process of solving of the problem, one has information about efficiency of the method applied at the present moment. To detect the efficiency of any optimization method at some stage of looking for a solution of the given problem, there is a need to perform one or several iterations. Because of this, to choose the method which is more appropriate for given stage of solving of the problem, the operation of switching from one method to another is usually repeated. Also, it is necessary to know about switching times. But this information can be easy obtained by tracking of the current method measuring parameters characterizing its convergence.

Thus principal problem of the multimethod's technology is the choice of the method which allows to continue effectively optimization process from the moment when convergence of current method was impaired.

## 1. The concept of paralell multimethod's optimization

Modern operational systems provide a solving of the given problem by an organization parallel computational flows for simultaneous computation by several methods. In each flow, one can realizes iterative process of one out of a collection of the methods. Thus the solving of a single problem can be performed by several methods. With multiprocessor's technology in hands, of course, it is convenient, for accomplishing of iteration of each method, to use individual processor.

After finding next approximation, each method is considered, for instance, evaluating an increment of the functional. More effective method is taken to continue the optimization. Next, obtained by this method approximation is passed to other methods as initial data to perform next iteration. Starting from this approximation, one or certain iterations are again performed by all methods. Among obtained approximations in hands again one takes this on which the functional has a smaller value.

Continue iterative process until the moment when, for obtained approximation, optimum criterion will be achieved. After that we find approximative solution of the problem under consideration. In this case the solution is found by multimethod's algorithm consisting of the sequence of the steps of different methods attached to optimization process to accelerate its convergence. The advantage of multimethod's algorithm in comparison with each method separately that it is more adequate in application to concrete problem. At each stage of looking for the solution, multimethod's algorithm make of use optimization method which is more suitable for peculiarities og the given problem (for example, the ravineness of function, specific character and the structure of constraints).

In graphic, the decrease of the functional $I(u)$, on the iterations of multimethod's algorithm is displayed by broken line which consists of the graphics of separate methods.

Fig. 1 shows the work of multimethod's algorithm in the case, when two methods $M_1$ and $M_2$ are used. The graphics given show the decrease of functional on the iterations of the methods $M_1$ and $M_2$. The graphic of decrease of functional on the iterations of multimethod's algorithm is the curve . It is constructed by using two graphics corresponding to the methods $M_1$ and $M_2$. Namely, the region BC is obtained by parallel translation of the region EL. According to this figure, the zero value of functional is achieved by $k_1$ iterations of the method $M_1$, while using of the method $M_2$ requires $k_2$ iterations.

Multimethod's algorithm works, up to $\bar{k}$-th iteration, by the method $M_1$ (the curve ), and next by the method $M_2$ (the curve ). The reason is that beginning from $\bar{k}$-th iteration, the velocity of decrease of functional in process of using of the method $M_2$ is greater. As a result, zero value of functional is achieved by multimethod's algorithm using $k^*$ iterations. This is considerably less than in the case if one uses the methods $M_1$ and $M_2$ separately.

