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PRIME CORDIAL LABELING ON ECCENTRIC GRAPH OF CYCLE AND PATH

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Abstract
A prime cordial labeling of a graph G with the vertex set V(G) is a bijection f:V(G)→{1,2,3,…..,|V(G)|} such that each edge uv is assigned the label 1 if gcd(f(u),f(v))=1 and 0 if gcd(f(u),f(v))>1, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. Here we consider the structure of eccentric graph of cycle and path, besides labeling the graphs with prime cordial labeling.

Mathematics Subject Classification(2010): 05C12, 05C78

Keywords
Eccentricity, Eccentric Graph, Cordial Labeling, Prime Cordial Labeling

1.Introduction

Graph labeling is a strong relation between numbers and structure of graphs. A useful survey to know about the numerous graph labeling methods is given by [Gallian,2012] . By combining the relatively prime concept in number theory and cordial labeling concept[Cahit,1987] in graph labeling, [Sundaram,2005] introduced the concept called prime cordial labeling.

[Vaidya,2010] as well as [Vaidya,2012] have discussed prime cordial labeling in the context of some graph operations. Prime cordial labeling for some cycle related graphs have been discussed by [Vaidya,2010]. [Vaidya,2011] have investigated many results on prime cordial labeling. Some authors in [Vaidya,2012] have proved that the wheel graph W_n admits prime cordial labeling for $n \geq 8$.

Based on the distance kind of property, the notion of an eccentric graph of a given graph has been considered and its properties has have been investigated [Akiyama,1985]. The eccentric graphs are indeed underlying graphs of eccentric digraphs which have also been well-investigated [Boland,2004 & Buckley,1990]. Here we consider the eccentric graph of path and cycle and obtain its structure.

We consider graphs which are only undirected. For unexplained notions and notations, we refer to [Akiyama,1985, Buckley,1990, West,2001]. A path in a graph G is a sequence of vertices u1, u2,…… un such that ui is adjacent to ui+1 for i, 1≤i≤n-1. The distance between two vertices u and v in a graph G is the length of the shortest path between u and v. The eccentric graph G_e of a graph G is defined as a graph having the same set of vertices as G with two vertices u and v adjacent in G_e if and only if either u is an eccentric vertex of v or v is an eccentric vertex of u.

The eccentricity e (u) of a vertex u in G, with vertex set V (G), is defined as

$$e(u) = \max_{v \in V(G)} d(u,v).$$

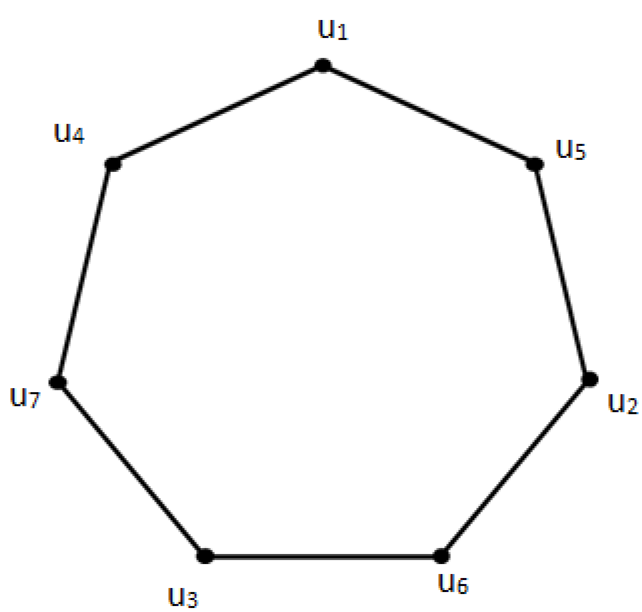
A vertex v is an eccentric vertex of a vertex u if d (u,v) = e (u). The set of all eccentric vertices of u is denoted by E (u). The set of all eccentric vertices of G is denoted by EP (G). The present work is aimed to investigate some new results on prime cordial labeling for eccentric graph of path and cycle.

