Mathematical Model for Curbing Liquor Habit through Rehabilitation

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Abstract

In this paper, a mathematical model has been developed to examine the impact of behavior of liquored individuals after coming from the Rehabilitation center. A liquored related illness forces the individuals to go to Rehabilitation center. Rehabilitation center has always put their helping hands forward to decrease the habit of liquoring in individuals and to increase the maximum number of quitters from liquor habit in our society. Basic Reproduction Number, stability analysis and its numerical simulation for the proposed problem has been carried out which proves that the Rehabilitation for the liquored is beneficial for improving individuals' health and consequently society.

Keywords: Liquored Dynamical System, Liquored related illness, Rehabilitation, Threshold, Quitters

1. Introduction

Liquor, which is also known as "alcohol or distilled beverages" is an alcoholic drink containing ethanol and is produced by distilling. In India, it is known by the term "hard liquor" to say distilled beverages from those that are weak. It is referred to as "spirit" if the beverage has at least 20% alcohol by volume or if it has no added sugar [9]. Liquor which is a dangerous enemy has been called as a "cruel ruler" and a "bad ruler" of human. It is said to be cruel ruler because it has been cruel to many people and has brought lots of trouble to human beings. Accidents and sad happening due to drinking of liquor is always highlighted in every day's newspaper. Alcohol has some of its advantage as it is being used in medicine, perfumes, industrial world and many more. But inside the body, it becomes a bad ruler. Individuals who intake liquor never know how harmful it is. Initially, they take little amount and gradually increases their amount in order to satisfy their cravings for it. This is how the liquor starts to rule the mind and body of individuals and hence becomes a habit of person.

Liquor habit makes reduces the resistance to many diseases. It has worse effect on brain, nerves, muscles, damages liver, other important organs of the body etc. and then the person becomes victim of one such disease. Along with this, it also interferes with work, makes person careless, causes accidents, tumble down many households and has also spoiled many bright panoramas, etc. Liquor habit is an enemy to health, happiness and success. It also wastes time and money both. It becomes difficult to recover from illness. It has been found by the life insurances companies that the death rate is higher for liquored habit person [7].

Individuals suffering from illness needs to get started on the road to recovery. Recovery is commonly a slow but sure process. Rehabilitation helps one to adapt the path for recovery, but one needs to build a new and meaningful life where one can stay liquor free for long term and liquor habit no longer prevails. Support is essential whether a person's join rehabilitation. It becomes easier to recover from liquor habit when a person gets encouragement, comfort, guidance, support from their family members, friends, healthcare provider, other which recovers liquor, many people from faith community, etc. Once a person quits drinking, particularly in the first six months' thirst for liquor is more strong. Rehabilitation prepares human being to face challenges, craving liquor and to deal with many stressful situations etc. [8]. It is not possible to overcome from the habit of liquor if one makes up his/her mind. One can bring change in their own at any point of time. In order to make a successful life one must never take first drink and also one who has to make up their firm decision for giving up liquor habit and has to quit themselves permanently from it.

Shah N.H. *et. al* (2015) [12] has done their research article entitled "Liquor Habit Transmission Model" in which they analyzed the population dynamics of liquor habit based on number of pegs taken by an individual in a day using an application of *SEIR* model.

In this paper, we will analyze how the rehabilitation can increase quitters from liquor habit similar to *SEIR* model. The mathematical model, notations along with its parametric values, basic reproduction number are formulated and discussed respectively in Section 2. Stability with the subsection of local and global stability are evaluated in Section 3.1 and 3.2 respectively of Section 3. Numerical analysis of the results is illustrated in Section 4. Section 5 comprises of Conclusion.

2. Mathematical Model

Here, we formulate a mathematical model for reduction of liquor habit through rehabilitation using *SEIR* model. The notations along with its description and parametric values are given in below Table1.

