

## **A PERIODIC REVIEW INVENTORY MODEL WITH RAMP TYPE DEMAND AND PRICE DISCOUNT ON BACKORDERS**

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**Abstract:** In this paper, we study a periodic review inventory model with ramp type demand. The study includes some features that are likely to be associated with certain types of inventory, like inventory of seasonal products, newly launched fashion items, electronic goods, mobile phones, etc. When stock on hand is zero, the inventory manager offers a price discount to customers who are willing to backorder their demand. The optimum ordering policy and the optimum discount offered for each backorder are determined by minimizing the total cost in a replenishment interval.

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**Key words and phrases:** Periodic review model; ramp type demand; shortage; price discount on backorder.

### **1. INTRODUCTION**

In traditional inventory models, it is generally assumed that the demand rate is independent of factors like stock availability, price of items, etc. However, in actual practice, the demand of newly launched products such as fashionable garments, electronic items, mobile phones etc. increases with time and later it becomes constant. This phenomenon is termed as ‘ramp type demand’. It is commonly observed in seasonable products, new brand of consumer goods. The demand for these items increases in its growth stage and then remains stable in its maturity stage. The inventory model with ramp type demand rate was proposed by Hill (1995) for the first time. He considered the inventory models for increasing demand followed by a constant demand. Mandal and Pal (1998) developed an order level inventory model for deteriorating items with

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ramp type demand. Wu et al. (1999) derived an EOQ model with Weibull deterioration rate, and the demand rate with a ramp type function of time. Giri et al. (2003) developed an economic order quantity model with Weibull deterioration distribution, shortages and ramp type demand. Deng et al. (2007) studied the inventory model for deteriorating items with ramp type demand rate. Skouri et al. (2009) developed an economic order quantity model with general ramp type demand rate, time dependent deterioration rate, and partial backlogging rate. Ahmed et al. (2013) proposed a new method for finding the EOQ policy, for an inventory model with ramp type demand rate, partial backlogging and general deterioration rate.

In classical inventory models with shortages, it is generally assumed that the unmet demand is either completely lost or completely backlogged. However, it is quite possible that while some customers leave, others are willing to wait till the fulfilment of their demand. In some situations, the inventory manager may offer a discount on backorders and/or reduction in waiting time to tempt customers to wait. Pan and Hsiao (2001) proposed a continuous review inventory model considering the order quantity and with negotiable backorders as decision variables. Ouyang et al. (2003) developed a periodic review inventory model with backorder discounts to accommodate more practical features of the real inventory systems. Chuang et al. (2004) discussed a distribution free procedure for mixed inventory model with backorder discount and variable lead time. Uthayakumar and Parvati (2008) considered a model with only first two moments of the lead time demand known, and obtained the optimum backorder price discount and order quantity in that situation. Pal and Chandra (2012) studied a deterministic inventory model with shortages. They considered only a fraction of the unmet demand is backlogged, and the inventory manager offers a discount on it. Pal and Chandra (2014) developed a periodic review inventory model with stock dependent demand, permissible delay in payment and price discount on backorders.

In this paper, we consider a periodic review inventory model with ramp type demand. The manager offers his customer a discount in case he is willing to backorder his demand when there is a stock-out. The paper is organized as follows. Assumptions and notations are presented in Section 2. In Section 3, the model is formulated and the optimal order quantity and backorder price discount determined. In Section 4, numerical examples are cited to illustrate the policy and

to analyse the sensitivity of the model with respect to the model parameters. Concluding remarks are given in Section 5.

## 2. NOTATIONS AND ASSUMPTIONS

To develop the model, we use the following notations and assumptions.

### Notations

$I(t)$  = inventory level at time point  $t$

$b$  = fraction of the demand backordered during stock out

$b_0$  = upper bound of backorder ratio

$K$  = ordering cost per order

$P$  = purchase cost per unit

$h$  = holding cost per unit per unit time

$s_1$  = backorder cost per unit backordered per unit time

$s_2$  = cost of a lost sale

$\pi$  = price discount on unit backorder offered

$\pi_0$  = marginal profit per unit

$T$  = length of a replenishment cycle

$T_1$  = time taken for stock on hand to be exhausted,  $0 < T_1 < T$

$S$  = maximum stock height in a replenishment cycle

$s$  = shortage at the end of a replenishment cycle

### Assumptions

1. The model considers only one item in inventory.
2. Replenishment of inventory occurs instantaneously on ordering, that is, lead time is zero.
3. Shortages are allowed, and a fraction  $b$  of unmet demands during stock-out is backlogged.
4. The demand rate  $R(t)$  is assumed to be a ramp type function of time  $t$

$$R(t) = D_0[t - (t - \mu)H(t - \mu)]$$

where  $D_0$  and  $\mu$  are positive constants and  $H(t - \mu)$  is the Heaviside's function defined as

$$\text{follows: } H(t - \mu) = \begin{cases} 1 & \text{for } t \geq \mu \\ 0 & \text{for } t < \mu \end{cases}$$