2.Eccentric graph of cycle C_n

Case1:- when n is odd.

Consider a cycle C_n (n odd) with n number of vertices and n number of edges. Then we will see that the eccentric graph of C_n is also a cycle C_n with different vertex name. Suppose the vertices of C_n are $u_1,u_2,u_3,\ldots\ldots,u_n$. then the eccentric graph of C_n has the vertices as $u_1,u_2,\ldots,u_{(n+1)/2}$ in the alternative vertices of the cycle with length n ,and the remaining vertices are numbered as $u_{(n+1)/2+1}$, $u_{(n+1)/2+2},\ldots..,u_n$ starting for next to u_1 and upto another side of u_1 respectively.

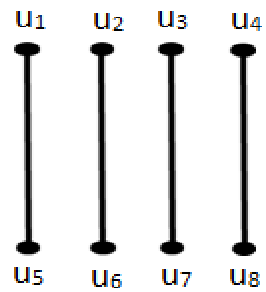
Illustration: Eccentric graph of C_7



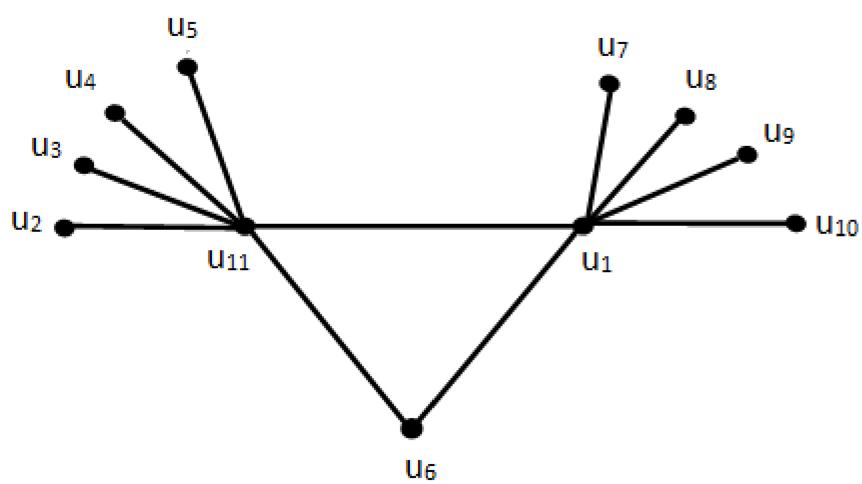
Case2:- when n is even.

Suppose the vertices of C_n (n even) are $u_1,u_2,\ldots..,u_n$. then the eccentric graph of C_n is a bipartite graph with the partition V_1 and V_2 where $V_1=\{u_1,u_2,\ldots.,u_{n/2}\}$ and $V_2=\{u_{n/2+1},u_{n/2+2},\ldots..,u_n\}$. in this bipartite graph each vertices u_i for i=1,2,…,n/2 are adjacent to exactly one vertex named as ui+n/2, i.e., one vertex of V1 is adjacent to only

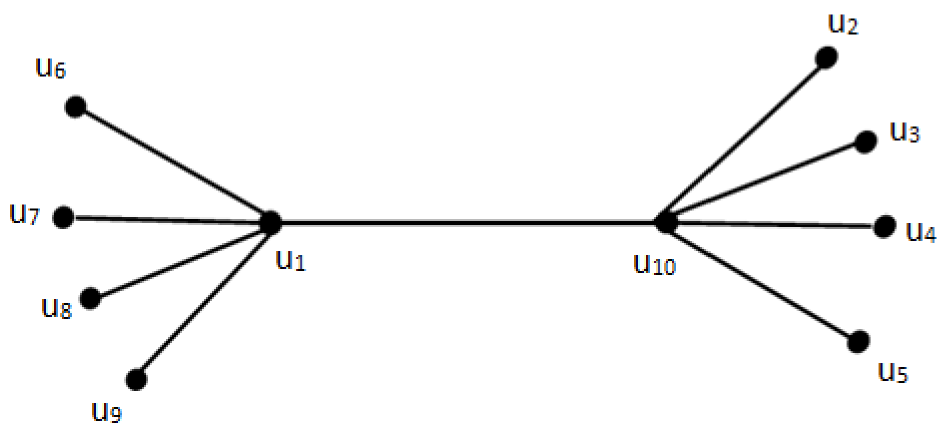
one vertex of V_2.
Illustration: Eccentric graph of C_8