Table1: Notations and its Parametric Values

Notations	Description	Parametric Values
$L(t)$	Number of individuals who are liquoring at some instant of time t	100
$I(t)$	Number of individuals who are suffering from liquor related illness at some instant of time t	40
$R(t)$	Number of individuals going for rehabilitation at some instant of time t	35
$Q(t)$	Number of individuals who quit liquor at some instant of time t	15
B	New Recruitment Rate	0.2
β	Rate of individuals suffering from illness due to liquor	0.20
γ	Rate of individuals joining rehabilitation due to illness	0.15
η	Rate of individuals who gets victim of disease during rehabilitation	0.06
δ	Rate of individuals who again starts liquoring after rehabilitation	0.09
α	Rate of individuals who quits liquor after rehabilitation	0.15
μ	Mortality Rate	0.2

Let $N(t)$ denotes the sample size of total human population at any instant of time t. Here, $N(t)$ is divided into four compartments $L(t), I(t), R(t)$ and $Q(t)$ which are individually described in above Table1. Thus, $N(t)=L(t)+I(t)+R(t)+Q(t)$. The schematic diagram to cure liquor habit through rehabilitation is shown in figure 1.

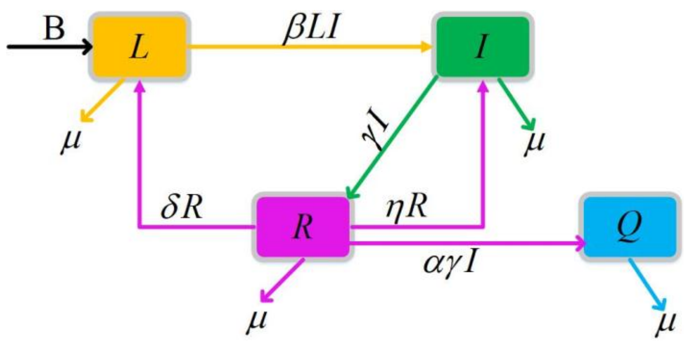

Figure1: Schematic Diagram to cure Liquor Habit through Rehabilitation

Person who has a habit of liquoring (L) becomes ill and suffers from any of the liquor related illness (I) at some stage of life which is defined by the rate β. Rate γ represents the individuals who go for rehabilitation to get cured of illness. As, rehabilitation does not ensure 100% curing of any severe stage of illness some of them again becomes victim of disease which is described by the rate η. δ is the rate who again starts liquoring and α is the rate who quits after rehabilitation respectively. Here, new recruitment rate (B) and mortality rate (μ) are assumed to be equal.

Now, from the above figure 1 a set of non-linear differential equations for curtailing liquor habit through rehabilitation has been constructed.

$$\frac{dL}{dt} = \mathrm{B} - \beta LI + \delta R - \mu L$$
$$\frac{dI}{dt} = \beta LI + \eta R - (\gamma + \mu)I$$
$$\frac{dR}{dt} = \gamma I(1-\alpha) - (\mu+\eta)R - \delta R$$
$$\frac{dQ}{dt} = \alpha\gamma I - \mu Q \tag{1}$$

with $N = L + I + R + Q$, $L > 0, I \geq 0, R \geq 0, Q \geq 0$

In system of equation (1), $N(t)$ is constant so we assume that $L(t)+I(t)+R(t)+Q(t)=1$. Also, as the variable Q does not appear in any of the first three equations from the set of equations (1) we consider the following subsystem of equations

$$\begin{aligned}\frac{dL}{dt} &= \mathrm{B} - \beta LI + \delta R - \mu L \\ \frac{dI}{dt} &= \beta LI + \eta R - (\gamma + \mu)I \\ \frac{dR}{dt} &= \gamma I(1-\alpha) - (\mu + \eta)R - \delta R\end{aligned} \tag{2}$$

On adding the above set of equations (2) we get

$$\frac{d}{dt}(L + I + R) = \mathrm{B} - \mu(L + I + R) - \alpha I \geq 0$$

This gives $\lim_{t\to\infty}\sup(L + I + R) \leq \frac{\mathrm{B}}{\mu} = 1$

So, the feasible region for (2) is

$$\Lambda = \left\{ \left(L, I, R\right) : L + I + R \leq 1, L > 0, I \geq 0, R \geq 0 \right\}$$

Thus, liquor free equilibrium of system (2) is $E_0 = \left(1, 0, 0\right)$.

Now, we are interested in calculating the basic reproduction number which is to be calculated using next generation matrix method [3], [4], [5], [13]. The next generation matrix method is defined as FV^{-1} where F and V both are Jacobian matrices of $\Im$ and v evaluated with respect to the individuals suffering from liquor related illness $\left(I\right)$ and the one joining rehabilitation $\left(R\right)$ at the point E_0.

