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Two-Layered Supply Chain with Quadratic Demand and Preservation Technology Investment for Time Dependent Deteriorating Item with Fixed-Life

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Abstract

This article focuses on a two-layered supply chain comprising of a single vendor and single buyer policy for a single product which deteriorates with respect to time. To decrease the deterioration of a product, a buyer spends capital on preservation technology to preserve the item. The objective is to maximize the total profit of the supply chain with respect to cycle time and investment for preservation technology. The manufacturer production process is imperfect; It may shift from an in control state to an out of control state at any random time during a production run. Vendor follows lot for lot policy for replenishment produced to the buyer. The model is supported with numerical examples and also by an established scenario of the model. Sensitivity analysis is done to assume decision-making insights.

Keywords: Two layered Supply Chain, Fixed Life-Time, Preservation Technology Investment, Lot-for-Lot Policy, Deterioration

1. INTRODUCTION

In this fast-paced world, business managers struggle for the survival and growth of their companies. Goyal (1976) was the first who Derived a model on supply chain coordination in a single-buyer and single-vendor scenario. Among the early researchers, Banerjee (1986) developed a joint economic lot size model for a single-buyer, single-vendor system with a lot-for-lot policy. Goyal (1988) extended Banerjee's model (1986) by reducing the lot-for-lot assumption. Goyal and Nebebe (2000) further generalized the idea and achieved a lower cost than the shipment policy adopted by Hill (1997).

Deterioration of goods like instable liquids, fruits, fresh vegetables, radioactive substances, medicine, blood etc. in form of direct decay or mutilation, continuing physical decay in course of time, or obsolescence is a natural phenomenon and highly impacts a player's inventory policies. Out of the numerous studies on deterioration items, only a few of them have considered fixed life-time issue of deteriorating items. Goyal and Giri (2001) studied a mathematical model for recent trends in the modeling of deteriorating inventory. Afterwards, Teng and Yang (2004) developed deterministic economic order quantity models with partial backlogging when demand and cost are fluctuating with time. Later, Yang (2005) established a model for comparison among various partial backlogging inventory models for deteriorating items. On the other hand, to reduce the deterioration, Hsu *et al.* (2010) investigated an inventory model with preservation technology investment to minimize the deterioration rate of inventory for constant demand. Dye and Hsieh (2012) studied an optimal replenishment policy for deteriorating items with effective investment in preservation technology. Hsein and Dye (2013) established a production inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time. Recently, Shah and Shah (2014) evaluated an inventory model for optimal cycle time and preservation technology investment for deteriorating items with price-sensitive stock-dependent on demand under inflation. Later on Shah, *et al.* (2014) developed optimal policies for deteriorating items with maximum lifetime and two-level trade credits.

In this paper, we analyze a supply chain that consists of a single vendor, single buyer and single item, when demand is dependent on time. The vendor is the manufacturer and faces the production of defective items. Here, consider an item deteriorating in nature. To reduce the deterioration rate, the investment in preservation technology is integrated. The trade-off is to be obtained between demand and investment in the preservation to maximize the profit of the supply chain.

The rest of the paper is organized as follows. Section 2 presents the notations and the assumptions that are used. Section 3 derives the mathematical model of the inventory problem. Section 4 establishes the proposed inventory model with numerical examples. This section also provides some decision-making insights. Finally, Section 5 gives conclusion and future research directions.

2. NOTATION AND ASSUMPTIONS

The proposed inventory problem is based on the following notations and assumptions.

2.1 Notation

$I_v(t)$	Inventory level at any time t of vendor
OC_v	Ordering cost per unit (in $)
HC_v	Holding cost per unit (in $)
TC_v	Average total cost of product (in $)
$DICV$	Vendor’s defective item cost per unit time (in $)
π_v	Total Profit of vendor (in $)
w	Unit wholesale price (in $)
p	Selling price of the buyer per unit (in $)
m	Fixed life-time of the product (in years)

$\theta(t)$	$=\dfrac{1}{1+m-t}$ deterioration rate at time t
$I_b(t)$	Inventory level at any time t of buyer
OC_b	Ordering cost per unit (in $)
HC_b	Holding cost per unit (in $)
π_b	Total Profit of Buyer (in $)
u	Preservation technology investment per unit time (in $)
$f(u)$	$=1-\dfrac{1}{1+\mu u}$ proportion of reduced deterioration item (in year), $\mu > 0$
π	Total profit of supply chain (in $)
T	Cycle Time (in years)
T_d	Time delay for the vendor to being (start) production (in years)
$g(t)$	Probability density function of the time t
$E(N)$	Expected number of defective items produced during a production run
$R(t)$	$=a\cdot\left(1+b\cdot t-c\cdot t^2\right)$; demand rate at time t
$P(t)$	$=\lambda\cdot R(t)$; production rate at time t, $\lambda > 1$
αP	Defective item production rate; $0\le\alpha\le 1$

2.2 Relations between Parameters

$p > C$

$0\le\theta<1$

$T\le m$

2.3 Assumptions

The following assumptions are made to develop the proposed model:

1. We consider a supply system consisting of a single – manufacturer and single – buyer to deal with single – item.
2. The demand rate, (say) $R(t)=a\cdot\left(1+b\cdot t-c\cdot t^2\right)$ is function of time, $a>0$ is scale demand, $0\le b<1$ denotes the linear rate of change of demand with respect to time, $0\le c<1$ denotes the quadratic rate of change of demand.
3. The production rate ($P(t)$) of the manufacturer is greater than the demand rate $R(t)$ of the buyer, which implies that the vendor has sufficient production capacity to meet the demand of the buyer.
4. Time horizon is infinite.
5. Shortages are not allowed.
6. Lead time is zero or negligible.

