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Probabilistic Assessment of a Mechanical Component

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Abstract

This paper presents the application of probabilistic methodology to a mechanical component. The probabilistic analysis approach assigns Probability Density Functions to sources of uncertainty and variation and then propagates the PDFs through a physics-based model to produce PDFs of model responses. This paper discusses how appropriate PDFs are selected for boundary condition uncertainty, model uncertainty, and manufacturing variation. A Latin Hypercube experimental design provides a series of design points that fill the entire design space. The Latin Hypercube is run through a physics-based model to relate model inputs with analysis outputs. With this data, the model is emulated with a Gaussian Process. The Gaussian Process serves as a fast running approximation of the physics-based model. The emulator is coupled with lifing equations for a Monte-Carlo analysis that yields probability distributions for model outputs. Furthermore, sensitivity analysis quantifies the relative effect of uncertainty and variation on part life. A jet engine turbine component is used as an example of the application of the general methodology.

Nomenclature

a, b ,c – thermal mechanical fatigue life equation coefficients
$k_{1 \ldots 5}$– b-spline coefficients
pf – profile factor
σ – standard deviation
R^2 – coefficient of determination
T_{local} – local temperature value at given radius
T_{avg} – the average temperature of the temperature profile
T_c– the cooling air temperature
T_g – the average hot gas path temperature
μ – mean\nominal

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Acronyms

CCE – Collaborative Computing Environment
CFD – Computational Fluid Dynamics
CVRMSE – Cross Validation Root Mean Squared Error
DFV – Design for Variation
DOE – Design of Experiments
FEM – Finite Element Model
HPT – High Pressure Turbine
IPT – Integrated Product Team
LCF – Low Cycle Fatigue
LPT – Low Pressure Turbine
MCMC – Markov Chain Monte-Carlo
MLEGP – Maximum Likelihood Estimate of Gaussian Processes
PDF – Probability Density Function
RCCA – Root Cause Corrective Action
TMF – Thermal Mechanical Fatigue
TMTF – Turning Mid Turbine Frame

Software Packages

ANSYS® (ANSYS® 2007)
Unigraphics® (Unigraphics® 2008)
Unigraphics Advanced Simlutation® (Unigraphics®, 2008)

1.0 Introduction

When designing a physical part, there is inherent uncertainty and variation that must be accounted for to ensure that the design meets the performance criteria. The models used to access the design may contain inaccurate parameters, biases, or stochastic error. Boundary conditions are typically uncertain because there is not sufficient data to accurately quantify the boundary conditions or boundary prediction models contain uncertainties. There is also manufacturing variation that affects final part performance in the field. Finally, there is usage and environmental uncertainty. While a nominal design may meet all the objectives, a manufactured part within tolerance may not. Clearly, all this uncertainty and variation must be considered during the design phases of a product. There are two approaches for handling this uncertainty and variation.

The deterministic approach generates a nominal design with sufficient margin such that the manufactured part will still meet the requirements even in a worst case scenario. Factors of safety are assigned to safety critical metrics and/or key performance criteria. With this approach, it is impossible to determine if there truly is sufficient margin. The uncertainty is not explicitly accounted for so the engineer is forced to rely on expert opinion and prior demonstrated capability. This can cause part performance predictions to be overstated. Furthermore, there is no way to calculate how much the margin or safety factor is adversely affecting the design in terms of other metrics. This can lead to a part that is over designed and over cost. While it is straightforward and easy to apply the deterministic approach, there are significant inherent.

An alternative to deterministic design is probabilistic design. Probabilistic design assigns Probability Density Functions (PDFs) to sources of design uncertainty and variation. Thus, the uncertainty is explicitly accounted for in the probabilistic approach. The probability distributions are then propagated through the analysis to produce PDFs of the responses. With these output PDFs, the probability that the design will meet its requirements can be calculated. Thus, there is no “sufficient margin” concern and opposing design requirements can be effectively traded and balanced. This allows the engineer and company to make the appropriate business decisions.

There are challenges associated with probabilistic design. First, probabilistic design requires more execution of the physics-based model than the deterministic design approach. This is a challenge because of schedule constraints and computing resources. Due to schedule, the models must be automated because it is infeasible to manually execute all of the physics-based analyses. This means that engineers must make engineering mode parametric and robust. Second, assigning accurate probability distributions that represent the true uncertainty is difficult. Correlated uncertainty makes this even more challenging. Finally, probabilistic design requires thousands of design points be analyzed for a Monte-Carlo analysis. Unless the physics-based analysis runs in seconds, which is typically not the case, the physics-based model must be replaced with an emulator. Building an emulator that accurately replicates the physics-based model is challenging when there are non-linearities in the design space and residual error in the physics-based model predictions.

This paper discusses the application of the Design for Variation (DFV) methodology developed by Reinman et al. (2012) to the design and analysis of a turbine jet engine component. Furthermore, the development of an automated engineering analysis workflow via a new propriety tool called Collaborative Computing Environment (CCE) is presented. The building of a robust parametric automated engineering analysis workflow is one of the largest challenges to implementing the DFV methodology especially when high fidelity, multi-disciplinary models need to be coupled together. Previous work in this area by Bunker (2009) and Heinze et al. (2012) only account for manufacturing variation. Boundary condition uncertainty and model uncertainty are also considered in this paper. Furthermore, the methodology is applied to a real world design with schedule and cost constraints.

2.0 General Methodology

This paper follows and summarizes the methodology described by Reinman et al. (2012). In this methodology, there are five steps for executing a DFV-enabled process.

Step 1) Define
Some customer requirements may be probabilistic while others are deterministic. All deterministic requirements must be converted to probabilistic requirements by including the customer and considering the consequence of failure.

Step 2) Analyze
The analysis processes must be DFV-enabled. This means that all models and analyses in the process, including both geometric and analytical models, are robust parametric and batch executable. Developing a DFV-enabled process is challenging and the ten elements of a DFV-enabled process are discussed later. This DFV-enabled process is used to identify root causes of performance variation and uncertainty.

Step 3) Solve
Identify the optimum design that satisfies the probabilistic customer requirements.

Step 4) Verify/Validate
Model input and output data from the real world might have already been collected and used to calibrate the model, but typically these data sets are too small to cover the full range of input variation. Once the optimal design has been obtained and the real-world version of the process moves into the validation phase, model input and output data should be collected to verify and validate previous variation and uncertainty assumptions.

