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Option pricing under Heston Regime-Switching Diffusion model with jumps

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Abstract. The purpose of this paper is to evaluate a European option under the Heston Regime-Switching model with Double Exponential jump, such that the jump component is modeled by a compound Poisson process and the Markov-switching levy measure. Since the value of the interest rate parameter can be switched between some regimes of economics, we consider the interest rate parameters as a Markov-switching process. We employ the Fast Fourier Transform (FFT) method for valuing a European option under this financial model.

Keywords: option pricing, Heston model, regime-Switching, Double exponential jump, Fast Fourier Transform algorithm

1. Introduction

The Heston model that was introduced Steven Heston (1993) [1], is one of the most popular stochastic volatility model for option pricing, the experimental data have-shown the distribution of risky asset returns has tails longer a normal distribution. A stochastic volatility model has correlated price and volatility, and the risk-free interest rate is constant. But the values of this parameter can be switched between regimes of economics. In this paper, we consider the parameters of interest rate as a Markov-switching process. The Markov regime switching models, which is known as regime-switching model, first was introduced by Hamilton (1989)[3]. In this model, the parameters such as rate of return, volatility, and risk-free interest rate are assumed to depend on a finite-state Markov chain, which represents different states of regimes, describing various randomly changing economic factors. After combining the models with Markov chain, the regime switching process can be tacked the effects of important events on the asset price behavior [6].

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Now we have a Heston regime-switching model, that wants to add a compound Poisson process which is a discontinuous component, because of having good results for model calibration, so that, the jump diffusion model in a way to capture important empirical features, such as volatility smile or skewness and secondly, jump processes an incomplete explanation of the financial market, meaning the real market cannot be perfectly hedged in a short time. Kou [5] introduced a jump diffusion model for option pricing, the jump size follows a double exponential with certain mixture probability, and his result showed this model is useful for option pricing. In this paper we propose a Heston double exponential jump diffusion model with regime-switching. The Heston diffusion component is given by Markov-switching process and the jump component is modeled by a compound Poisson process with Markov-switching levy measure, the statistical properties of pure jump process with Markov switching compensator are taken from Elliott and Osakwe [8], Liu and Nguen [2], showed regime switching with jumps, in Heston stochastic volatility models, controls the dynamics of stock price, and can be fit the real market. Then, by adopting methodology of Lui, Zhang and Yin [6] and according to Wang, Haron, Jing and Huan [7], we use a fast Fourier transform approach to option pricing for regime-switching model of underlying asset process. First, we gained Fourier transform of option price with the joint characteristic function of sojourn time in explicit form for two-state Markov chains, those are the solutions for m-state case of m-dimensional differential. Then, the numerical results of option pricing by FFT method for two-state of regimes are reported. Note that, through in this paper we will deal with risk-neutral space (Ω, F, P), and all expectations are taken by respect to the risk-neutral measure P.

2. Heston stochastic volatility model

Heston(1993) proposed the following model, the underlying asset price is as follows

$$
\begin{aligned}
dS_t &= rS_t dt + \sqrt{V_t}\, dW_t \\
dV_t &= k'(\theta - \mathrm{V}_t)dt + \sigma \sqrt{V_t}\, dW_t'
\end{aligned}
$$

where $\{V_t\}_{t\geq 0}$ and $\{r_t\}_{t\geq 0}$ are the volatility and interest rate process, respectively, and $\{W_t\}_{t\geq 0}$, $\{W_t'\}_{t\geq 0}$ are correlated Brownian motion process (with correlation parameter ρ). The correlation coefficient $\rho \in (-1,1)$, i.e., $dW_t dW_t' = \rho dt$. $\{V_t\}_{t\geq 0}$ is square root mean reverting process, which is first used by (Cox, Ingersoll & Ross 1985), with long-run mean θ, and rate of reversion k', σ is referred to the volatility of volatility. All parameters $(r, k', \theta, \sigma, \rho)$ are time and state homogeneous. It is often convenient to write

$$
W_t = \rho W_t' + \sqrt{1-\rho^2}\hat{W}_t,
$$

where $\hat{W}_t$ is a standard Brownian motion independent of W_t' .

