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Retransmission of Corrupted and Lost Frames in a Computer Communication Network: A Queuing Theoretic Approach

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Abstract

In this paper, a finite capacity single server Markovian feedback queuing model with retention of reneging customer is applied in computer communication networks. The concept of retention of reneging customer is considered as a tracker to trace lost frame during transmission. We think of a tracer in transmission link which provide a facility

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to trace a lost frame and there are probable chances that a lost frame is traced. Feedback is taken as retransmission of a corrupted frame. The steady state solution of the model is obtained. The qualities of service measures are derived. The effect of probability of retention of reneging frame on the average buffer size is studied. The numerical results show that the average number of frames in buffer increases steadily whenever the probability of retention of reneging frames increases.

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1 Introduction

In order to handle flow control and error situations there are three protocols viz. Stop-and-Wait Automatic Repeat Request (ARQ), Go-Back-N ARQ and Selective Repeat ARQ. Stop-and-Wait ARQ, which is simplest among all protocols, the sender transmits a frame and then waits till it receives positive acknowledgement (ACK). If acknowledgement gets lost/delayed than the sender will not receive any ACK and therefore he will resend the frame after timeout, the receiver has the two copies of same frame but he will discard one as the sequence number for both will be same. If frame gets lost/corrupted than sender will not get any ACK and after timeout he will resend the frame.

In go-back-N ARQ, the sender sends the frames continuously without waiting for acknowledgement. If acknowledgement gets lost/delayed then the next expected frame is send. In frame lost/corrupted case the sender will not get any acknowledgement and so he will send the next frame but it is discarded as it is out of order the receiver is silent and will discard all subsequent frames until it receives the one it is expecting. After receiving expected frame out of order frames which were previously discarded are again retransmitted

The selective-repetitive ARQ scheme also accepts out of order frame and send negative acknowledgement (NAK) for the frame which it has not received, therefore in this case we avoid unnecessary transmission by sending only frames that are corrupted or lost in between the transmission process.

In computer communication, the transmission of protocol data unit is sometimes repeated due to occurrence of an error. Error control involves both error detection and error correction. It is necessary because errors are inevitable in data communication, in spite of the use of better equipment and reliable transmission media based on the current technology. To detect and correct corrupted frames, we need to add redundancy bits to our data frame. When the frame arrives at the receiver site, it is checked for error. Let p_1 be the probability of retransmission of a frame which gets corrupted during transmission and q_1 be the complimentary probability that the frame is not corrupted. We consider feedback as retransmission of a corrupted frame i.e. after getting corrupted frame by the receiver, the sender retransmit the corrupted frame again for service with probability p_1 or may not be retransmitted with complimentary probability. The corrupted frame when retransmitted is referred to as feedback frame.

Rework in industrial operations is an example of a queue with feedback. In case of feedback, after getting partial or incomplete service, customer retries for service. It represents customer dissatisfaction because of inappropriate quality of service. [Takacs, 1948] studies queue with feedback to determine the stationary process for the queue size and the first two moments of the distribution function of the total time spent in the system by a customer. [Davignon and Disney, 1973] study single server queues with state dependent feedback. [Santhakumaran and Thangaraj, 2000] consider a single server feedback queue with impatient and feedback customers. They study $M/M/1$ queueing model for queue length at arrival epochs and obtain result for stationary distribution, mean and variance of queue length. [Thangaraj and Vanitha, 2009] obtain transient solution of $M/M/1$ feedback queue with catastrophes using continued fractions. The steady-state solution, moments under steady state and busy period analysis are calculated. [Ayyappan et al. 2010] study $M/M/1$ retrial queueing system with loss and feedback under non-pre-emptive priority service by matrix geometric method.

In data communication network, each frame in the queue will wait a certain length of time for his service to begin by router. If it has not begun by then, it will leave the queue and get lost. The frame lost times (reneging) follows exponential distribution with parameter . This process in queueing theory is termed as reneging of a frame. Queuing with customer impatience is frequently used in call centres, packet switched data communications networks, hospitals, insurance and business etc. The concept of customer impatience appeared in the queuing theory in the work of Haight in 1957. He considered a model of balking for M/M/1 queue in which there is a greatest queue length at which an arrival would not balk. This length was a random variable whose distribution was same for all customers. [Haight, 1959] studied a queue with reneging in which he studied the problem like how to make rational decision while waiting in the queue, the probable effect of this decision etc. [Ancker Gafarian, 1963] study $M(M(/1)N)$ queuing system with balking and reneging and perform its steady state analysis. [Ancker and Gafarian, 1963] also obtain results for a pure balking system (no reneging) by setting the reneging parameter equal to zero.

