AMO - Advanced Modeling and Optimization, Volume 17, Number 2, 2015

SOME NEW FIBONACCI DIVISOR CORDIAL GRAPHS

Poulomi Ghosh [1], Anita Pal [2]

[1, 2] Department of Mathematics, NIT Durgapur, 713209, West Bengal, India

[1] e-mail: poulomighosh.ju@gmail.com

[2] e-mail: anita.buie@gmail.com

Abstract
Let G=(V,E) be a (p,q) graph and f: V→{F_1,F_2,…..,F_p} be a bijection,where F_i is the i-th Fibonacci number. For each uv assign the label 1 when f(u) divides f(v) or f(v) divides f(u) and 0 otherwise. Then f is called a Fibonacci Divisor Cordial Labeling if the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. If a graph has a Fibonacci divisor cordial labeling, then it is called Fibonacci divisor cordial graph. In this paper, we prove that the graphs obtained by switching of arbitrary vertex of cycle and wheel, duplication of arbitrary vertex of cycle, degree splitting graph of path are Fibonacci divisor cordial graphs. We also prove that the Umbrella graph and the bistar $B_{n,n}$ are also Fibonacci divisor cordial graphs.

AMS subject classification: 05C78

Key Words: Cordial Labeling, Divisor Cordial Labeling, Fibonacci Divisor Cordial Labeling.

1. INTRODUCTION

We begin with simple, finite, connected and undirected graph G=(V,E) with p vertices and q edges. For standard terminology and notations related to graph theory we refer to [Harary, 1972] while for number theory we refer to [Burton, 1980] and graph labeling, we refer to [Gallian, 2013]. We will provide brief summary of definitions and other information which are necessary for present investigations.

Definition 1.1: If the vertices of a graph are assigned values subject to certain condition(s) then it is known as graph labeling.

Definition 1.2: A mapping f:V→{0,1} is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f.

Notation 1.3: If for an edge e=uv, the induced edge labeling $f^*: E \rightarrow \{0,1\}$ is given by $f^*(e)=|f(u)-f(v)|$. Then $v_f(i)$ = number of vertices having label i under f and $e_f(i)$ = number of edges having label i under f*.

Two of the most important types of labeling are called graceful and harmonious. A third important type of labeling, which contains aspects of both of the other two, is called cordial labeling and was introduced by [Cahit, 1990].

Definition 1.4: Let G=(V,E) be a (p,q) graph. Let $f:V \rightarrow \{0,1\}$ and for each edge uv, assign the label $|f(u)-f(v)|$. Then the binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.A graph G is cordial if it admits cordial labeling.

Graph labeling [4] is a strong communication between number theory [Burton,1980] and structure of graphs[Harary,1972]. By combining the Fibonacci number and divisibility concept in number theory and cordial labeling concept in Graph Labeling, R. Sridevi et al.[Sridevi,2013] introduced a new concept called Fibonacci Divisor cordial labeling.

Definition 1.5: Let a and b be two integers. If a divides b means that there is a positive integer k such that b=ka. It is denoted by a/b.

Definition 1.6: The Fibonacci numbers can be defined by the linear recurrence relation satisfying the following conditions:

$$F_n = \begin{cases} 0 & \text{if } n=0 \\ 1 & \text{if } n=1 \\ F_{n-1} + F_n & \text{if } n>1 \end{cases}$$

This generates the infinite sequence of integers beginning 0,1,1,2,3,5,8,13,21,34,55,89,……..

Theorem 1.7: [Burton,1980] F_m/F_n iff m/n , where $m,n \geq 3$.

R. Varatharajan et al.[varatharajan,2011], have introduced the notion of divisor cordial labeling:

Definition 1.8: Let G=(V,E) be a simple graph and $f:V \rightarrow \{1,2,\ldots\ldots,|V|\}$ be a bijection. For each edge uv, assign the label 1 if either f(u)/f(v) or f(v)/f(u) and the label 0 otherwise. f is called a Divisor cordial labeling if $|e_f(0)-e_f(1)| \leq 1$. A graph with a divisor cordial labeling is called a divisor cordial graph.