7. The proportion of reduced deterioration rate, $f(u)$, is assumed to be a continuous increasing and concave function of investment u on preservation technology ,i.e. $f'(u)>0$ and $f''(u)>0$. WLOG, assume $f(0)=0$.
8. The instantaneous rate of deterioration is $\theta(t)=\dfrac{1}{1+m-t}$, $0\le t\le T\le m.$ for any finite value of m, we have $\theta(t)<1$. If $m\to\infty$ then $\theta(t)\to 0$ i.e. the item is non-deteriorating.
9. As the manufacturer's production rate is greater than the buyer's demand rate, the vendor may start production with a time delay (T_d) in each production cycle.

3. MATHEMATICAL MODEL

We derive the mathematical model for supply chain considering deteriorating items depends on the fixed life-time with preservation technology under quadratic demand. The proportion of reduced deterioration rate $f(u)$ will take place only when the manufacturer / buyer wishes to invest 'u' in the preservation technology.

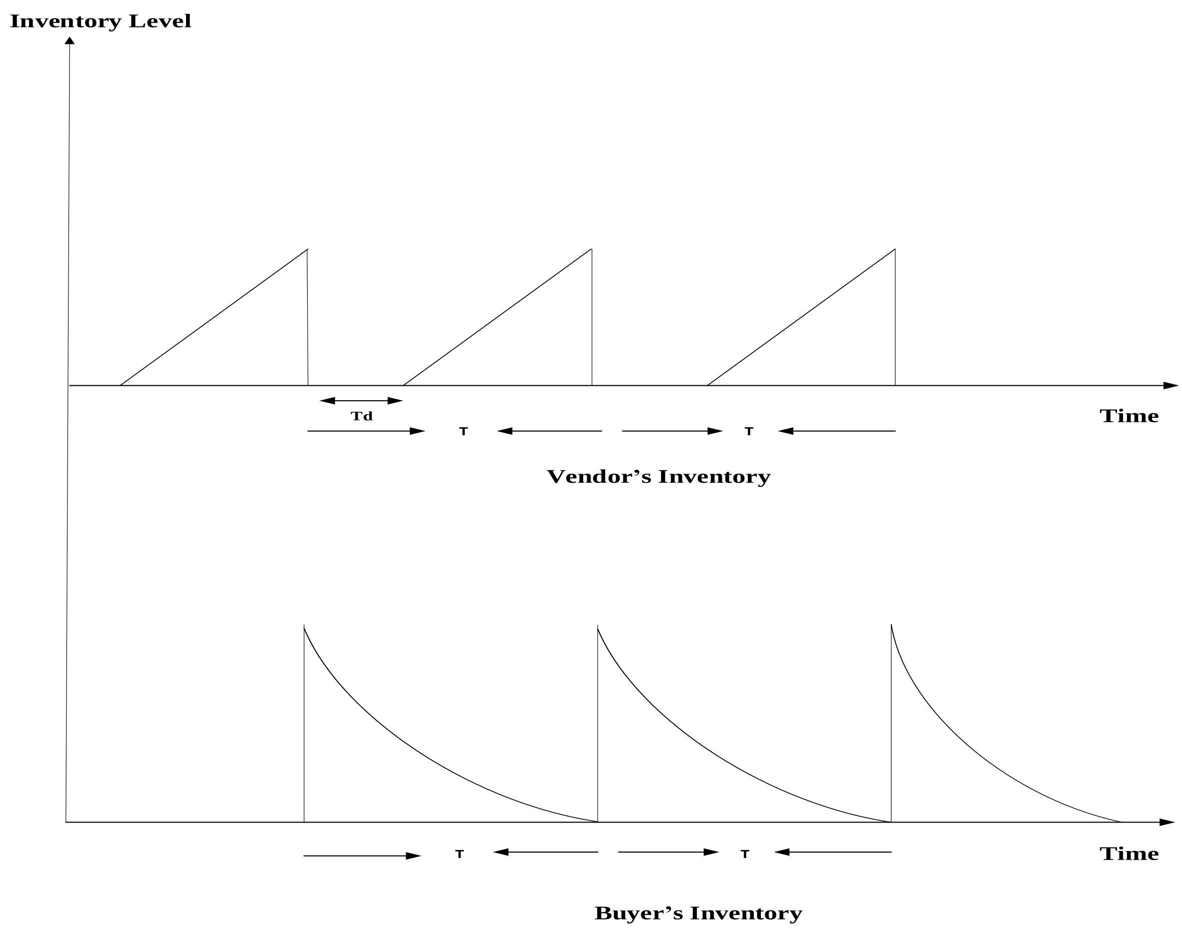

Figure 1. A Schematic Diagram of the Vendor-Buyer Inventory System

3.1 Buyer’s Inventory Model

A schematic diagram of the buyer’s inventory level at time t during a cycle of length T is given by