Let $X = (I, R, L)$

$$\therefore \frac{dX}{dt} = \Im(X) - v(X)$$

where $\Im(X)$ denotes the rate of new liquors and $v(X)$ denotes the rate of transfer of liquor habit which is given as

$$\Im(X)=\begin{bmatrix}\beta LI\\0\\0\end{bmatrix} \text{ and } v(X)=\begin{bmatrix}(\gamma+\mu)I-\eta R\\(\eta+\mu)R-\gamma(1-\alpha)I+\delta R\\-\mathrm{B}+\beta LI-\delta R+\mu L\end{bmatrix}$$

Now, the derivative of $\Im$ and v at liquor free equilibrium point E_0 gives matrices F and V of order 3×3 defined as

$$F=\left[\frac{\partial\Im_i(E_0)}{\partial X_j}\right]\;V=\left[\frac{\partial v_i(E_0)}{\partial X_j}\right] \text{ for } i,j=1,2,3$$

$$\text{Hence, } F=\begin{bmatrix}\beta&0&0\\0&0&0\\0&0&0\end{bmatrix} \text{ and } V=\begin{bmatrix}(\gamma+\mu)&-\eta&0\\-\gamma(1-\alpha)&\eta+\mu+\delta&0\\\beta L&-\delta&\mu\end{bmatrix}$$

where V is non-singular matrix. Thus, the basic reproduction number R_0 which is the spectral radius of matrix FV^{-1} is given as

$$R_0=\frac{\beta(\eta+\mu+\delta)}{\alpha\eta\gamma+\delta\gamma+\delta\mu+\eta\mu+\gamma\mu+\mu^2}$$

On equating the set of equations (2) equal to zero, an endemic equilibrium point defined as smoking present equilibrium point (E^*) is obtained which is as follows:

Liquor present equilibrium is $E^*=(L^*,I^*,R^*)$

where $L^*=\dfrac{\mathrm{B}+\delta R}{\mu+\beta I}$, $R^*=\dfrac{\gamma(1-\alpha)I}{\eta+\mu+\delta}$, $I^*=\dfrac{\mathrm{B}(\eta+\mu+\delta)(R_0-1)}{R_0\left[\mu^2+\mu(\eta+\mu+\delta)+\alpha\gamma(\eta+\delta)\right]}$

3. Stability Analysis

In this section, the local and global stability at E_0 and E^* using the linearization method and matrix analysis are to be studied.

3.1 Local Stability

Theorem 3.1.1: (stability of E_0) If $R_0<1$ then the smoking free equilibrium point (E_0) of system (2) is locally asymptotically stable and if $R_0>1$ then it is unstable.

Proof: At point E_0, the Jacobian matrix of the system (2) is

$$J(E_0)=\begin{bmatrix} -\mu & -\beta & \delta \\ 0 & \beta-(\gamma+\mu) & \eta \\ 0 & \gamma(1-\alpha) & -(\delta+\eta+\mu) \end{bmatrix}$$

The characteristic polynomial for the above matrix is

$$\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0$$

where

$a_1=\eta+3\mu+\delta-\beta+\gamma>0$ which is obvious.

$$\begin{aligned} a_2 &= \alpha\eta\gamma-\beta\delta-\beta\eta-2\mu\beta+\delta\gamma+2\delta\mu+2\eta\mu+2\gamma\mu+3\mu^2 \\ &= (\alpha\eta\gamma+\delta\gamma+\delta\mu+\eta\mu+\gamma\mu+\mu^2)+\delta\mu+\eta\mu+\gamma\mu+2\mu^2-\beta(\delta+\eta+\mu)-\mu\beta \\ &= [\alpha\eta\gamma+\delta\gamma+\delta\mu+\eta\mu+\gamma\mu+\mu^2-\beta(\delta+\eta+\mu)]+\mu\left[\delta+\eta+\gamma+2\mu-\beta\right] \\ &= \alpha\eta\gamma+\delta\gamma+\delta\mu+\eta\mu+\gamma\mu+\mu^2\left[1-R_0\right]+\mu\left[\delta+\eta+\gamma+2\mu-\beta\right] \end{aligned}$$

If $R_0<1$ then $a_2>0$.

$$\begin{aligned} a_3 &= \mu\left[\alpha\eta\gamma-\beta\delta-\beta\eta-\mu\beta+\delta\gamma+\delta\mu+\eta\mu+\gamma\mu+\mu^2\right] \\ &= \mu\left[-\beta(\delta+\eta+\mu)+\alpha\eta\gamma+\delta\gamma+\delta\mu+\eta\mu+\gamma\mu+\mu^2\right] \\ &= \mu\left(\alpha\eta\gamma+\delta\gamma+\delta\mu+\eta\mu+\gamma\mu+\mu^2\right)\left[1-R_0\right] \end{aligned}$$

If $R_0<1$ then $a_3>0$.