Step 5) Sustain
Once the model has been validated, data collection and testing continue, but the purpose of the data collection changes to ensuring that the model remains consistent with the real world. At this time, efforts are made to stabilize the important causes of process performance variation that are under manufacturing control and to monitor those that are not.

A fully DFV-enabled physics-based model according to Reinman et al. (2012) has the following ten elements within six categories. The elements and categories are summarized as:

Model Preparation

1. A robust parametric physics-based model

Model Input Variability and Uncertainty Quantification

2. Process for retrieving data needed to quantify variability and uncertainty in model inputs
3. Process for performing statistical analysis/developing statistical model of input data
    a. Preserve correlations and physical meaning

Model Sensitivity Analysis

4. Process for generating a matrix of space-filling computer experiments (model runs) for emulator development
5. Process for running the physics-based model at the space-filling design points
6. Process for building and validating the model emulator and performing a variance-based sensitivity analysis (Saltelli et al. 2001)

Model Calibration

7. Process for determining what experimental/field data are required for model calibration and measurement uncertainty (amount and characteristics to be measured)
8. Process for performing Bayesian model calibration (Kennedy and O'Hagan 2001): calibrate, potentially bias correct, and assess residual variation

Uncertainty Analysis

9. Process for generating a Monte-Carlo sample and driving though
    a. Parametric model (if fast enough)
    b. Model emulator
    c. Calibrated and potentially biased corrected emulator model

Enable Practice

10. Update local engineering standard work (work instructions and criteria) and local training. Establish a process to ensure the model is capable early on and over time.

One particularly challenging element is the creation of a robust parametric physics-based model. In the example discussed in this paper, models from different engineering disciplines need to be coupled together in an automated multi-disciplinary engineering analysis workflow. A new decentralized approach was developed to facilitate the development of a robust parametric physics-based model. A decentralized approach keeps the building, testing, and workflow execution control with the discipline engineers that traditionally own a particular piece of the overall workflow. This parallelizes the development of the workflow and accelerates workflow debugging because the discipline experts are directly responsible for failures and bugs. A tool called CCE was developed to allow engineering teams to collaboratively build and execute automated multi-disciplinary engineering analysis workflows.

CCE works by first placing all parts of the automated workflow in a revision management system where each discipline owns a branch. The engineers are then responsible for making their piece of the workflow parametric and batch enabled. Once that is achieved, CCE parallelizes the execution of multiple design points by submitting and retrieving cases from a compute cluster. Furthermore, CCE provides methods to generate a Design of Experiments (DOE) and to summarize the results across multiple runs into a single location. To tie the pieces of the workflow together, CCE automates the passing of files between the different branches of the revision management system. Execution of each piece of the workflow is controlled by the owning engineer. During execution of the workflow, each piece of the workflow is executed as an entire batch.

To illustrate the real world application of the general methodology, an example problem using a jet engine component is provided. This problem contains all the elements and steps of a DFV-enabled process and illustrates the application of CCE to build an automated workflow.

3.0 Example Problem

A Turning Mid-Turbine Frame (TMTF) is in the gas path between the High Pressure Turbine (HPT) and Low Pressure Turbine (LPT) of a jet engine. The purpose of a TMTF is to allow access to the shaft for a bearing while minimizing aerodynamic loses. The TMTF also protects the bearing structure from high gas path temperatures. It is challenging to design a TMTF due to harsh boundary conditions, model uncertainty, and multiple objectives. The figure below shows a representative engine cross section with a TMTF.

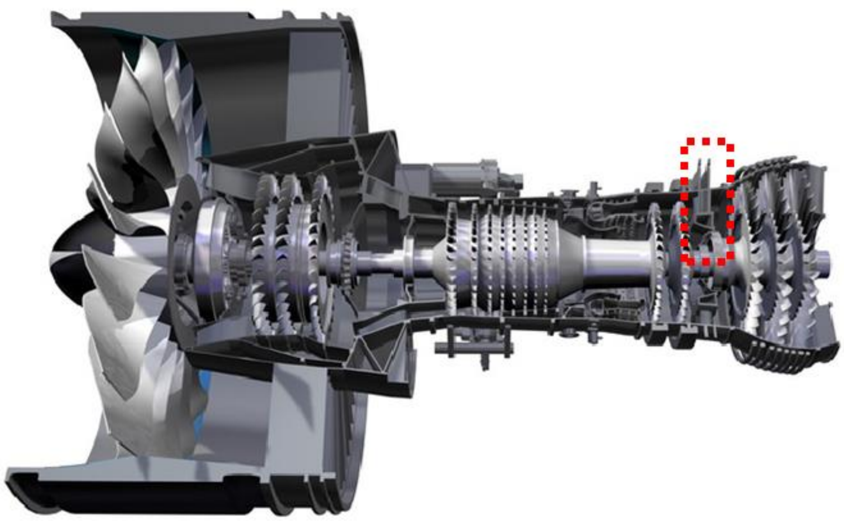

Figure 1: PW1000G Cross Section with Highlighted TMTF Region (http://www.a320neo.com/pratt-whitney-pw1000g.php)

When designing a TMTF or any mechanical part, the goal is to ensure that production hardware meets the requirements in the production engines. This is a difficult problem because there is a great deal of uncertainty. For example, the gas path temperature profile is an important driver of Thermal Mechanical Fatigue (TMF) and Low Cycle Fatigue (LCF) life. The actual gas path temperature profile that is seen by the hardware in the engine is unknown. In many instances, available gas path temperature profile data is from engines with different configurations. Models that are used to predict the gas path temperature profile have inherent uncertainty. Furthermore, the models that are used to predict the stress, strain, metal temperature, and life have inherent uncertainty. The models may contain inaccurate parameters, biases, or stochastic residual error. Finally, manufacturing variation effects final part performance. While the nominal design may meet the objectives, a manufactured part within tolerance may not meet requirements. Clearly, it is difficult to meet all of the requirements without needlessly increasing manufacturing cost given the high degree of uncertainty.

3.1 Customer Requirements

The TMTF component requirements were communicated from the customer to the Integrated Product Team (IPT). Engine level requirements such as weight, performance, part level requirements were defined. The TMTF had requirements for cost, weight, TMF life, LCF life, oxidation life, pressure loss, reliability, secondary flow, and schedule. In addition, the LPT had a module level efficiency requirement. The TMTF contributes to the overall LPT efficiency.