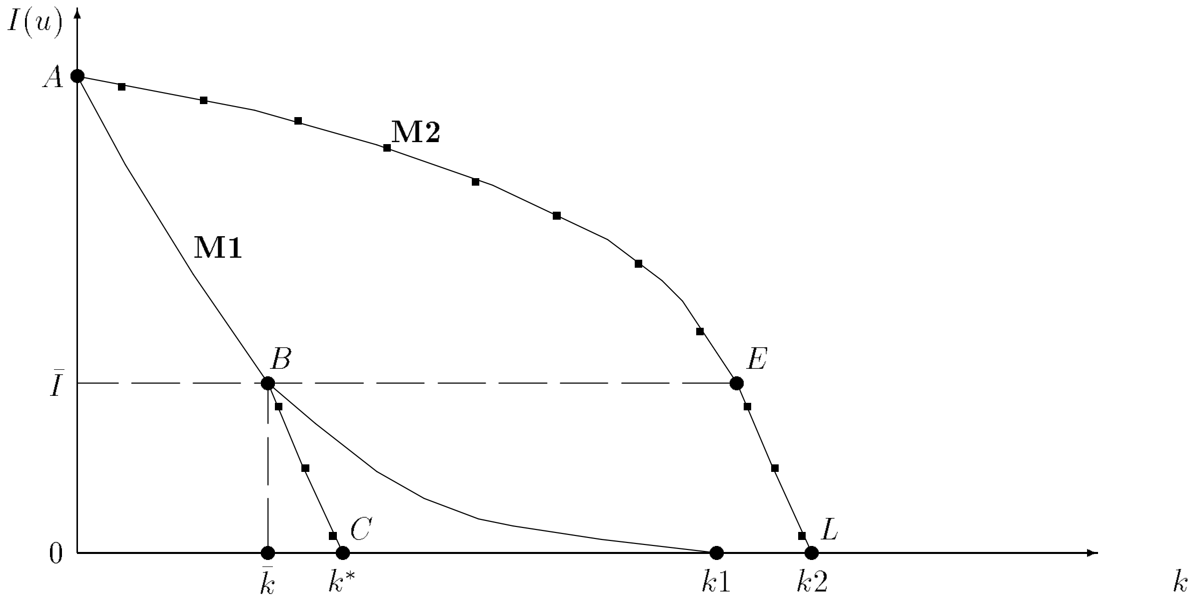


Figure 1: Graphics of decrease of functional on the iterations of the methods $M1, M2$ and multimethod's algorithm.

## 2. Parallel computations in the first-order methods

Using different criterions for choosing closest optimization method and also organizing in different ways parallel computations on the method's iterations, one can obtain several different combnations of algorithmes for solving a single problem. Moreover, it can be constructed such multimethod's algorithms which no contains repeated computations on the iterations of different optimization methods. For example, in the methods of gradient type, laborious computations of the gradient, requiring an integration of the adjoint system, should be performed only once, then, obtained gradient should be used in the iterative formulas of all methods. In this case, computational expanditures on one step of multiflow's algorithm are considerably reduced. Moreover, a realization of the step by any of the methods is accomplished by using the same approximately obtained values. Then all optimization algorithmes are applied as if to the one and the same approximate model. Thus the criterion of the new approximation is defined only by the optimization methods. Otherwise, because of computational errors, the same parameters used to estimate a convergence of the methods could have different values what can lead to improper choice of the best method.

**2.1. The problem with terminal conditions.** Let us consider optimal control problem provided by conditions in the form of the equalities at the right trajectory end. Controlled process is described by the system

$$\dot{x} = f(x, u, t),\ x(t_0) = x_0,\ t \in T = [t_0, t_1],\ x(t) \in R^n,\ u(t) \in R^r \tag{1}$$

with terminal conditions

$$I_j(u) = \varphi^j(x(t_1)) = 0,\ j = \overline{1,m}. \tag{2}$$

The control is constrained through

$$u(t) \in U, \tag{3}$$

where $U$ is bounded closed set in $R^r$. Vector-functions $f(x,u,t)$ are assumed to be differentiable w.r.t. to $x$ and $u$ and continuous w.r.t. $t$; $\varphi^j(x)$, $j = \overline{1,m}$ are assumed to be continuously differentiable w.r.t. $x$ functions.

It is required to find among controls fulfilled by (3), such one which provides the validity of conditions (2), for controlled process (1) and, on the other hand, provides the minimum of the functional

$$I_0(u) = \varphi^0(x(t_1)), \tag{4}$$

where $\varphi^0(x)$ is continuously differentiable function.

