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Extension of Fixed Charge Bulk Transportation Problem

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ABSTRACT
In this paper a fixed charge bulk transportation problem is discussed in which only n' out of n destinations ($n' < n$) are to be served in bulk by the given m sources ($m \leq n'$). This is assumed that the transportation is done in parallel from all sources and each destination receive its demand from a single source but a source can serve to more than one destination. The purpose is to find the optimal grouping of source-destination pair which minimizes the total cost i.e the bulk cost and the fixed cost. In order to find optimal grouping, lexi-search approach has been used. A heuristic is proposed to find out the starting upper bound.

KEYWORDS
Fixed Charge Transportation Problem(FCTP); Bulk Transportation Problem (BTP); Lexi-Search

AMS CLASSIFICATION
MSC 90B06; 90C05; 90C08; 90C10

Introduction

A Variant of transportation problem is Fixed Charge Transportation Problem (FCTP) in which both variable cost and fixed cost are present. In case of classical transportation problems, our aim is to find least expensive flow of the material between source-destination pair. But in FCTP, the purpose is to find that schedule flow of material through which the total cost is minimum which is the sum of the variable and fixed cost. This fixed cost which may be the renting cost of a vehicle, arrival charges at airport, set up charges in manufacturing a product etc. is independent of the quantity shipped.

Mathematically FCTP can be stated as:

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$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij} x_{ij} + f_{ij} y_{ij})$$

Subject to

$$\begin{aligned}
&\sum_{j=1}^{n} x_{ij} = a_i, && i = 1, 2, 3, \ldots, m \\
&\sum_{i=1}^{m} x_{ij} = b_j, && j = 1, 2, 3, \ldots, n \\
&x_{ij} \geq 0 \qquad \forall (i, j) \\
&y_{ij} = \begin{cases} 1 & x_{ij} > 0 \\ 0 & x_{ij} = 0 \end{cases} \\
&\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad a_i, b_j, c_{ij}, f_{ij} \geq 0
\end{aligned}$$

Here,
$I = \{1, 2, \ldots, m\}$: number of sources
$J = \{1, 2, \ldots, n\}$: number of destinations
a_i = Availability at each source, b_j = Requirement at each destination
x_{ij} = the quantity transported to destination j from source i
c_{ij} = transportation cost involved when a unit is supplied to destination j from source i
$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$; This shows the case of balanced transportation problem
f_{ij} = fixed cost involved when a unit is supplied to destination j from source i

Over the last few decades numerous methods have been proposed to find the solution of FCTP either implicitly or explicitly(e.g, [1], [2], [4], [6], [7], [18], [21], [25]). It was also found that the optimal solution of FCTP lies on the boundary of the feasible region and is one of the extreme point, with some other properties ([19], [20], [31]). Balinski [12] solved FCTP by making it as an integer program. Later on, many computational studies have been claimed to find optimal solution. But among those methods only two methods viz., branch and bound method([29], [22]) and ranking of extreme points([24], [27]) are given. The ranking method requires analyzing almost the whole distribution problem whereas branch-and-bound method shows exponential behaviour. But these methods are restricted by limitation on computer time that is why many researchers have turned to the heuristic approach for finding the solution of FCTP([3], [5], [8], [13], [14], [15], [17], [26], [30]). A paradox in FCTP is also discussed by S.R. Arora and Anu Ahuja [9].

Another variant of transportation problem is Bulk Transportation Problem (BTP) in which the homogeneous material is transported in bulk between the source-destination pair. In cost minimizing transportation problem our aim is to find minimum cost and every unit which is transported depends on the quantity being transported. But in some situations the cost behaves as a bulk cost because it become independent of the transported quantity which results to the Bulk Transportation Problem.