The goals for the IPT were to allow for more aerodynamic analysis iterations within the schedule, quantify the effect of temperature profile uncertainty, increase the part life, and improve the LPT module efficiency. Typically, TMTF IPTs are able to manually execute roughly a dozen analyses in six months and within these analyses, only one to two aerodynamic iterations would be executed.

Due to the challenging nature of the design problem and schedule, not every output could be probabilistically analyzed. For the TMTF, weight, LCF life, and TMF life were dealt with probabilistically while all other requirements were handled deterministically. These outputs were selected because they were the most critical and had the most difficult requirements to satisfy.

3.2 Model Preparation

The TMTF was a full wheel cast part consisting of fourteen airfoils. Two airfoils with cyclic symmetry boundary conditions were analyzed. This modeling technique captures the necessary physics while reducing computational solution time. Thirty-three input variables were parametric and thirty-three outputs were tracked. For the thirty-three input variables, twenty-four were geometric, five defined the temperature profile, and four parameters characterized the TMF life model. The geometry is shown in the figure below.

Figure 2: Geometry

These parameters were selected because they were expected to be important drivers of stress, temperature, and life in the life limiting locations. This decision was based on previous sensitivity analyses and engineering experience.

Design space limits were assigned to geometric parameters based on expert elicitation. It was decided, that the limits would far exceed typical manufacturing tolerances because the goal was to find a nominal geometry that was robust to variation and

uncertainty. The details of the temperature profile and TMF life model parameters will be discussed later in the paper. TMF life and LCF life were tracked at every fillet location for each airfoil in the model for a total of thirty-two life outputs. The last output was the weight of the part.

The engineering analysis workflow consisted of the following analyses: 3d geometry generation, gas path temperature profile generation, external thermal boundary condition generation and application, internal thermal boundary condition generation and application, thermal meshing, thermal Finite Element Model (FEM) solution, structural meshing, structural FEM solution, TMF life calculation, and LCF life calculation. The engineering workflow is shown in the figure below.

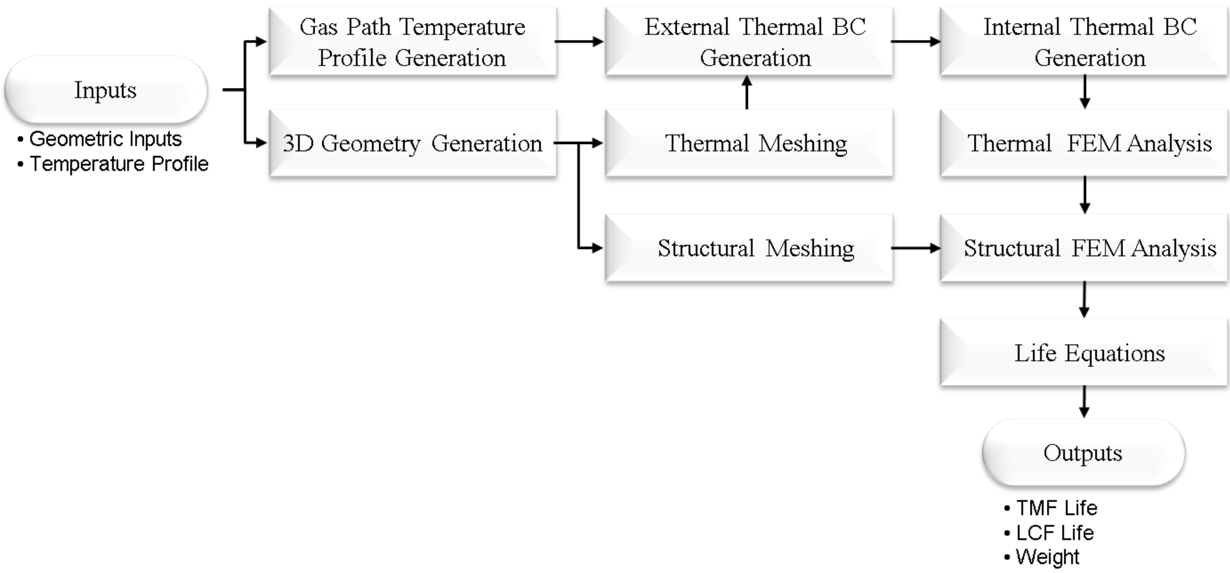

Figure 3: Automated Engineering Analysis Workflow

The 3-dimensional TMTF geometry was modeled and meshed using Unigraphics and Unigraphics Advanced Simulation. The weight of the part was also calculated in Unigraphics. Two separate meshes were created for the analysis, one thermal and one structural. Transient Internal and external boundary conditions were generated via proprietary Pratt & Whitney software. The thermal and structural analyses were solved in ANSYS. During the transient structural solution, the stress, strain, and metal temperature were stored at the life limiting time point for the life limiting regions. These values were then used to calculate TMF life and LCF life via a proprietary Pratt & Whitney lifing system.

All of the analyses were coupled together in an automated engineering analysis workflow via the CCE tool. Each piece of the workflow was parametric with the ability to be executed on a compute cluster. Once the pieces of the workflow were finished, the IPT collaboratively built and executed the automated workflow. The development of the automated workflow eliminated a major hurdle of coupling models and analyses together to create a parametric physics-based model. CCE enabled the automated workflow because each engineer had ownership over their own piece of the workflow. This means they were responsible for building, testing, and controlling the execution of their piece of the workflow. Furthermore, CCE ties the analysis files to a revision management system. This allowed automation experts to work with the IPT since files could easily be shared and kept in sync. CCE allowed the IPT to build, test, and execute the workflow while satisfying schedule constraints.

The automated workflow was executed at 870 discrete design points. A Latin Hypercube (Fang et al. 2006) was generated to fill the design space. There were twenty-nine input variables in the Latin Hypercube. The team selected thirty runs per input variable because this was expected to sufficiently fill the design space to produce an accurate emulator. The automated workflow achieved a 60.9% success rate. Due to schedule constraints, the IPT was not able to improve the robustness of the models any further. The biggest causes of model failure were structural meshing and parametric geometry. The structural meshing was difficult due to refined sub models and maintaining boundary conditions at the cut plane boundaries. The parametric geometry would fail due to complex endwall contouring not updating in the Unigraphics model. A Root Cause Corrective Action (RCCA) is proposed in the sustain section of this paper to address this issue.

3.3 Model Input Variability and Uncertainty Quantification

Each geometric variable was assumed to follow a normal distribution and each parameter was assumed to be independent. The standard deviation associated with each distribution was selected based on expert opinion. The goal of the analysis was to determine the mean of the distribution. The normal distribution and independence assumptions were not validated due to time constraints. Ideally, manufacturing data from other similar parts that use the same manufacturing process would be used to test the normal and independent assumptions.

