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Heterogeneous firms' responses to political uncertainty with business-cycle fluctuations in profitability

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Abstract

In this paper we complement the results of the Pastor and Veronesi's model of heterogeneity in the firms' exposures to government policy (PV-H) by incorporating a variation of the profitability rate into the model. The difference of the level of stock prices between high and low exposure firms is an increasing function of economic conditions and with an increase in profitability rate it tends to decrease in strong conditions when the fraction of total wealth produced by high exposure firms is large. The difference of risk premiums between heterogeneous firms is countercyclical. Higher uncertainty of profitability rate tends to increase (decrease) this difference (where firms with more exposure own a larger share of the total wealth) in very bad (good) conditions. The correlation between each pair of stocks is more in firms with higher level of exosure to political uncertainty under each state of the economy.

Keywords: Heterogeneous firms, political uncertainty, uncertainty of profitability rate, business-cycle.

1 Introduction

Political stability plays a critical role in the economic growth and development in any country. Lack of stability in government policies can be a crucial contributing factor to financial crisis. Firms' stock prices respond to government economic and non-economic policies such as changes to environmental regulations, taxation, spending programmes, presidential elections, to name a few. Different firms may have different exposures to government policies which lead to different pricing quantities. It is wise for any firm to ask how will government policy affect its stock prices.

However, even though it is clear that firms' stock prices react to economic and political news, several questions arise regarding responses that different firms

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with business-cycle fluctuations in profitability reveal to political uncertainty. For instance, how does the uncertainty of the profitability rate affect the equity risk premiums in heterogeneous firms?What effects do economic conditions have on the level of stock prices and the equity risk premiums in high and low exposure firms?Or if firms are more exposure does the correlation between each pair of their stocks is more in different economic conditions?Also, is there a relationship between level of stock prices in different firms and profitability uncertainty?Finally, can we infer that the difference of risk premiums between high and low exposure firms is lower when economic conditions are strong and higher when conditions are poor?

PV-H makes use of the assumption that profitability in every time step of length dt evolves stochastically at a constant rate and government policy is the only economic variable affecting it. In general, the profitability rate in each firm fluctuates over time through many of variations.

In this paper, we study the PV-H in which we assume profitability rate has a business-cycle variation, instead of being an observable constant, to answer the above questions. Specifically, we assume that profitability rate is generated by a mean-reverting process. Our key assumption is that the mean reverting profitability rate is non-observable. Agents learn about it through the observation of realized profitability and another noisy signal. The main objective of the paper is to characterize key pricing quantities for different firms and study heterogeneous firms' behaviors for different values of the uncertainty of the additional signal.

We find a nonlinear increasing pattern in difference of the level of stock prices between high and low exposure firms. We also show that high-exposure firms with low capital share earn larger difference in the level of stock prices, measured using market-to-book ratio, than do firms that own a high share of the total wealth and feature high exposure to government uncertain policy. Holding everything else equal, an increase in uncertainty of profitability signals lowers the difference of market-to-book ratios between heterogeneous firms in strong economic conditions when high exposure firms possess a large share of aggregate capital in the economy.

The documented state-dependence pattern in difference of risk premiums between high and low exposure firms confirms the implications of the general equilibrium model in Pastor and Veronesi's earlier work [1], where they argue that the composition of the equity risk premium depends on economic conditions. In addition, this difference is larger when high-exposure firms own a high share of the total wealth. More importantly, in this case, it tends to increase on higher values of uncertainty about profitability signals in dire economic conditions and works in the opposite direction to the strong economic conditions. We also show that the difference of risk premiums between heterogeneous firms is countercyclical: higher in dire times and lower in strong times.

We also find that high-exposure firms with high or low share of the total wealth in the economy carry higher correlation value in their stocks than for low-exposure ones under each state of the economy. The relation between the precision of profitability signals and the correlation between the stock returns in high or low exposure firms is generally ambiguous.

Pastor and Veronesi [2] investigate heterogeneous firms' responses to political uncertainty without business-cycle variation in profitability. We achieve some realistic empirical predictions by incorporating this variation into PV-H. Only a small set of remarkable theoretical works explain the link between political or economic uncertainty and financial markets (Pastor and Veronesi [1,2,3], Ulrich [4], Croce et al. [5]). Recent papers including empirical studies of this link are Chang et al. [6], Gulen and Ion [7], Julio and Yook [8], Gao and Qi [9], Brogaard and Detzel [10], Durnev [11], Liu et al. [12].

2 The model

We begin this section by describing the PV-H in which the profitability rate tends to move to the mean value over time. In Section 2.2. , we use a learning technique namely the Kalman-Bucy filter that helps to access information about the profitability rate. Then we design the government's decision rule in this extended model.

2.1 Economic environment

Similar to Pastor and Veronesi [2], we consider an economy with a finite horizon $[0, T]$ and a continuum of all-equity firms $i \in [0, 1]$. Let B_t^i denote firm i's capital at time t. Suppose firms are divided into N sectors where N is finite. each sector can fluctuate with a government policy more than or less than the all of the sectors and that's why we call "government beta" a measure of government's relative risk. Those fluctuations have a reference value which is equal to one. Each sector is characterized by a nonnegative beta. Profitability in each firm follows a stochastic process given by:

$$\frac{dB_t^i}{B_t^i} = (\mu_t + \beta^n g_t)dt + \sigma dZ_t^n + \sigma_1 dZ_t^i, \quad n = 1, 2, \ldots, N \tag{1}$$

where (σ, σ_1) are observable constants, dZ_t^n is a sector-specific shock, dZ_t^i is a firm-specific shock and all Brownian motions are independent of each other. The profitability rate μ_t is time-varying and evolves according to a mean-reverting process:

$$d\mu_t = \theta(\bar{\mu} - \mu_t)dt + \sigma_\mu dZ_{\mu,t} \tag{2}$$

where $\bar{\mu}$ denotes average profitability, θ denotes the speed of mean reversion and σ_μ is the uncertainty without having any information about μ_t which is called prior uncertainty. Also $dZ_{\mu,t}$ is a Brownian motion independent of all others in the economy. The current goverment policy impact g_t is a simple step function of time:

$$g_t = \begin{cases} g^{old}, & t \le \tau \\ g^{old}, & t > \tau \quad \text{if there is no policy change} \\ g^{new}, & t > \tau \quad \text{if there is a policy change} \end{cases} \tag{3}$$

where $0 < \tau < T$ is an exogenously given time-invariant constant.
Firms are owned by a continuum of identical investors who maximize expected utility derived from terminal wealth. For all $j \in [0,1]$, investor j's utility function is given by:

$$u(W_T^j) = \frac{(W_T^j)^{1-\gamma}}{1-\gamma}, \tag{4}$$

where W_T^j is investor j's wealth at time T and $\gamma > 1$ is the coefficient of relative risk aversion. Stocks pay liquidating dividends at time T.
The government solves

$$\max\{E_\tau[\frac{W_T^{1-\gamma}}{1-\gamma}|\text{no policy change}], E_\tau[C\frac{W_T^{1-\gamma}}{1-\gamma}|\text{policy change }]\}, \tag{5}$$

where $W_T = B_T = \int_0^1 B_T^i di$ is the final value of aggregate capital and C is the political cost incurred by the government if a new policy is introduced. The value of C is randomly drawn at time τ from a lognormal distribution centered at $C = 1$:

$$c \equiv \log(C) \sim N(-\frac{1}{2}\sigma_c^2, \sigma_c^2)$$

where C is independent of the Brownian motions in equations (1), (2). We refer to σ_c as political uncertainty. Political uncertainty introduces an element of surprise into policy changes, resulting in stock price reactions at time τ. The government maximizes the investors' welfare on average (because $E(C) = 1$), but it also deviates from this objective in a random fashion.