3. Eccentric graph of path P_n
Case1:-when n is odd.
Consider a path P_n (n odd) with n number of vertices and (n-1) number of edges. Suppose the vertices of P_n are $u_1, u_2, u_3, \ldots\ldots, u_n$. Then we will see that the eccentric graph of P_n is a graph with a cycle of three vertices named $u_1, u_{(n+1)/2}, u_n$. And the vertices u_1 and u_n each have (n-3)/2 pendant edges. Hence u_1 have the adjacent edges $u_{(n+1)/2}$, $u_{(n+1)/2+1}, \ldots\ldots, u_n$ and u_n has adjacent edges $u_1, u_2, \ldots.., u_{(n-1)/2}$.
Illustration: Eccentric graph of path P_{11}.



Case2:- when n is even.
Suppose the vertices of P_n (n even) are $u_1, u_2, u_3, \ldots\ldots, u_n$. Eccentric graph of path P_n is a graph with n vertices and (n-1) edges in which u_1 and u_n are adjacent and both have (n-2)/2 pendant edges. Thus u_1 has n/2 adjacent named $u_{n/2+1}$, $u_{n/2+2}, \ldots.., u_n$ and u_n has also n/2 adjacent vertices named $u_1, u_2, \ldots.., u_{n/2}$.
Illustration: Eccentric graph of path P_{10}.



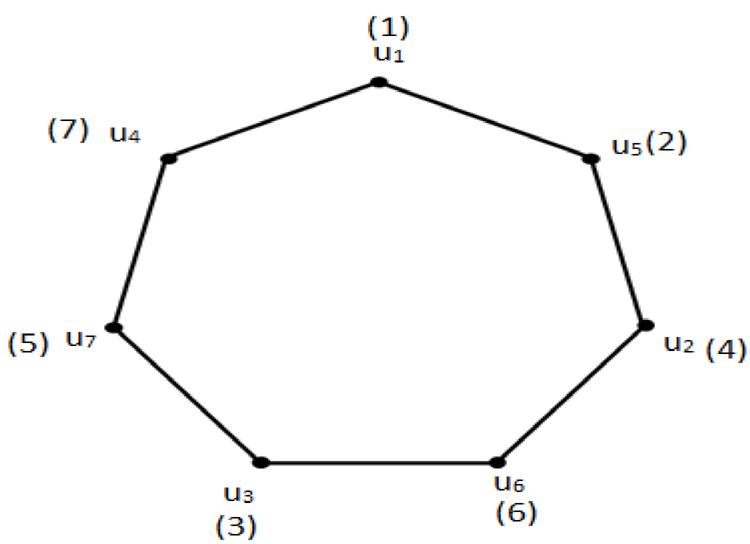
3.Prime Cordial Labeling of eccentric graph of cycle with odd length

Now we will label the eccentric graph of a cycle C_n with odd length. Consider a labeling $f:V(C_n)\rightarrow\{1,2,3,\ldots..,|V(C_n)|\}$. First suppose $f(u_1)=1$, $f(u_{(n+1)/2+1})=2$, $f(u_2)=4$, $f(u_{(n+1)/2+2})=8$ and next ((n-1)/2-4) vertices are labeled with the even numbers from 10 to (n-1). Then the next vertex has label 6 and after that the remaining vertices are labeled with the odd numbers starting from 3 to n. So, from the corresponding edge labeling we will get $e_{f^*}(0) = (n-1)/2$ and $e_{f^*}(1) =(n+1)/2$.

Thus $|e_{ve^*}(1)-e_{f^*}(0)|= |(n+1)/2-(n-1)/2|=1$.