Definition 1.9: Let G=(V,E) be a simple (p,q) graph and $f:V \rightarrow \{F_1, F_2,\ldots,F_p\}$, where F_i is the i^{th} Fibonacci number, be a bijection. For each edge uv, assign the label 1 if either f(u)/f(v) or f(v)/f(u) and the label 0 otherwise. f is called a Fibonacci Divisor Cordial Labeling if $|e_f(0)-e_f(1)| \leq 1$.

A graph with a Fibonacci Divisor Cordial labeling is called a Fibonacci Divisor Cordial graph.

Definition 1.10: A vertex switching G_v of a graph G is obtained taking a vertex v of G, removing all the edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.11: Duplication of a vertex v_k of a graph G produces a new graph G_1 by a vertex $v_{k'}$ with $N(v_k) = N(v_{k'})$.

Definition 1.12: Let G=(V,E) be a graph with $V=S_1 U S_2 U \ldots\ldots U S_t U T$ where each S_i is a set of vertices having at least two vertices of the same degree and T = V-(¿i=1¿tSi¿ . The degree splitting graph of G denoted by DS(G) is obtained from G by adding vertices $w_1, w_2, \ldots\ldots, w_t$ and joining to each vertex of S_i for $1 \leq i \leq t$.

Definition 1.13: A graph obtained from a fan by joining a path of length m, P_m to a middle vertex of a path P_n in fan F_n . It is denoted by U(m,n) and it is called an Umbrella.

2. MAIN RESULTS

Theorem 2.1: The graph obtained by switching of an arbitrary vertex in cycle C_n is a Fibonacci Divisor Cordial Graph.

Let $v_1, v_2, \ldots\ldots, v_n$ be the successive vertices of C_n, and G_v denotes the graph obtained by switching of vertex v of G. Without loss of generality let the switched vertex be v_1. We note that $|V(G_{v1})| = n$ and $|E(G_{v1})| = 2n - 5$. We define, f: $V(G_1) \rightarrow \{F_1, F_2, \ldots\ldots, F_n\}$ as follows:

$$f(v_1) = F_1$$

$$f(v_i) = F_i \ ; \ 2 \leq i \leq n.$$

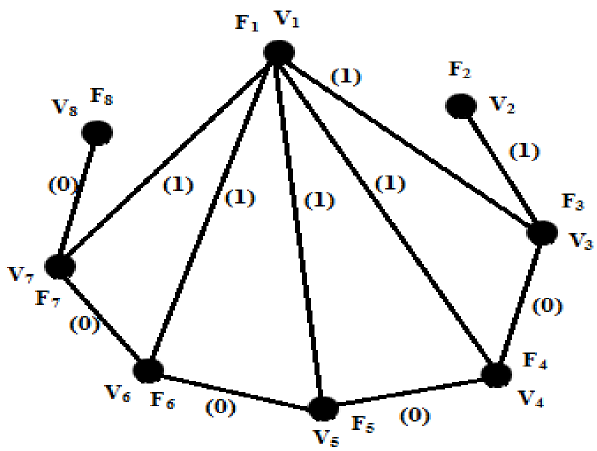
since the switched vertex v_1 is labeled as 1, the (n-5) edges incident to 1 receive label 1 and also the edge v_2v_3 receives label 1. Other edges receives 0 as the consecutive integers does not divide each other. Therefore , $|e_f(0)| = n-3$

And $|e_f(1)| = n-2$.

So, $|e_f(0) - e_f(1)| = 1$.

Hence the graph obtained by switching of an arbitrary vertex in cycle C_n is a divisor cordial graph.

Illustration: Consider the graph obtained by switching of vertex v_1 in cycle C_8.



Theorem 2.2: The graph obtained by switching of a rim vertex in wheel W_n , $n \geq 4$ is a Fibonacci Divisor Cordial Graph.