3. Heston Double exponential jump diffusion model

Suppose N_t is a Poisson process (with intensity $\lambda > 0$) which doesn't depend on the diffusion component. In Heston double exponential jump diffusion model, the stock price is modeled by an exponential Levy process as is follows

$$S_t = S_0 e^{Z_t}$$

so that stock price process S_t , $0 \le t \le T$ is written as an exponential of levy process Z_t , $0 \le t \le T$. Kou's levy model [5], is a continuous diffusion process which is added a compound Poisson process to making discontinuous jump process. The risk neutral dynamics is given by a similar way in the Black-Scholes model for the logarithm of the asset price

$$Z_t = (r - \frac{1}{2}V_t - \lambda k)t + \sqrt{V_t}W_t + \sum_{n=1}^{N_t} Y_n ,$$

where N_t is a Poisson process with intensity λ , $\{Y_n\}$ is a sequence of independent identically distributed nonnegative random variables, such that Y_n has an asymmetric double exponential distribution with the density

$$f_Y(y) = p\alpha e^{-\alpha y} 1_{\{y \ge 0\}}(y) + (1-p)\beta e^{\beta y} 1_{\{y<0\}}(y),$$

where $0 < p < 1$, $\alpha > 1, \beta > 0$. Here the condition $\alpha > 1$ is imposed to ensure the asset price S_t has finite expectation. Note that, $\{N_t\}$, $\{W_t\}$, and $\{Y_n\}$ are independent. According to Elliot and Osakwe [8], If γ be the random measure which selects the random jump times and random jump sizes $x = Z_s - Z_{s-}$.

We have

$$Z_t = (r - \frac{1}{2}\mathrm{V}_t)t + \sqrt{V_t}W_t + \int_0^t \int_{-\infty}^{+\infty} x\,\gamma(dx, ds) - \int_0^t \int_{-\infty}^{+\infty} (e^x - 1)\kappa(x)dxds$$

Under the probability P , the statistically properties of Z 's determined by its compensator [8]. In double exponential jump diffusion model, measure $\kappa(x)$ is as follows [7]

$$\kappa(x) = \lambda\left[p\alpha e^{-\alpha x} 1_{\{x \ge 0\}}(x) + (1-p)\beta e^{\beta x} 1_{\{x<0\}}(x)\right].$$

4. Heston double exponential jump diffusion model with regime-switching

Let $X = \{X_t, t \ge 0\}$ be a Markov chain, homogeneous continuous-time, independent of the jump process with a finite state space $\{e_1, e_2, ..., e_N\}$, which is a set of unit vectors with $e_i = (0,,...,0,1,0,...,0)^* \in \mathrm{R}^m$, where $*$ denotes transpose of a vector or matrix. The state of the chain X represents different state of an economy, and also let the generator matrix of X be $\Pi = \{\pi(ji)\}$, $1 \le i, j \le N$, the entry of the π(ji) matrix represents the rates of which the process X jumps from state i to state j , that are transition rates. Thus process has a rate matrix Π , and if $q_t(j) = P\{X_t = e_j\}$ is the probability of being in state j at time t, so that $q_t = (q_t(1), q_t(2), ..., q_t(\mathrm{m}))$. Then Π satisfy $\frac{dq_t}{dt} = \Pi q_t$. Then, we can write X as follows

$$X_t = X_0 + \int_0^t \Pi X_s ds + M_t,$$

where $M = \{M_t\}_{t \geq 0}$ is a martingale with respect to the filtration generated by X, and its R^m – valued. Now consider a financial model having a risk-free asset and some risky asset. The risk-free asset is either a bank account or risk-free bond. The instantaneous market interest rate $\{r(t, X_t)\}_{t \in [0,T]}$ is given by $r_t := r(t, X_t) = \langle r, X_t \rangle$ where $r := (r(1), ..., r(m))$, $r(i) > 0$ for each $j = 1, 2, ..., m$ and $\langle .,. \rangle$ denotes the inner product in R^m. The price process $\{B_t\}$ of Bank account (or risk-free bond) is now written by