The thermal profile was the 1-dimensional radial gas path temperature profile in front of the TMTF. Proprietary Pratt & Whitney models used the thermal profile as an input to determine the full thermal boundary conditions that were applied to the TMTF. There were three data sets available to quantify the uncertainty in the gas path temperature profile. Unfortunately, these data were from different engines that had different combustor designs, different HPT airfoils counts, and different core sizes. While applying this data to the TMTF was an extrapolation, it was the most applicable engine data available. Each data set contained temperature measures at fixed circumferential and radial positions for multiple time points and power conditions.

The goal for the uncertainty quantification was to model the engine-to-engine uncertainty and apply it to the worst temperature profile. Because this was a single cast part, the worst profile at any circumferential location was used in the model. If one airfoil fails, the entire part fails. The analysis system was designed to model the worst temperature profile and subsequently scale the thermal results to different power conditions. While the gas path temperature profile shape certainly changes with different power conditions, it was assumed to be fixed for this analysis because the analysis tools could not handle analyzing different temperature profiles at different power conditions. Hence, the goal was to quantify the uncertainty of engine to engine variation in the worst temperature profile, regardless of circumferential location.

The 1-dimensional radial gas path temperature profile was typically modeled as a non-dimensional profile factor (pf) instead of an absolute temperature. The formula for profile factor was:

$$pf = \frac{T_{local} - T_{avg}}{T_g - T_c} \tag{1}$$

where T_{local} was the local temperature value at given radius, T_{avg} was the average temperature of the temperature profile, T_g was the average hot gas path temperature produced by the combustor, and T_c was the cooling air temperature from upstream of the combustor.

Each engine dataset had temperature profiles at multple circumferential locations at many time points. Since the worst profile was being modeled, the data needed to be processed to find the worst profile, regardless of circumferential location, for every time point. Thus, an equation that relates temperature profile to min part life was developed to determine the worst temperature profile.

To develop the equation, a Latin Hybercube Sample (Fang et al. 2006) of one hundred temperature profiles was generated and run through the full analysis. Each profile was generated using a b-spline with five independent coefficients (Ramsay et al. 2009). Once the thermal, structural, and life analyses were complete, each profile had an associated TMF life and LCF life at each fillet location. From analyzing the data, the TMF life was more limiting than the LCF life. Therefore, the LCF life was eliminated and the minimum TMF life was used. Using this data, an equation was developed that relates b-spline coefficients (k) and profile factor (pf) to the TMF life. Figure 4 shows the results of the residual analysis.

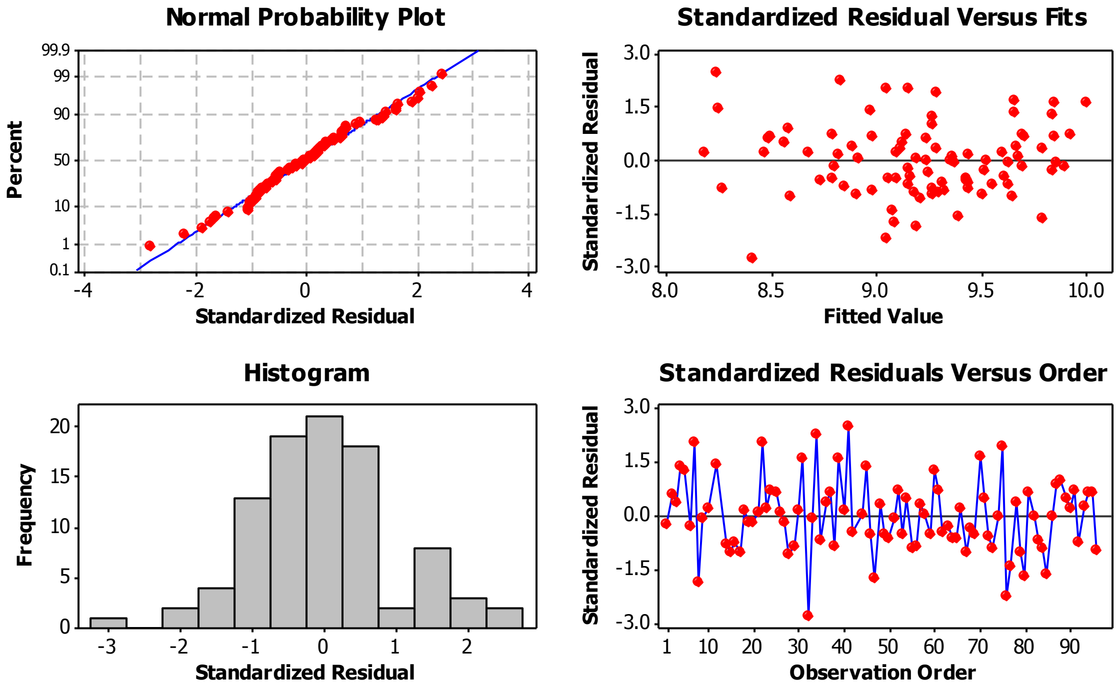

Figure 4: Residual Plots for TMF life

The R^2 value for the equation was 99.1%. The residual analysis shows that the residuals are normally distributed and independent of the fitted values. Given the high R^2 value and the residual analysis, it was suitable to use this equation to determine the worst profile at a given time point.

For each time point of each engine dataset, the worst profile was determined via the TMF life equation. With this dataset of worst temperature profiles, a Bayesian hierarchical model fitting procedure was executed to develop a statistical sample of temperature profiles. This statistical sample represents the engine to engine variation in the worst temperature profile. The sample was then centered on a computationally derived worst profile. The computationally derived worst profile was treated as a nominal worst profile.

3.4 Model Emulation and Sensitivity Analysis

Due to the large number of outputs, a reduction in the number of outputs was necessary. This reduced the emulator building and execution time and allowed schedule constraints to be met. There were some regions that were obviously not life limiting when compared to other regions. Furthermore, the life results of many regions were highly correlated. In many of these instances, one region always had lower life than another. In these cases, the higher life region was eliminated. For this problem, the thirty-three outputs were reduced to ten, seven TMF regions, two LCF regions, and weight.