Mathematically BTP can be stated as:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} g_j x_{ij} \leq d_i \qquad i \in (1, 2, 3, \ldots, m)$$

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j \in (1, 2, 3, \ldots, n)$$

$$x_{ij} = \begin{cases} 1 & \text{if } j\text{th destination is supplied by } i\text{th source} \\ 0 & \text{otherwise} \end{cases}$$

Where
d_i : availability at the ith source
g_j : requirement at the jth destination
c_{ij} : transportation cost from source i to destination j and is independent of the quantity shipped

Various authors developed branch and bound technique to solve BTP ([11], [16], [28]). Sundara Murthy [23] solved this problem with some additional restriction that each destination fulfill its demand from a single source but a source can serve to more than one destination depending upon its capacity. A lexi search technique is used for this purpose which works efficiently over branch and bound method. A Fixed Charge Bulk Transportation (FCBTP) have combined the features of both fixed charge transportation problem and bulk transportation problem. But in real world some situations arise when instead of serving all the destinations only a few destinations can serve our purpose. Hence, In this paper an algorithm is developed to find the optimal grouping using lexi search approach which is based on ([10], [23]) and ensures the optimality in a limited number of steps. The algorithm starts with the initial upper bound on the objective value which is near to the optimal solution. The mathematical model of the problem is shown in Section 1. Section 2 consists of some definitions and results based on which an algorithm is presented in Section 3. A numerical illustration is shown in Section 4 and computational details for some random problems are shown in Section 5. Some conclusions based on the study are given towards the end in Concluding Remarks.

1. Problem Description

1.1. *Fixed Charge Bulk Transportation Problem (FCBTP)*

The fixed charge bulk transportation problem differs from classical transportation problem because in this fixed cost combined with the bulk cost which makes the objective a step function. Here, the set $I = \{1, 2, \ldots, m\}$ denotes the number of sources and $J = \{1, 2, \ldots, n\}$ the number of destinations. The availability at any source i is repesented by d_i & the requirement at any destination j is denoted by g_j. Similarly, c_{ij}& f_{ij} defines the bulk cost & fixed cost from source i to destination j respectively($\forall i \in I, j \in J$). The total fixed charge bulk cost is denoted by Z. There is an assumption that demand of each destination is fulfilled by a single source but a source can supply to any number of destination depending upon its capacity. Our aim is to find the optimal feasible solution which gives the minimum total cost. As the quantity is transported in bulk, therefore x_{ij} is defined as

$$x_{ij} = \begin{cases} 1 & If i\text{th source serves } j\text{th destination} \\ 0 & \text{otherwise} \end{cases}$$

But in real world sometimes a situation arise in which instead of serving all the destinations we want to serve only a few destinations because that fulfill our purpose of optimality. It means from the given n destinations only $n'(< n)$ destinations are to be served. Hence the problem is to find the group of source-destination pair which yield the minimum value of objective function.

Mathematical Model:

$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}x_{ij} + f_{ij}y_{ij})$$

Subject to

$$\sum_{j\in J} g_j x_{ij} \leq d_i, \quad i \in I \qquad \cdots\cdots\cdots (i)$$

$$\sum_{i\in I} x_{ij} = 1, \quad j \in J \qquad \cdots\cdots\cdots (ii)$$

$$\sum_{j\in J}\sum_{i\in I} x_{ij} = n', \qquad \cdots\cdots\cdots (iii)$$

$$x_{ij} = 0 \text{ or } 1 \quad \forall (i,j) \in IXJ \qquad \cdots\cdots\cdots (iv)$$

$$y_{ij} = \begin{cases} 1 & x_{ij} > 0 \\ 0 & x_{ij} = 0 \end{cases} \qquad \cdots\cdots\cdots (v)$$

A solution which satisfies equation (i),(ii),(iii),(iv),(v) give rise to be a feasible solution and a feasible solution which minimizes the total cost is called optimal feasible solution.

2. Theoretical Development

Some Definitions and Results

Notation
Z_u : starting upper bound of objective function value
I_u : index of available sources
J_u : index of served destinations
$\uplus$: Augmentation
$\cap$:Negation of augmentation
D : column vector consisting the entries of fixed bulk matrix in nondecreasing order of their values.
e : index set of the entries in D
E_r : index set of the unserved entries in D
J' : $J - J_u$: the set of unserved destinations

Fixed Bulk Matrix An mn matrix which is formed by taking sum of the bulk cost and fixed cost matrix defines a Fixed Bulk Matrix. It is denoted by FB.