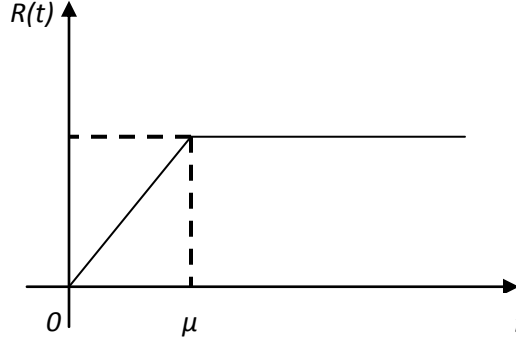


Figure 1: The ramp type demand rate

5. The time taken for stock on hand to be exhausted ( $T_I$ ) is greater than  $\mu$ .
6. During the stock-out period, the backorder fraction  $b$  is directly proportional to the price discount  $\pi$  offered by the inventory manager. Thus,

$$b = \frac{b_0}{\pi_0} \pi, \text{ where } 0 \leq b_0 \leq 1, 0 \leq \pi \leq \pi_0$$

### 3. MODEL FORMULATION

The planning period is divided into reorder intervals, each of length  $T$  units. Orders are placed at time points  $0, T, 2T, 3T, \dots$ , the order quantity being just sufficient to bring the stock height to a certain maximum level  $S$ .

Depletion of inventory occurs due to demand during the period  $(0, T_1)$ ,  $T_1 < T$ , and in the interval  $(T_1, T)$  shortage occurs, of which a fraction  $b$  is backlogged. Hence, the variation in inventory level with respect to time is given by

$$\begin{aligned} \frac{d}{dt} I(t) &= -D_0 t, & \text{if } 0 < t < \mu \\ &= -D_0 \mu, & \text{if } \mu < t < T_1 \\ &= -b D_0 \mu, & \text{if } T_1 < t < T \end{aligned}$$

Since  $I(0) = S$  and  $I(T_1) = 0$ , we get

$$\begin{aligned}
 I(t) &= -\frac{D_0 t^2}{2} + S, \quad \text{if } 0 < t < \mu \\
 &= D_0 \mu (T_1 - t), \quad \text{if } \mu < t < T_1 \\
 &= b D_0 \mu (T_1 - t), \quad \text{if } T_1 < t < T
 \end{aligned}$$

Hence,

$$\begin{aligned}
 S &= \frac{D_0 \mu}{2} (2T_1 - \mu) \\
 s &= b D_0 \mu (T - T_1)
 \end{aligned}$$

Then,

Ordering cost during a cycle (OC) =  $K$

Holding cost of inventories during a cycle (HC)

$$= h \int_0^{T_1} I(t) dt = \frac{D_0 \mu^2 h}{2} (2T_1 - \mu) + \frac{D_0 \mu h}{2} \left( (T_1 - \mu)^2 - \frac{\mu^2}{3} \right)$$

Backorder cost during a cycle (BC)

$$= -s_1 \int_{T_1}^T I(t) dt = \frac{s_1 b D_0 \mu}{2} (T - T_1)^2$$

Lost sales cost during a cycle (LC) =  $(1 - b) D_0 \mu (T - T_1)$

Purchase cost of inventory during a cycle is (PC)

$$= P \left( \frac{D_0 \mu}{2} (2T_1 - \mu) + b D_0 \mu (T - T_1) \right)$$

Hence, the cost per unit length of a replenishment cycle is given by

$$\begin{aligned}
 C(T_1, T, b) &= \frac{1}{T} [\text{OC} + \text{HC} + \text{BC} + \text{LC} + \text{PC}] \\
 &= \frac{1}{T} \left( K + D_0 \mu (T - T_1) (b(P - s_2) + s_2) + \frac{D_0 \mu}{2} \left( P(2T_1 - \mu) + hT_1^2 - \frac{h\mu^2}{3} + s_1 b (T - T_1)^2 \right) \right) \\
 &= \frac{N(T_1, T, b)}{T}
 \end{aligned}$$

The optimal values of  $T_1$ ,  $T$  and  $b$ , which minimize  $C(T_1, T, b)$ , must satisfy the following equations:

$$(1 - b)(P - s_2) + hT_1 - s_1 b (T - T_1) = 0 \quad (3.1)$$

$$D_0 \mu (s_1 b (T - T_1) + b(P - s_2) + s_2) = C(T_1, T, b) \quad (3.2)$$

$$T - T_1 = \frac{2(s_2 - P)}{s_1} \quad (3.3)$$

#### 4. NUMERICAL ILLUSTRATION AND SENSITIVITY ANALYSIS

Since it is difficult to find closed form solutions to the sets of equations (3.1) - (3.3), we numerically find solutions to the equations for given sets of costs using the statistical software MATLAB. The following tables show the change in optimal inventory policy with change in a model parameter, when the other parameters remain fixed.