Emulators were created for strain, stress, and temperature and life equations were then used directly. The life equations were easy to program and ran quickly so there was no need to emulate life. Furthermore, emulating life values was difficult due to the inherent nonlinearities. The emulators were created via the R (R Development Core Team 2010) package MLEGP (Dancik 2010). MLEGP stands for Maximum Likelihood Estimate of Gaussian Processes. For this problem, this approach delivered the best accuracy while maintaining execution time requirements when compared to other available emulation software. The emulator error was higher than the desired goal of +/- 4%. Emulator error is quantified through the Cross Validation Root Mean Squared Error (CVRMSE) statistic. The goal of +/- 4% was derived from the typical uncertainty in the FEM due to mesh size. Per past experience, this level of uncertainty was deemed to be acceptable. The likely cause of the emulator error was the 39.1% failure rate in the Latin Hypercube runs. The design space was not adequately filled to generate a quality emulator. However, the emulators were still trend wise accurate for the important drivers and emulator uncertainty was accounted for in the final uncertainty analysis. Actual versus predicted plots for a subset of the outputs are shown below.

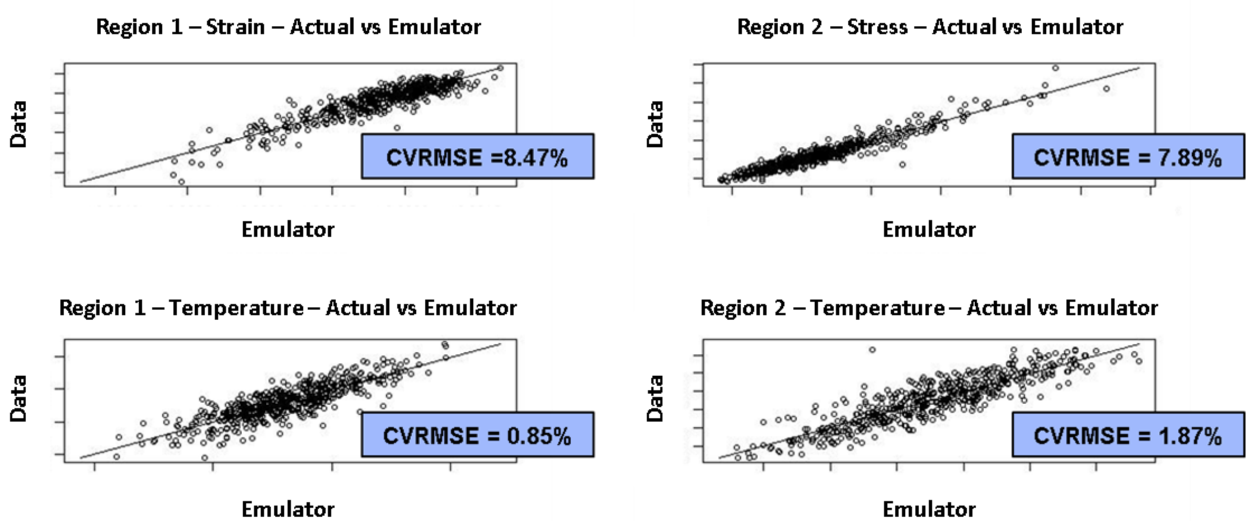

Figure 5: Actual versus Predicted Plots for Emulators

Regions 1 and 2 in figure 5 are two regions that could be life limiting. Figure 5 shows that the emulators are highly accurate for temperature based on the CVRMSE but are less accurate for strain and stress. However, the emulators are acceptable to make trend wise decisions and the emulator error will be accounted for in the final uncertainty analysis. For the sensitivity results, the emulator error was the largest contributor to the variance in the life output as shown in figure 6. This emulator error could cause the weight of the final part to be higher to ensure that the life metric would be met. Life may need to be increased at the expense of weight to account for the large degree of emulator error.

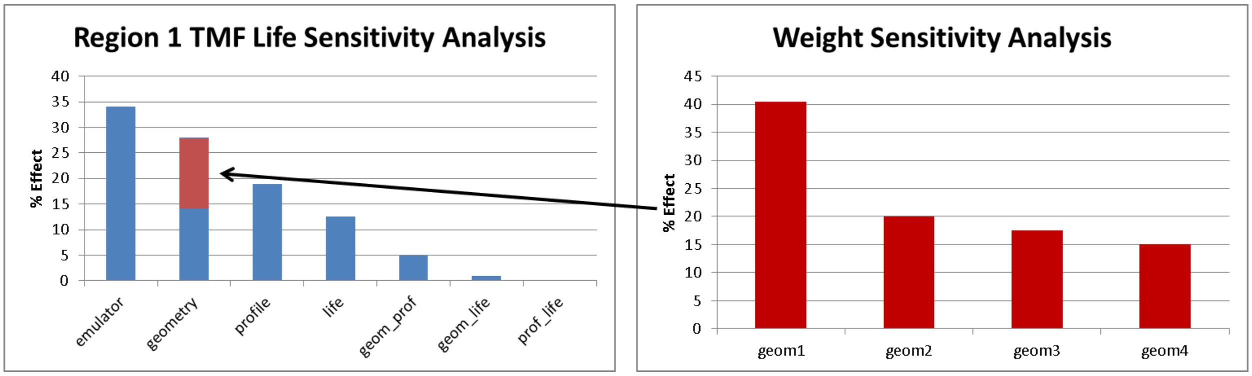

Figure 6: Sensitivity Analysis Results

These sensitivity results were the global sensitivity results. The entire range of the geometric values was used to calculate the sensitivity. A uniform distribution was assumed for the geometric parameters. In the left hand sensitivity analysis, geom_prof, geom_life, and prod_life represent interaction effects of geometry and profile, geometry and life, and profile and life, respectively. The full profile and life uncertainty was captured. Intuitively, these results made sense based on expert opinion. This shows that the selected nominal design was the most important factor towards determining the life of the part. Furthermore, there was minimal interaction between the parameters. Thus, the same engineering design would be optimal regardless of the temperature profile and life coefficients. This shows that the trend wise accurate emulators could be used to find the optimal design. However, the actual life results of the optimal design point would have to be determined by the full computational analyses.

The sensitivity results revealed a beneficial situation. Half of the geometric factors that drove life did not drive weight. The right hand sensitivity analysis in figure 6 shows the major drivers of weight and the left hand sensitivity analysis shows the level of impact those parameters have on life. Thus, there was an opportunity to increase the life of the part without increasing weight because there were parameters that affected life but not weight. Once all of those parameters were optimized, the IPT would have to increase the weight of the part to increase life. Furthermore, the uncertainty in the life result due to emulator error would require an increase to the nominal life of the part to ensure that the life metric was met.