Let v be the apex vertex and v_1 ,v_2 , …… , v_n be the rim vertices of wheel W_n. Let G_{v1} denote the graph obtained by switching of a rim vertex v_1 of $G = W_n$. We note that , $|V(G_{v1})| = n+1$ and $|E(G_{v1})| = 3(n-2)$. We define f as follows :

$$f(v_1) = F_1$$

f (v) = Fibonacci number with the indices, largest prime number which is less than or equal to (n+1).

and the other vertices i.e. , from v_2 to v_n are labeled as follows

$F_2 , F_{2.2} , F_{2.2^2} , \ldots\ldots, F_{2.2^{k1}}$

$F_3 , F_{3.2} , F_{3.2^2} , \ldots\ldots, F_{3.2^{k2}}$

$F_5 , F_{5.2} , F_{5.2^2} , \ldots\ldots, F_{5.2^{k3}}$

…. ….. ….. ….. …….

where $F_{(2m-1).2^{km}} \leq F_n$ and $m \geq 1$, $k_m \geq 0$.

The apex vertex is labeled with the Fibonacci number having indices the largest prime number which is less than or equal to (n+1) , so trivially (n-2) edges receive 0. Only the edge vv_2 will receive 1.

Since $f(v_1) = F_1$, so (n-3)adjacent edges will receive 0. Also we see that the consecutive adjacent vertices having the labels Fibonacci numbers with even indices and consecutive adjacent vertices having labels Fibonacci numbers with odd and even indices contribute 1 to each edge. Similarly, the consecutive adjacent vertices having the labels Fibonacci numbers with odd indices and consecutive adjacent vertices having labels Fibonacci numbers with even and odd indices contribute 0 to each edge. Since, F_m/F_n iff m/n, where m, $n \geq 3$,

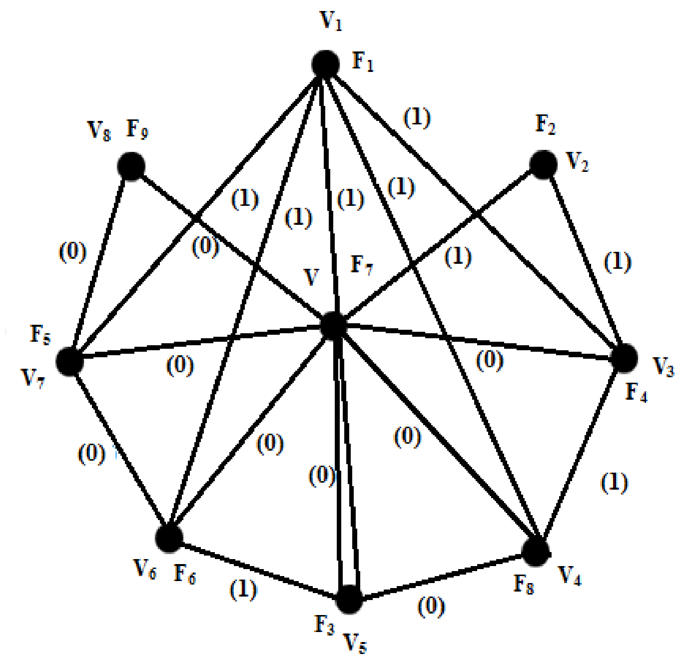
$$e_f(1) = (3n-5)/2 \text{ and } e_f(0) = (3n-7)/2, \text{ if } n \text{ is odd}$$

$$e_f(1) = e_f(0) = 3(n\text{-}2)/2, \text{ if n is even.}$$

Therefore, $|e_f(0) – e_f(1)| \leq 1$.

Hence, the graph obtained by switching of a rim vertex in wheel W_n , $n\geq 4$ is a divisor cordial graph.

Illustration: Consider the graph obtained by switching of vertex v_1 in wheel W_8.



Theorem 2.3: The graph obtained by duplication of an arbitrary vertex in cycle C_n ($n\geq 5$) is a Fibonacci Divisor Cordial Graph.

