# Political uncertainty and stock prices 

Sara Barati, Karim Ivaz<br>Department of Applied Mathematics, University of Tabriz, Tabriz, Iran.<br>sarabarati50@gmail.com


#### Abstract

In this paper we depict the results of the Pastor and Veronesi's model of firms' exposures to government uncertain policy. The level of stock prices is an increasing function of economic conditions. The risk premium is low when the government policy is poor and it is high when the government policy is favorable. The relation of the correlation between each pair of stocks with economic conditions is ambiguous.


Keywords: Political uncertainty, stock prices, economic conditions.

## 1 Introduction

Political stability plays a critical role in the economic growth and development in any country. Lack of stability in government policies can be a crucial contributing factor to financial crisis. Firms' stock prices respond to government economic and non-economic policies such as changes to environmental regulations, taxation, spending programmes, presidential elections, to name a few. It is wise for any firm to ask how will government policy affect its stock prices.

In an influential paper, Pastor and Veronesi [2] analyze how government policy and its inherent uncertainty may affect stock prices. They develop a general equilibrium model in which firm profitability follows a stochastic process whose drift is affected by the prevailing government policy. The policy's impact on the mean is uncertain and all agents learn in a Bayesian way about this impact by observing the realizations of profitability. In their framework, the government may decide to change its policy, it has to sustain some political costs which are randomly drawn at time $\tau$.

The current manuscript depicts some of the results that Pastor and Veronesi [2] derived for the case in which all of the financial quantities vary with economic conditions. The novelty of the current work consists in considering economic conditions variable as an important quantity in firms stock price variations similar to what has been done by Pastor and Veronesi [1] for the case of multiple policies.

[^0]we consider an economy with a finite horizon $[0, T]$ and a continuum of allequity firms $i \in[0,1]$. Let $B_{t}^{i}$ denote firm $i$ 's capital at time $t$. Profitability in each firm follows a stochastic process given by:
\[

$$
\begin{equation*}
\frac{d B_{t}^{i}}{B_{t}^{i}}=\left(\mu+g_{t}\right) d t+\sigma d Z_{t}+\sigma_{1} d Z_{t}^{i} \tag{1}
\end{equation*}
$$

\]

where $\left(\mu, \sigma, \sigma_{1}\right)$ are observable constants, $d Z_{t}$ is a shock, $d Z_{t}^{i}$ is a firm-specific shock and all Brownian motions are independent of each other. The current goverment policy impact $g_{t}$ is a simple step function of time:

$$
g_{t}=\left\{\begin{array}{ll}
g^{\text {old }}, & t \leq \tau  \tag{2}\\
g^{\text {old }}, & t>\tau \\
g^{\text {new }}, & t>\tau
\end{array} \quad\right. \text { if there is no policy change }
$$

where $0<\tau<T$ is an exogenously given time-invariant constant.
Firms are owned by a continuum of identical investors who maximize expected utility derived from terminal wealth. For all $j \in[0,1]$, investor $j$ 's utility function is given by:

$$
\begin{equation*}
u\left(W_{T}^{j}\right)=\frac{\left(W_{T}^{j}\right)^{1-\gamma}}{1-\gamma} \tag{3}
\end{equation*}
$$

where $W_{T}^{j}$ is investor $j$ 's wealth at time $T$ and $\gamma>1$ is the coefficient of relative risk aversion. Stocks pay liquidating dividends at time $T$.
The government solves

$$
\begin{equation*}
\max \left\{E_{\tau}\left[\left.\frac{W_{T}^{1-\gamma}}{1-\gamma} \right\rvert\, \text { no policy change }\right], E_{\tau}\left[\left.C \frac{W_{T}^{1-\gamma}}{1-\gamma} \right\rvert\, \text { policy change }\right]\right\}, \tag{4}
\end{equation*}
$$

where $W_{T}=B_{T}=\int_{0}^{1} B_{T}^{i} d i$ is the final value of aggregate capital and $C$ is the political cost incurred by the government if a new policy is introduced. The value of $C$ is randomly drawn at time $\tau$ from a lognormal distribution centered at $C=1$ :

$$
c \equiv \log (C) \sim N\left(-\frac{1}{2} \sigma_{c}^{2}, \sigma_{c}^{2}\right)
$$

where $C$ is independent of the Brownian motions in equations (1), (2). We refer to $\sigma_{c}$ as political uncertainty. Political uncertainty introduces an element of surprise into policy changes, resulting in stock price reactions at time $\tau$. The government maximizes the investors' welfare on average (because $E(C)=1$ ), but it also deviates from this objective in a random fashion.