$$dB_t = r_t B_t dt, \quad B_0 = 1.$$

Now from [7] suppose for each j we have compensator measure $\kappa_j(x)dxds$

where

$$\kappa_j(x) = \lambda(j)\left[p(j)\alpha(j)e^{-\alpha(j)x}1_{\{x \geq 0\}}(x) + (1 - p(j))\beta(j)e^{\beta(j)x}1_{\{x<0\}}(x)\right]$$

Pure jump component of process Z has the exhibition $\int_0^t \int_{-\infty}^{+\infty} x\,\gamma(dx, ds)$. Now, suppose the compensator of γ is $\kappa(dx, ds)$, where

$$\kappa(dx, ds) = \sum_{j=1}^{m} \langle X_{s-}, \mathrm{e}_j \rangle \kappa_j(x)dxds, \tag{1}$$

The logarithmic return of stock price is as follows

$$Z_t = \ln(\frac{S_t}{S_0})$$

Then, we have

$$\begin{aligned} Z_t &= \int_0^t (r_t - \frac{1}{2}V_t)ds + \int_0^t \sqrt{V_t}dW_t + \int_0^t \int_{-\infty}^{+\infty} x\,\gamma(dx, ds) \\ &- \sum_{j=1}^{m} \int_0^t \int_{-\infty}^{+\infty} \langle X_{s-}, \mathrm{e}_j \rangle (e^x - 1)\kappa_j(x)dxds \end{aligned} \tag{2}$$

Let

$$Z_t^1 = \int_0^t (r_t - \frac{1}{2}V_t)ds + \int_0^t \sqrt{V_t}dW_t$$

$$Z_t^2 = \int_0^t \int_{-\infty}^{+\infty} x\,\gamma(ds,ds) - \sum_{j=1}^{m} \int_0^t \int_{-\infty}^{+\infty} \langle X_{s-}, \mathrm{e}_j \rangle (e^x - 1)\kappa_j(x)dxds$$

Such that

$$Z_t = Z_t^{(1)} + Z_t^{(2)} \tag{3}$$

Then, we consider European call option on $S(t)$ with strike price $K > 0$, and maturity $T > 0$. Under the risk-neutral pricing principle, the option price $C(K)$ is as follows

$$C(K) = E\left[\exp(-\int_0^T r_t dt)(S_0 e^{Z_T} - K)^+\right] \tag{4}$$

Let $k = \ln(\frac{K}{S_0})$, then (4) can be written as

$$C(K) = S_0 E\left[\exp(-\int_0^T r_t dt)(e^{Z_T} - e^k)^+\right] \tag{5}$$

5. FFT approach for European option pricing

FFT approach provides a quick method for option pricing, which is used when the characteristic function of initial stock price process is available. Note that, when k goes to $-\infty$, $C(k)$ dose not decay to 0, for this reason, option pricing function is not square integrable, and we cannot directly take its Fourier transform. To obtain a square integrable function, Carr and Madan [9] introduced an extra exponential term to get the modified function, with this definition, modified option price in k, is square integrable over $(-\infty,\infty)$, and Fourier transform can be defined.

Now, from Liu, Zhang and Yin [6], we define a modified price function $c(k)$ as follows

$$c(k) = e^{\eta k}\frac{C(k)}{S_0}, \quad -\infty < k < \infty \tag{6}$$

where $\eta > 0$ is damping factor, and positive number. In the following we will derive an explicit formula for the Fourier transform of $c(k)$.