$\therefore$ By Routh Hurwitz criteria [6], if $R_0<1$ then it is locally asymptotically stable at E_0 and if $R_0>1$ then it is unstable.

Lemma 3.1.2: (stability of E^*) Let K be a real matrix of order 3×3. If $tr(K)$, $\det(K)$ and $\det(K^{[2]})<0$ then all the eigen values of the matrix K have negative real parts.

Proof: On linearizing the set of equations (2) at point $E^*=(L^*,I^*,R^*)$ the Jacobian matrix of the system (2) is obtained as follows:

$$J(E^*)=\begin{bmatrix} -\left(\mu+\beta I^*\right) & -\beta L^* & \delta \\ \beta I^* & -\dfrac{\eta R^*}{I^*} & \eta \\ 0 & \gamma(1-\alpha) & -(\delta+\eta+\mu) \end{bmatrix}$$

$$\begin{aligned} \therefore\ trace\left(J\left(E^*\right)\right) &= -\beta I^* - 2\mu - \frac{\eta R^*}{I^*} - \eta - \delta \\ &< 0 \end{aligned}$$

$$\begin{aligned} \det\left(J\left(E^*\right)\right) &= -\left[I^* \begin{Bmatrix} L^*\beta^2\left(\delta+\eta+\mu\right)+\beta\delta\gamma\left(\alpha-1\right)+\beta\eta\gamma\left(\alpha-1\right) \\ +R^*\beta\eta\left(\delta+\eta+\mu\right)+\eta\gamma\mu\left(\alpha-1\right)+\dfrac{R^*\eta\mu}{I^*}\left(\delta+\eta+\mu\right) \end{Bmatrix} \right] \\ &= -\frac{\eta R^*\mu}{I^*}\left(\delta+\eta+\mu\right)+\eta\gamma\mu\left(1-\alpha\right)-\eta R^*\beta\left(\delta+\eta+\mu\right)+\eta I^*\beta\gamma\left(1-\alpha\right) \\ &\quad -\beta^2 L^* I^*\left(\delta+\eta+\mu\right)+I^*\beta\delta\gamma\left(1-\alpha\right) \\ &= -\eta\gamma\mu\left(1-\alpha\right)+\eta\gamma\mu\left(1-\alpha\right)-\eta I^*\beta\gamma\left(1-\alpha\right)+\eta I^*\beta\gamma\left(1-\alpha\right)-\beta^2 L^* I^*\left(\delta+\eta+\mu\right) \\ &\quad +\beta\delta\left(\delta+\eta+\mu\right)R^* \\ &= \beta\left(\delta+\eta+\mu\right)\left[-\beta L^* I^*+\delta R^*\right] \\ &= \beta\mu\left(\delta+\eta+\mu\right)\left[-\mathrm{B}+\mu L^*\right] \\ &= \beta\mu\left(\delta+\eta+\mu\right)\left[-1+L^*\right] \\ &< 0 \end{aligned}$$

Now, the second additive compound matrix of $J\left(E^*\right)$ [1], [11] which is given by $J^{[2]}\left(E^*\right)$ is as follows:

$$J^{[2]}\left(E^*\right)=\begin{bmatrix} -\left(\mu+\beta I^*+\dfrac{\eta R^*}{I^*}\right) & \eta & -\delta \\ \gamma(1-\alpha) & -\left(\delta+\eta+2\mu+\beta I^*\right) & -\beta L^* \\ 0 & \beta I^* & -\left(\dfrac{\eta R^*}{I^*}+\delta+\eta+\mu\right) \end{bmatrix}$$