2.2 Learning

The investors do not observe the true value of the profitability rate μ_t but they learn about it by observing the realized profitability and an external unbiased signal of the form:

$$de_t = \mu_t dt + \sigma_e dZ_{e,t} \tag{6}$$

where σ_e is a known constant and $dZ_{e,t}$ is the standard Brownian motion uncorrelated with all others. Agents solve a Bayesian learning problem which leads to asset pricing implications. The following result holds.

Lemma 2.1 *Suppose that the time t information set is given by $\mathcal{F}_t$. Before observing any signals, the prior distribution for μ_t and g at time 0 is jointly normal:*

$$\begin{pmatrix} \mu_0 \\ g \end{pmatrix} \sim N\left(\begin{matrix} \bar{\mu} \\ 0 \end{matrix}, \begin{pmatrix} \sigma_\mu^2 & 0 \\ 0 & \sigma_g^2 \end{pmatrix}\right)$$

The agents' inference at time $t < \tau$ of μ and g has a jointly Gaussian distribution given by:

$$\begin{pmatrix} \mu_t \\ g \end{pmatrix} \sim N\left(\begin{matrix} \hat{\mu}_t \\ \hat{g}_t \end{matrix}, \begin{pmatrix} \hat{\sigma}_{\mu,t}^2 & \hat{\sigma}_{\mu g,t} \\ \hat{\sigma}_{\mu g,t} & \hat{\sigma}_{g,t}^2 \end{pmatrix}\right)$$

where the posterior means follow:

$$d\hat{\mu}_t = \theta(\bar{\mu} - \hat{\mu}_t)dt + \sigma^{-1}(\hat{\sigma}^2_{\mu,t}1_N + \beta\hat{\sigma}_{\mu g,t})'d\hat{Z}_{A,t} + \sigma_e^{-1}\hat{\sigma}^2_{\mu,t}d\hat{Z}_{B,t}$$

$$d\hat{g}_t = \sigma^{-1}(\hat{\sigma}_{\mu g,t}1_N + \beta\hat{\sigma}^2_{g,t})'d\hat{Z}_{A,t} + \sigma_e^{-1}\hat{\sigma}_{\mu g,t}d\hat{Z}_{B,t}$$

and the posterior variances and covariances follow:

$$\frac{d\hat{\sigma}^2_{\mu,t}}{dt} = -2\theta\hat{\sigma}^2_{\mu,t} + \sigma^2_\mu - \sigma^{-2}(\hat{\sigma}^2_{\mu,t}1_N + \beta\hat{\sigma}_{\mu g,t})'(\hat{\sigma}^2_{\mu,t}1_N + \beta\hat{\sigma}_{\mu g,t}) - \sigma_e^{-2}(\hat{\sigma}^2_{\mu,t})^2$$

$$\frac{d\hat{\sigma}_{\mu g,t}}{dt} = -\theta\hat{\sigma}_{\mu g,t} - \sigma^{-2}(\hat{\sigma}^2_{\mu,t}1_N + \beta\hat{\sigma}_{\mu g,t})'(\hat{\sigma}_{\mu g,t}1_N + \beta\hat{\sigma}^2_{g,t}) - \sigma_e^{-2}\hat{\sigma}^2_{\mu,t}\hat{\sigma}_{\mu g,t}$$

$$\frac{d\hat{\sigma}^2_{g,t}}{dt} = -\sigma^{-2}(\hat{\sigma}_{\mu g,t}1_N + \beta\hat{\sigma}^2_{g,t})'(\hat{\sigma}_{\mu g,t}1_N + \beta\hat{\sigma}^2_{g,t}) - \sigma_e^{-2}\hat{\sigma}^2_{\mu g,t}$$

Here, $d\hat{Z}_{A,t}$ and $d\hat{Z}_{B,t}$ are new Brownian motions which reflect expectation errors:

$$d\hat{Z}_{A,t} = \sigma^{-1}(\frac{dB_t}{B_t} - E(\frac{dB_t}{B_t}|\mathcal{F}_t))$$

$$d\hat{Z}_{B,t} = \sigma_e^{-1}(de_t - E(de_t|\mathcal{F}_t)).$$

Also β is the $N \times 1$ vector of government betas and 1_N is the column vector of order N whose components all equal to one.

Proof. See Theorem 10.2 of [13].

We state some intuitive properties from the above filter. First, if the profitability rate mean-reverts faster, so that θ is large. Second, the inferred profitability rate $\hat{\mu}_t$ mean-reverts to the unconditional mean $\bar{\mu}$ with the same speed that the true profitability rate μ_t does. Unlike the true profitability rate, however the process for $\hat{\mu}_t$ has time-varying volatility (through $\hat{\sigma}^2_{\mu,t}, \hat{\sigma}_{\mu g,t}$). Under the investors' information set $\mathcal{F}_t$, profitability in each sector evolves as:

$$\frac{dB^n_t}{B^n_t} = (\hat{\mu}_t + \beta^n\hat{g}_t)dt + \sigma d\hat{Z}^n_{A,t} \qquad n = 1, 2, \ldots, N$$

2.3 The government's policy decision

Turn now to the government's decision rule in this extended model. Exploiting (5), we obtain the following proposition:

Proposition 2.2 *Let*

$$x = \int_\tau^T \mu_t dt + \beta g(T-\tau) + \sigma(Z_T - Z_\tau),$$

and $\phi(x|a, b, c, d, f, t)$ denotes the multivariate normal density with parameters (μ_x, σ^2_x) in the following:

$$\mu_x = \bar{\mu}(T-t) + (a-\bar{\mu})Q(t,T) + c(T-t)\beta$$

$$\sigma_x^2 = \{(b - \frac{\sigma_\mu^2}{2\theta})Q^2(t,T) + \frac{\sigma_\mu^2}{\theta^2}(T-t-Q(t,T))\}1_{N\times N} + \beta\beta' d(T-t)^2 + \sigma^2 I_{N\times N}(T-t)$$
$$+(\beta 1_N' + 1_N\beta')(T-t)Q(t,T)f,$$

where

$$Q(t,T) = \frac{1-e^{-\theta(T-t)}}{\theta},$$

and we also denote an identity matrix and a matrix where every element is equal to one by $I_{N\times N}$ and $1_{N\times N}$, respectively.
A policy change occurs at time τ if and only if:

$$c < \underline{c}(w_\tau, \hat{\mu}_\tau, \hat{\sigma}^2_{\mu,\tau}, \hat{g}_\tau, \hat{\sigma}^2_{g,\tau}, \hat{\sigma}_{\mu g,\tau}, \tau)$$

where

$$\underline{c}(w_\tau, \hat{\mu}_\tau, \hat{\sigma}^2_{\mu,\tau}, \hat{g}_\tau, \hat{\sigma}^2_{g,\tau}, \hat{\sigma}_{\mu g,\tau}, \tau) = \log(\frac{\Phi(w_\tau, \hat{\mu}_\tau, \hat{\sigma}^2_{\mu,\tau}, \hat{g}_\tau, \hat{\sigma}^2_{g,\tau}, \hat{\sigma}_{\mu g,\tau}, \tau)}{\Phi(w_\tau, \hat{\mu}_\tau, \hat{\sigma}^2_{\mu,\tau}, 0, \sigma_g^2, 0, \tau)}).$$

Here

$$\Phi(w_\tau, \hat{\mu}_\tau, \hat{\sigma}^2_{\mu,\tau}, \hat{g}_\tau, \hat{\sigma}^2_{g,\tau}, \hat{\sigma}_{\mu g,\tau}, \tau) = \int_{R^N} (\sum_{n=1}^N w_\tau^n e^{x^n})^{1-\gamma} \phi(x|\hat{\mu}_\tau, \hat{\sigma}^2_{\mu,\tau}, \hat{g}_\tau, \hat{\sigma}^2_{g,\tau}, \hat{\sigma}_{\mu g,\tau}, \tau)dx,$$

$$\Phi(w_\tau, \hat{\mu}_\tau, \hat{\sigma}^2_{\mu,\tau}, 0, \sigma_g^2, 0, \tau) = \int_{R^N} (\sum_{n=1}^N w_\tau^n e^{x^n})^{1-\gamma} \phi(x|\hat{\mu}_\tau, \hat{\sigma}^2_{\mu,\tau}, 0, \sigma_g^2, 0, \tau)dx,$$

with

$$w_\tau^n = \frac{B_\tau^n}{B_\tau}, \quad n = 1, 2, \ldots, N.$$

Proof. see Appendix.