Let $F_T = \sigma\{X_t,\ 0 \le t \le T\}$, be the σ -algebra generated by the Markov chain X_t , $0 \le t < T$, and $f_T(t)$ be the conditional density function of Z_T given F_T , now define

$$R_T = \int_0^T r_t dt. \tag{7}$$

Then, the Fourier transform of $c(k)$ is calculated as follows

$$\psi(u)=\int_{-\infty}^{\infty} e^{iuk} c(k)dk = \frac{E\left[e^{-R_T}\phi_T(u-i(1+\eta))\right]}{\eta^2+\eta-u^2+i(1+2\eta)u}, \tag{8}$$

where

$$\begin{aligned}\phi_T(u) &= E[e^{iuZ_T^{(1)}} \mid F_T]E[e^{iuZ_T^{(2)}} \mid F_T] \\ &= \phi_T^{(1)}(u)\phi_T^{(2)}(u)\end{aligned} \tag{9}$$

is the conditional characteristic function of Z_T given F_T, from [10] the characteristic function of Heston model as follows:

$$\phi_T^{(1)}(u)=\exp(A(u)+C(u)+D(u)) \tag{10}$$

$$A(u)=iu(z_0+R_T)$$

$$C(u)=\frac{2\xi(u)\left(1-e^{-g(u)T}\right)V_0}{2g(u)-\left(\mathrm{g}(u)-\omega(u)\right)\left(1-e^{-g(u)T}\right)}$$

$$D(u)=\frac{-k'\theta}{\sigma^2}\left[2\log\left(\frac{2g(u)-\left(\mathrm{g}(u)-\omega(u)\right)\left(1-e^{-g(u)T}\right)}{2g(u)}\right)+\left(\mathrm{g}(u)-\omega(u)\right)T\right]$$

$$\xi(u)=-\frac{1}{2}(u^2+iu)$$

$$\mathrm{g}(u)=\sqrt{\omega^2(u)-2\sigma^2\xi(u)}$$

$$\omega(u)=\mathrm{k}'-\rho\sigma ui$$

where V_0 is initial variance.

Now, let us consider

$$T_j=\int_0^T \left\langle X_t, e_j\right\rangle dt, \quad j=1,\dots,m \tag{11}$$

be the sojourn time of Markov chain X_t in state e_j during in interval $[0,T]$.

Then $\sum_{j=1}^{m} T_j = T$. It follows that

$$R_T=\sum_{j=1}^{m-1}\left(r(j)-r(m)\right)T_j + r(m)T, \tag{12}$$

Denote $\upsilon = u-i(1+\eta)$, by putting (12) in (10) we have

$$e^{-R_T}\phi_T^{(1)}(\upsilon)=\exp\left(B_1(u)T+C(u)+D(u)\right)\exp\left(i\sum_{j=1}^{m-1}A_1(u,j)T_j\right),$$

where for $j=1,\ldots,m-1$

$$A_1(u,j)=\upsilon\left[r(j)-r(m)\right]+i\left[r(j)-r(m)\right]$$

$$B_1(u)=i\upsilon\left(r(m)\right)-r(m)$$

$$C(u)=i\upsilon(z_0)+\frac{2\xi(\upsilon)\left(1-e^{-g(\upsilon)T}\right)V_0}{2g(\upsilon)-\left(\mathrm{g}(\upsilon)-\omega(\upsilon)\right)\left(1-e^{-g(\upsilon)T}\right)}$$

$$D(u)=\frac{-k'\theta}{\sigma^2}\left[2\log\left(\frac{2g(\upsilon)-\left(\mathrm{g}(\upsilon)-\omega(\upsilon)\right)\left(1-e^{-g(\upsilon)T}\right)}{2g(\upsilon)}\right)+\left(\mathrm{g}(\upsilon)-\omega(\upsilon)\right)T\right]$$