**Table 1:** Showing the optimal inventory policy for different values of  $h$ , when  $K = 500$ ,  $D_0 = 100$ ,  $P = 5$ ,  $\mu = 0.25$ ,  $s_1 = 6$  and  $s_2 = 7$ .

$h$	$T_l$	$T$	$b$	$C(T_l, T, b)$
1	5.80	6.47	0.2545	270.04
4	2.62	3.28	0.3038	386.63
5	2.28	2.95	0.3700	410.51
8	1.70	2.37	0.5672	464.98
10	1.47	2.14	0.6916	492.50

**Table 2:** Showing the optimal inventory policy for different values of  $s_1$ , when  $K = 500$ ,  $D_0 = 100$ ,  $P = 5$ ,  $\mu = 0.25$ ,  $h = 4$  and  $s_2 = 7$ .

$s_1$	$T_l$	$T$	$b$	$C(T_l, T, b)$
2	1.96	3.96	0.3328	320.81
6	2.62	3.28	0.3038	386.63
8	2.73	3.23	0.3319	397.75
14	2.88	3.17	0.2613	413.19
15	2.90	3.16	0.2168	414.63

**Table 3:** Showing the optimal inventory policy for different values of  $s_2$ , when  $K = 500$ ,  $D_0 = 100$ ,  $P = 5$ ,  $\mu = 0.25$ ,  $h = 4$  and  $s_1 = 6$ .

$s_2$	$T_l$	$T$	$b$	$C(T_l, T, b)$
7	2.62	3.28	0.3038	386.63
14	2.67	5.67	0.3333	392.16
17	3.05	7.05	0.5841	429.75
19	3.34	8.01	0.5750	459.03
21	3.65	8.99	0.7995	490.43

**Table 4:** Showing the optimal inventory policy for different values of  $P$ , when  $K = 500$ ,  $D_0 = 100$ ,  $h = 4$ ,  $\mu = 0.25$ ,  $s_1 = 6$  and  $s_2 = 7$ .

$P$	$T_I$	$T$	$b$	$C(T_I, T, b)$
1	2.46	4.46	0.3330	271.28
5	2.62	3.28	0.3038	386.63
10	3.66	3.16	0.9668	616.09
12	4.15	3.32	0.9874	714.66
15	5.01	3.68	0.9945	875.67
17	6.01	4.34	0.9945	991.57
21	7.06	4.72	0.9977	1230.52

Let us consider the following model parameters:  $K = 500$ ,  $D_0 = 100$ ,  $P = 5$ ,  $\mu = 0.25$ ,  $h = 4$ ,  $s_1 = 6$  and  $s_2 = 7$ . Table 5 gives the percentage change in the total cost over an inventory cycle with change in the cost parameters.

**Table 5:** Percentage change in total cost with change in the model parameters

Parameter		% change in total cost	Parameter		% change in total cost
Name	Value		Name	Value	
$h$	1	-30.15	$s_1$	2	-17.02
	5	6.18		4	-5.19
	8	20.27		8	2.88
	10	27.38		11	5.38
	13	35.99		14	6.87
$P$	1	-29.83	$s_2$	14	1.43
	7	25.16		17	11.15
	10	59.35		19	18.73
	12	84.84		21	26.85
	17	156.47		23	35.36

The above tables show that, for other parameters remaining constant,

- (a) both  $T_I$  and  $T$  are decreasing in  $h$ , but increase as  $s_2$  increase;
- (b)  $b$ , and hence  $\pi$ , decreases with increase in  $s_1$ , but increases with  $h$ ,  $s_2$  and  $P$ ;
- (c)  $T_I$  is increasing in  $s_1$ ,  $s_2$  and  $P$  while  $T$  is decreasing in  $h$  and  $s_1$ .

The above observations indicate that, with the aim to minimizing total cost, the policy should be to maintain high inventory level for low holding costs but high backorder cost, lost sales cost and

purchase cost. Also, higher the backorder cost, lower should be the price discount offered and for higher lost sales cost, higher price discount should be offered.

## **5. CONCLUSION**

The paper studies a periodic review inventory model with ramp type demand allowing shortages. The study includes some features that are likely to be associated with certain types of inventory in real life, like inventory of seasonal products, newly launched fashion items, electronic goods, mobile phones, etc. A fraction of the demand is backlogged, and the inventory manager offers a discount to each customer who is ready to wait till fulfilment of his demand. The optimum ordering policy and the optimum discount offered for each backorder are determined by minimizing the total cost in a replenishment interval. Through numerical study, it is observed that for low backorder cost, it is beneficial to the inventory manager to offer the customers high discount on backorders.

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