Here w_τ^n is the relative size of sector n at time τ, where "size" is measured as the fraction of total wealth produced by each sector ($n = 1, 2, \ldots, N$) at time τ. Also we denote by p_τ the probability of a policy change at time τ.

3 Stock prices

In this section, we characterize the stock price level, the equity risk premium, the volatility of stock return and the correlation between each pair of stocks within heterogeneous firms in the economy.

Assuming complete markets, standard arguments imply that the state price density is uniquely given by

$$\pi_t = \frac{1}{\lambda} E(B_T^{-\gamma}|\mathcal{F}_t)$$

where λ is the Lagrange multiplier from the utility maximization problem of the representative investor. Firm i's stock is a claim on the firm's liquidating dividend at time T which is equal to B_T^i. Thus the market value of stock i is given by the standard pricing formula:

$$M_t^i = E[\frac{\pi_T}{\pi_t} B_T^i | \mathcal{F}_t]$$

Now we prepare the stock pricing results before time τ, which are the focus of this paper.

Proposition 3.1 *The stochastic discount factor at time $t < \tau$ is given by:*

$$\pi_t = B_t^{-\gamma} e^{\frac{\gamma\sigma^2}{2}(T-\tau)} \Omega(w_t, \hat{\mu}_t, \hat{\sigma}_{\mu,t}^2, \hat{g}_t, \hat{\sigma}_{g,t}^2, \hat{\sigma}_{\mu g,t}, t)$$

where

$$\Omega(w_t, \hat{\mu}_t, \hat{\sigma}_{\mu,t}^2, \hat{g}_t, \hat{\sigma}_{g,t}^2, \hat{\sigma}_{\mu g,t}, t) = E\{(\frac{B_\tau}{B_t})^{-\gamma}(p_\tau \Omega(w_\tau, \hat{\mu}_\tau, \hat{\sigma}_{\mu,\tau}^2, 0, \sigma_g^2, 0, \tau)$$

$$+(1-p_\tau)\Omega(w_\tau, \hat{\mu}_\tau, \hat{\sigma}_{\mu,\tau}^2, \hat{g}_\tau, \hat{\sigma}_{g,\tau}^2, \hat{\sigma}_{\mu g,\tau}, \tau))|\mathcal{F}_t\}$$

Here

$$\Omega(w_\tau, \hat{\mu}_\tau, \hat{\sigma}_{\mu,\tau}^2, \hat{g}_\tau, \hat{\sigma}_{g,\tau}^2, \hat{\sigma}_{\mu g,\tau}, \tau) = \int_{R^N} (\sum_{n=1}^N w_\tau^n e^{x^n})^{-\gamma} \phi(x | \hat{\mu}_\tau, \hat{\sigma}_{\mu,\tau}^2, \hat{g}_\tau, \hat{\sigma}_{g,\tau}^2, \hat{\sigma}_{\mu g,\tau}, \tau) dx$$

$$\Omega(w_\tau, \hat{\mu}_\tau, \hat{\sigma}_{\mu,\tau}^2, 0, \sigma_g^2, 0, \tau) = \int_{R^N} (\sum_{n=1}^N w_\tau^n e^{x^n})^{-\gamma} \phi(x | \hat{\mu}_\tau, \hat{\sigma}_{\mu,\tau}^2, 0, \sigma_g^2, 0, \tau) dx$$

The dynamics of the stochastic discount factor at time $t < \tau$ is given by:

$$\frac{d\pi_t}{\pi_t} = -\sigma_{\pi,A,t}' d\hat{Z}_{A,t} - \sigma_{\pi,B,t} d\hat{Z}_{B,t}$$

where

$$\sigma_{\pi,A,t} = \gamma\sigma w_t - \frac{1}{\Omega}\frac{\partial\Omega}{\partial\hat{\mu}_t}\sigma^{-1}(\hat{\sigma}_{\mu,t}^2 1_N + \beta\hat{\sigma}_{\mu g,t}) - \frac{1}{\Omega}\frac{\partial\Omega}{\partial\hat{g}_t}\sigma^{-1}(\hat{\sigma}_{\mu g,t} 1_N + \beta\hat{\sigma}_{g,t}^2) - \frac{\sigma}{\Omega}(w_t \odot (\frac{\partial\Omega}{\partial w_t} - w_t' \frac{\partial\Omega}{\partial w_t})),$$

$$\sigma_{\pi,B,t} = -\frac{1}{\Omega}\frac{\partial\Omega}{\partial\hat{\mu}_t}(\sigma_e^{-1}\hat{\sigma}_{\mu,t}^2) - \frac{1}{\Omega}\frac{\partial\Omega}{\partial\hat{g}_t}(\sigma_e^{-1}\hat{\sigma}_{\mu g,t}).$$

Here "$\odot$" denotes "element-by-element" multiplication.

Proposition 3.2 *The "market-to-book" ratio for each firm in sector m ($m = 1, 2, \ldots, N$) at time $t < \tau$ is given by*

$$\frac{M_t^{i,m}}{B_t^{i,m}} = e^{-\frac{\sigma^2}{2}(T-\tau)} \frac{\Phi^m(w_t, \hat{\mu}_t, \hat{\sigma}_{\mu,t}^2, \hat{g}_t, \hat{\sigma}_{g,t}^2, \hat{\sigma}_{\mu g,t}, t)}{\Omega(w_t, \hat{\mu}_t, \hat{\sigma}_{\mu,t}^2, \hat{g}_t, \hat{\sigma}_{g,t}^2, \hat{\sigma}_{\mu g,t}, t)}$$

where

$$\Phi^m(w_t, \hat{\mu}_t, \hat{\sigma}_{\mu,t}^2, \hat{g}_t, \hat{\sigma}_{g,t}^2, \hat{\sigma}_{\mu g,t}, t) = E\{(\frac{B_\tau}{B_t})^{-\gamma}\frac{B_\tau^{i,m}}{B_t^{i,m}}(p_\tau \Phi^m(w_\tau, \hat{\mu}_\tau, \hat{\sigma}_{\mu,\tau}^2, 0, \sigma_g^2, 0, \tau)$$

$$+(1-p_\tau)\Phi^m(w_\tau,\hat{\mu}_\tau,\hat{\sigma}^2_{\mu,\tau},\hat{g}_\tau,\hat{\sigma}^2_{g,\tau},\hat{\sigma}_{\mu g,\tau},\tau))|\mathcal{F}_t\}$$