$$\xi(\upsilon)=-\frac{1}{2}(\upsilon^2+i\upsilon)$$

$$\mathrm{g}(\upsilon)=\sqrt{\omega^2(\upsilon)-2\sigma^2\xi(\upsilon)}$$

$$\omega(\upsilon)=\mathrm{k}'-\rho\sigma\upsilon i$$

From [7] denoted by

$$\varphi_j(u)=\lambda(j)\left\{\frac{p(j)\alpha(j)}{\alpha(j)-iu}+\frac{[1-p(j)]\beta(j)}{\beta(j)+iu}-1-iuJ(j)\right\}$$

where

$$J(j)=\frac{[1-p(j)]\beta(j)}{1+\beta(j)}-\frac{p(j)\alpha(j)}{1-\alpha(j)}-1$$

and for $j=1,\ldots,m-1$ the characteristic function of $\phi_T^{(2)}(u)$ from [7] and [8] we have:

$$\begin{aligned}\phi_T^{(2)}(u)&=\exp\left(\varphi_m(u)T\right)\exp\left(\sum_{j=1}^{m}\left(\varphi_j(u)-\varphi_m(u)\right)T_j\right)\\&=\exp\left(B_2(u)T\right)\exp\left(i\sum_{j=1}^{m-1}A_2(u,j)T_j\right),\end{aligned}$$

where

$$A_2(u,j)=\big(\lambda(j)-\lambda(m)\big)i-\upsilon\big[\lambda(j)J(j)-\lambda(\mathrm{m})J(\mathrm{m})\big]$$
$$-i\,\lambda(j)\left\{\frac{p(j)\alpha(j)}{\alpha(j)-i\,\upsilon}+\frac{[1-p(j)]\beta(j)}{\beta(j)+i\,\upsilon}\right\}$$
$$-i\,\lambda(j)\left\{\frac{p(\mathrm{m})\alpha(\mathrm{m})}{\alpha(\mathrm{m})-i\,\upsilon}+\frac{[1-p(\mathrm{m})]\beta(\mathrm{m})}{\beta(\mathrm{m})+i\,\upsilon}\right\}$$

$$B_2(u)=-\lambda(m)[1+i\,\upsilon J_m]+\lambda(m)\left\{\frac{p(m)\alpha(m)}{\alpha(m)-i\,\upsilon}+\frac{[1-p(m)]\beta(m)}{\beta(m)+i\,\upsilon}\right\}$$

So we have

$$e^{-R_T}\phi_T(\upsilon)=\exp\big(B(u)T+C(u)+D(u)\big)\exp\left(i\sum_{j=1}^{m-1}A(u,j)T\right) \tag{13}$$

where

$$A(u,j)=A_1(u,j)+A_2(u,j),$$
$$B(u,j)=B_1(u,j)+B_2(u,j)$$

Substituting (13) into (8), we obtain

$$\psi(u)=\frac{\exp\big(B(u)T+C(u)+D(u)\big)E\left[\exp\left(i\sum_{j=1}^{m-1}A(u,j)T_j\right)\right]}{\eta^2+\eta-u^2+i(1+2\eta)u} \tag{14}$$

Therefore, the specification of $\psi(u)$ reduces the computation of the characteristic function of the random vector $(T_1,...,T_{m-1})^*$.

Now, by adopted methodology of Liu, Zhang and Yin [6] we have

1) **Two-state case**: Let m=2 and suppose the generator of the Markov chain X be given by

$$Q=\begin{pmatrix}-\pi_1 & \pi_1\\ \pi_2 & -\pi_2\end{pmatrix},\qquad \pi_1,\pi_2>0$$

where π_1 is the jump rate from state 1 to state 2, and π_2 is the jump rate from state 2 to state 1, in this case, we want to obtain the characteristic function of T_1 , that is the sojourn time is state 1.