Hence the graph is Prime Cordial for this case.

Illustration:



Prime Cordial Labeling of Eccentric Graph of C_7

4.**Prime Cordial Labeling of eccentric graph of cycle with even length**

Now we will label the eccentric graph of a cycle C_n with even length. Consider a labeling $f:V(C_n)\rightarrow\{1,2,3,\ldots..,|V(C_n)|\}$. For this purpose we have to consider the vertices as $u_1,u_{n/2+1},u_2,u_{n/2+2},\ldots.u_i,u_{n/2+i},\ldots..,u_{n/2},u_{n/2+n/2}$. Then let, $f(u_1)=1,f(u_{n/2+1})=2,f(u_2)=4,f(u_{n/2+2})=8$ and after that we will label the vertices with the even numbers starting from 10 to n and the next vertex will be labeled with 6. After that we will label the vertices with the odd numbers stating from 3 to (n-1).Then from the corresponding edge labeling we will have two cases:

(i) n=2m where m is an odd number and

(ii) n=2m where m is an even number.

Case (i):- When n=2m where m is an odd number.

Then $e_{f^*}(0) = (n-2)/2$ and $e_{f^*}(1) =n/2$.

Thus, $|e_{f^*}(1)-e_{f^*}(0)|=|n/2-(n-2)/2|=1$.

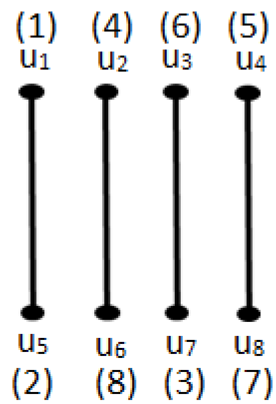
Case (ii):- When n=2m where m is an even number.

Then $e_{f^*}(0) =e_{f^*}(1) =n/2$.

Thus $|e_{f^*}(1)-e_{f^*}(0)|=|n/2-n/2|=0$.

Hence the eccentric graph of a cycle C_n when n is even, is a prime cordial graph.

Illustration:



Prime Cordial Labeling of Eccentric Graph of cycle C_8.

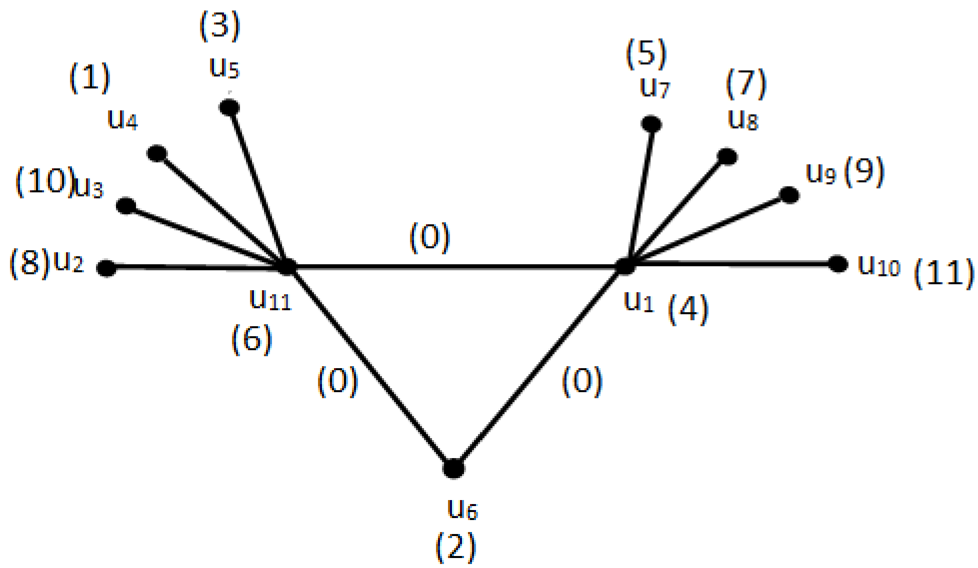
5.**Prime Cordial Labeling of eccentric graph of path with odd length**