3.5 Model Calibration

The TMF life model was a nonlinear regression equation derived from experimental data. The two inputs were strain and temperature. The TMF life equation had three uncertain coefficients: a, b, and c that represent the intercept and the coefficients on strain and temperature. A Bayesian regression was performed to determine the full posterior distributions of a, b, c and the square root of the residual variance along with the associated Maximum a Posteriori estimates based on the experimental data. To facilitate numerical stability, the Markov Chain Monte-Carlo (MCMC) was executed on the natural logarithm of TMF life instead of TMF life directly. For a detailed description of Bayesian regression theory, see Gelman et al. 2003. The Bayesian regression was executed using the R package R2WinBUGS (Sturtz et al. 2005). The full posterior distributions of the life equation coefficients and the residual variance are shown in figure 7.

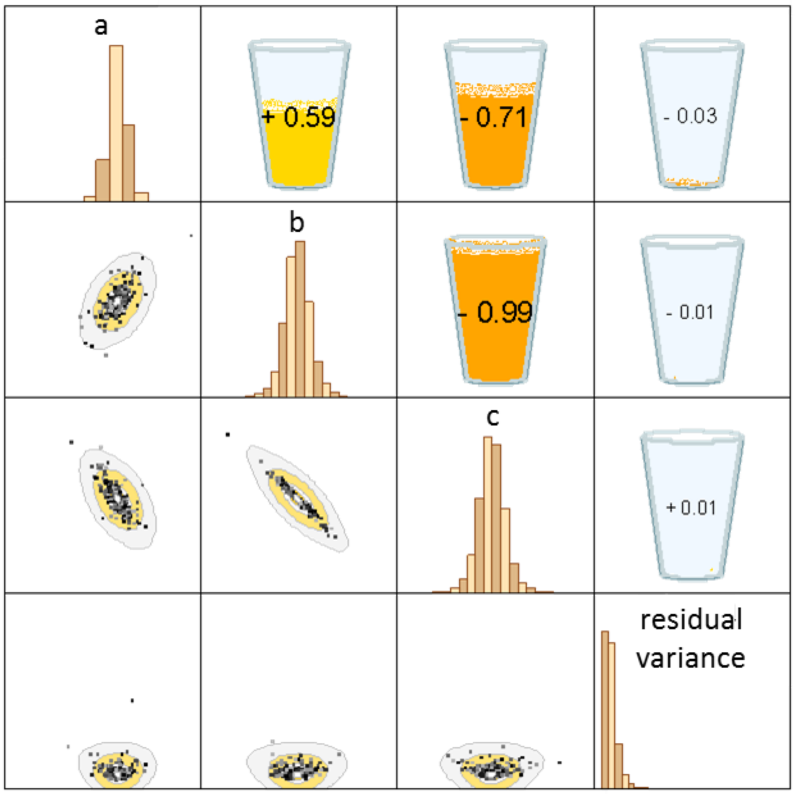

Figure 7: Beer and Eggs Correlation Plot (Schnute et al. 2014) of TMF Life Equation Parameter Distributions

The eggs in the lower left hand corner are a contour plot that shows the correlation between parameters. The beer in the upper right hand corner shows the strength and direction of the correlation. Visually, the residual variance follows the chi-squared distribution, as is expected for the regression. The correlation plot of the posterior distributions showed that the life equation parameters were highly correlated. Thus, the MCMC samples were used directly in the TMTF probabilistic life assessment. The regression uncertainty in the TMF life equation was large relative to the lives predicted by the equation. This was large contributor to the final TMF life prediction uncertainty for the TMTF.

3.6 Uncertainty Analysis

The objective of the uncertainty analysis was to propagate all sources of uncertainty and variation through the computational analysis to calculate probabilistic distributions on all of the desired outputs. To do this, a Monte-Carlo analysis was executed for a given nominal design point. First, a nominal design point was selected and distributions were calculated for each of the geometric variables. Each geometric variable had a normal distribution and was independent of the other geometric parameters. Second, a sample of temperature profiles was generated. With this information, emulators were used to predict stress, strain, temperature, and weight. With each emulator prediction, a nugget error term was added to the prediction to account for the emulator error. Third, LCF and TMF life were calculated from the emulator stress, strain, and temperature values. For TMF life, a sample of life equation coefficients was generated to calculate the sample of TMF life. The LCF and TMF life samples were created for the nine regions of interest on the TMTF. Finally, the minimum LCF and TMF lives across all of the regions were saved to produce probabilistic distributions for the minimum LCF and TMF lives. These distributions along with the probabilistic weight distribution constituted the objective of the uncertainty analysis. This procedure is illustrated in figure 8. It is important to note that the uncertainty analysis was computationally feasible because of the speed of the emulators.