Reneging is an important feature in various situations like call centres ([Garnett et al. 2002], [Kawanishi, 2008], and telecommunication networks ([Baccelli and Hebuterne, 1981], [Zohar et al. 2002]) where usually calling customer hangs up before service agent and thus gets reneged. In health care applications ([Brahimi and Worthington, 1991], [Kopach et al. 2008], the patients (customers) who leave the emergency rooms in hospitals without been seen are also considered as reneged customers. Kidney transplant waiting system can be considered as a queue with reneging where reneging occurs because a customer that is waiting for a kidney may die. [Kumar and Sharma, 2012] study Markovian feedback queuing system with retention of reneged customers. They obtain the steady state solution of the model and derived various performance measures.

Keeping in mind the adverse effect of queuing delay in transmission process and retransmission of corrupted frames in computer communication network, we modelled frames loss in queuing delay as reneging customers, retransmission of corrupted frame as feedback customers and tracing lost frame for receiver as the retention of reneging customers.. A reneging frame can be traced by a tracer and there are probable chances that a lost frame is traced. Thus, a reneging frame (lost frame) can be traced (retained) in the buffer with some probability (say, q_2) or it may not be traced with probability $p_2(= 1 - q_2)$. [Kumar and Sharma, 2012] study Markovian feedback queuing system with retention of reneged customers. They obtain the steady state solution of the model and derived various performance measures.

In this paper we extend the work of [Kumar and Sharma, 2012] and applied the phenomena of feedback customer as retransmission of corrupted frame and tracing a lost frame as retention of reneging frames in computer communication network. After getting corrupted frame by the receiver, the sender retransmits the corrupted frame again for service with probability p_1 or may not be retransmitted with complimentary probability. The frame loss in transmission process is modelled as reneging frames and probable chances of tracing lost frames by a tracer is considered as the retention of reneging frames in the buffer. We study the impact of probability of retention of reneging frames and explain numerically that with the increase in probability of retention of reneging frames, the frame loss during transmission decreases simultaneously.

Rest of the paper is arranged as follows: model is described in section 2, the differential difference equations of the model are provided in section 3 and the solution of model is in section 4 and in section 5, the quality of service measures and numerical results are provided in section 6, the paper is concluded in section 7.

2 Model Description

We extend the work of [Kumar and Sharma, 2012] where an $M/M/1/N$ queue with instantaneous Bernoulli feedback with reneged customers and retention of reneged customers is considered. The capacity of the system is taken as finite. Frames arrive at the receiver end one by one according to Poisson stream with arrival rate λ. There is one receiver which receives all arriving frames. Receiving times are independently and identically distributed exponential random variables with parameter μ. After getting corrupted frame by the receiver, the sender retransmits the corrupted frame again for service with probability p_1 or may not be retransmitted with complimentary probability. The corrupted frame when retransmitted is referred to as feedback frame. The frames both newly queued frames and those that are corrupted are received in order in which they join the buffer for transmission purpose. We do not distinguish between the regular frames and feedback frames. The frames are received according to first come, first served rule. The frames in queue (regular frame or feedback frame) may get lost. In fact, each frame, upon arrival, has an individual timer, which follows an exponential distribution with parameter ξ. This time is reneging time of an individual frame after which frame either may not be traced by the tracer with probability p_2 and thus gets lost or it may be traced with complimentary probability say q_2 and is transmitted successfully. A system of difference differential equations satisfied by the $M/M/1/N$ feedback queue with reneged customers and retention of reneged customers are modelled as a continuous time Markov chain (CTMC). Let $X(t) : tR^+$ be the number of customers in the system at time t. Let $P_n(X(t) = n), n = 0, 1, 2, .., N$ be the state probabilities that there are n frames in the buffer at time t.

3 Chapman-Kolmogorov forward differential-difference equations

Based on above assumptions the Chapman-Kolmogorov forward differential-difference equations of the model are

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu q_1 P_1(t) \tag{1}$$

$$\frac{dp_n(t)}{dt} = -[\lambda+\mu q_1+(n-1)\xi p_2]P_n(t)+(\mu q_1+n\xi p_2)P_{n+1}(t)+\lambda P_{n-1}(t) \quad 1 \leq n \leq N-1 \tag{2}$$

$$\frac{dP_N(t)}{dt} = \lambda P_{N-1}(t) - [\mu q_1 + (N-1)\xi p_2]P_N(t) \quad n = N \tag{3}$$

The steady-state solution of these equations are obtained in the next section.The probability P_0 implies that there is no frame in the buffer i.e. the transmission link does not have any frame to transmit at that time and P_n is the probability that there are n frames in the buffer.