$$\begin{aligned}\det\left(J^{[2]}\left(E^*\right)\right) &= -\left[\beta^2\left(L\beta+\delta+\eta+\mu\right)\right]I^{*2}\\ &-\left[\beta\left(L\beta\mu+R\beta\eta-\alpha\delta\gamma+\delta^2+2\delta\eta+\delta\gamma+4\delta\mu+\eta^2+4\eta\mu+3\mu^2\right)I^*\right]\\ &-\left[\begin{array}{l}LR\beta^2\eta+2R\beta\eta\delta+2R\beta\eta^2+4R\beta\eta\mu+\alpha\delta\gamma\eta+\alpha\eta^2\gamma+\alpha\eta\gamma\mu\\ +\delta^2\mu-\delta\eta\gamma+2\delta\eta\mu+3\delta\mu^2-\eta^2\gamma+\eta^2\mu-\eta\gamma\mu+3\eta\mu^2+2\mu^3\end{array}\right]\\ &-\frac{1}{I^*}\left[R^2\beta\eta^2+R\alpha\eta^2\gamma+R\delta^2\eta+2R\delta\eta^2+4R\delta\eta\mu+R\eta^3-R\eta^2\gamma+4R\eta^2\mu+4R\eta\mu^2\right]\\ &-\frac{1}{I^{*2}}\left[R^2\delta\eta^2+R^2\eta^3+2R^2\eta^2\mu\right]\\ &<0\end{aligned}$$

Hence, E^* is locally asymptotically stable by above lemma.

3.2 Global Stability

Theorem 3.2.1: (stability of E_0) If $\beta \le \gamma\alpha$ then E_0 is globally asymptotically stable.

Proof: Consider the Lyapunov function

Consider a Lyapunov function

$$\begin{aligned}L &= I+R\\ \therefore \frac{dL}{dt} &= -\left(\gamma+\mu\right)I+\beta LI+\eta R+\gamma\left(1-\alpha\right)I-\left(\eta+\mu\right)R-\delta R\\ &= -\gamma I-\mu I+\beta LI+\eta R+\gamma I-\gamma\alpha I-\eta R-\mu R-\delta R\\ &= -\mu\left(I+R\right)+I\left(\beta-\gamma\alpha\right)-\delta R\\ &\le 0\end{aligned}$$

We have $\frac{dL}{dt}<0$ for $\beta L \le \gamma\alpha$

But we have noted that $L<1$ so $\frac{dL}{dt}<0$ for $\beta \le \gamma\alpha$ and $\frac{dL}{dt}=0$ when $T_B = M = 0$.

$\therefore$ By LaSalle's Invariance Principle [10], E_0 is globally asymptotically stable.

Theorem 3.2.2: (stability of E^*) Consider a piecewise smooth vector field $g\left(L,I,R\right)=\left\{g_1\left(L,I,R\right),g_2\left(L,I,R\right),g_3\left(L,I,R\right)\right\}$ on Λ^* that satisfies the condition

$(Curl\,g)\cdot\vec{n}<0$, $g\cdot f=0$ inside Λ^*,where $f=(f_1, f_2, f_3)$is a Lipschitz continuous field inside Λ^*, $\vec{n}$ is a normal vector to Λ^* and $Curl\, g=\left(\frac{\partial g_3}{\partial I}-\frac{\partial g_2}{\partial R}\right)\hat{i}-\left(\frac{\partial g_3}{\partial L}-\frac{\partial g_1}{\partial R}\right)\hat{j}+\left(\frac{\partial g_2}{\partial L}-\frac{\partial g_1}{\partial I}\right)\hat{k}$.

Then, the system of differential equations $L=f_1, I=f_2, R=f_3$ has no homoclinic loops, periodic solutions and oriented phase polygons inside Λ^* [2].

Proof: Suppose $\Lambda^*=\left\{(L,I,R): L+\left(\frac{\mu+\alpha\gamma}{\mu}\right)I+R=1, L>0, I\geq 0, R\geq 0\right\}$. Also, it can easily be proved that $\Lambda^*$is subset of Λ, $\Lambda^*$is positively invariant and endemic equilibrium E^*belongs to Λ^*. Let f_1, f_2 and $f_3$represents the right-hand side of equations in set of equations (2) respectively. Using $L+\left(\frac{\mu+\alpha\gamma}{\mu}\right)I+R=1$ to write f_1, f_2 and $f_3$in the equivalent forms, we get