$$\frac{dI_b(t)}{dt}+\theta(t)\big(1-f(u)\big)I_b(t)=-a\left(1+b\cdot t-c\cdot t^2\right)$$

with $I_b(T)=0$. Solving equation (1) with the boundary condition, we get,

$$I_b(t)=(1+m-t)^X\cdot a\left(\begin{array}{l}\left(\frac{(1+m-t)^{(-X+1)}}{(-X+1)}+\frac{(1+m)^{(-X+1)}}{(-X+1)}\right)-\left(\frac{(1+m-T)^{(-X+1)}}{(-X+1)}+\frac{(1+m)^{(-X+1)}}{(-X+1)}\right)\\ +b\left(\begin{array}{l}\frac{t\cdot(1+m-t)^{(-X+1)}}{(-X+1)}+\frac{(1+m-t)^{(-X+2)}}{(-X+1)\cdot(-X+2)}-\frac{(1+m)^{(-X+2)}}{(-X+1)\cdot(-X+2)}\\ -\frac{T\cdot(1+m-T)^{(-X+1)}}{(-X+1)}-\frac{(1+m-T)^{(-X+2)}}{(-X+1)\cdot(-X+2)}+\frac{(1+m)^{(-X+2)}}{(-X+1)\cdot(-X+2)}\end{array}\right)\\ +c\left(\begin{array}{l}-\frac{t^2\cdot(1+m-t)^{(-X+1)}}{(-X+1)}-\frac{2\cdot t\cdot(1+m-t)^{(-X+2)}}{(-X+1)\cdot(-X+2)}-\frac{2\cdot(1+m-t)^{(-X+3)}}{(-X+1)\cdot(-X+2)\cdot(-X+3)}\\ +\frac{2\cdot(1+m)^{(-X+3)}}{(-X+1)\cdot(-X+2)\cdot(-X+3)}+\frac{T^2\cdot(1+m-T)^{(-X+1)}}{(-X+1)}+\frac{2\cdot T\cdot(1+m-T)^{(-X+2)}}{(-X+1)\cdot(-X+2)}\\ +\frac{2\cdot(1+m-T)^{(-X+3)}}{(-X+1)\cdot(-X+2)\cdot(-X+3)}-\frac{2\cdot(1+m)^{(-X+3)}}{(-X+1)\cdot(-X+2)\cdot(-X+3)}\end{array}\right)\end{array}\right)$$

where, $X=\dfrac{1}{1+\mu\cdot u}$ and $I_{b0}=I_b(0)$.

Now, the total cost per unit time of buyer is sum of

Setup cost : $OC_b=\dfrac{A_b}{T}$

Holding cost : $HC_b=\dfrac{h_b}{T}\cdot\int_0^T I_b(t)\,dt$

Investment for Preservation Technology : $PTI=u$

The average total cost : $TC_b=HC_b+OC_b+PTI$

Therefore, the total profit per unit time for buyer is $\pi_b=\dfrac{p}{T}\cdot\int_0^T R(t)\,dt-\dfrac{w\cdot I_{b0}}{T}-TC_b$

3.2 Vendor's Model

We assume that the vendor begins production at a time delay T_d and the production process may shift from an in-control state to an out-of-control state at a random time $t\in[T_d,T]$. If

$E(N)$ denotes the expected number of defective items produced during a production run then

$$E(N)=\int_{T_d}^{T}\alpha\cdot P(T-t)\,g(t)\,dt$$

where, $P(T-t)=\lambda\cdot R(T-t)$.

The change in inventory level at the vendor's end is the same as the net change in production rate and the defective item removal rate. Thus, the rate of change of inventory level can be described by the following differential equation

$$\frac{dI_v(t)}{dt}=P(t)-\frac{E(N)}{T-T_d},\ T_d\le t\le T$$

with, $I_v(T_d)=0$. Solving the above differential equation together with the boundary condition, we get,

$$I_v(t)=\lambda\cdot a\cdot\left(t+\frac{1}{2}bt^2-\frac{1}{3}ct^3\right)-\left(\frac{\frac{1}{3}\frac{\alpha\lambda ac\left(T^3-T_d{}^3\right)}{T-T_d}+\frac{1}{2}\frac{\alpha\lambda a\left(-b-2cT\right)\left(T^2-T_d{}^2\right)}{T-T_d}+\alpha\lambda a\left(1+bT+cT^2\right)}{T-T_d}\right)t$$

$$-\lambda\cdot a\cdot\left(T_d+\frac{1}{2}bT_d{}^2-\frac{1}{3}cT_d{}^3\right)+\left(\frac{\frac{1}{3}\frac{\alpha\lambda ac\left(T^3-T_d{}^3\right)}{T-T_d}+\frac{1}{2}\frac{\alpha\lambda a\left(-b-2cT\right)\left(T^2-T_d{}^2\right)}{T-T_d}+\alpha\lambda a\left(1+bT+cT^2\right)}{T-T_d}\right)T_d$$

Now, the total cost per unit time of vendor is comprised by

Setup cost :	$OC_v=\frac{A_v}{T}$
Holding cost :	$HC_v=\frac{h_v}{T}\cdot\int_{T_d}^{T}I_v(t)\,dt$
Defective item Cost :	$DICv=\frac{C\,E(N)}{T}$
The average total cost :	$TC_v=HC_v+OC_v+DICv$

Therefore, the total profit per unit time for vendor is $\pi_v=\frac{w-C}{T}-I_v(T)-TC_v$

3.3 Supply Chain Model

The total profit of the whole supply chain is

$$\pi(T,u)=\pi_v(T,u)+\pi_b(T,u)$$

In the next section, we discuss uniform distribution and exponential distribution for defective items and study the supply chain with theoretical values for the inventory parameters.