4 Solution of the Model

In steady state $\lim_{t\to\infty} P_n(t) = P_n$ and therefore $\frac{dP_n(t)}{dt} = 0$ as $t \to \infty$ Thus, the steady state equations corresponding to (1)-(3) are as follow

$$0 = -\lambda P_0 + \mu q_1 P_1 \tag{4}$$

$$0 = -[\lambda+\mu q_1+(n-1)\xi p_2]P_n+(\mu q_1+n\xi p_2)P_{n+1}+\lambda P_{n-1} \quad 1 \leq n \leq N-1 \tag{5}$$

$$0 = \lambda P_{N-1} - [\mu q_1 + (N-1)\xi p_2]P_N \quad n = N \tag{6}$$

Solving iteratively equations (4)-(6), we get

$$P_n = \prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2} P_0 \tag{7}$$

P_n is the steady-state (long-run) probability that there are n frames in the buffer. Using the normalization condition, $\sum_{n=0}^{N} P_n = 1$,we get

$$P_0 = \frac{1}{1 + \sum_{n=1}^{N} \prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2}} \tag{8}$$

The probabilities P_0 and P_n in equation (7) and (8) are used to compute the quality of service measures.

5 Quality of Service Measures

In this section, some important quality of service measures viz. expected number of frames in the buffer, expected number of frames received successfully by the receiver, average number of frame lost due to reneging, average number of reneging frames retained or traced by the tracer for receiver are derived. Numerical results are also obtained to highlight the effect of tracing the lost frames on number of successfully received frames by the receiver.

(i) Expected Number of Frames in the Buffer (L_s)

$$L_s = \sum_{n=1}^{N} n[\prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2}]P_o \tag{9}$$

(ii) Expected Number of Frames Received Successfully ($E(F.R.S.)$)

$$E(F.R.S.) = \mu q_1 \sum_{n=1}^{N} [\prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2}]P_o \tag{10}$$

(iii) Average Rate of Frame lost due to Reneging (R_r)

$$R_r = \sum_{n=1}^{N} (n-1)\xi p_2[\prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2}]P_o \tag{11}$$

(iv) Average Rate of Reneging Frames Retained (R_R)

$$R_R = \sum_{n=1}^{N} (n-1)\xi q_2[\prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2}]P_o \tag{12}$$

where P_0 is given in (9).

(v) The Expected Number of customers Served, E(Customer Served)

$$E(\text{Customer Served}) = \sum_{n=1}^{N} n\mu q_1 P_n$$

$$E(\text{Customer Served}) = \sum_{n=1}^{N} n\mu q_1 [\prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2}] P_0 \qquad (13)$$

Where P_0 is computed in (8).

6 Numerical Results

Here we provide numerical results for different quality of measures. We study the variation in quality of measures with respect to probability of retention of reneging frames q_2. When $N = 10, \lambda = 4, \mu = 5q_1 = 0.7$ and $\xi = 0.4$ we have the following observations:

$S.No.$	q_2	L_s	$E(F.R.S)$	R_r	R_R
1	0	3.19502	3.026383	0.691991	0
2	0.1	3.35689	3.051796	0.657991	0.07311
3	0.2	3.54053	3.07879	0.621483	0.155371
4	0.3	3.74972	3.107471	0.581355	0.249152
5	0.4	3.98880	3.137923	0.536085	0.35739
6	0.5	4.26255	3.170179	0.483629	0.483629
7	0.6	4.575926	3.204186	0.421322	0.631983
8	0.7	4.933527	3.239754	0.345824	0.806922
9	0.8	5.338653	3.27649	0.253192	1.012766
10	0.9	5.791887	3.313729	0.139194	1.252746
11	1	6.289261	3.350488	0	1.527534

Table 1: Quality of Service Measures Vs. Probability of Retention of Reneging Frames (q_2)

Table-1 indicates that as we increase the probability of retaining a reneging frame there is a simultaneous increase in the average number of frames received successfully by the receiver, the average number of frames lost due to reneging deceases and average number of reneging frames reneging frames retained increases. Here, the average rate of frame loss

due to reneging decreases subsequently on increasing the probability of retaining the reneging frames q_2. Thus, one can study the effect of different probabilities of retaining reneging frames on the different service measures.
When $q_2 = 0$, there is no retention of reneging frames. This is the case of an $M/M/1/N$ queueing model with reneging and corrupted or feedback frames and therefore $R_R = 0$. Also, when $q_2 = 1, R_r = 0$, that is, all lost frames are retained or traced in this case. Moreover when $q_1 = 1$, we have the case of $M/M/1/N$ queuing model with reneging frames. In absence of the probability of tracing a lost frame or tracer i.e. $q_2 = 0$, our model reduces to one with reneging and corrupted frames.

7 Conlusion

In this paper, we study a computer communication situation in which the frames in the transmission link either get lost due to reneging or may get corrupted due to error in transmission and a finite capacity single server Markovian feedback queuing model with reneging frames and retention of reneging frames is considered. The steady-state solution of the model and some quality of service measures are derived.
The numerical results show that the average number of frames reaching receiver completely and successfully increases steadily as the probability of retaining the reneging frames increases. The model results may be useful in modeling computer communication networks and service processes involving frame loss and retransmission of corrupted frames.

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