Here

$$\Phi^m(w_\tau,\hat{\mu}_\tau,\hat{\sigma}^2_{\mu,\tau},\hat{g}_\tau,\hat{\sigma}^2_{g,\tau},\hat{\sigma}_{\mu g,\tau},\tau)=\int_{R^N}(\sum_{n=1}^{N}w^n_\tau e^{x^n})^{-\gamma}e^{x^m}\phi(x|\hat{\mu}_\tau,\hat{\sigma}^2_{\mu,\tau},\hat{g}_\tau,\hat{\sigma}^2_{g,\tau},\hat{\sigma}_{\mu g,\tau},\tau)dx$$

$$\Phi^m(w_\tau,\hat{\mu}_\tau,\hat{\sigma}^2_{\mu,\tau},0,\sigma^2_g,0,\tau)=\int_{R^N}(\sum_{n=1}^{N}w^n_\tau e^{x^n})^{-\gamma}e^{x^m}\phi(x|\hat{\mu}_\tau,\hat{\sigma}^2_{\mu,\tau},0,\sigma^2_g,0,\tau)dx$$

Proposition 3.3 *Stock return process for each firm i in sector m ($m=1,2,\dots N$) at time $t<\tau$ is given by*

$$\frac{dM^{i,m}_t}{M^{i,m}_t}=\mu_{m,t}dt+\sigma'_{m,A,t}d\hat{Z}_{A,t}+\sigma_{m,B,t}d\hat{Z}_{B,t}+\sigma_1 dZ^i_t,$$

where

$$\sigma_{m,A,t}=\sigma j_m+(\frac{1}{\Phi^m}\frac{\partial\Phi^m}{\partial\hat{\mu}_t}-\frac{1}{\Omega}\frac{\partial\Omega}{\partial\hat{\mu}_t})(\sigma^{-1}(\hat{\sigma}^2_{\mu t}1_N+\beta\hat{\sigma}_{\mu g,t}))$$

$$+(\frac{1}{\Phi^m}\frac{\partial\Phi^m}{\partial\hat{g}_t}-\frac{1}{\Omega}\frac{\partial\Omega}{\partial\hat{g}_t})(\sigma^{-1}(\hat{\sigma}_{\mu g,t}1_N+\beta\hat{\sigma}^2_{g,t}))+\frac{\sigma}{\Phi^m}(w_t\odot(\frac{\partial\Phi^m}{\partial w_t}-w'_t\frac{\partial\Phi^m}{\partial w_t}))$$

$$-\frac{\sigma}{\Omega}(w_t\odot(\frac{\partial\Omega}{\partial w_t}-w'_t\frac{\partial\Omega}{\partial w_t})),$$

$$\sigma_{m,B,t}=(\frac{1}{\Phi^m}\frac{\partial\Phi^m}{\partial\hat{\mu}_t}-\frac{1}{\Omega}\frac{\partial\Omega}{\partial\hat{\mu}_t})(\sigma_e^{-1}\hat{\sigma}^2_{\mu t})+(\frac{1}{\Phi^m}\frac{\partial\Phi^m}{\partial\hat{g}_t}-\frac{1}{\Omega}\frac{\partial\Omega}{\partial\hat{g}_t})(\sigma_e^{-1}\hat{\sigma}_{\mu g,t})$$

Here j_m denotes the m-th column of an m-by-m identity matrix and

$$\mu_{m,t}=\sigma_{\pi,A,t}\sigma_{m,A,t}+\sigma_{\pi,B,t}\sigma_{m,B,t}.$$

Proposition 3.4 *The correlation between the stock returns of firms i and j in each sector ($m=1,2,\dots,N$) at time $t<\tau$ is given by:*

$$\rho^{ij}_t=\frac{\sigma'_{m,A,t}\sigma_{m,A,t}+\sigma^2_{m,B,t}}{\sigma'_{m,A,t}\sigma_{m,A,t}+\sigma^2_{m,B,t}+\sigma^2_1}$$

4 A two-sector example

In this section we assume that there are two sectors in the economy to illustrate the asset pricing results of political uncertainty in different firms. The high and low beta firms are characterized by H and L respectively such that their betas average equals one. Firms with greater and lesser government exposures are located in sectors H and L with government betas $\beta^H>1$ and $\beta^L<1$ respectively.

Table 1 shows the parameter choices that we apply to calibrate the model. We set $\beta^H=1.8$ and $\beta^L=0.2$. Also we denote the relative size of sectors H and L at time $t=5$ by w^H_t and w^L_t, respectively. We use the variable $\hat{\mu}_t+\hat{g}_t$ to highlight economic conditions. All quantities are computed at time $t = 5$ which is midway between time 0 and time τ.

We illustrate the difference of the market-to-book ratios between high and low exposure firms in Figure 1. The solid and the dashed line in Figure 1 show the difference $\frac{M_t^H}{B_t^H} - \frac{M_t^L}{B_t^L}$ when high exposure firms own a high and low level of aggregate capital in the economy, respectively. As the figure shows, the quantity $\frac{M_t^H}{B_t^H} - \frac{M_t^L}{B_t^L}$ increases monotonically with economic conditions. Higher values of $\hat{\mu}_t + \hat{g}_t$ increase stock prices due to an increase in expected profitability. The slope of this increase is always higher in good economic conditions than in the conditions when the economy is weak. When $\hat{\mu}_t + \hat{g}_t$ is very low, the old policy is likely to be replaced at time τ (i. e. , $p_\tau \approx 1$). Therefore, $d\hat{Z}_{A,t}$ shocks are temporary and have a small effect on the level of stock prices. In contrast, when $\hat{\mu}_t + \hat{g}_t$ is high, the old policy is likely to be retained (i. e. , $p_\tau \approx 0$). Therefore $d\hat{Z}_{A,t}$ shocks are permanent and have a large effect on the level of stock prices. As a result the relation between $\frac{M_t^H}{B_t^H} - \frac{M_t^L}{B_t^L}$ and $\hat{\mu}_t + \hat{g}_t$ is steeper in good economic conditions. Note that under our assumption on relative wealth which firms own at time $t = 5$, the quantity $\frac{M_t^H}{B_t^H} - \frac{M_t^L}{B_t^L}$ is depressed in state where high exposure firms possess a high capital share in the economy: $w_t^H = 0.8$. When high exposure firms own a high share of the total wealth, their capital covaries more closely with aggregate capital B_t and this makes the larger sector firms riskier. Hence $\frac{M_t^H}{B_t^H} - \frac{M_t^L}{B_t^L}$ is expected to shrink when the capital share in H sector is high. An increase in profitability uncertainty lowers the expected profitability in strong economic conditions and will eventually decrease $\frac{M_t^H}{B_t^H} - \frac{M_t^L}{B_t^L}$ when $w_t^H = 0.8$.

Figures 2 and 3 present the non-linear dependence of the difference $\mu_t^H - \mu_t^L$ on economic conditions when $w_t^H = 0.8$ and $w_t^H = 0.2$, respectively. As figures show, the difference of risk premiums between heterogeneous firms tends to be lower in strong economic conditions and higher in very bad times. In poor economic conditions, uncertainties in the economy increase and investors are afraid of taking risks. Therefore to attract them to risky assets, expected risk premiums should increase during poor economic times. The solid line in Figure 2 (for which $w_t^H = 0.8$) shows that in very bad economic conditions ($\hat{\mu}_t + \hat{g}_t = 0$), the difference $\mu_t^H - \mu_t^L$ is higher in the higher values of uncertainty about profitability rate. When $\sigma_e \to \infty$, the signal (6) is imprecise. Thus $\hat{g}_t$ and $\hat{\mu}_t$ are perfectly correlated. As a result, a larger σ_e increases volatilities in stocks when conditions are poor.