The gradients of functionals $I_j(u)$, $j = \overline{0,m}$, in terms of the functions $H^j(\psi_j, x, u, t) = \psi_j'(t)f(x,u,t)$ and adjoint system

$$\dot{\psi}_j = -f_x(x,u,t)'\psi_j(t),\ \psi_j(t_1) = -\varphi_x^j(x(t_1)) \tag{5}$$

are given by the formula

$$\bigtriangledown I_j(u) = -H_u^j(\psi_j, x, u, t),\ j = \overline{0,m}.$$

Gradient procedure of the minimization of the functional (4) without taking into account the constraints (2) and (3) is given through relation

$$u^{k+1} = u^k - \alpha_k \bigtriangledown I_0(u^k), \tag{6}$$

where $\alpha_k$ are choosing, for example, from condition of fastest decrease of the functional $I_0(u)$. The solution of the problem with terminal conditions (2) without constraints (3), one can find by parallel application of the linearization and penalization methods given, for example, in [Tyatyushkin, 1992]. When using penalization method, penalty functional which consists of the functions (2) and (4), is minimized with the help of gradient procedure (6). Making of use multimethod's technology, one can also simplify each iteration of the linearization method such that its working time will be close to the penalization method one. Then the algorithm will be as follows:

1) for given $u^k(t), t \in T$, the system (1) is integrated; in the integration points, phase coordinates of trajectory $x^k(t)$ are memorized.

2) $m+1$ flows are organized, for parallel integration of adjoint system provided by different initial conditions $\psi_j(t_1) = -\varphi_x^j(x(t_1)), j = \overline{0,m}$;. In process of integration, the solutions $\psi_j(t)$ are used to construct linear system of algebraic equations

$$\sum_{i=1}^{m} \left( \int_{t_0}^{t_1} H_u^{j'} H_u^i dt \right) \lambda_i = I_j(u^k) - \int_{t_0}^{t_1} H_u^{j'} H_u^0 dt,\ \ j = \overline{1,m}. \tag{7}$$

3) after solving the system, one finds a values of the variables $\lambda_i$, $i = \overline{1,m}$;

4) one constructs new approximation of the control

$$u^{k+1} = u^k + \alpha_k \delta u^k, \quad \delta u = H_u^0 + \sum_{i=1}^m \lambda_i H_u^i,$$

where the parameters $\alpha_k$ satisfy inequality

$$I_0(u^k + \alpha_k \delta u^k) + \beta I_{j_0}(u^k + \alpha_k \delta u^k) \le I_0(u^k) + \beta I_{j_0}(u^k) - \varepsilon \int_{t_0}^{t_1} \delta u' \delta u dt,$$

$$0 < \varepsilon < 1, \quad j_0 = \arg \max_{1 \le j \le m} |I_j(u^k)|, \quad \beta = \sum_{i=1}^m |\lambda_i|.$$

¿From this algorithm, as particular case, (for $m = 0$), we have the gradient method (6).

**2.2. The problem with constraints on control.** Let us turn our attention to the discussion of the algorithmes being destinated for solving of the problems provided by constraints on control, but with free right end. Suppose that for some $u^k(t) \in U, t \in T$, one finds any solution of the system (1) $x^k(t), t \in T$. Setting in (5) $j = 0$, integrate adjoint system from $t = t_1$ to $t = t_0$ when $u = u^k(t), x = x^k(t)$. Calculate, on its solution $\psi^k = \psi^0(t)$, the cotrol using the maximum principle.

$$\bar{u}^k(t) = \arg \max_{u \in U} H(\psi^k, x^k, u, t), \quad t \in T,$$

and find the value of scalar function

$$w_k(\bar{u}(t), t) = H(\psi^k, x^k, \bar{u}, t) - H(\psi^k, x^k, u^k, t), t \in T.$$

Let $t = \tau_k$ the maximum point of this function being on $T$. Then necessary optimum condition of the control $u^k$ will be

$$w_k(\bar{u}^k(\tau_k), \tau_k) = 0.$$

In the case when, for given $u^k$ and obtained $x^k, \psi^k, \bar{u}^k$, maximum principle

$$w_k(\bar{u}^k(\tau_k), \tau_k) > 0,$$

is broken then one can realize iteration of the method [Vasil'ev, 1981] to improve $u^k$.