4. NUMERICAL EXAMPLE AND SENSITIVE ANALYSIS

4.1 Uniform Distribution

Example 1. Taking, $g(t)=\dfrac{1}{T-T_d}$, $a=100$ units, $b=0.05$, $c=0.10$, $\lambda=1.2$, $T_d=0.3$ year, $\alpha=0.05$, $A_b=\$50$ per order, $A_v=\$60$ per order, $h_b=\$1.2$ /unit/year, $h_v=\$0.6$ /unit/year, $C=\$40$ per unit, $w=\$80$ per unit, , $\mu=0.3$, $p=300$, $m=1$ year, in approximate units. The optimal solution is given in Table 1.

Table 1. Optimal Solution with and without Preservation Technology

Uniform Distribution	T (year)	u (\$)	π (\$)
With Preservation Technology	1.138234317	94.71695403	24318.33888
Without Preservation Technology	0.7006960231	--	22012.48187

Table 1 shows that the effectiveness of preservation technology investment is imperative. For the model without preservation technology, it is observed in table 1 that players should take less time to sell products due to the higher rate of deterioration. Consequently, the cycle time becomes shorter. The concavity behaviour of the objective function $\pi(T,u)$ with respect to T and u is shown in Figure 2 and 3.

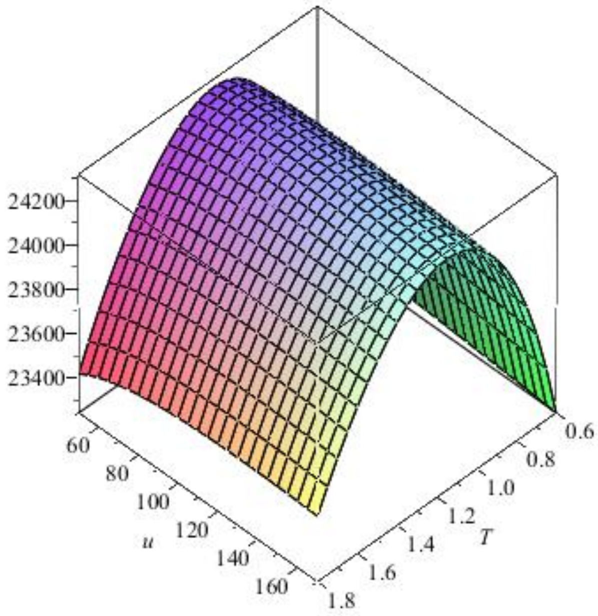

Figure 2. Concavity Behaviour of the Profit Function
(Uniform distribution with Preservation Technology)

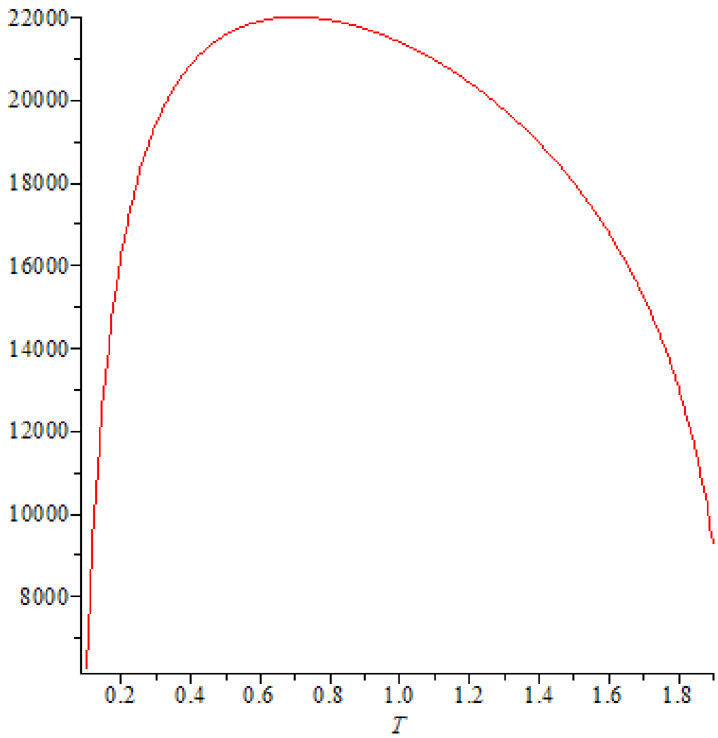

Figure 3. Concavity Behaviour of the Profit Function
(Uniform distribution without Preservation Technology (i.e. $f(u)=0$))

Sensitivity analysis

With the inventory parameters as given in Example 1, the sensitivity analysis is carried out by changing one variable at a time as $-20\%, -10\%$, 10% and 20%. The variations in total profit of the supply chain are presented in Figure 4.

Selling price and scale demand rapidly increases total profit. The linear rate of demand, production rate and fixed life-time slowly increases total profit of the supply chain and rest of the parameters like, purchase cost, holding costs, ordering cost of two players, time delay for the vendor and wholesale price decreases total profit. Almost similar trend is observed for the total profit in without preservation.