Take initial state $X_0=e_j$, define

$$\Phi_{e_j}(\theta,T)=E\left[e^{i\theta T_1}\,|\,X_0=e_j\right],\qquad j=1,2.$$

Then following from [6] we can obtain the $\phi_{e_1}(\theta,T)$ and $\phi_{e_2}(\theta,T)$ satisfying the following system of integral equations

$$\Phi_{e_1}(\theta,T)=e^{i\theta T}e^{-\pi_1 T}+\int_0^T e^{i\theta t}\phi_{e_2}(\theta,T-t)\pi_1 e^{-\pi_1 t}dt$$
$$\Phi_{e_2}(\theta,T)=e^{-\pi_2 T}+\int_0^T \phi_{e_1}(\theta,T-t)\pi_2 e^{-\pi_2 t}dt$$

by taking Laplace transform, we obtain the following system of algebraic equations, and then by solving equation yields, and taking inverse Laplace transform, we have

$$\Phi_{e_1}(\theta,T)=\frac{1}{s_1-s_2}\Big((s_1+\pi_1+\pi_2)e^{s_1T}-(s_1+\pi_1+\pi_2)e^{s_2T}\Big),$$
$$\Phi_{e_2}(\theta,T)=\frac{1}{s_1-s_2}\Big((s_1+\pi_1+\pi_2-\mathrm{i}\theta)e^{s_1T}-(s_1+\pi_1+\pi_2-\mathrm{i}\theta)e^{s_2T}\Big),$$

where S_1 and S_2 are two roots of the equation follows

$$s^2+(\pi_1+\pi_2-i\theta)s-i\theta)s-i\theta\pi_2=0$$

The Fourier transform (14) in this case is given by

$$\psi(u)=\frac{\exp\big[B(u)T+C(u)+D(u)\big]\Phi_{e_j}\big(A(u,1),T\big)}{\eta^2+\eta-u^2+i(1+2\eta)u}. \tag{15}$$

2) General case (m>2). From [7] and by lemma A.1 of Buffington and Elliot [4] the characteristic function of the random vector $(T_1,...,T_{m-1})^*$ is given by

$$E\left[\exp\left(i\sum_{j=1}^{m-1}\theta_j T_j\right)\right]=\left\langle\exp\big[\Pi+i\,diag(\theta_1,\theta_2,...,\theta_{m-1},0)\big]X_0,1\right\rangle, \tag{16}$$

where $1=(1,...,1)^*\in R^m$.

By substituting $\theta_j=A(u,j)$ into (14) and using the result in (16), we obtain Fourier transform $\psi(u)$, which can be used in the inverse transform to specify the option price.

6. FFT algorithm

In this section we used the approach introduced by Carr and Madan [9], the discrete Fourier transform of a given sequence $\{f_j\}_{j=0}^{N-1}$, From [7] as follows

$$F_l=\sum_{j=0}^{N-1}e^{-i(\frac{2\pi}{N})l_j}f_j\,,l=0,1,...,N-1 \tag{17}$$

the modified option price $c(k)$ can be obtained by the inverse Fourier transform

$$c(k)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-iuk}\psi(u)du=\frac{1}{\pi}\int_0^{\infty}e^{-iuk}\psi(u)du, \tag{18}$$

The $\psi(u)$ is a transform function, and the option price according to (6), $C(k) = e^{-\eta k} S_0 c(k)$. Put $u_j = h(j-1), \quad j = 0,1,...,\text{N}-1$, where h is the grid size in the variable u. Then (17) can be approximated by the following summation

$$c(k) \approx \frac{1}{\pi} \sum_{j=0}^{N-1} e^{-iu_j k} \psi(u_j) h$$

Next, let ζ be the grid size in k and choose a grid along the modified log strike price k as $k_l = -\frac{N}{2}\zeta + \zeta l \ , l = 0,1,...,N-1$.