The point set, where maximum principle is broken, denote by

$$T_\varepsilon = \{t \in T : \; w_k(\bar{u}^k(t), t) \ge \varepsilon w_k(\bar{u}^k(\tau_k), \tau_k)\}, \quad \varepsilon \in [0, 1].$$

Observe, that at $\varepsilon = 0$ we have $T_\varepsilon^k = T$, while at $\varepsilon = 1$ the set $T_\varepsilon^k$ consists of the maximum points of the function $w_k(u(t), t)$.

Varying $\varepsilon$, one can find such its value, for which the control

$$u_\varepsilon^k = \begin{cases} \bar{u}^k(t), & t \in T_\varepsilon, \\ u^k(t), & t \in T \backslash T_\varepsilon, \end{cases} \tag{8}$$

provides a least value of the purposeful functional $I_0(u)$, i.e.

$$\varepsilon_k = \arg \min_{\varepsilon \in [0,1]} I_0(u_\varepsilon^k).$$

When looking for $\varepsilon_k$, one can use several flows for simultaneous integration of the system (1) with controls (8), corresponding to different values of $\varepsilon \in [0,1]$. In addition, at $t = t_1$ we have different phase space points $x^k_\varepsilon(t_1)$ and pertinent values $I_0(u^k_\varepsilon) = \varphi(x^k_\varepsilon(t_1))$. Choosing the smallest value of the fuctional $I_0(u^k_\varepsilon)$, verify the inequality $I_0(u^k_\varepsilon) < I_0(u^k)$. In the case if one holds, we set $u^{k+1} = u^k_\varepsilon$. Otherwise, one can continue the subdivision of $\varepsilon$ and find the values of functional for the following its values.

By virtue of structure of the controls, generated by the iteration formula (8), the relaxationness of an algorithm can be impaired still before obtaining the control satisfying maximum principle. Because of this, to continue optimization process, there is a need to apply another algorithm, on iteration of which controls constructed not only with boundary points, but also with interior ones w.r.t. the set $U$ as well. For example, to restore the covergence as usual succed with the help of construction using convex combination of two control

$$u^{k+1}(t) = u^k(t) + \alpha[\bar{u}^k(t) - u^k(t)], \ \alpha \in [0,1]. \tag{9}$$

The calculations by the formulas (8) and (9), one can perform simultaneously choosing from obtained approximations such $u^{k+1}$ to which corresponds the smallest value of the functional. In the case, if the values of the functional are compared within several iterations, then as criterion to compare the efficiency of the methods (8) and (9), one should use the values of an increase of the functional, obtained on neighboring iterations of each methods.

In actual practice, it is established that application of the variations of two type, namely, "horizontal" (8) and "vertical" (9), allows us to avoid an appearence of such effect like "stiking of control to the boundaries", which is peculiar to the algorithmes based on maximum principle [Vasil'ev, 1981, Lyubushin, 1983].

In the case, in iteration formula (9), one derives the control $\bar{u}^k(t)$ using linearized maximum principle

$$\bar{u}^k(t) = \arg\max_{u \in U} H_u(\psi^k, x^k, u, t)' u(t), \quad t \in T, \tag{10}$$

iterations will be of conditional gradient method. It is evident, that for systems which is linear in control, the control function (10) coincides with that deduced from the maximum principle. A further algorithm of improving of control, one can obtain substituting, in the iteration formula (8), the interval $T_\varepsilon$ for the following one:

$$T^k_\varepsilon = [\tau_k - \varepsilon(\tau_k - t^k_0), \tau_k + \varepsilon(t^k_1 - \tau_k)], \ \ \varepsilon \in [0,1], \tag{11}$$

where $t^k_0, t^k_1$ are the nearest left and right discontinuity points of the function $w(\bar{u}^k(t), t)$.