The variations in preservation technology investment are exhibited in Figure 5. Increase in scale demand and wholesale price gives huge positive impact on investment in preservation. Whereas linear rate of demand, production rate and ordering cost increases investment in preservation gradually. On the other side, holding cost for both players, purchase cost, selling price and fixed life-time decreases investment in preservation. A similar trend is observed in without preservation.

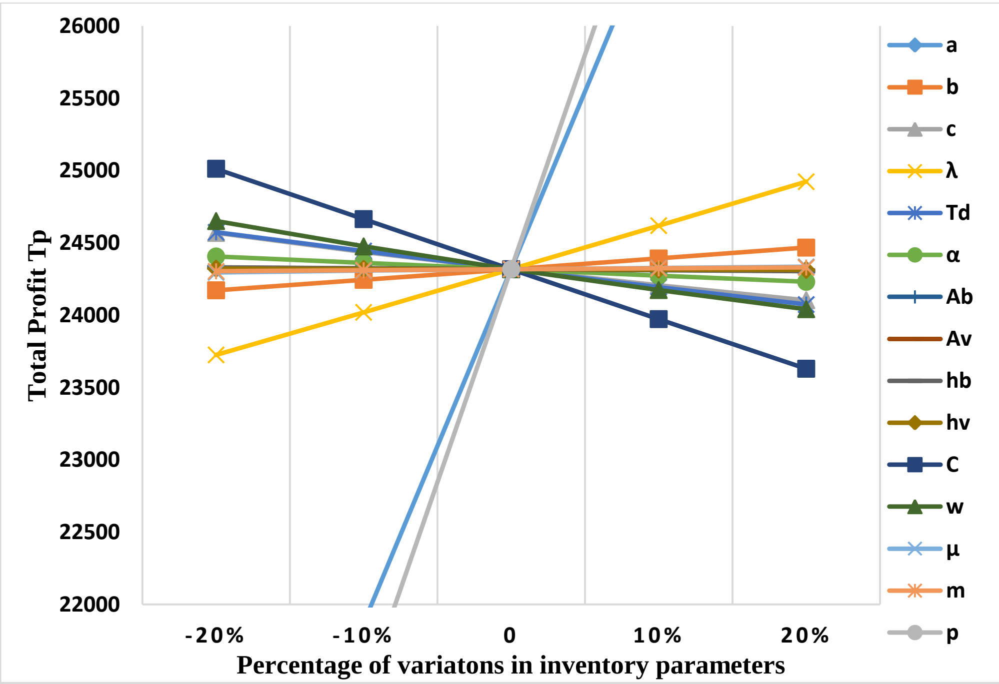

Figure 4. Variations in Total Profit (π) (with preservation technology)

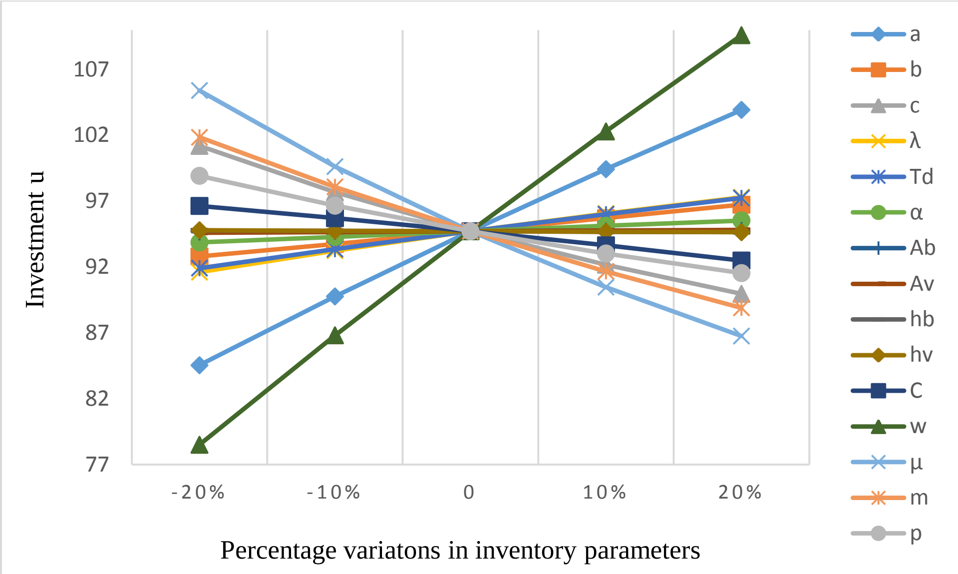

Figure 5. Variations in Investment on Preservation Technology (u) (with preservation technology)

As shown in Figure 6, an increase in wholesale price gives large positive impact on cycle time. Linear rate of demand, fixed life-time, time delay for vendor, production rate and ordering cost increases cycle time gradually whereas the rest of the inventory parameters decrease the cycle time when we consider preservation technology. Selling price gives a great negative effect on cycle time. Also, a similar observation is observed in cycle time when players do not use preservation technology.