Figure 4 plots the value of $\mu_t^H - \mu_t^L$ in the high capital share state (solid line) and that in the low capital share state (dashed line). This value is higher when capital share is high ($w_t^H = 0.8$). Higher capital share in H sector enhances the difference $\mu_t^H - \mu_t^L$ through an increase in risk of this sector. In strong economic conditions (for which $\hat{\mu}_t + \hat{g}_t = 0.2$), for higher values of profitability uncertainty (σ_e), the value of $\mu_t^H - \mu_t^L$ is lower when $w_t^H = 0.8$. A larger σ_e decreases the expected stock returns in strong economic conditions.

Figures 5 and 6 show that firms with high correlation between their stock returns are typically firms with high exposure to government uncertain policy.

Intuitively, high correlation states in high exposure firms are associated with high risk premium ($\mu_t^H > \mu_t^L$).

5 Conclusion

What the findings do imply is, indeed, that the role of economic conditions as an important factor affecting on key pricing quantities in different firms can not be elaborately assessed without simultaneously considering heterogeneous firms and business cycle variation in profitability.

From an economic point of view, Our study gives a more realistic extension of PV-H in which two economic state variables capture a better proxy for economic conditions. With this theoretical extension, we can improve our economic intuition of heterogeneous firms' exposures to political uncertainty and make some new empirical predictions.

This supplementary investigation with incorporating business-cycle variation into PV-H also contributes to a deeper understanding of the behavior of different firms in response to political uncertainty.

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Appendix.

This appendix contains the detailed explanations af items that we refer to in the text.

Proof of Proposition 2.2
Using the mean-reverting process in (2), we have

$$\mu_T = \bar{\mu} + e^{-\theta(T-\tau)}(\mu_\tau - \bar{\mu}) + \sigma_\mu \int_\tau^T e^{-\theta(T-s)} dZ_{\mu,s}, \tag{7}$$

$$\int_\tau^T \mu_t dt = \frac{-1}{\theta}(\mu_T - \mu_\tau - \theta\bar{\mu}(T-\tau) - \sigma_\mu(Z_{\mu,T} - Z_{\mu,\tau})). \tag{8}$$

therefore

$$E(\int_\tau^T \mu_s ds|\mathcal{F}_\tau) = \bar{\mu}(T-\tau) + (\hat{\mu}_\tau - \bar{\mu})Q(\tau,T),$$

where

$$Q(\tau,T) = \frac{1-e^{-\theta(T-\tau)}}{\theta}.$$

Using (8) we get

$$var(\int_\tau^T \mu_t dt|\mathcal{F}_\tau) = \frac{1}{\theta^2}(var(\mu_T - \mu_\tau|\mathcal{F}_\tau) + \sigma_\mu^2(T-\tau) - 2\sigma_\mu cov((\mu_T - \mu_\tau, Z_{\mu,T} - Z_{\mu,\tau})|\mathcal{F}_\tau)). \tag{9}$$

Also we can reformulate (7) in the following equivalent form

$$\mu_T - \mu_\tau = \bar{\mu}(1 - e^{-\theta(T-\tau)}) + \mu_\tau(e^{-\theta(T-\tau)} - 1) + \sigma_\mu \int_\tau^T e^{-\theta(T-s)} dZ_{\mu,s}.$$

This allows us to write

$$var(\mu_T - \mu_\tau|\mathcal{F}_\tau) = \hat{\sigma}_{\mu,\tau}^2(1 - e^{-\theta(T-\tau)})^2 + \sigma_\mu^2 \int_\tau^T e^{-2\theta(T-s)} ds =$$

$$\hat{\sigma}_{\mu,\tau}^2(1 - e^{-\theta(T-\tau)})^2 + \frac{\sigma_\mu^2}{2\theta}(1 - e^{-2\theta(T-\tau)}),$$

$$cov((\mu_T-\mu_\tau, Z_{\mu,T}-Z_{\mu,\tau})|\mathcal{F}_\tau)) = cov((\mu_T-\mu_\tau, \int_\tau^T dZ_{\mu,s})|\mathcal{F}_\tau)) =$$

$$\sigma_\mu \int_\tau^T e^{-\theta(T-s)}ds = \frac{\sigma_\mu}{\theta}(1-e^{-\theta(T-\tau)}).$$

Hence, we get the following result by substituting the above equalities in (9):

$$var(\int_\tau^T \mu_t dt|\mathcal{F}_\tau) = \frac{1}{\theta^2}\hat{\sigma}^2_{\mu,\tau}(1-e^{-\theta(T-\tau)})^2 + \frac{\sigma^2_\mu}{2\theta^3}(1-e^{-2\theta(T-\tau)}) +$$

$$\frac{\sigma^2_\mu}{\theta^2}(T-\tau) - \frac{2\sigma^2_\mu}{\theta^3}(1-e^{-\theta(T-\tau)})$$

We finally obtain

$$var(\int_\tau^T \mu_t dt|\mathcal{F}_\tau) = (\hat{\sigma}^2_{\mu,\tau} - \frac{\sigma^2_\mu}{2\theta})Q^2(\tau,T) + \frac{\sigma^2_\mu}{\theta^2}(T-\tau-Q(\tau,T))$$

For the remainder of the proof, we refer the reader to [2].

Joint distribution

The joint distribution of the stochastic variables $b^1_\tau, b^2_\tau, \hat{g}_\tau, \hat{\mu}_\tau$ conditional on the information available at time $t<\tau$ is given by:

$$\begin{pmatrix} b^1_\tau \\ b^2_\tau \\ \hat{g}_\tau \\ \hat{\mu}_\tau \end{pmatrix} \sim N\left(\begin{pmatrix} E(b^1_\tau) \\ E(b^2_\tau) \\ E(\hat{g}_\tau) \\ E(\hat{\mu}_\tau) \end{pmatrix}, \begin{pmatrix} V(b^1_\tau) & C(b^1_\tau,b^2_\tau) & C(b^1_\tau,\hat{g}_\tau) & C(b^1_\tau,\hat{\mu}_\tau) \\ C(b^1_\tau,b^2_\tau) & V(b^2_\tau) & C(b^2_\tau,\hat{g}_\tau) & C(b^2_\tau,\hat{\mu}_\tau) \\ C(b^1_\tau,\hat{g}_\tau) & C(b^2_\tau,\hat{g}_\tau) & V(\hat{g}_\tau) & C(\hat{\mu}_\tau,\hat{g}_\tau) \\ C(b^1_\tau,\hat{\mu}_\tau) & C(b^2_\tau,\hat{\mu}_\tau) & C(\hat{\mu}_\tau,\hat{g}_\tau) & V(\hat{\mu}_\tau) \end{pmatrix}\right)$$

where

$$E(b^n_\tau) = b^n_t + \bar{\mu}(\tau-t) + (\hat{\mu}_t-\bar{\mu})Q(t,\tau) + (\tau-t)(\beta^n\hat{g}_t - \frac{\sigma^2}{2}) \qquad n=1,2$$

$$E(\hat{g}_\tau) = \hat{g}_t$$

$$E(\hat{\mu}_\tau) = \bar{\mu} + (\hat{\mu}_t-\bar{\mu})e^{-\theta(\tau-t)}$$

$$V(b^n_\tau) = (\hat{\sigma}^2_{\mu,t} - \frac{\sigma^2_\mu}{2\theta})Q^2(t,\tau) + \frac{\sigma^2_\mu}{\theta^2}(\tau-t-Q(t,\tau)) + (\beta^n)^2\hat{\sigma}^2_{g,t}(\tau-t)^2$$

$$+\sigma^2(\tau-t) + 2\beta^n(\tau-t)Q(t,\tau)\hat{\sigma}_{\mu g,t}, \qquad n=1,2$$