Alphabet Table This is a $3\,X\,mn$ row matrix which is formed by an arrangement of mn entries in three rows. The first row denoted by e shows the index set of the entries in D, second row denoted by D consisting of the mn elements of FB matrix when they are organised in non-decreasing order of their values, and the third row represented by AB which is consisting of the positions of the entries in FB matrix and denoted by an ordered pair of values. Any y^{th} entry in AB is an ordered pair represented by $(at(y,1), at(y,2))$, where $at(y,1)$ represent the row of the FB matrix in which the y^{th} entry of D lies, and the corresponding column is denoted by $at(y,2)$. So, for any $y < z$ the corresponding cost is $c_{at(y,1)at(y,2)} + f_{at(y,1)at(y,2)} \leq c_{at(z,1)at(z,2)} + f_{at(z,1)at(z,2)}$.

Partial Word Let Pw be the partial word of length r.
$Pw = (at(y_1,1)at(y_1,2), at(y_2,1)at(y_2,2)\ldots\ldots, at(y_r,1)at(y_r,2))$
or $Pw = ((i_1,j_1),(i_2,j_2),\ldots\ldots,(i_r,j_r)), r \leq n$
X^{Pw} represents the partial solution corresponding to the partial word Pw and consists of solutions for r served destinations whereas $(r+1, r+2, \ldots, n')$ destinations are still to be served. Each partial word Pw of lenght r can be considered as a guide or a leader of the group of words. These word are so generated that their contribution to the objective function is in decreasing order. If at any stage let a partial word of length r is under study, $r \leq n'$ i.e $Pw = (at(y_1,1)at(y_1,2),\ldots\ldots, at(y_r,1)at(y_r,2))$ then it means that all the partial word which start with $(at(y,1)at(y,2))$ where, $1 \leq y \leq y_1 - 1$ have not generated value better than Z_u. It is to be noted that we may have that $at(y_p,1) = at(y_q,1)$, for some p and q belonging to $\{1,2,3,.....,r\}$ but for Pw to contain a word w, $at(y_p,2) \neq at(y_q,2)\,\forall\, p\ and\ q \in \{1,2,3,....,r\}$ s.t $p \neq q$. Contribution of Pw to the objective function is denoted by

$$Z(X)^{Pw} = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij} + f_{ij})x_{ij} : x_{ij} = 1)$$

Theorem 2.1. *If a partial word given by*
$Pw = ((i_1, j_1), (i_2, j_2), \cdots, (i_r, j_r))$
$= ((at(y_1, 1), at(y_1, 2)), \cdots\cdots, (at(y_r, 1), at(y_r, 2))), r \leq n'$,
is such that $\forall y_r \in E_r$ *either* $at(y_r, 2) \in J_u$ *or* $Z(X^{Pw}) \geq Z_u$ *where* Z_u *is the current upper bound of objective function value, then*
$\overline{Pw} = ((at(y_1, 1), at(y_1, 2)), (at(y_2, 1), at(y_2, 2)), \cdots, (at(y_{r-1}, 1), at(y_{r-1}, 2)))$
can't generate a word with the better value of the objective than Z_u.

Proof. Assume that Pw be a partial word of length r for which one of the following holds

(1) For all $y_r \in E_r$ either $at(y_r, 2) \in J_u$.
(2) $Z(X^{Pw}) \geq Z_u$,

It means that either the destination $at(y_r, 2)$ has already been served or supply to the destination $at(y_r, 2)$ from sources $at(y_r, 1)\, \forall y_r \in E_r$ when $r - 1$ destinations $at(y_1, 2), at(y_2, 2), \cdots, at(y_{r-1}, 2)$ are being served by the sources $at(y_1, 1), at(y_2, 1), \cdots, at(y_{r-1}, 1)$ would not have the corresponding objective value $< Z_u$. This indicate that the partial word
$\overline{Pw} = ((at(y_1, 1), at(y_1, 2)), (at(y_2, 1), at(y_2, 2)), \cdots, (at(y_{r-1}, 1), at(y_{r-1}, 2)))$
would not produce a word corresponding to which $Z(X^{Pw}) < Z_u$. $\square$