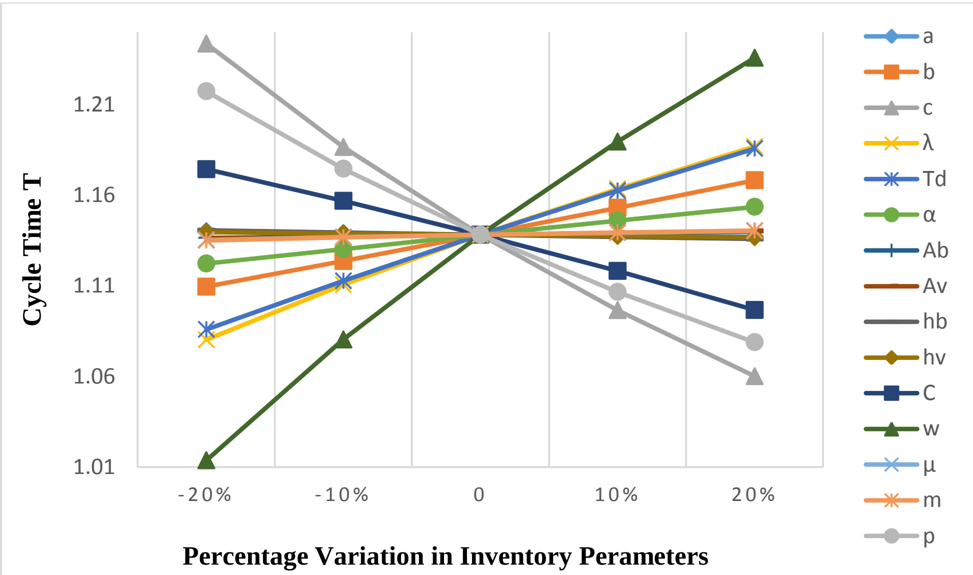

Figure 6. Variations in Cycle Time (T) (with preservation technology)

4.2 Exponential Distribution

Taking, $g(t)=\beta\cdot e^{-\beta(T-T_d)}$, $a=100$ units, $b=0.05$, $c=0.10$ $\lambda=1.2$, $T_d=0.3$ year, $\alpha=0.05$, $A_b=\$50$ per order, $A_v=\$60$ per order, $h_b=\$1.2$ /unit/year, $h_v=\$0.6$ /unit/year, $C=\$40$ per unit, $w=\$80$ per unit, , $\mu=0.3$, β=0.3, $p=300$ $m=1$ year, in approximate units. We obtain the optimal conclusions for exponential shift distributions as given in Table 2.

Table: 2 Optimal Results with and without Preservation Technology

Exponential Distribution	T (year)	u (\$)	π (\$)
With Preservation Technology	1.049873287	89.97512237	24687.82654
Without Preservation Technology	0.6325313384	-	22668.80854

The concavity behaviour of the objective function $\pi(T,u)$ with respect to T and u is shown in figures 7 and 8.

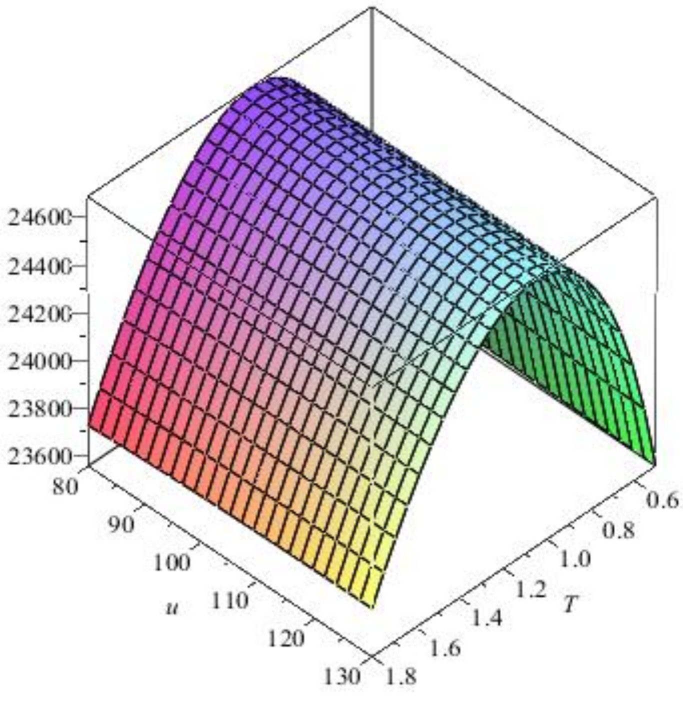

Figure 7. Concavity Behaviour of the Profit Function
(Exponential distribution with Preservation Technology)