$$f_1(L,I)=\mathrm{B}-\beta LI+\delta R-\mu L$$
$$f_1(L,R)=\mathrm{B}-\beta L\left[(1-R-L)\left(\frac{\mu+\alpha\gamma}{\mu}\right)\right]+\delta R-\mu L$$
$$f_2(L,I)=\beta LI+\eta R-(\gamma+\mu)I$$
$$=\beta LI-(\gamma+\mu)I+\eta\left[1-L-\left(\frac{\mu+\alpha\gamma}{\mu}\right)I\right]$$
$$f_2(I,R)=\beta LI+\eta R-(\gamma+\mu)I$$
$$=\beta I\left[1-R-\left(\frac{\mu+\alpha\gamma}{\mu}\right)I\right]I+\eta R-(\gamma+\mu)I$$
$$f_3(L,R)=\gamma I(1-\alpha)-(\mu+\eta)R-\delta R$$
$$=\gamma(1-\alpha)\left[(1-R-L)\left(\frac{\mu+\alpha\gamma}{\mu}\right)\right]-(\mu+\eta)R-\delta R$$
$$f_3(I,R)=\gamma I(1-\alpha)-(\mu+\eta)R-\delta R$$

Suppose $g=(g_1, g_2, g_3)$ be a vector field such that

$$g_1 = \frac{f_3(L,R)}{LR} - \frac{f_2(L,I)}{LI}$$
$$= -\frac{\eta}{L} + \frac{\gamma(1-\alpha)\mu}{LR(\mu+\alpha\gamma)} - \frac{\gamma(1-\alpha)\mu}{L(\mu+\alpha\gamma)} - \frac{\gamma(1-\alpha)\mu}{R(\mu+\alpha\gamma)} - \frac{\delta}{L} + \frac{\gamma}{L} - \beta - \frac{\eta}{LI} + \frac{\eta}{I} + \frac{\eta}{L}\left(\frac{\mu+\alpha\gamma}{\mu}\right)$$
$$g_2 = \frac{f_1(L,I)}{LI} - \frac{f_3(I,R)}{IR}$$
$$= \frac{\mathrm{B}}{LI} - \beta + \frac{\delta R}{LI} + \frac{\eta}{I} - \frac{\gamma(1-\alpha)}{R} + \frac{\delta}{I}$$
$$g_3 = \frac{f_2(I,R)}{IR} - \frac{f_1(L,R)}{LR}$$
$$= -\frac{\gamma}{R} + \frac{\beta L}{R} + \frac{\eta}{I} - \frac{\mathrm{B}}{LR} - \frac{\delta}{L} + \frac{\beta}{R}\left(\frac{\mu}{\mu+\alpha\gamma}\right) - \beta\left(\frac{\mu}{\mu+\alpha\gamma}\right) - \frac{\beta L}{R}\left(\frac{\mu}{\mu+\alpha\gamma}\right)$$

As the alternate form of f_1, f_2 and f_3 are equivalent

$$g \cdot f = g_1 f_1 + g_2 f_2 + g_3 f_3 = 0$$

Normal vector $\vec{n} = \left(1, \frac{\mu+\alpha\gamma}{\mu}, 1\right)$

$$Curl\,\vec{g} = \left[\frac{-\eta}{I^2} - \frac{\delta}{LI} - \frac{\gamma(1-\alpha)}{R^2}\right]\hat{i} - \left[\frac{\beta}{R} + \frac{\mathrm{B}}{L^2R} + \frac{\delta}{L^2} - \frac{\beta}{R}\left(\frac{\mu}{\mu+\alpha\gamma}\right)\right]\hat{j} + \left[-\frac{\mathrm{B}}{L^2I} - \frac{\delta R}{L^2I} - \frac{\eta}{L^2I} + \frac{\eta}{I^2}\right]\hat{k}$$

$$(Curl\,\vec{g})\cdot\vec{n} = -\frac{\delta}{LI} - \frac{\mu+\alpha\gamma}{L^2R} - \frac{\delta(\mu+\alpha\gamma)}{L^2M} - \frac{\gamma(1-\alpha)}{LR^2} - \frac{\mathrm{B}}{L^2I} - \frac{\delta R}{L^2I} - \frac{\eta}{LI^2} - \frac{\beta\alpha\gamma}{\mu R}$$
$$< 0$$

So, the system (2) has no homoclinic loops, periodic solutions and oriented phase polygons in the interior of Λ^*.

$\therefore E^*$ is globally asymptotically stable in the interior of Λ^*.

4. Numerical Simulation

In this section, numerical results with their interpretation are described which will help us to know the impact of rehabilitation for curing liquor habit.