$$V(\hat{g}_\tau) = \int_t^\tau \sigma^{-2}(\hat{\sigma}_{\mu g,s}1_N + \beta\hat{\sigma}^2_{g,s})'(\hat{\sigma}_{\mu g,s}1_N + \beta\hat{\sigma}^2_{g,s})ds + \int_t^\tau (\sigma_e^{-2})(\hat{\sigma}^2_{\mu g,s})ds$$

$$V(\hat{\mu}_\tau) = \int_t^\tau \sigma^{-2}(\hat{\sigma}^2_{\mu,s}1_N + \beta\hat{\sigma}_{\mu g,s})'(\hat{\sigma}^2_{\mu,s}1_N + \beta\hat{\sigma}_{\mu g,s})ds + \int_t^\tau \sigma_e^{-2}(\hat{\sigma}^2_{\mu,s})^2 ds$$

$$C(\hat{\mu}_\tau,\hat{g}_\tau) = \int_t^\tau \sigma^{-2}(\hat{\sigma}^2_{\mu,s}1_N + \beta\hat{\sigma}_{\mu g,s})'(\hat{\sigma}_{\mu g,s}1_N + \beta\hat{\sigma}^2_{g,s})ds + \int_t^\tau \sigma_e^{-2}\hat{\sigma}^2_{\mu,s}\hat{\sigma}_{\mu g,s}ds$$

$$C(b^n_\tau,b^m_\tau) = (\hat{\sigma}^2_{\mu,t} - \frac{\sigma^2_\mu}{2\theta})Q^2(t,\tau) + \frac{\sigma^2_\mu}{\theta^2}(\tau-t-Q(t,\tau)) + (\beta^n+\beta^m)((\tau-t)Q(t,\tau)\hat{\sigma}_{\mu g,t})$$

Table 1: Parameter choices

σ_g	σ_c	$\bar{\mu}$	σ	σ_1	T	τ	γ	θ	σ_μ
0.02	0.10	0.10	0.05	0.10	20	10	5	0.35	0.02

$$+\beta^n \beta^m (\tau - t)^2 \hat{\sigma}^2_{g,t} \qquad n = 1, m = 2$$

$$C(b^n_\tau, \hat{g}_\tau) = \int_t^\tau (\hat{\sigma}_{\mu g,t} + \beta^n \hat{\sigma}^2_{g,t}) dt, \qquad n = 1, 2$$

$$C(b^n_\tau, \hat{\mu}_\tau) = \int_t^\tau (\hat{\sigma}^2_{\mu,t} + \beta^n \hat{\sigma}_{\mu g,t}) dt, \qquad n = 1, 2$$

Here

$$Q(t, \tau) = \frac{1 - e^{-\theta(\tau - t)}}{\theta}.$$

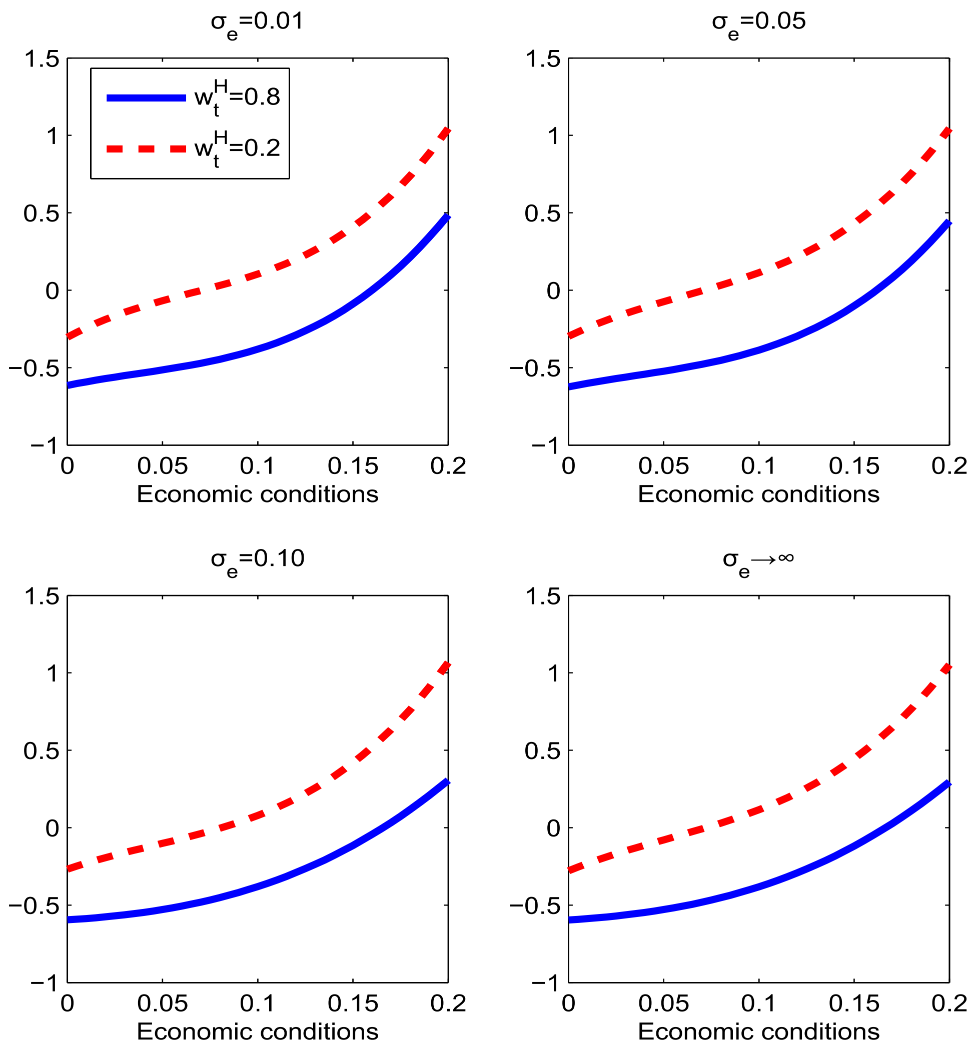

Figure 1: The difference of the level of stock prices between high and low exposure firms described by $(\frac{M_t^H}{B_t^H} - \frac{M_t^L}{B_t^L})$.