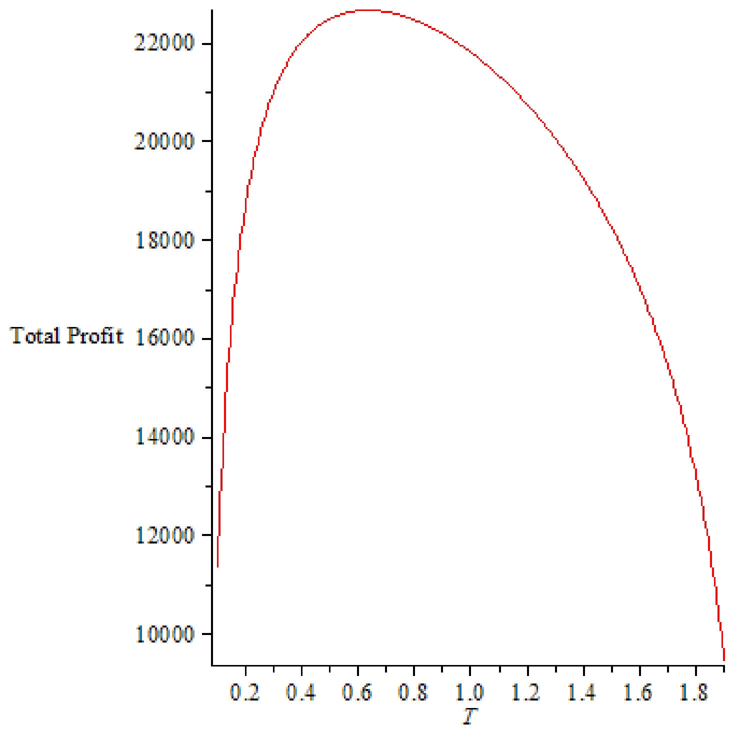

Figure 8. Concavity Behaviour of The profit Function
(Exponential distribution without Preservation Technology (i.e. $f(u)=0$))

The observations are almost similar to those as for uniform distribution for Figures 9 -11.

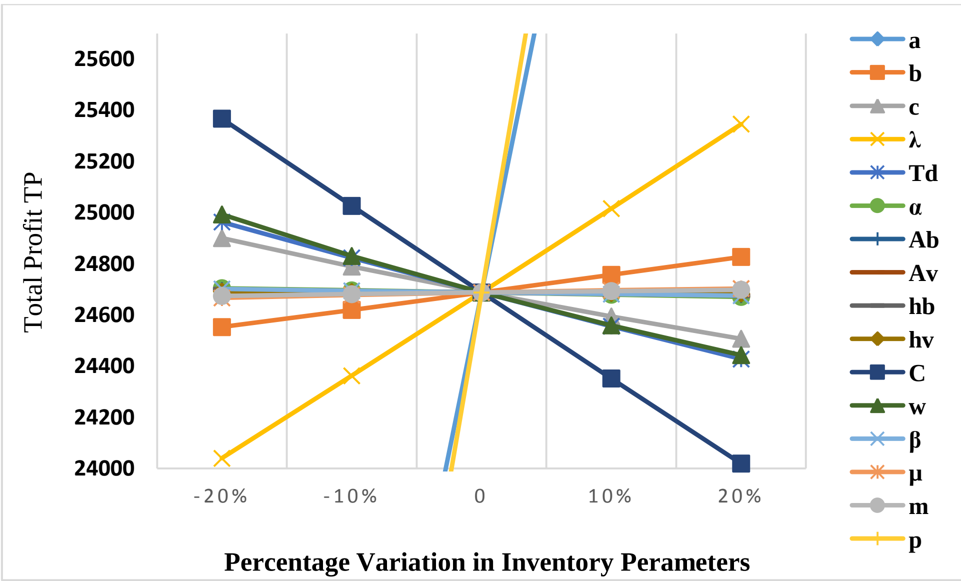

Figure 9. Variations in Total Profit (π) (with preservation technology)

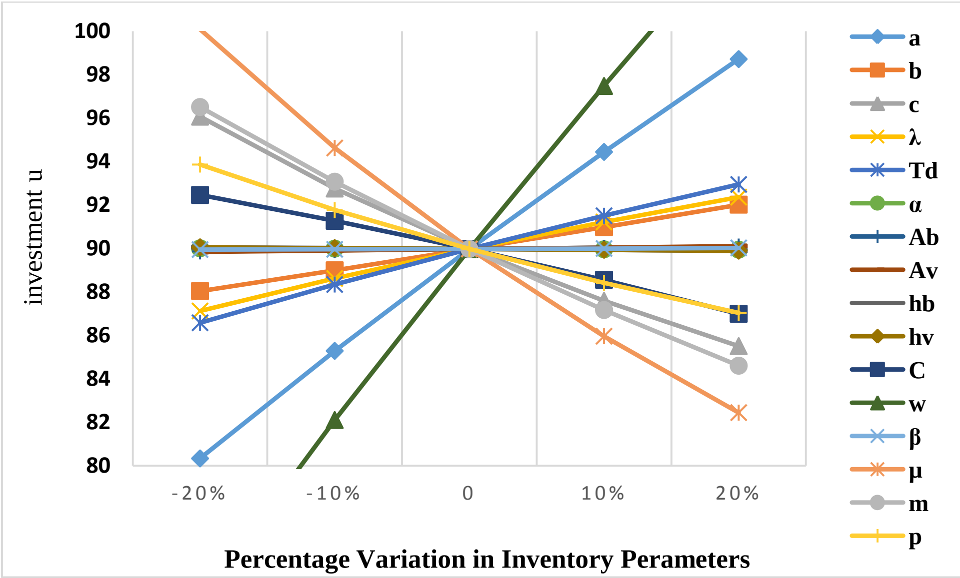

Figure 10. Variations in Investment on Preservation Technology (u) (with preservation technology)

