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INEXTENSIBLE FLOWS OF NEW RULED SURFACES GENERATED BY FOCAL CURVE ACCORDING TO BISHOP FRAME

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Abstract. In this paper, it is studied that inextensible flow of ruled surfaces which is generated by focal curve according to Bishop frame in $\mathbb{R}^3$. Then, it is given that some properties of these surface.

1. Introduction

It is well known that many nonlinear phenomena in physics, chemistry and biology are described by dynamics of shapes, such as curves and surfaces. The time evolution of a curve or surface generated by its corresponding flow in $\mathbb{E}^3$. The flow of a curve or surface is said to be inextensible if, in the former case, the arclength is preserved, and in the latter case, if the intrinsic curvature is preserved. Physically, inextensible curve and surface flows are characterized by the absence of any strain energy induced from the motion. A piece of paper carried by the wind, can be described by inextensible curve and surface flows. Such motions arise quite naturally in a wide range of physical applications. Also, inextensible curve and surface flows arise in the context of many problems in computer vision and computer animation, [6].

Curved surfaces can be expressed in terms of Gaussian curvature, the product of curvature in orthogonal directions. A flat surface growing isotropically, such as a uniformly expanding disk, maintains zero Gaussian curvature. However, if marginal regions grow more slowly than the center, the disk will adopt a cup shape that has positive Gaussian curvature. Conversely, if the marginal areas grow more rapidly, the disk will buckle to form a shape with a wavy edge, such as a saddle, with negative Gaussian curvature. Moreover, much of the modern global theory of complete minimal surfaces in three dimensional Euclidean space has been affected by the work of Osserman during the 1960's. Recently, many of the global questions arose in this classical subject, [2, 7,8].

The focal curve of an immersed smooth curve $\gamma(s)$ in $\mathbb{R}^3$, consists of the centres of its osculating spheres. This curve may be parametrised in terms of the Frenet frame of γ, as $C_\gamma(s) = (\gamma + c_1\mathbf{N} + c_2\mathbf{B})(s)$, where the coefficients c_1 and c_2 are smooth functions that we call the focal curvatures of γ .

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In this paper, we deal with inextensible flow of ruled surfaces which is generated by Focal curve in $\mathbb{R}^3$.

2. Preliminaries

A parametric curve $\gamma(s)$ is a curve on a surface in $\mathbb{R}^3$ that has a constant s or t-parameter value. In this paper, $\gamma'(s)$ denotes the derivative of $\gamma(s)$ with respect to arc length parameter s and we assume that $\gamma(s)$ is a regular curve. For every point of $\gamma(s)$, if $\gamma''(s) \neq 0$, the set $\{\mathbf{T}(s), \mathbf{N}(s), \mathbf{B}(s)\}$ is called the Frenet frame along $\gamma(s)$, where the unit tangent, principal normal, and binormal vectors of the curve at the point $\gamma(s)$, respectively. Derivative formulas of the Frenet frame is governed by the relations

$$\frac{d}{ds}\begin{bmatrix} \mathbf{T}(s) \\ \mathbf{N}(s) \\ \mathbf{B}(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa(s) & 0 \\ -\kappa(s) & 0 & \tau(s) \\ 0 & -\tau(s) & 0 \end{bmatrix}\begin{bmatrix} \mathbf{T}(s) \\ \mathbf{N}(s) \\ \mathbf{B}(s) \end{bmatrix},$$

where $\kappa(s)$ and $\tau(s)$ are called the curvature and torsion of the curve $\gamma(s)$, respectively

The Bishop frame or parallel transport frame is an alternative approach to defining a moving frame that is well defined even when the curve has vanishing second derivative. One can express parallel transport of an orthonormal frame along a curve simply by parallel transporting each component of the frame. The tangent vector and any convenient arbitrary basis for the remainder of the frame are used. The Bishop frame is expressed as

$$\begin{aligned} \mathbf{T}' &= k_1\mathbf{M}_1 + k_2\mathbf{M}_2, \\ \mathbf{M}_1' &= -k_1\mathbf{T}, \\ \mathbf{M}_2' &= -k_2\mathbf{T}, \end{aligned} \tag{2.2}$$

where we shall call the set $\{\mathbf{T}, \mathbf{M}_1, \mathbf{M}_2\}$ as Bishop trihedra and k_1 and k_2 as Bishop curvatures, [5].