In the process of finding of the value of the parameter $\varepsilon_k$, provided a convergence of the algorithm, one can apply the procedure above with making of use parallel computations. This way to construct the interval provides blowing down of its ends towards the point $\tau_k$, in the case, if the function $w(\bar{u}^k(t), t)$ is constant within some neighborhood of the point $\tau_k$, preserving by this a convergence of the algorithm [Vasil'ev, 1981].

**2.3. The block-scheme of multimethod's algorithm work.** Summurizing above we see that applying various iteration procedures and making of use different rules to construct the sets of varying of control, we obtain the collection of algorithm, each of them works effectively enough only in a definite situation. Thus, in the process of finding of optimal control, it is necessary to include several algorithmes.

Organizing parallel computations to realize some collection of algorithmes and applying the selection procedure to take the best approximation after simultaneous iterations by all methods, we are able to find effectively optimal control by multimethod's algorithm.

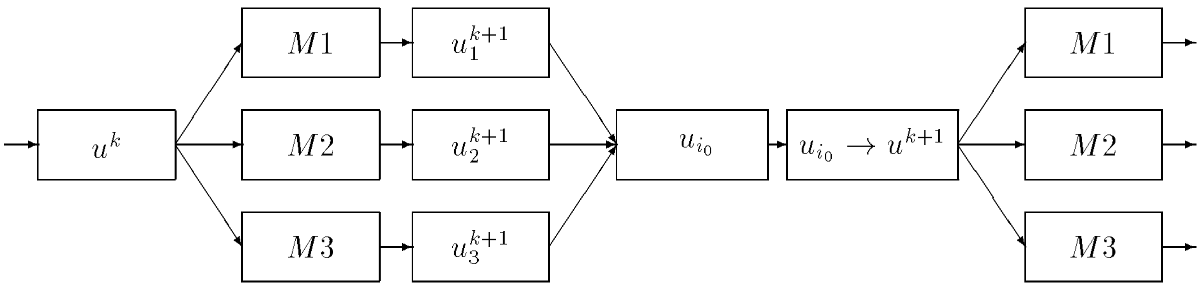


Figure 2: The scheme of realization of $(k+1)$th iteration by multimethod's algorithm using three methods $M1$, $M2$ and $M3$.

Figure 2 informs us about work of multimethod's algorithm, in the case when one uses three methods. In this, the block of selection of the best approximation finds $u_{i_0}$ from a largest value of increment of the functional obtained on $(k+1)$th iteration

$$u_{i_0} = \arg \max_{i \in (1,2,3)} (I(u^k) - I(u_i^{k+1})).$$

This approximation is passed to all methods $u_i^{k+1} = u_{i_0}$ to perform next iteration.

It should be noted that, that from the common methods collection, for solving another problem, it can be generated another multimethod's algorithm which would be more adequate, because of taking into account the peculiarities of this problem.

## 3. Computational aspects in the process of realization of multimethod's algorithm

**3.1. The solving of adjoint problem.** Most labor-consuming operation performed on each step of all first-order algorithmes is numerical integration of original and adjont system of differential equations provided by some control. The solving of adjoint system on the iterations is used to find either the value of Hamiltonian or both to perform the calculation of gradients of functionals. Thus numerical integraton of this system is accomplished on each step of any one of the first-order methods. Since multimethod's technology provides parallel accomplishing the steps by all methods, then obtained by single integration, the solution of adjoint system is used in all iteration formulas simultaneously.