Remark 1. Let $Pw = (at(y_1, 1)at(y_1, 2), at(y_2, 1)at(y_2, 2), \ldots, at(y_r, 1)at(y_r, 2))\,, r \leq n'$ be a partial word for which $Z(X)^{Pw} < Z_u$ and $\sum_{i \in I} d_i{}^u \geq \sum_{j \in J'} g_j$ then the current partial word may generate a word with better objective value than Z_u.

Remark 2. If $\forall y_r \in E_r$ either $at(y_r, 2) \in J_u$ or $Z(X^{Pw}) \geq Z_u$ then the partial word is rejected as it would not generate a better feasible solution and the allocation of some or all the the first $r - 1$ destinations must be altered.
Let $Pw = (at(y_1, 1)at(y_1, 2), at(y_2, 1)at(y_2, 2), \ldots, at(y_{r-1}, 1)at(y_{r-1}, 2))$ be a partial word s.t $at(y_r, 2) \in$J$_u$ or $Z(X^{Pw}) \geq Z_u$. Then this partial word must undergo some alteration i.e we try to find the possibility of choosing a destination $at(y, 2), y_{r-1} < y \leq mn$. If $|E_r| < n' - (r - 1)$, then we try to find the possibility of choosing different destination at $(r - 2)th$ position and so on.

Remark 3. Let us assume that the next $(n' - r)$ elements from Alphabet Table are arranged in non decreasing order of their value of the objective function. Let $J'^{(n'-r)}$ be the index set of the first $(n' - r)$ destinations s.t $\sum_{j \in J'^{(n'-r)}} g_j > \sum_{i \in I} d_i^u$, then the partial word Pw is rejected.

Remark 4. Let $Pw = (at(y_1, 1)at(y_1, 2), at(y_2, 1)at(y_2, 2), \ldots, at(y_r, 1)at(y_r, 2))\,, r \leq n'$ s.t $|E_{r+1}| < n' - |Pw|$, then the current partial word would not generate a word of length n' and hence the partial word
$\overline{Pw} = ((at(y_1, 1), at(y_1, 2)), (at(y_2, 1), at(y_2, 2)), \cdots, (at(y_{r-1}, 1), at(y_{r-1}, 2)))$ must be altered.

Theorem 2.2. *If* $Pw = (at(y_1, 1), at(y_1, 2))$ *is the partial for which any one of the following holds*

(i) $Z(X^{Pw}) \geq Z_u$
(ii) $|E_2| < n' - 1$

then Z_u is the optimal value of objective function.

Proof. If for a partial word Pw, $Z(X^{Pw}) \geq Z_u$ then this partial word is rejected as it can not generate a word with objective value $< Z_u$. Also as $c_{at(y_1,1)at(y_1,2)} + f_{at(y_1,1)at(y_1,2)} \leq c_{at(z,1)at(z,2)} + f_{at(z,1)at(z,2)} \forall z > y_1$, it follows that $Z(X^{\widetilde{Pw}}) \geq Z_u$ $\forall$ partial word $\widetilde{Pw} = (at(z,1), at(z,2))$. Therefore all partial word $\widetilde{Pw} = (at(z,1), at(z,2)), z > y_1 \,\&\, z \in E_2$ can not generate a word with the objective value $< Z_u$. If condition (ii) holds then from (Ref. Remark 4), Pw can not contain a word of length n'. As $|E_2| < n' - 1$, this implies that $mn - z < n' - 1, \forall z \in E_2$. Hence $Pw = ((at(z,1)at(z,2))$ can not produce a word of length n' because the words are so generated that their contribution to the objective function is in decreasi ng order. Then any word which derived from the partial word $(at(y,1), at(y,2)), y < y_1$ can not generate a value better than Z_u. Therefore, as we are not able to find a word with a better value of the objective so the optimal value of the objective function is Z_u. □