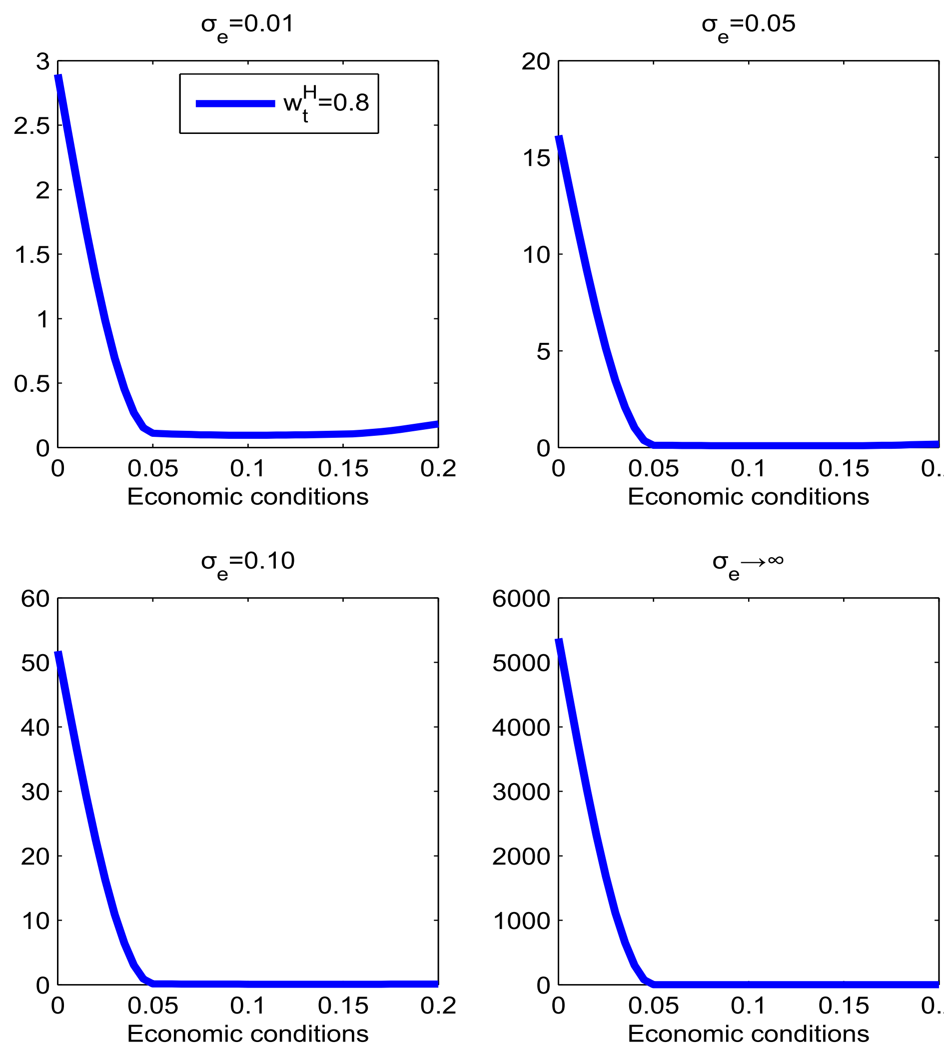

Figure 2: The difference of risk premiums between high and low exposure firms described by $(\mu_t^H - \mu_t^L)$.

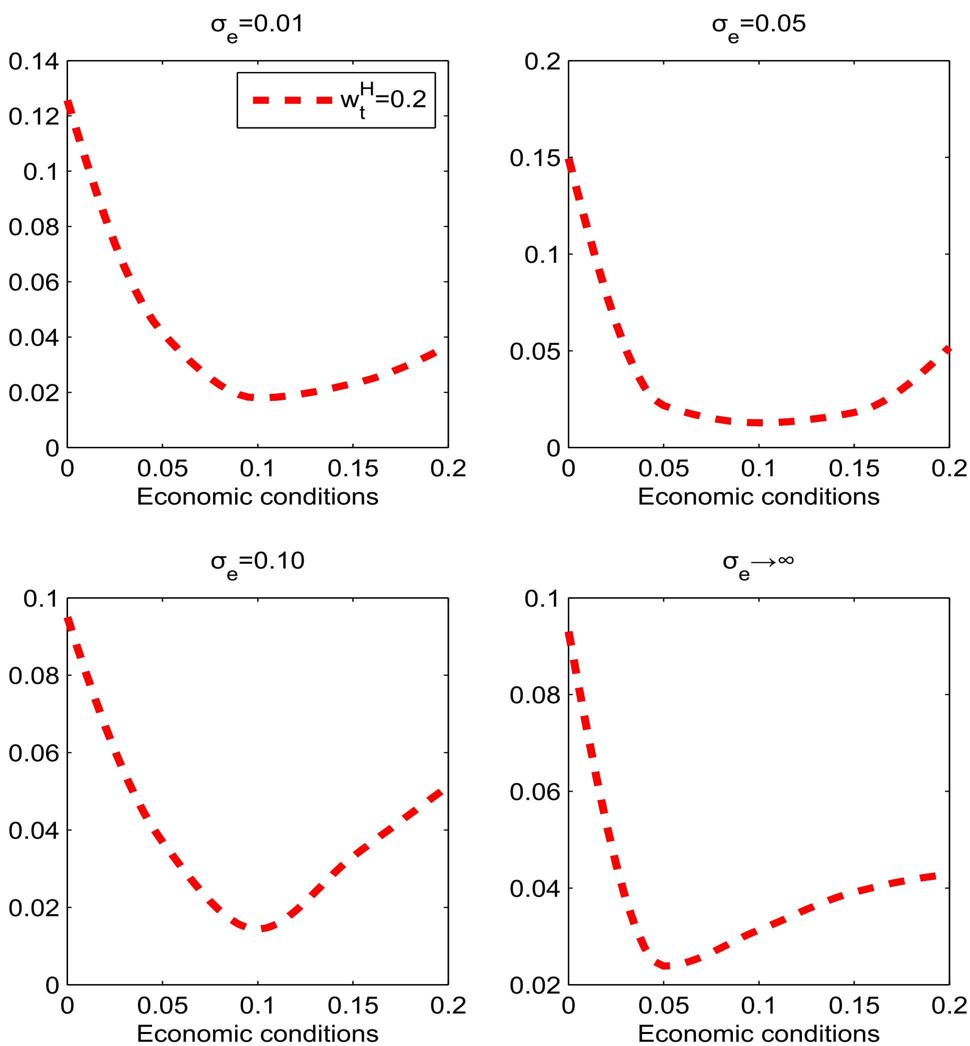

Figure 3: The difference of risk premiums between high and low exposure firms described by $(\mu_t^H - \mu_t^L)$.

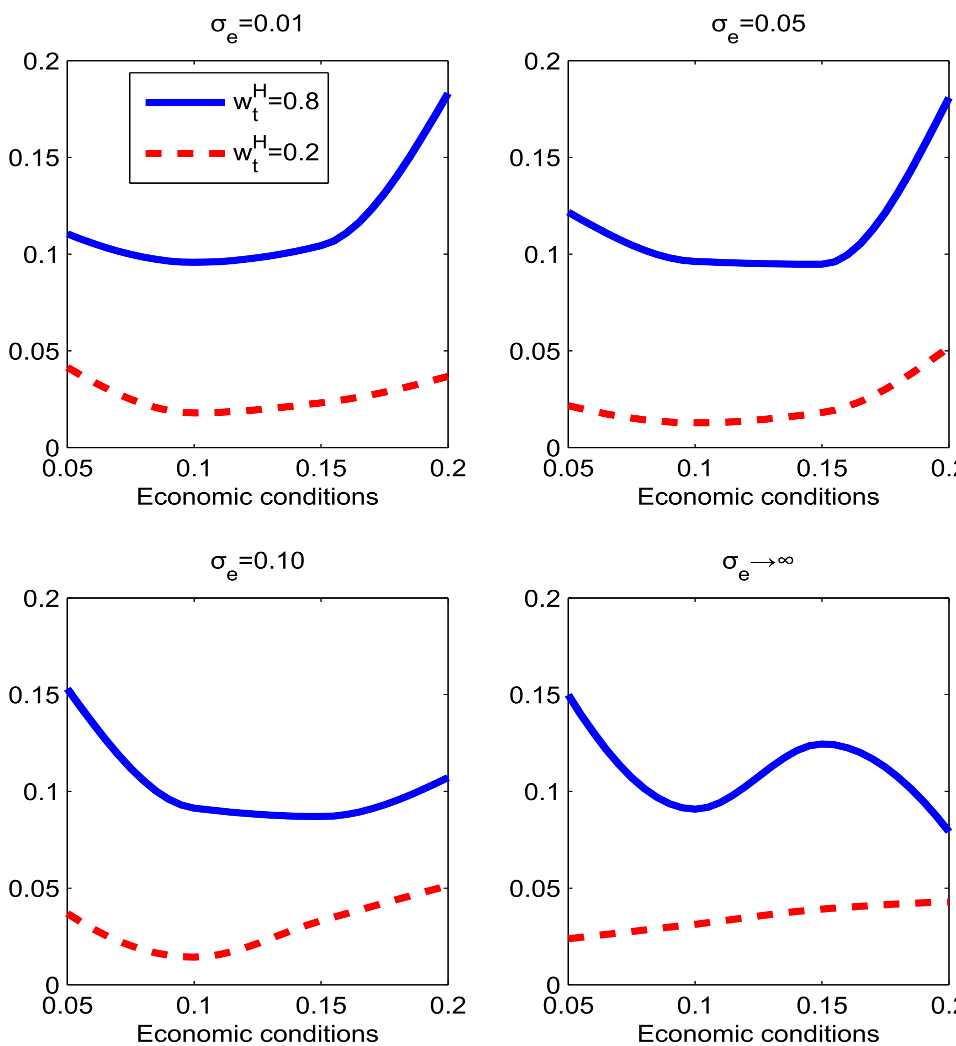

Figure 4: The comparison of the difference of risk premiums between high and low exposure firms when $w_t^H = 0.8$ and $w_t^H = 0.2$.