**3.2. The computation of method's step.** The realization of each step of the first-order method needs to find the value of method's step $\alpha$ ($\varepsilon$), for which new obtained approximation provides the smallest value of purposeful functional. The search of such value of $\alpha$ requires multiple integration of initial system (1). Constructing, by given iteration formula, at different values of $\alpha$ of control $u^{\alpha}$, and integrating initial system, at $u = u^{\alpha}$, one obtains various trajectories $x^{\alpha}(t), t \in T$, and finds pertinent values of the functional $I(u^{\alpha})$. Applying one-dimensonal searh method, one can, by several such recalculations, to find such approximate value of $\alpha = \alpha_k$, which provides the minimum of $I(u^{\alpha})$ w.r.t parameter $\alpha$. If, in the process, the inequality $I(u^{\alpha_k}) < I(u^k)$ holds, the control $u^{\alpha_k}$ is taken as new approximation $u^{k+1}$. Otherwise, iteration process, using this method, come to an end.

To recover a convergence of the method one correct calculations, namely, diminish the integration step, raise the accuracy of one-dimensonal search and so on. By multimethod's algorithm, one can continue the process of improvement of control by switching on another method, while the condition for termination of its work is impossibility to garantee relaxation by none of the methods entering in given collection.

To find the value of $\alpha_k$ while using multimethod's algorithm, one can use also another scheme of searching. Its essence consists of the fact that in the process of subdivision of $\alpha$, for example, by cutting in two, with each its value, it accomplishes test step by all methods. Since, in many algorithmes, for finding $\alpha_k$, it accomplishes subsequent subdivision in two, untill the inequality $I(u^{\alpha_k}) < I(u^k)$ will be hold, then, for any fixed $\alpha$, one can perform the checking of validity of this inequality simultaneously for all methods. In addition, with given value of $\alpha$, in correspondence with iteration formula of each method, it is constructed the control $u^{\alpha}$ integrated the system (1). On its solution, it is calculated the functional $I(u^{\alpha})$. If, in the process, for some value of parameter $\alpha$, for some of the methods, required inequality will be hold, then $u^{\alpha_k}$, obtained by this method, should be taken as new approximation to be passed to all methods for continuation of iteration process.

**3.3.** As parameters used to estimate the efficiency of iteration process, one can take, for instance, the velocity of reduction of the residuals in optimization conditions, the value of increment of the purposeful functional or the extent of violation of some important, for example, in the physical sense, constraints. The acuraccy of calculations of these parameters provides the proper choice of the method and opportune transition to another optimizaton method.

**3.4. The methods of appromximation of control.** The principle error in model of control problem bringing in the process of numerical solving is due to discrete approximation of controlled dynamical system and tabular representation of control function.

The methods of numerical integration allow to perform the discretization of the system with given acuraccy, but provided that control functions are continuous and its values can be defined for arbitrary $t \in T$. However, in actual practice, piecewise constant approximaton of control functions is often applied and its values are defined only in given sites of the temporal network. These values are changed in iteration process, depending on optimization method to decrease minimized functional or to reduce the residuals for given terminal conditions. The values of control functions between the points, which are necessary, for example, to apply the numerical methods Runge—Kutta type, as usual are taken to be equal either the value in the nearest left point or both are defined by linear interpolation by the values in left and right points. In this process, the error of numerical integration can be considerably extended. Then, on obtained numerical solution of the system, the values of parameters, which are used to choose the method, could be incorrect.

To diminish the errors of calculation of trajectory, one can condense the temporal network in order to increase the number of desired values of control which implies to solve optimization problem of big size. In the case if required control is enough smooth, then, with some number of points and with the help of interpolation fofmulas, one can provide admissible acuraccy of its approximation. However, in general case, if control functions are discontinuous, for example of relay type, this approach may turn out to be low-efficient, since, in this case the condensation of network affects on the approximation accuracy only in the neighbourhood of the switching points. Because of this, in the process of solving of problems, optimal control of which has relay type, it is necessary to use the procedure of switching times correction to provide the approximation for control functions with prescribed accuracy. In the absence of this procedure, the control is found in the form of the array of numbers defined on temporal network. Such control may differ in significant ways from optimal one both by the value of purposeful functional and by accuracy of opimal conditions fulfillment

**3.5. The providing of accuracy for choosing of the method.** The application of procedure selection which implies that of multimethod's algorithm could be justified only in the case if, by all methods, the same way to approximate the control are used. Also, it is required that all methods are provided by the same accuracy of integration of the systems. In this case, by all optimizaton methods the same finite dimensional problem obtained by discretization is solved, while the values of parameters used to choose the method give more correct estimate of efficiency of algorithmes, providing by dong so, correct choosing of the method to continue iteration process. As a result, approximate solution is found by using lesser number of iterations, in comparison if each method of given collection used separately. This is because at each stage of solving the problem under consideration, more effective algorithm is applied to be more adequate in this situation.