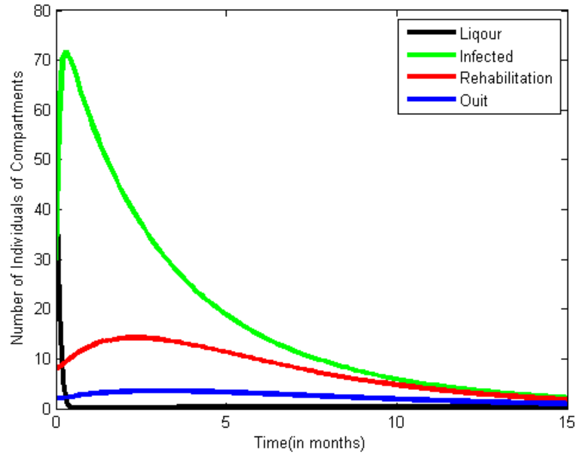

Figure 2: Transmission of individuals in each compartments

Figure 2 shows that in the initial stage liquor decreases which means that illness due to liquor is increasing simultaneously which is seen in the figure 2 above. This increase in illness pushes the individuals for rehabilitation which is increasing in first few months but this decreases and becomes stable after few months which describes that it pushes maximum number of individuals to quit, but as rehabilitation decreases quitters are also decreasing.

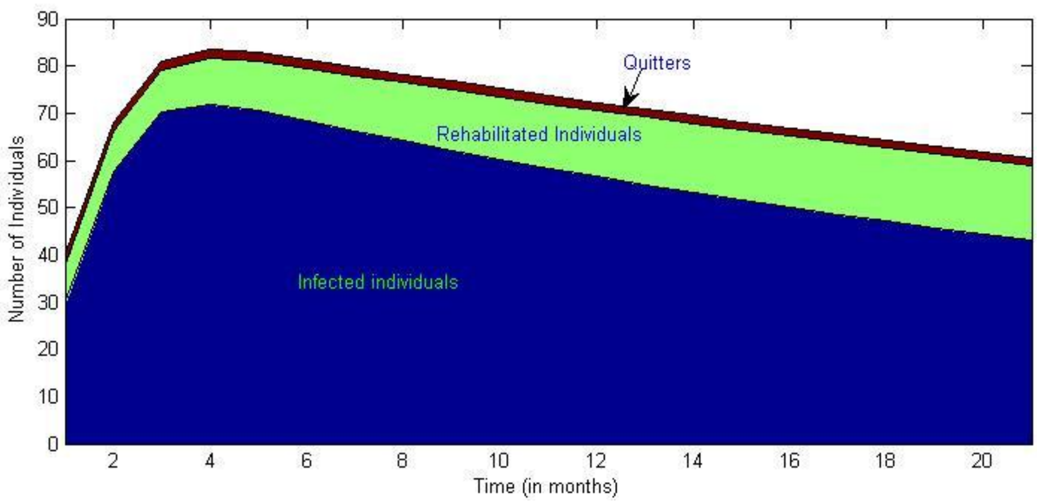

Figure 3: Transmission of Infected, Rehabilitation, Quitters individuals

Figure 3 shows that illness caused due to liquor habit has made them to adapt a track of quitting themselves from it by joining rehabilitation and has proved to be successful.

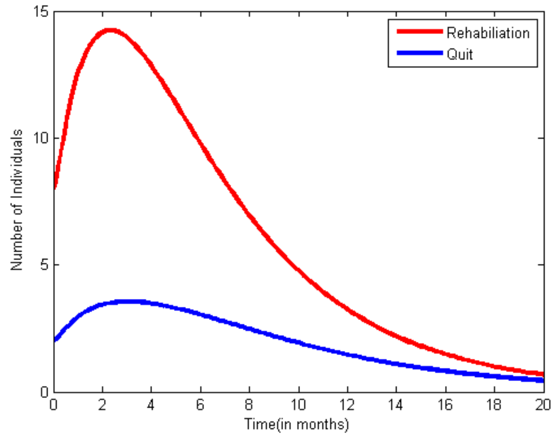

Figure 4: Transmission of Rehabilitation and Quitters individuals

Figure 4 shows that the impact of rehabilitation is truly high to increase the number of quitters from liquor habit. Rehabilitation has played a vital role in reducing liquor habits from the individuals of our society.