Theorem 2.3. *Let* $Pw = (at(y_1,1)at(y_1,2), \ldots, (at(y_i,1)at(y_i,2), \ldots, at(y_r,1)at(y_r,2))$, $r \leq n'$ *be the partial word of length r which is obtained from successive augmentation from the partial word* $\overline{Pw} = (at(y_1,1)at(y_1,2), \ldots, (at(y_i,1)at(y_i,2))$. *Let* $I_u = \{at(y_i,1)\}$, *where* $at(y_i,1) \neq at(y_p,1), p = i+1, \ldots, r$ *&* $Z(X^{Pw}) = \sum_{j \in J_u} (c_{at(y_i,1)j} + f_{at(y_i,1)j} : x_{at(y_i,1)j} = 1)$. *Let* $(c_{at(y_i,1)j} + f_{at(y_i,1)j}) \in J - J_u$ *be arranged in non decresing order of their values and* $J'^{(n'-r)}$ *be the index of the first* $(n'-r)$ *cost in this ordering. If* $Z(X^{Pw}) \geq Z_u - \sum_{J'^{(n'-r)}} (c_{at(y_i,1)j} + f_{at(y_i,1)j})$, *then the partial word* $\overline{Pw}$ *is rejected.*

Proof. As we are given that $I_u = \{at(y_i,1)\}$, where $at(y_i,1) \neq at(y_p,1), p = i+1, \ldots, r$ & $Z(X^{Pw}) = \sum_{j \in J_u} (c_{at(y_i,1)j} + f_{at(y_i,1)j} : x_{at(y_i,1)j} = 1)$. So, $Z(X^{Pw}) \geq Z_u - \sum_{J'^{(n'-r)}} (c_{at(y_i,1)j} + f_{at(y_i,1)j})$ and it means that $Z(X^{Pw}) + \sum_{J'^{(n'-r)}} (c_{at(y_i,1)j} + f_{at(y_i,1)j})_{at(y_i,1)j} \geq Z_u$. It implies that any word which is generated from the patial word $\overline{Pw}$ would not have the value less than Z_u. Hence this partial word $\overline{Pw}$ can't generate a better word and must be rejected.

□

3. Algorithm

In the algorithm partial word will be updated using the following two ways.

A-I(Augmentation) Assume that the partial word represented by Pw is such that

$Pw = ((at(y_1,1), at(y_1,2)), (at(y_2,1), at(y_2,2)), \ldots, (at(y_r,1), at(y_r,2)))$,
$Z(X^{Pw}) < Z_u$ and is augmented with the $(r+1)th$ ordered pair, say

$(at(y_{r+1},1), at(y_{r+1},2))$, i.e source $at(y_{r+1},1)$ is chosen to serve $at(y_{r+1},2)$ destination. Then the partial word Pw is updated as follows:
$Pw = Pw \biguplus (at(y_{r+1},1), at(y_{r+1},2))$
$d^u_{at(y_{r+1},1)} = d^u_{at(y_{r+1},1)} - g_{at(y_{r+1},2)}$
$I_u = I_u \bigcap \{at(y_{r+1},1)\}$ if $d^u_{at(y_{r+1},1)} < g_j \;\forall\, j \in J'$. Otherwise, I_u remains the same.
$J_u = \{at(y_1,2), at(y_2,2), \cdots, at(y_r,2)\} \biguplus \{at(y_{r+1},2)\}$, $J = J - \{at(y_{r+1},2)\}$,
Set $x_{at(y_{r+1},1)at(y_{r+1},2)} = 1$

A-II(Negation) Assume that the partial word represented by Pw is such that
$Pw = (at(y_1,1), at(y_1,2), (at(y_2,1), at(y_2,2)), \ldots, (at(y_r,1), at(y_r,2)))$
for which one of the following holds:

(1) $Z(X^{Pw}) \geq Z_u$

(2) Any one of the conditions mentioned in (Ref. Remarks 2, 3, 4) or Theorem 2.1 holds

Then the partial word Pw is updated as follows:
$Pw = Pw \bigcap (at(y_r,1), at(y_r,2))$
$d^u_{at(y_r,1)} = d^u_{at(y_r,1)} + g_{at(y_r,2)}$
If $at(y_r,1)$ is not an element of I_u, then $I_u = I_u \biguplus (at(y_r,1))$. Otherwise, I_u remains the same. $J_u = \{at(y_1,2), at(y_2,2), \cdots, at(y_r,2)\} \bigcap \{at(y_r,2)\}$, $J = J + \{at(y_r,2)\}$,
Set $x_{at(y_r,1)at(y_r,2)} = 0$.

3.1. *Method to find initial upper bound Z_u*

Construct Alphabet Table AB, $J_u = \phi, Pw = \phi, J = \{1, 2 \ldots, n\}$,
$I = \{1, 2, \ldots, m\}, e = 1, N = 1$, go to Step (i).

Step (i) If $at(e,1) \notin I_u$ or $at(e,2) \in J_u$ or $d_{at(e,1)} < g_{at(e,2)}$, then go to Step (iii). else update as in (A-I), find Pw, go to Step (ii).

Step (ii) If $N < n'$ and $I_u \neq \phi$ then set $N = N + 1$ and $e = e + 1$ and then go to Step (i) else go to Step (iv). If $N = n'$ we will get a word w and hence update $Pw = w$, $Z_u = Z(X^w)$ and go to Step (v).

Step (iii) If $e < mn$, set $e = e + 1$ and then go to Step(i) else go to Step (iv).

Step (iv)
(a) If $Pw \neq \phi$
If $e < mn$ then update as in (A-II), find $e = y_N$ and set $e = e + 1$ and then go to Step (i).
If $e = mn$, then update as explained in (A-II) and set $N = N - 1$ then find $e = y_N$ set $e = e + 1$ and go to Step (i).

(b) If $Pw = \phi$, set $e = e + 1$ and then go to Step (i).

Step (v) Hence Z_u is the initial upper bound of objective function value. Stop.

3.2. *Main Algorithm*

Step 0 Find AB, Z_u(Ref. Section 3.1) and let the corresponding word is denoted by w i.e
$w = (at(y_1, 1), at(y_1, 2) \ldots (at(y_{n'}, 1), at(y_{n'}, 2)), J_u = \phi$, $Pw = \phi$,
$J = \{1, 2 \ldots, n\}, I = \{1, 2, \ldots, m\}$, $e = 1$ and $N = 1$, set $Z_{\text{opt}} = Z_u$ and go to Step 1.

Step 1 If $at(e, 1) \notin I_u$ or $at(e, 2) \in J_u$ or $d_{at(e,1)} < g_{at(e,2)}$, then go to Step 5.
else update as in (A-I), find Pw, go to Step 2.

Step 2 (a) If $N < n'$, for this Pw
$$\sum_{j \in J'^{(n'-r)}} g_j > \sum_{i=1}^{m} d_i^u \text{ then go to Step 3.}$$
Or if $|E_{r+1}| < n' - |Pw|$,
(i) $|Pw| = 1$, Go to Step 7.
(ii) $|Pw| > 1$, then update as in (A-II) and set $N = N - 1, e = y_{N-1}$ and then go to Step 3.
else go to Step 4.

(b) If N=n', go to Step 4.

Step 3 (a) If $Pw \neq \phi$
If $e < mn$ then update as in (A-II), find $e = y_N$ and set $e = e + 1$ and go to Step 1.
If $e = mn$, then update as explained in (A-II) and set $N = N - 1$ then find $e = y_N$, set e=e+1, go to Step 1.

(b) If $Pw = \phi$, set $e = e + 1$ and go to Step 1.