## 4. The numerical experiments.

Let us present the results of the numerical experiments exhibiting application of multimethod's technology. Two examples are treated: test problem of minimization of Rosenbroke's function and the problem of optimal control of the rocket flight.

**4.1. Example 1.** It is required to find the minimum of Rosenbroke's function

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

with constraint

$$(x_1 - 2)^2 + (x_2 - 2)^2 \leq 1$$

and initial approximation $x^0 = (2, 1)$.

It is known that absolute minimum of this function is achieved in the point (1,1). It is equal to 0. In the problem under consideration, the point (1,1) is not permissible. Therefore the zero value of purposeful functional is not attained.

Numerical solving of this problem was performed by the method of conditional gradient, by the method of gradient projection and also by multimethod's algorithm which contains these two methods. The smallest value of the function equal to 0.0358, for prescribed accuracy $\varepsilon = 10^{-4}$ was attained using 240, 341 and 170 iterations, respectively. Minimum point is $x^* = (1.189, 1.415)$.

**4.2. Example 2.** In this case, controlled process is described by the system

$$\begin{aligned}
\dot{x}_1 &= -c\rho x_1^2[1.174 - 0.9\cos u_1] - g\sin x_2/(1+x_3)^2 \\
\dot{x}_2 &= 0.6c\rho x_1^2 \sin u_1 + x_1\cos x_2/[R(1+x_3)] - g\cos x_2/[x_1(1+x_3)^2] \\
\dot{x}_3 &= x_1 \sin x_2/R \\
\dot{x}_4 &= x_1\cos x_2/[1+x_3],
\end{aligned}$$

where $\rho = 0.002704\exp(-4.26Rx_3)$, $R = 209$, $g = 0.00032172$, $c = 26600$.

It is given initial data $x(0) = (0.36,\ -0.141372,\ 0.019139,\ 0.0)$ and terminal conditions $x(t_k) = (0.27,\ 0.0,\ 0.011962,\ \mathit{unbounded})$, while the parameter $t_k$ is not fixed.

It is required to find such control $u(t), t \in [0, t_k]$ and such smallest value of parameter $t_k$ which garantee the fulfillment of terminal conditions, while the functional

$$I(u) = \int_0^{t_k} x_1[\exp(-4.26Rx_3)]^{1/3}dt$$

must attain smallest value.

This problem was solved by linearization method above and also by the method of projected Lagrangian, on iteration of which the nonlinear Lagrangian with linearizable constraint is minimized [Tyatyushkin, 1993]. Approximate solution (the accurasy w.r.t. boundary conditions is $10^{-3}$) was found using 282 and 215 iterations, respectively. The solution with the same accuracy while using multimethod's algorithm insluding these two methods was found using 164 iterations. The value of parameter $t_k$ equal 72.412, while the functional $I(u)$ attains the value equal to 24.59.

### 5. Conclusion

Thus, we can coclude, that for each problem under consideration there exists appropriate sequence of steps using different methods which provides more effective search of optimal control. In multimethod's algorithmes the construction of such sequence is accomplished automatically using some given criterion estimating the efficiency of optimization process at each stage of solving of the problem. The base for application of the techology described above is the packages of applied software, for example, [Zholudev, 1989, Tyatyushkin, 1989, 1992, 1993], which include the methods of first- and second-order, for solving optimal control problems with constraints of different types.

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