Therefore,

$$c(k_l) \approx \frac{1}{\pi} \sum_{j=0}^{N-1} e^{-ijlk\zeta} e^{ij(\frac{N}{2})h\zeta} \psi(jh)h, \qquad l = 0,1,...,N-1$$

By setting $h\zeta = \frac{2\pi}{N}$, we have

$$c(k_l) \approx \frac{h}{\pi} \sum_{j=0}^{N-1} e^{-ijl(\frac{2\pi}{N})} e^{ij\pi} \psi(jh), \qquad l = 0,1,...,N-1$$

We are going to obtain an accurate integration with longer value of h and for this purpose, we implement Simpson's rule weighting into our summation. By Simpson's rule weighting, we write the call price as

$$c(k_l) \approx \frac{h}{\pi} \sum_{j=0}^{N-1} e^{-ijl(\frac{2\pi}{N})} e^{ij\pi} \psi(jh) \frac{1}{3}\left[3 + (-1)^{j+1} - \chi_j\right] \tag{19}$$

where χ_j is Kronecker delta function as follows

$$\chi_j = \begin{cases} 1 & for\ j = 0 \\ 0 & for\ j \neq 0 \end{cases}$$

The summation in (19) is an exact application of FFT. By comparing (19) with (17) can be seen easily, that $\{c(k_l)\}_{l=0}^{N-1}$ can be obtained by taking the Fourier transform of the sequence $\left\{ e^{ij\pi} \psi(jh) \frac{1}{3}\left[3 + (-1)^{j+1} - \chi_j\right] \right\}_{l=0}^{N-1}$.

7. Numerical result using FFT

In this section, we use fast Fourier transform (FFT) for valuing a European option under mentioned financial model, to do this, we choose the number of grid points N=4096 (2^{12}). That is, we call FFT producer to calculate 4096 option prices altogether (each one with a different strike price and other parameters are the same). The grid size along the log strike price k is set to be $\zeta = 0.01$ consequently, $h = 0.1534$. We choose damping factor η to be $\eta = 1.0$ .the initial asset price $S_0 = 100\$$. To simplify analysis, we assume the regime switching Markov chain has two states.

Example 7.1. (Low initial volatility $V_0 = 0.05$) The Heston parameters in this example are, $k' = 2$, $\theta = 0.04$, $\sigma = 0.1$, $\rho = 0.5$, and all options have maturity $T = 1$ (year). For two-state Markov chain model the parameters are given by $\pi(1) = 20$, $\pi(2) = 30$, $r(1) = 0.05$, $r(2) = 0.1$, $\lambda(1) = 57$, $\lambda(2) = 74$, $\alpha(1) = 35$, $\alpha(2) = 30$, $\beta(1) = 33$ $\beta(2) = 35$, $p(1) = 0.429$, $p(2) = 0.571$. Large jump rates $\pi(1)$ and $\pi(2)$ are chosen so that the system switches frequently during the life of the options. In each case $(X_0 = e_1, X_0 = e_2)$.

Table 7.1: Option pricing using FFT with low initial volatility

$Ln(K/S_0)$	H $X_0 = e_1$	HDEJD $X_0 = e_1$	H $X_0 = e_2$	HDEJD $X_0 = e_2$
-0.3	30.914	33.523	30.980	33.600
-0.2	23.765	28.107	23.839	28.196
-0.1	16.405	22.871	16.477	22.962
0	9.767	18.001	9.826	18.095
0.1	4.884	13.664	4.923	13.757
0.2	2.041	9.977	2.060	10.064
0.3	0.2179	6.994	0.223	7.020

In table 7.1 the option prices from Heston regime-switching double exponential jump diffusion model are compared to Heston regime switching model without jump. This table reports the results for 7 call option with different strike prices (from deep-in-the-money to at-the-money and to deep-out-of-money) which are obtained by using FFT. Column one lists the log strike (The strike) for the options. Column two and four, list option prices of Heston regime-switching model for both $X_0 = e_1$ and $X_0 = e_2$, respectively, and column three and five option price of Heston regime-switching double exponential jump diffusion (DEJD) for $X_0 = e_1$ and $X_0 = e_2$

Example 7.2: (High initial volatility $V_0 = 0.4$) In this example all the parameters are the same as example 7.1, except initial volatility, that is, $V_0 = 0.4$, numerical results as follows