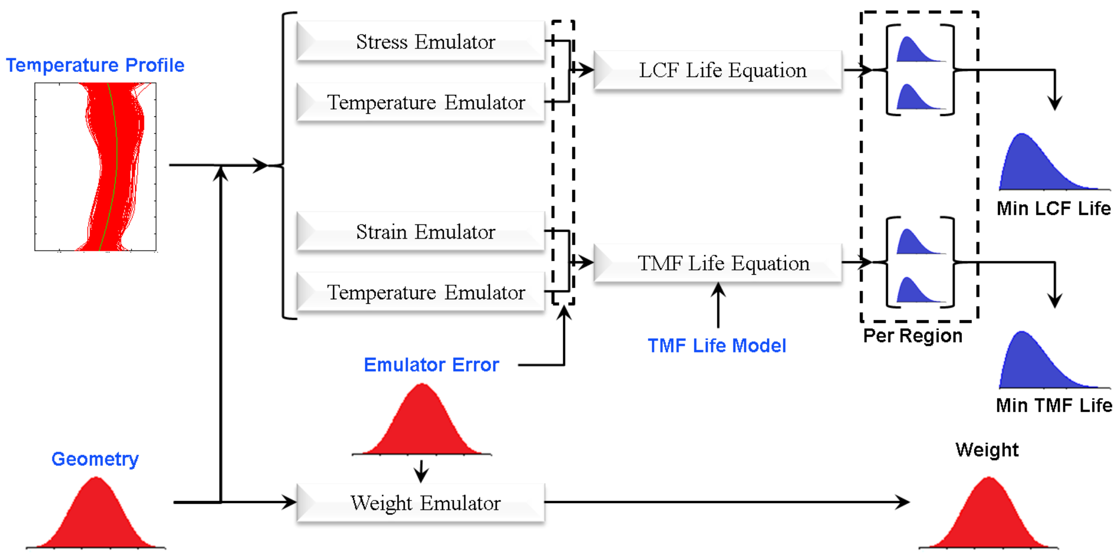

Figure 8: Uncertainty Analysis Procedure

3.7 Solve – Probabilistic Solution

The goal of the TMTF IPT was to find a nominal design that was robust to variation and uncertainty, such that, the design objectives were met. Thus, an optimizer was wrapped around the uncertainty analysis to determine the nominal part design (μ). The standard deviations (σ) were fixed because there was no output in the analysis, such as, cost or manufacturability to prevent the optimizer from driving the standard deviation to zero. The objective for the optimization was to minimize the mean weight while satisfying the TMF and LCF life constraints. The R function optim (Nash and Ravi 2011) and the algorithm L-BFGS-B algorithm (Byrd et al. 1995) were used for the optimization. The optimization procedure is illustrated in figure 9.

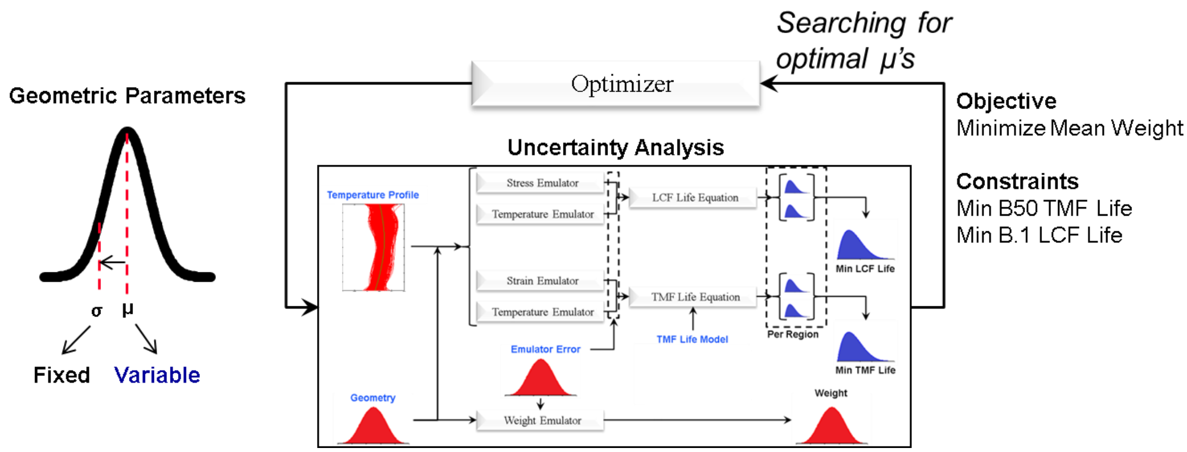

Figure 9: Optimization Procedure

The life equation used for TMF life predicts actual TMF life while the LCF life equation predicts the B.1 LCF life. B.1 LCF life was the life of a 0.1 percentile part. The weight equation predicted the actual weight. The probabilistic solutions are shown in figure 10.

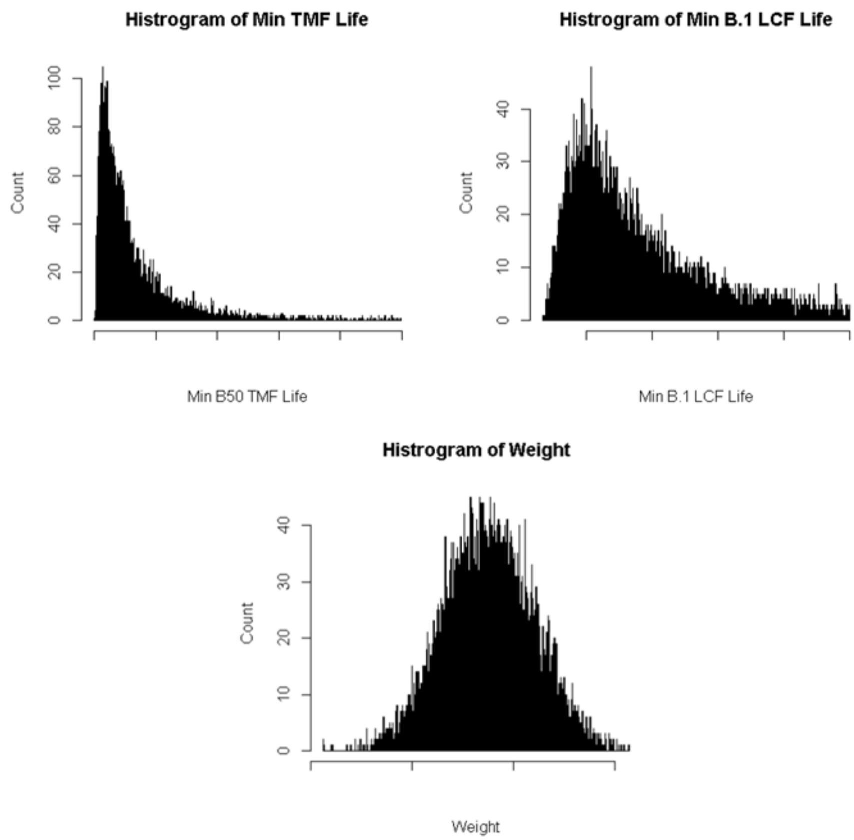

Figure 10: Probabilistic Results for TMF Life, LCF Life, and Weight

The probabilistic requirements for TMF life and LCF life were met. Figure 10 shows the distribution of minimum B50 TMF life and the minimum B.1 LCF life. 50% of the lives are below the B50 life and 0.1% are below the B.1 life. The IPT was able to access the probability of satisfying the TMF life and LCF life requirements by using these distributions. The factors that increased TMF life also increased LCF life. Between the TMF and LCF life, the TMF life requirement was more difficult to satisfy. However, the TMF life metric was met even after accounting for uncertainty. Because of this, weight was able to be significantly reduced. This was an advantage of the automation and the probabilistic approach. The optimization was able to minimize weight and meet the life constraints since the driving parameters were partially decoupled. In a manual deterministic approach, the engineer would not have known how the parameters were decoupled. The automated, probabilistic approach yielded a significant benefit.