Only a small set of remarkable theoretical works explain the link between political or economic uncertainty and financial markets (Pastor and Veronesi [1,2,3], Ulrich [4], Croce et al. [5]). Recent papers including empirical studies of this link are Chang et al. [6], Gulen and Ion [7], Julio and Yook [8], Gao and Qi [9], Brogaard and Detzel [10], Durnev [11], Liu et al. [12].

## 2 Learning

In this section, we use a learning technique namely the Kalman-Bucy filter that helps to access information about the government policy. Then we design the government's decision rule in this model. Agents solve a Bayesian learning problem which leads to asset pricing implications. The following result holds.

Lemma 2.1 Suppose that the time $t$ information set is given by $\mathcal{F}_{t}$. Before observing any signals, the prior distribution for $g$ at time 0 is normal:

$$
g \sim N\left(0, \sigma_{g}^{2}\right)
$$

The agents' inference at time $t<\tau$ has a Gaussian distribution given by:

$$
g \sim N\left(\widehat{g}_{t}, \widehat{\sigma}_{g, t}^{2}\right)
$$

where the posterior mean follows:

$$
d \widehat{g}_{t}=\widehat{\sigma}_{t}^{2} \sigma^{-1} d \widehat{Z}_{t}
$$

and the posterior variance follows:

$$
\widehat{\sigma}_{t}^{2}=\frac{1}{\frac{1}{\sigma_{g}^{2}}+\frac{1}{\sigma^{2}} t} .
$$

Here, $d \widehat{Z}_{t}$ is a new Brownian motion which reflects expectation error:

$$
d \widehat{Z}_{t}=\frac{d s_{t}-E_{t}\left(d s_{t}\right)}{\sigma}
$$

Proof. See Theorem 10.2 of [13].
Under the investors' information set $\mathcal{F}_{t}$, profitability evolves as:

$$
\frac{d B_{t}}{B_{t}}=\left(\mu+\widehat{g}_{t}\right) d t+\sigma d \widehat{Z}_{t}
$$

### 2.1 The government's policy decision

Turn now to the government's decision rule in this extended model. Exploiting (4), we obtain the following proposition:

Proposition 2.2 A policy change occurs at time $\tau$ if and only if:

$$
\widehat{g}_{\tau}<\underline{g}(c),
$$

where

$$
\underline{g}(c)=-\frac{\left(\sigma_{g}^{2}-\widehat{\sigma}_{\tau}^{2}\right)(\gamma-1)(T-\tau)}{2}-\frac{c}{(T-\tau)(\gamma-1)} .
$$

Proof. see [2].
We denote by $p_{\tau}$ the probability of a policy change at time $\tau$.

## 3 Stock prices

In this section, we characterize the stock price level, the equity risk premium, the volatility of stock return and the correlation between each pair of stocks in the economy.

Assuming complete markets, standard arguments imply that the state price density is uniquely given by

$$
\pi_{t}=\frac{1}{\lambda} E\left(B_{T}^{-\gamma} \mid \mathcal{F}_{t}\right)
$$

where $\lambda$ is the Lagrange multiplier from the utility maximization problem of the representative investor. Firm $i$ 's stock is a claim on the firm's liquidating dividend at time $T$ which is equal to $B_{T}^{i}$. Thus the market value of stock $i$ is given by the standard pricing formula:

$$
M_{t}^{i}=E\left[\left.\frac{\pi_{T}}{\pi_{t}} B_{T}^{i} \right\rvert\, \mathcal{F}_{t}\right] .
$$

Now we prepare the stock pricing results before time $\tau$, which are the focus of this paper.

Proposition 3.1 The stochastic discount factor at time $t<\tau$ is given by:

$$
\pi_{t}=B_{t}^{-\gamma} \Omega\left(\widehat{g}_{t}, t\right)
$$

where

$$
\begin{gathered}
\Omega\left(\widehat{g}_{t}, t\right)=p_{t}^{y e s} G_{t}^{y e s}+\left(1-p_{t}^{n o}\right) G_{t}^{n o} \\
G_{t}^{y e s}=e^{-\gamma \mu(T-t)-\gamma \widehat{g}_{t}(\tau-t)+\frac{\gamma^{2}}{2}\left((T-\tau)^{2} \sigma_{g}^{2}+(\tau-t)^{2} \widehat{\sigma}_{t}^{2}\right)+\gamma(1+\gamma) \frac{\sigma^{2}}{2}(T-t)}, \\
G_{t}^{n o}=e^{-\gamma\left(\mu+\widehat{g}_{t}\right)(T-t)+\frac{\gamma^{2}}{2}(T-t)^{2} \widehat{\sigma}_{t}^{2}+\gamma(1+\gamma) \frac{\sigma^{2}}{2}(T-t)} .
\end{gathered}
$$