Table 7.2: Option pricing using FFT algorithm with high initial volatility

$Ln(K/S_0)$	H $X_0 = e_1$	HDEJD $X_0 = e_1$	H $X_0 = e_2$	HDEJD $X_0 = e_2$
-0.3	34.323	37.317	34.376	37.385
-0.2	29.190	32.796	29.240	32.867
-0.1	24.232	28.382	24.281	28.456
0	19.600	24.163	19.643	24.237
0.1	15.414	20.215	15.453	20.289
0.2	11.770	16.606	11.804	13.677
0.3	8.715	13.383	8.742	13.400

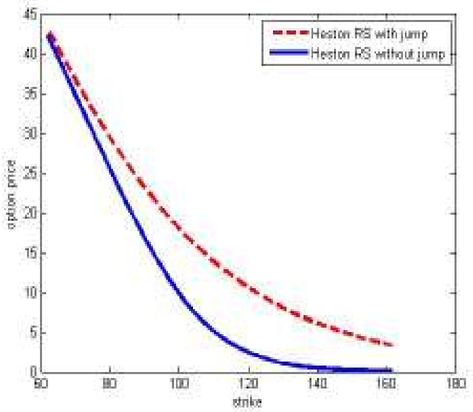

Figure1. Prices curve of Heston Regime-switching model with jump and without jump, by low initial volatility $V_0 = 0.05$

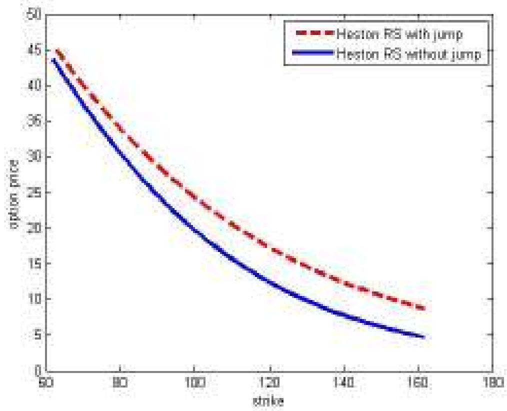

Figure 2. Prices curve of Heston Regime-switching model with jump and without jump, by high initial volatility $V_0 = 0.4$

By comparing tables 7.1, 7.2 and figures, corresponding to prices curve, we realize two points; first, option prices are under the influence of initial volatility, which means by growing from 0.05 to 0.4, option prices increase with respect to the strike prices [11]. Second, call option prices of Double Exponential jump model (DEJD) respecting to strike price are higher than without jump models, because the jump increases the risk premium [7].

Example 7.3: In this example we consider three maturity, monthly (T=0.1), trimester (T=0.25), and semiannual (T=0.5), then compare the option price for two regime of state, in a Heston regime switching double exponential jump diffusion model (HDEJD). We see that by decreasing of the maturity time, the option price will be decreased. All parameters except T are the same example 7.1.

Table 7.2: Option pricing using FFT algorithm with different maturity times.

$Ln\left(\frac{K}{S_0}\right)$	T=0.5		T=0.25		T=0.1	
	Regime1	Regime2	Regime1	Regime2	Regime1	Regime2
-0.3	29.649	29.733	26.564	27.647	26.450	26.524
-0.2	23.503	23.601	20.711	20.814	18.933	19.026
-0.1	17.656	17.766	14.173	14.301	11.408	11.547
0	12.463	12.579	8.632	8.766	5.264	5.460
0.1	8.210	8.321	4.603	4.738	1.769	1.941
0.2	5.022	5.119	2.136	2.238	0.452	0.540
0.3	2.845	2.921	0.865	0.938	0.095	0.126

8. Conclusions

In this paper, we considered the Heston double exponential jumps diffusion model with regime switching where the values of the interest rate parameter can be switched between some regimes of economic. To do this, we considered the interest rate parameter as a Markov switching process. Then, a European option prices under Heston regime switching model, are compared in two cases, with double exponential jump and without jump, for two initial volatility values, and various maturity times.

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