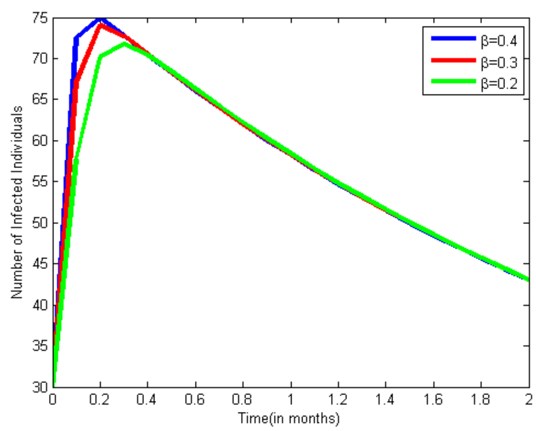

Figure 5: Impact on infected individuals for different values of β

Figure 5 shows that on increasing the value of β which is the transmission rate of occurring illness for liquor habit individuals increase the infected individuals which is obvious as more number of liquor habit individuals increases, illness due to it in individuals will definitely increase.

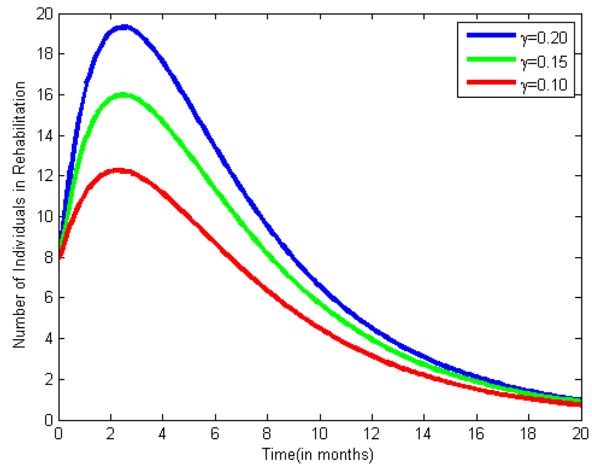

Figure 6: Impact on individuals in rehabilitation for different values of γ

Figure 6 shows that increase in the rate of individuals joining rehabilitation due to suffering of illness increases the number of individuals from approximately 12 to 20 in rehabilitation center to get cured of it.

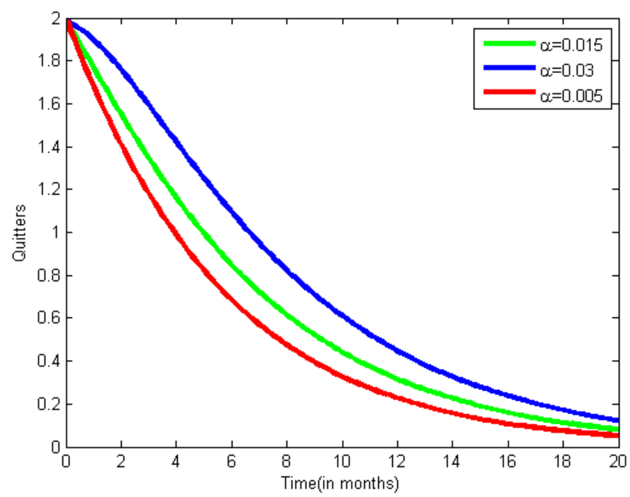

Figure 7: Impact on Quitters for different values of α

Figure 7 shows the effect for different values of α which is the rate of individuals who quit themselves from the habit of liquoring through rehabilitation. Increasing value of α increases the time span of individuals in quitting themselves which is obvious as individual need time to get rid of their habit. Rehabilitation or any center surely increases quitters but every men consumes time.

5. Conclusion

Here, a mathematical model has been formulated as a system of nonlinear ordinary differential equations to study the impact of rehabilitation on individuals in curing liquor habit. System has proved to be locally and globally asymptotically stable at both equilibrium points i.e. liquor free equilibrium point and liquor existence equilibrium point. A basic reproduction number has been calculated at a liquor free equilibrium point and is equal to 0.61600 which shows that 61% of the individuals in the society reduce their liquor habit through rehabilitation. Numerical Simulation has been carried out to examine the results of compartments which interprets that liquor related illness can be cured earlier by joining rehabilitation in the initial phase of the disease and an individual gets motivated to quit themselves from this rigorous habit of liquoring but will take sufficient time to quit completely. Liquor which is a splendid servant should be used only outside the body it should not be allowed to make a ruler inside the body. Every individual in the society should give their foremost steps in building our society free from liquor.

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