Here

$$
\begin{aligned}
P_{t}^{y e s} & =N\left(\underline{g}(0) ; \widehat{g}_{t}-\gamma \widehat{\sigma}_{t}^{2}(\tau-t)+\frac{\frac{\sigma_{c}^{2}}{2}}{(T-\tau)(1-\gamma)}, \widehat{\sigma}_{t}^{2}-\widehat{\sigma}_{\tau}^{2}+\frac{\sigma_{c}^{2}}{(T-\tau)^{2}(1-\gamma)^{2}}\right) \\
P_{t}^{n o} & =N\left(\underline{g}(0) ; \widehat{g}_{t}-\gamma\left(\widehat{\sigma}_{t}^{2}(T-t)-(T-\tau) \widehat{\sigma}_{\tau}^{2}\right)+\frac{\frac{\sigma_{c}^{2}}{2}}{(T-\tau)(1-\gamma)}, \widehat{\sigma}_{t}^{2}-\widehat{\sigma}_{\tau}^{2}+\frac{\sigma_{c}^{2}}{(T-\tau)^{2}(1-\gamma)^{2}}\right) .
\end{aligned}
$$

The dynamics of the stochastic discount factor at time $t<\tau$ is given by:

$$
\frac{d \pi_{t}}{\pi_{t}}=-\sigma_{\pi, t} d \widehat{Z}_{t}
$$

where

$$
\sigma_{\pi, t}=\gamma \sigma-\frac{1}{\Omega\left(\widehat{g}_{t}, t\right)} \frac{\partial \Omega\left(\widehat{g}_{t}, t\right)}{\partial \widehat{g}_{t}} \widehat{\sigma}_{t}^{2} \sigma^{-1}
$$

Note that $N(x ; a, b)=p(c<x)$ where $c$ is an stochastic variable with mean a and variance $b$.

Proposition 3.2 The "market-to-book" ratio for each firm at time $t<\tau$ is given by

$$
\frac{M_{t}^{i}}{B_{t}^{i}}=\frac{\Phi\left(\widehat{g}_{t}, t\right)}{\Omega\left(\widehat{g}_{t}, t\right)},
$$

where

$$
\begin{gathered}
\Phi\left(\widehat{g}_{t}, t\right)=\bar{p}_{t}^{y e s} K_{t}^{y e s}+\left(1-\bar{p}_{t}^{n o}\right) K_{t}^{n o}, \\
K_{t}^{y e s}=e^{(1-\gamma) \mu(T-t)+(1-\gamma) \widehat{g}_{t}(\tau-t)+\frac{(1-\gamma)^{2}}{2}\left((T-\tau)^{2} \sigma_{g}^{2}+(\tau-t)^{2} \widehat{\sigma}_{t}^{2}\right)-(1-\gamma) \gamma \frac{\sigma^{2}}{2}(T-t)}, \\
K_{t}^{n o}=e^{(1-\gamma) \mu(T-t)+(1-\gamma) \widehat{g}_{t}(T-t)+\frac{(1-\gamma)^{2}}{2} \widehat{\sigma}_{t}^{2}(T-t)^{2}-(1-\gamma) \gamma \frac{\sigma^{2}}{2}(T-t)} .
\end{gathered}
$$

Here
$\bar{p}_{t}^{y e s}=N\left(\underline{g}(0) ; \widehat{g}_{t}+(1-\gamma) \widehat{\sigma}_{t}^{2}(\tau-t)+\frac{\frac{\sigma_{c}^{2}}{2}}{(T-\tau)(1-\gamma)}, \widehat{\sigma}_{t}^{2}-\widehat{\sigma}_{\tau}^{2}+\frac{\sigma_{c}^{2}}{(T-\tau)^{2}(1-\gamma)^{2}}\right)$,
$\bar{p}_{t}^{n o}=N\left(\underline{g}(0) ; \widehat{g}_{t}+(1-\gamma)\left(\widehat{\sigma}_{t}^{2}(T-t)-(T-\tau) \widehat{\sigma}_{\tau}^{2}\right)+\frac{\frac{\sigma_{c}^{2}}{2}}{(T-\tau)(1-\gamma)}, \widehat{\sigma}_{t}^{2}-\widehat{\sigma}_{\tau}^{2}+\frac{\sigma_{c}^{2}}{(T-\tau)^{2}(1-\gamma)^{2}}\right)$.
Proposition 3.3 Stock return process for each firm $i$ at time $t<\tau$ is given by

$$
\frac{d M_{t}^{i}}{M_{t}^{i}}=\mu_{M, t} d t+\sigma_{M, t} d \widehat{Z}_{t}+\sigma_{1} d Z_{t}^{i}
$$

where

$$
\begin{gathered}
\sigma_{M, t}=\sigma+\left(\frac{\frac{\partial \Phi\left(\widehat{g}_{t}, t\right)}{\partial \widehat{\widehat{g}}_{t}}}{\Phi\left(\widehat{g}_{t}, t\right)}-\frac{\frac{\partial \Omega\left(\widehat{g}_{t}, t\right)}{\partial \widehat{\widehat{g}}_{t}}}{\Omega\left(\widehat{g}_{t}, t\right)}\right) \widehat{\sigma}_{t}^{2} \sigma^{-1}, \\
\mu_{M, t}=\sigma_{\pi, t} \sigma_{M, t} .
\end{gathered}
$$