Step 4 Find $Z(X^{Pw})$ and if
(a) $Z(X^{Pw}) < Z_u$,
(i) If $N = n'$, go to Step 6.
(ii) If $N < n'$ and conditions mentioned in Theorem 2.3 holds then
If $|Pw| = 1$, go to Step 7.
If $|Pw| > 1$, update as in (A-II). Find $e = y_N$ and set $N = N - 1$ if $N > 1$ else set $N = 1$ and then go to Step 3, else go to Step 6.
(b) $Z(X^{Pw}) \geq Z_u$, $N > 1$ then update as in (A-II) set $N = N - 1, e = y_N$ and go to Step 3. If $y_r = mn$ then update as in (A-II). Set $N = N - 1$, $e = y_N$, go to Step 3. If $N = 1$ then go to Step 7.

Step 5 If $e < mn$ and Theorem 2.3 holds, go to Step 3 else set $e = e + 1$ and go to Step 1. If $e = mn$, go to Step 3.

Step 6 If $N < n'$, set $N = N + 1$ and $e = e + 1$ and then go to Step 1. If $N = n'$ we found a word w and hence update $w = Pw$, $Z_u = Z(X^w)$. If $Z_u < Z_{\text{opt}}$ then set $Z_{\text{opt}} = Z_u$ and go to Step 1 else go to Step 7.

Step 7 Stop. Z_u is the optimal solution of FCBTP for some $n' < n$.

4. Numerical Illustration

Consider a Fixed Charge Bulk Transportation Problem having three sources and five destinations. It is given that demands of only four destinations are to be fulfilled.

Table 1. The entries of each cell in the upper left corner shows the bulk cost and lower right corner shows the fixed cost of transportation. The availabilities and requirement at each source and destination is 7,8,9 & 3,5,4,6,2 respectively.

10 / 50	9 / 60	11 / 30	7 / 20	8 / 30	7
11 / 20	10 / 30	13 / 50	14 / 60	12 / 40	8
8 / 40	6 / 50	9 / 80	10 / 30	13 / 80	9
3	5	4	6	2	

Fixed Bulk Matrix

Table 2.

60	69	41	27	38
31	40	63	74	52
48	56	89	40	93

Alphabet Table:-

Step 0 (Initialization) Construct alphabet table AB, Find $Z_u = 187$ (Ref. Section 3.1) and the corresponding word is denoted by w i.e
$w = ((1,4),(2,1),(2,2),(3,3)), J_u = \phi, Pw = \phi,$
$J = \{1,2,3,4,5\}, I = \{1,2,3\}, e = 1, N = 1$, go to Step 1.

Table 3.

e	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	27	31	38	40	40	41	48	52	56	60	63	69	74	89	93
AB	(1,4)	(2,1)	(1,5)	(2,2)	(3,4)	(1,3)	(3,1)	(2,5)	(3,2)	(1,1)	(2,3)	(1,2)	(2,4)	(3,3)	(3,5)

Step 1 Find $at(1,1) = 1 \in I_u, at(1,2) = 4 \notin J_u$ then update as in (A-I). Set $Pw = \{(1,4)\}, d_1^u = 7 - 6 = 1$, $\mathrm{J}_u = \{4\}$, go to Step 2.

Step 2 None of the condition satisfies then go to Step 4.

Step 4 Find $Z(X^{Pw}) = 27 < Z_u$, go to Step 6.

Step 6 $N = 1 < n'$, Set $N = 2, e = 2$ and go to Step 1.

Step 1 Find $at(2,1) = 2 \in I_u, at(2,2) = 1 \notin J_u$ then update as in (A-I). Set $Pw = \{(1,4),(2,1)\}, d_2^u = 5$, $J_u = \{4,1\}$, go to Step 2.

Step 2 None of the condition satisfies then go to Step 4.

Step 4 Find $Z(X^{Pw}) = 58 < Z_u$, go to Step 6.

Step 6 $N = 2 < n'$, Set $N = 3, e = 3$ and go to Step1.