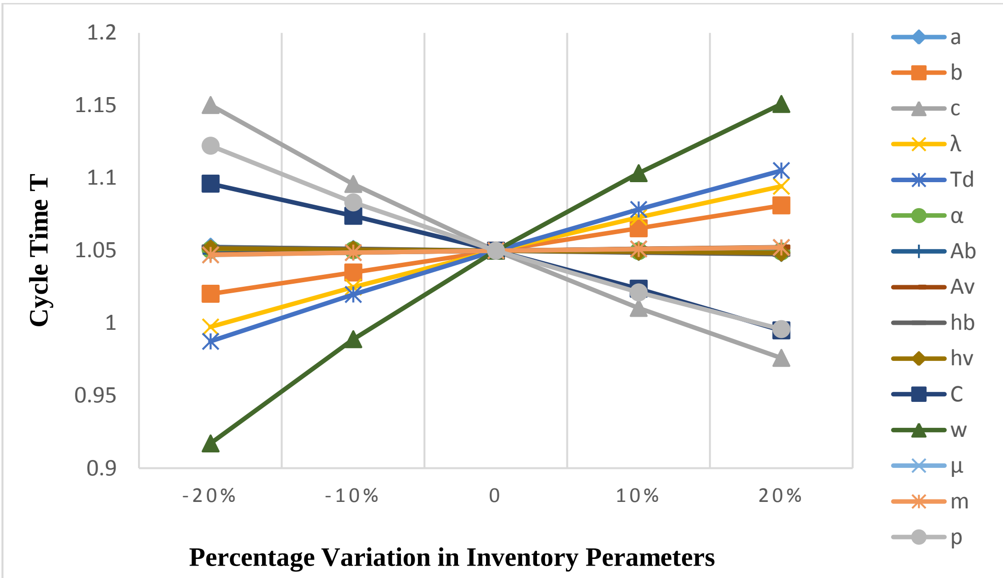

Figure 11. Variations in Cycle Time (T) (with preservation technology)

Finally, from Table 1, Table 2 and Figure 12, it is observed that when players use preservation, total profit of supply chain is maximum. When players use uniform distribution and preservation technology, supply chain profit is 9.48% greater as compared to that without preservation technology investment. Also, when players use exponential distribution and preservation technology, the supply chain's profit increases by 8.18% as compared to that without preservation investment.

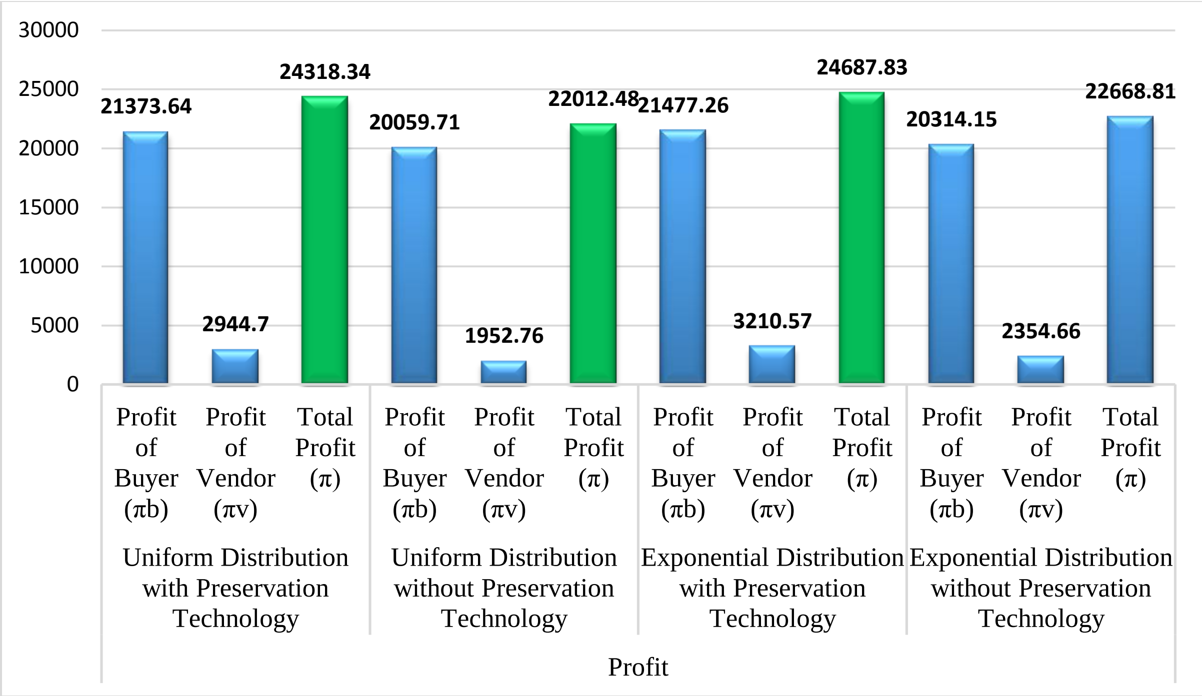

Figure 12. Comparison of Total Profits of Supply Chain with and without Preservation

5. CONCLUSION

In this paper, we deliberated a two-layered inventory model for an item deteriorating in nature under preservation technology investment, with quadratic demand and defective production. The total profit of the supply chain with respect to cycle time and preservation investment is maximized. For numerical examples, Supply chain attains the maximum profit and carry-out sensitivity analysis with respect to inventory parameters. Players study the effectiveness of preservation investment on the optimal supply chain decisions. Current research has several possible extensions such as the model can be further generalized by allowing shortages, price discount for the defective items can be considered, one can study multi layered supply chains, etc.

6. Acknowledgement

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