Proposition 3.4 The correlation between the returns of any pair of stocks at time $t<\tau$ is given by:

$$
\rho_{t}=\frac{\sigma_{M, t}^{2}}{\sigma_{1}^{2}+\sigma_{M, t}^{2}}
$$

## 4 Model evaluation

In this section we depict the role of economic conditions $\widehat{g}_{t}$ as an important factor affecting on key pricing quantities. We input the parameters using Table 1. Then with varying $\widehat{g}_{t}$, we can plot all of the asset pricing quantities. Note that we use MATLAB program as the tool for calculating all of the necessary financial quantities in the model.

From Figure 1, we see that the market-to-book ratio is an increasing function of $\widehat{g}_{t}$. When the government has a policy with an unfavorable impact on stock prices, It is more likely to change it's policy. therefore the shocks related to government policy are temporary and have a small impact on stock prices. On the contrary, In good economic conditions, the government tends to retain it's


Figure 1: The key pricing quantities
policy. Thus, the shocks are permanent and have a large impact on the level of stock prices.

We plot the risk premium as a function of the government policy $\widehat{g}_{t}$ at time $t=5<\tau$. As the figure shows, the risk premium in good economic conditions is larger than the bad conditions. Because the $\widehat{g}_{t}$ shocks have a large effect on stock prices. With a longer-lasting effect, The risk premium associated with exposure to those shocks is larger as well.

We display the correlation between each pair of stocks in different economic conditions at time $t=5$ in Figure 1.

Table 1: Parameter choices

| $\sigma_{g}$ | $\sigma_{c}$ | $\mu$ | $\sigma$ | $\sigma_{1}$ | T | $\tau$ | $\gamma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.02 | 0.10 | 0.10 | 0.05 | 0.10 | 20 | 10 | 5 |

## 5 Conclusions and future study

In this paper we show that the value of stocks fluctuate in response to economic conditions. We analyze how government policy as the only economic variable and its inherent uncertainty may affect stock prices. We consider the profitability rate as a constant variable in this paper. Assuming the profitability rate $\mu$ as an stochastic variable, the authors recommend extending study to another important topic of financial economics namely business cycle.

## References

[1] Pastor, L. , Veronesi, P., 2013. Political uncertainty and risk premia, Journal of Financial Economics 110, 520-545.
[2] Pastor, L. , Veronesi, P., 2012. Uncertainty about government policy and stock prices, The Journal of Finance 67, 1219-1264.
[3] Kelly, B. , Pastor, L. , Veronesi, P., 2014. The Price of political uncertainty: theory and evidence from the option market, The Journal of Finance (forthcoming).
[4] Ulrich, M., 2013. How does the bond market perceive government interventions?, Working paper, Columbia university.
[5] Croce, M. M. , Kung, H. , Nguyen, T. T., Schmid, L. , 2012. Fiscal policies and asset prices, Review of Financial Studies 25, 2635-2672.
[6] Chang, T., Chen, W. Y. , Gupta, R. , Nguyen, D. K. , 2015. Are stock prices related to the political uncertainty index in OECD countries? Evidence from the bootstrap panel causality test, Economic Systems 39, 288-300.
[7] Gulen, H. , Ion, M. , 2015. Policy uncertainty and corporate investment, Review of Financial Studies.
[8] Julio, B. , Yook, Y., 2012. Political uncertainty and corporate investment cycles, The Journal of Finance 67, 45-83.
[9] Gao, P., Qi, Y., 2013. Political uncertainty and public financing costs: Evidence from US municipal bond markets, (forthcoming).
[10] Brogaard, J. , Detzel, A. , 2015. The asset-pricing implications of government economic policy uncertainty, Management Science 61, 3-18.
[11] Durnev, A., 2012. The real effects of political uncertainty, Working paper, University of lowa.
[12] Liu, L. X. , Shu, H. , Wei, K. C. J. , 2015. The Impacts of Political Uncertainty on Asset Prices: Evidence from a Natural Experiment, Working paper.
[13] Liptser, R. S. , Shiryaev, A. N. , 1997. Statistics of random processes: I, II, Springer-Verlag, New York.


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