Consider a labeling $f:V(P_n)\rightarrow\{1,2,3,\ldots..,|V(P_n)|\}$. Suppose $f(u_1)=4$, $f(u_{(n+1)/2})=2$, $f(u_n)=6$ and the remaining (n-3) pendant edges are labeled with the colors 8,10,12,…..,(n-1),1,3,5,……,n respectively. Thus the (n-3)/2-2 adjacent vertices of u_n will be labeled with the even numbers from 8 to (n-1) and the remaining two by the color 1 and 3. And the (n-3)/2 adjacent vertices of u_1 will be labeled with the odd numbers from 5 to n respectively. Then the corresponding edge labeling will give $e_{f^*}(0) = (n+1)/2$ and $e_{f^*}(1) = (n-1)/2$.

Thus $|e_{f^*}(0)-e_{f^*}(1)|= |(n+1)/2-(n-1)/2|=1$.

Hence the eccentric graph of a path P_n when n is odd, is a prime cordial graph.

Illustration:



Prime Cordial Labeling of Eccentric Graph of Path P_{11}

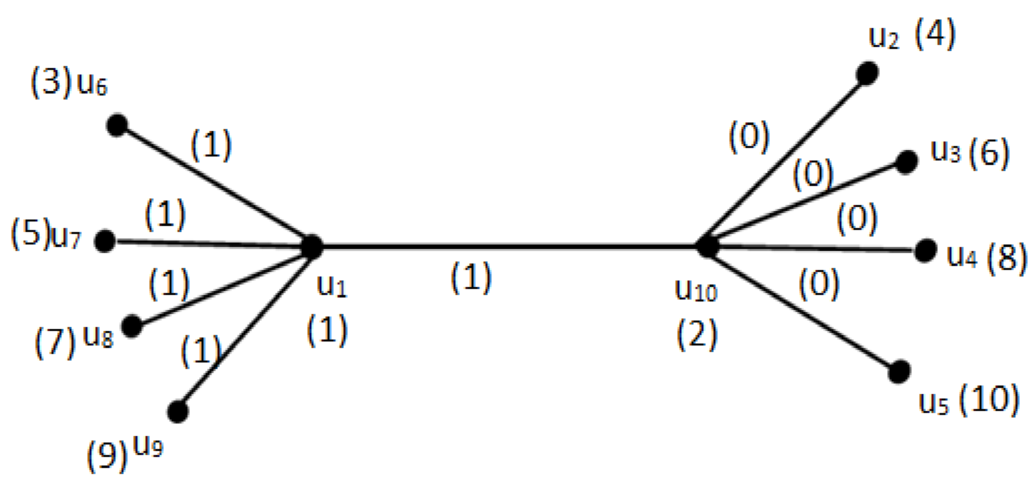
6.**Prime Cordial Labeling of eccentric graph of path with even length**

Consider a labeling $f:V(P_n)\rightarrow\{1,2,3,\ldots,|V(P_n)|\}$. Suppose f $(u_1)=1$, $f(u_n)=2$. Then the adjacent vertices of u_1 other than u_n are labeled with the odd numbers starting from 3 to (n-1) and the adjacent vertices of u_n other than u_1 are labeled with the even numbers starting from 4 to n. Then the corresponding edge labeling will give $e_{f^*}(0) = (n-2)/2$ and $e_{f^*}(1) = n/2$.

Thus $|e_{f^*}(1)-e_{f^*}(0)|=1$.

Hence the eccentric graph of a cycle P_n when n is even, is a prime cordial graph.

Illustration:



Prime Cordial Labeling of Eccentric Graph of Path P_{10}

7.Conclusion

Eccentric graphs of other special kinds of graphs such as interval graphs [Paul,2014], permutation and bipartite permutation graphs [Paul,2014] and so on can be investigated. It will also be of interest to examine the role of eccentric graphs of the type considered here in graph based probabilistic models. Here we have contributed some new results by investigating prime cordial labeling for eccentric graph of cycle and path.

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