Step 1 Find $at(3,1) = 1 \notin I_u, at(3,2) = 5 \notin J_u$ then go to Step 5.

Step 5 If $e = 3 < mn$ then set $e = e + 1 = 4$ and then go to step 1.

Step 1 Find $at(4,1) = 2 \in I_u, at(4,2) = 2 \notin J_u$ then update as in (A-I). Set $Pw = \{(1,4),(2,1),(2,2)\}, J_u = \{4,1,2\}, d_2^u = 0$, go to Step 2.

Step 2 None of the condition satisfies then go to Step 4.

Step 4 Find $Z(X^{Pw}) = 98 < Z_u$, go to Step 6.

Step 6 $N = 3 < n'$, Set $N = 4, e = 5$ and go to Step1.

Step 1 Find $at(5,1) = 3 \in I_u, at(5,2) = 4 \in J_u$ then go to Step 5.

Step 5 If $e = 5 < mn$ then set $e = e + 1 = 6$ and go to step 1.

Step 1 Find $at(6,1) = 1 \notin I_u, at(6,2) = 3 \notin J_u$ then go to Step 5.

Step 5 If $e = 6 < mn$ then set $e = e + 1 = 7$ and go to step 1.

Step 1 Find $at(7,1) = 3 \in I_u, at(7,2) = 1 \in J_u$ then go to Step 5.

Step 5 If $e = 7 < mn$ then set $e = e + 1 = 8$ and go to step 1.

Proceeding likewise we reach a stage when $N = n'$

Step 6 $N = 4 = n'$
Set $w = Pw = ((2,1),(1,5),(2,2),(3,4))$ & $Z_u = Z(X^{Pw}) = 149$.

This implies that we get a word $w = ((2,1),(1,5),(2,2),(3,4))$ and the corresponding objective is the optimal value which is better than the initial upper bound. Hence, the optimal value of the fixed charge bulk transportation problem is obtained using lexi-search approach in which out of five destinations only four destinations are to be served. The run time for the same problem is also found using MATLAB which is equal to 0.041662 seconds.

5. Computational Details

The algorithm has been coded in MATLAB and successfully verified for random generated FCBTP of different sizes. Implementation is done on Intel Processor i5 with 2.40 gigahertz, 4 gigabyte RAM on 64 - bit window operating system. Table 4 shows the computational behaviour of the algorithm for some class of different sizes.

Table 4. Average run time (taken over 1000 instances) of FCBTP for randomly generated problem of different sizes using MATLAB.

Source(m)	Destination(n)	Destinations to be served (n')	Run Time(sec)
10	10	8	0.041771
10	20	15	0.057594
20	20	18	0.087178
20	30	25	0.129657
30	30	28	0.187992
30	40	35	0.279782
40	40	38	0.389773
40	50	45	0.526476
50	50	48	0.632340
50	60	55	0.818588
60	60	58	1.044372
60	70	65	1.293830
70	70	68	1.555681
70	80	75	1.947334
80	80	78	2.323009
80	90	85	2.981653
90	90	88	3.475649
90	100	95	4.179205

Concluding Remarks

(1) An exact method to find solution of fixed charge bulk transportation problem is proposed in which out of the total n destinations only n' destinations ($n' < n$) using lexi search technique are serverd and converge to the optimal solution in limited steps because
  (a) The highest generated words are n^m.
  (b) Each new word created gives a more tight bound on the ideal estimation of the optimal solution.
  (c) Every Partial word generated in the process which yields the value greater than the initial upper bound are rejected.
  (d) Infeasible partial word are also rejected whenever encountered in the process, therefore infeasible words are never generated.

(2) The algorithm has been coded in MATLAB and runs successfully for a variety of test problems. This test problems have been generated randomly following a uniform distribution for all the instances i.e, availabilities, demands, bulk cost, fixed cost, sources and destinations.

(3) The proposed study can also be extended to a more generalized case in which there are bounds on the quantity to be transported from each source.

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