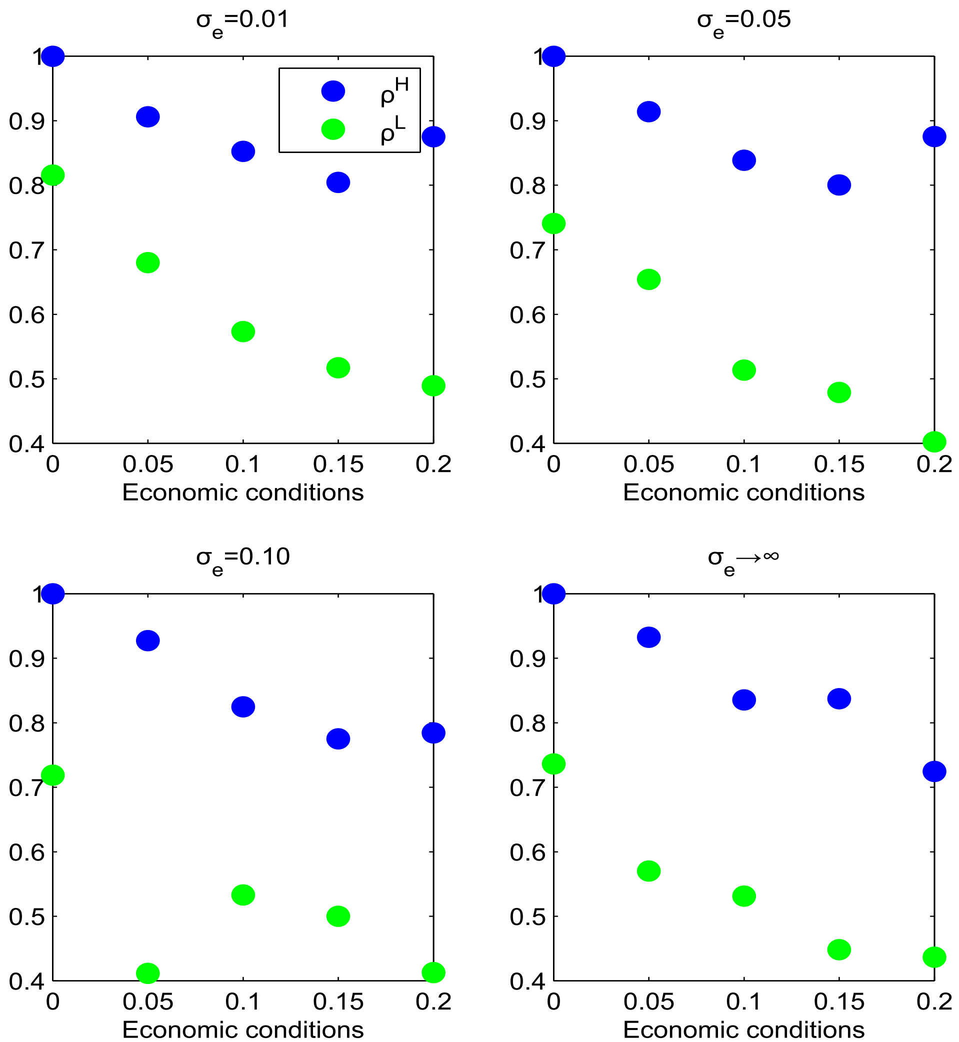

Figure 5: The correlation between stock returns in high and low exposure firms when $w_t^H = 0.8$.

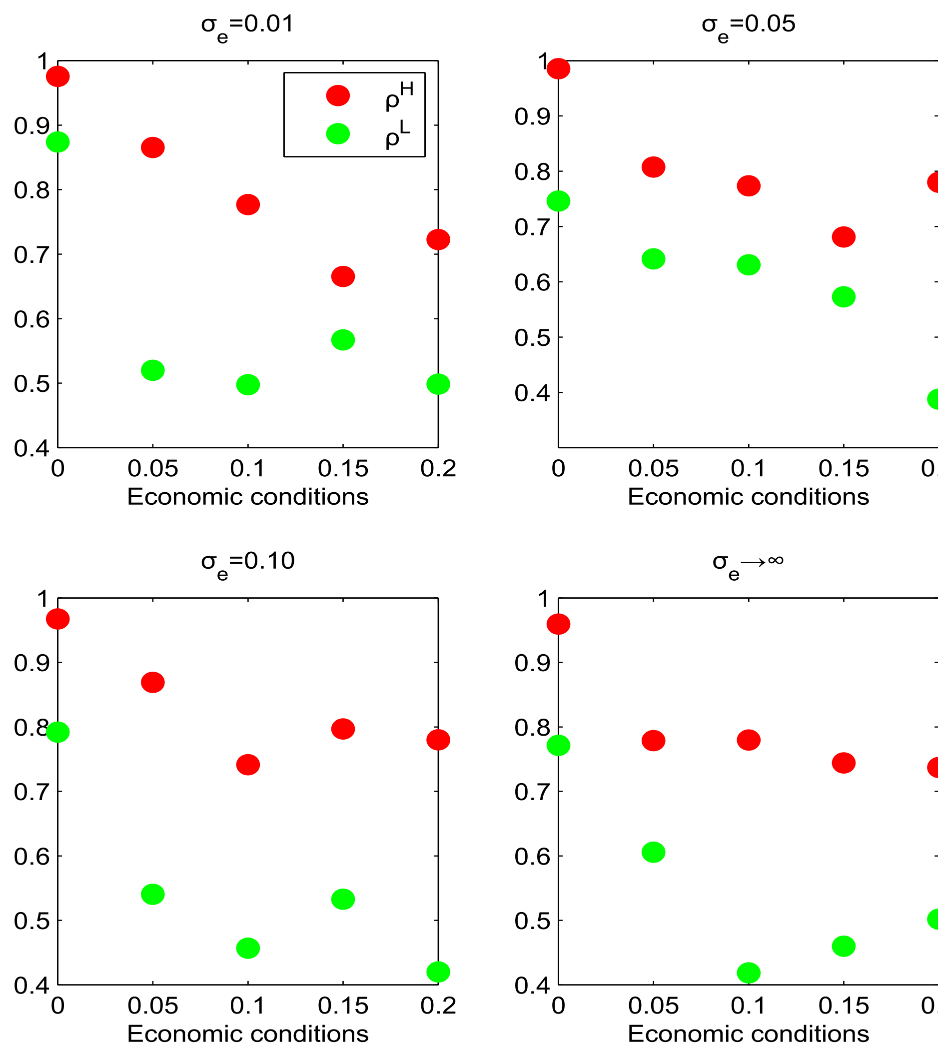

Figure 6: The correlation between stock returns in high and low exposure firms when $w_t^H = 0.2$.