3.8 Verify and Validated Assumptions

At this point, the model accounted for variation and uncertainty in the model inputs, as well as uncertainty in the model itself. The model has also been exercised to determine how probabilistic design requirements can be achieved. This process has depended on the process model being an accurate representation of the real world. As the part enters the manufacturing stage, all the variation and uncertainty assumptions and results should be validated.

The sensitivity results in figure 6 shows that emulator error was the largest driver of life uncertainty. A RCCA should be conducted to determine the cause of the emulator uncertainty. For the TMTF, one likely cause is the high failure rate of the analyses due to lack of robust parametric geometry and meshing. The analysis should then be re-executed using emulators with reduced uncertainty. The 2nd biggest driver of uncertainty was the geometric variation. The geometry parameters were assumed to be independent and normally distributed. To validate this assumption, the first sets of hardware should be measured. The 3rd biggest driver of life uncertainty was the thermal temperature profile. The temperature profile uncertainty sample should be updated when engine data becomes available. Furthermore, high fidelity CFD analysis can be used for a more accurate prediction of the gas temperature profile. Finally, highly instrumented engine tests should be used to validate the CFD model. The 4th largest drive of uncertainty was the life equations. Additional TMF life data should be gathered to reduce the life equation parameter uncertainty. Furthermore, physics-based TMF life prediction can be used to obtain a more accurate life prediction. Once these sources of uncertainty and variation have been verified and validated, the nominal design can be ensured to be robust.

3.9 Sustain

Once the model has been validated, data collection and testing continue, but the purpose of the data collection changes to ensuring that the model remains consistent with the real world. At this time, efforts are made to stabilize the important causes of process performance variation that are under manufacturing control and to monitor those that are not. For geometric parameters, important parameters will be Key Product Characteristics. Less important parameters could have their tolerances expanded to improve manufacturability or reduce costs if supported by an uncertainty analysis. The temperature profile uncertainty should be tied back to geometric variables in other parts via a combined combustor-turbine gas path uncertainty analysis. Once the important drivers of temperature profile are discovered, they can be appropriately controlled and monitored. Finally, Statistical Process Control can be applied with periodic material tests to ensure that material properties do not significantly change over time.

4.0 Conclusions

The probabilistic approach yielded significant benefits for the design of the TMTF. All key sources of variation and uncertainty were quantified to ensure that the part would meet the design criteria. Sensitivity analysis revealed the important input drivers of life and weight. Emulators were generated because they were much more computationally efficient and their ability to be used in Monte-Carlo analysis and optimization. A full uncertainty analysis within an optimization yielded a final design that significantly reduced the weight while meeting the life metrics.

The key challenges were building a robust parametric geometry model, robust meshing, and building quality emulators. For the TMTF, it was challenging to get the complex 3-dimensional geometry to automatically and consistently update for new geometric values. In addition, it was difficult to robustly mesh the geometry. This was related to the quality of the geometry being produced but there were also issues like maintaining cyclic boundary conditions that were specific to meshing. It is expected that parametric geometry and robust meshing will be challenging for other complex parts as well. Finally, it was hard to build quality emulators for stress and strain. If the design space were more adequately filled, then more accurate emulators could have been generated. However, it is expected that stress and strain will continue to be more challenging to emulate for complex parts.

If the automated workflow and probabilistic approach were not employed, it was unlikely that the IPT would have found a design that satisfied all the criteria. The IPT would not have been able to explore the entire design space. Only ten to twenty analyses could have been completed within the schedule. If the team were to find a viable solution in the handful of cases, the IPT would have little confidence that the manufactured part would satisfy life requirements in a real engine. Factors of safety would have to be assumed and it would be difficult to determine if the factors of safety were sufficient without explicitly accounting for sources of variation and uncertainty. Clearly, the automated workflow coupled with the probabilistic analysis yielded significant benefits.

References

ANSYS® (2007). ANSYS Mechanical Version 11.0, ANSYS, Canonsburg, PA.

Bunker, R. S. (2009). “The Effect of Manufacturing Tolerances on Gas Turbine Cooling”. *ASME: Journal of Turbomachinery*, 131, 1 – 11.

Byrd, R. H., Lu, P. and Nocedal, J.. A Limited Memory Algorithm for Bound Constrained Optimization, (1995), SIAM Journal on Scientific and Statistical Computing , 16, 5, pp. 1190-1208.

Dancik, G. M. (2010). mlegp: Maximum Likelihood Estimates of Gaussian Processes. R package version 3.1.1. http://CRAN.R-project.org/package=mlegp

Fang, K., Li, R., Sudjianto, A. (2006). Design and Modeling for Computer Experiments. Chapman & Hall/CRC, Boca Raton, Florida.

Kennedy, M. C. O’Hagan, A. (2001). “Bayesian Calibration of Computer Models (with Discussion)”. *Journal of Royal Statistical Society*, Series B 63, 425-464.

Gelman, A., Carlin, J.B., Stern, H.S., Rubin, D.B. (2003). Bayesian Data Analysis, 2nd ed. Boca Raton, FL: Chapman & Hall=CRC.

Heinze, K., Schrape, S., and Voigt, M. (2012). "Probabilistic endurance level analyses of compressor blades" *CEAS Aeronaut J*, 3(1), 55 – 65.

Nash, J. C., Ravi Varadhan R. (2011). Unifying Optimization Algorithms to Aid Software System Users: optimx for R. Journal of Statistical Software, 43(9), 1-14. URL http://www.jstatsoft.org/v43/i09/.

R Development Core Team (2010). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL http://www.R-project.org/.

Ramsay, J. O., Hooker, G., Graves, S. (2009). Functional Data Analysis with R and MATLAB. Spring Science+Business Media, LLC, New York, NY.

Reinman, G. et al. (2012). "Design for Variation". *Quality Engineering*, 24:2, 317-345.

Saltelli, A., Chan, K., Scott, E. M. (2001). Sensitivity Analysis. John Wiley & Sons Ltd., New York, NY.

Schnute, J. T., Couture-Beil, A., Haigh, R., Kronlund, A. R., Boers, N (2014). PBSmodeling: GUI Tools Made Easy: Interact with Models, Explore Data, Give Dynamic Presentations. R package version 2.15.0. http://cran.r-project.org/web/packages/PBSmodelling/index.html

Sturtz, S., Ligges, U., and Gelman, A. (2005). R2WinBUGS: A Package for Running WinBUGS from R. Journal of Statistical Software, 12(3), 1-16

Unigraphics® (2008). Version NX 6.0.5.2, Siemens, Munich, Germany.