The differential geometry of space curves is a classical subject which usually relates geometrical intuition with analysis and topology. For an unit speed curve γ , the focal curve $C_\gamma(s)$ is the centers of the osculating spheres of γ. Because of the center of any sphere tangent to at a point lies on the normal plane to γ at that point, the B- focal curve of γ according to Bishop frame may be parameterized as follows:

$$C_\gamma(s) = \gamma + p\mathbf{M}_1 + \frac{1-pk_1}{k_2}\mathbf{M}_2.$$

Definition 2.1. *A surface evolution $\phi(s,u,t)$ is its flow $\dfrac{\partial \phi}{\partial t}$ are said to be inextensible if its first fundamental form $\{E,F,G\}$ satisfies*

$$\frac{\partial E}{\partial t} = \frac{\partial F}{\partial t} = \frac{\partial G}{\partial t} = 0.$$

This definition states that the surface $\phi(s,u,t)$ is, for all time t, the isometric image of the original surface $\phi(s,u,t_0)$ defined at some initial time t_0. For a developable surface, $\phi(s,u,t)$ can be physically pictured as the parametrization of a waving flag. For a given surface that is rigid, there exists no nontrivial inextensible evolution.

3. Inextensible Flow Ruled Surfaces Generated By Focal Curve According To Bishop Frame

Theorem 3.1. *Let $F(s,u,t)$ be one-parameter family of surface $F(s,u)$ which is parameterized*

$$F(s,u,t) = \gamma(s,t) + p\mathbf{M}_1(s,t) + \frac{1-pk_1}{k_2}\mathbf{M}_2(s,t) + u\mathbf{T}(s,t). \tag{3.1}$$

If $k_1 = ck_2$, then one-parameter family of $F(s,u,t)$ is developable and minimal ruled surface.

Proof. The first derivative of the surface (3.1), we have

$$F_s = \mathbf{u}(k_1\mathbf{M}_1 + k_2\mathbf{M}_2) \tag{3.2}$$

and

$$F_u = \mathbf{T}. \tag{3.3}$$

Then, components of the first fundamental form are

$$\begin{aligned} E &= u^2(k_1^2 + k_2^2), \\ F &= 0, \\ G &= 1. \end{aligned} \tag{3.4}$$

Second derivatives of the surface are

$$\begin{aligned} F_{ss} &= u(k_1'\mathbf{M}_1 + k_2'\mathbf{M}_2 - (k_1^2 + k_2^2)\mathbf{T}), \\ F_{su} &= k_1\mathbf{M}_1 + k_2\mathbf{M}_2, \\ F_{uu} &= 0. \end{aligned} \tag{3.5}$$

The unit normal vector field of the surface is

$$\mathbf{U} = \frac{1}{\sqrt{k_1^2 + k_2^2}}(-k_1\mathbf{M}_2 + k_2\mathbf{M}_1) \tag{3.6}$$

Then, components of the second fundamental form are

$$h_{11} = \frac{u}{\sqrt{k_1^2 + k_2^2}}(k_1'k_2 - k_2'k_1), \tag{3.7}$$

$$h_{12} = h_{22} = 0. \tag{3.8}$$

So, the mean curvature of the surface

$$H = \frac{1}{2u(k_1^2 + k_2^2)^{3/2}}(k_1'k_2 - k_2'k_1). \tag{3.9}$$

If the surface is minimal, we have

$$k_1 = ck_2, \tag{3.10}$$

where c is constant of integration.

From equations (3.5), (3.7) and (3.8),

$$
\begin{aligned}
K &= \frac{h_{11}h_{22} - h_{12}^2}{EG - F^2} \\
&= 0.
\end{aligned}
$$

Theorem 3.2. *Let $F(s,u,t)$ be one-parameter family of developable surface $F(s,u)$ which is parameterized*

$$F(s,u,t) = \gamma(s,t) + p\mathbf{M}_1(s,t) + \frac{1-pk_1}{k_2}\mathbf{M}_2(s,t) + u\mathbf{T}(s,t).$$

If $\dfrac{\partial F}{\partial t}$ is inextensible, then

$$\frac{\partial k_1}{\partial t}k_1 + \frac{\partial k_2}{\partial t}k_2 = 0. \tag{3.11}$$

Proof. The proof is clear from equations (3.4).

Corollary 3.3. *Let $F(s,u,t)$ be one-parameter family of developable and minimal surface $F(s,u)$ which is parameterized*

$$F(s,u,t) = \gamma(s,t) + p\mathbf{M}_1(s,t) + \frac{1-pk_1}{k_2}\mathbf{M}_2(s,t) + u\mathbf{T}(s,t).$$

If $\dfrac{\partial F}{\partial t}$ is inextensible, then $k_1(s,t) = \xi(s)$, where $\xi(s)$ is a smoothh function.

Proof. The proof is obviously obtain from equations (3.10), (3.11).

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