Let u_1, u_2, u_3, ………., u_n be the vertices of the cycle C_n. Let G be the graph obtained by duplicating an arbitrary vertex of C_n. Without loss of generality, let this vertex be u_1. Then $E(G) = \{E(C_n) , e' , e''\}$ where $e' = u_1'u_2$ and $e'' = u_nu_1'$ and $V(G) = \{V(C_n) , u_1'\}$. Hence $|V(G) =n+1$ and $|E(G)| = n+2$.

The other vertices are labeled in the following order

$F_2, F_{2.2}, F_{2.2^2}$, ……………. , $F_{2.2^{k1}}$

$F_3, F_{3.2}, F_{3.2^2}$, ……………. , $F_{3.2^{k2}}$

$F_5, F_{5.2}, F_{5.2^2}$, ……………. , $F_{5.2^{k3}}$ (I)

….. …… …… …… …… ……

….. …… …… …… ……. ……

where $(2m -1)2^{km} \leq n$ and $m\geq 1$, $k_m > 0$. We observe that $(2m\text{-}1)2^a$ divides $(2m -1)2^b$; $(a < b)$ and $(2m -1)2^{ki}$ does not divide $(2m+1)$.

Case I: n is even and n ≥ 6

We label the vertices as follows :

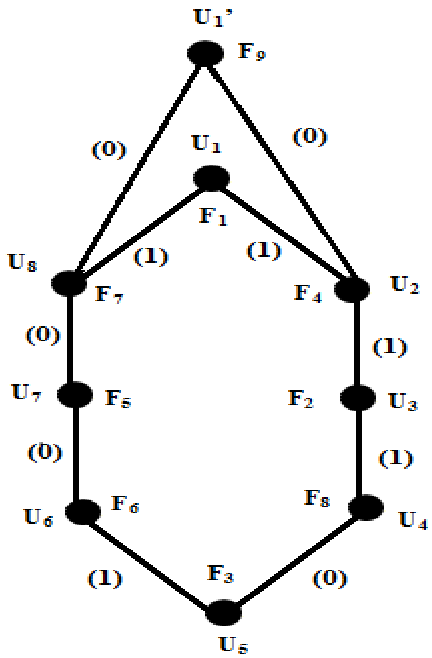
$f(u_1) = F_1$, $f(u_1') = F_{n+1}$

The cycle is labeled as in (I) only by interchanging the position of F_2 and F_4 i.e. , $f(u_2) = F_4$ and $f(u_3) = F_2$. Then (n+2)/2 edges receive the label 1 and (n-2)/2 edges receive the label 0. The two edges $v_1'v_2$ and $v_1'v_n$ contribute 0.

Therefore , $e_f(1) = (n+2)/2$

And $e_f(0) = (n-2)/2 + 2 = (n+2)/2$.

Illustration: Consider the graph obtained by duplication of vertex u_1 in cycle C_8.



Case I: n is odd

Subcase I: Consider the case when n = 5.

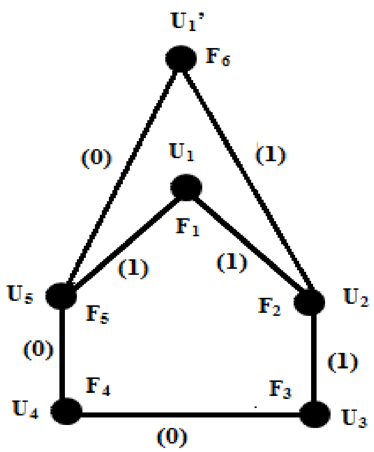
We label the vertices as follows

$f(u_i) = F_i$ and $f(u_1') = F_{i+1}$

So, $e_f(1) = 4$ and $e_f(0) = 3$.

$|e_f(1) – e_f(0)| = 1$.

Illustration: Consider the graph obtained by duplication of vertex u_1 in cycle C_5.



Subcase II: n is not a multiple of 3(n ≥ 7)

We label the vertices as follows

$f(u_1) = F_1$, $f(u_1') = F_3$

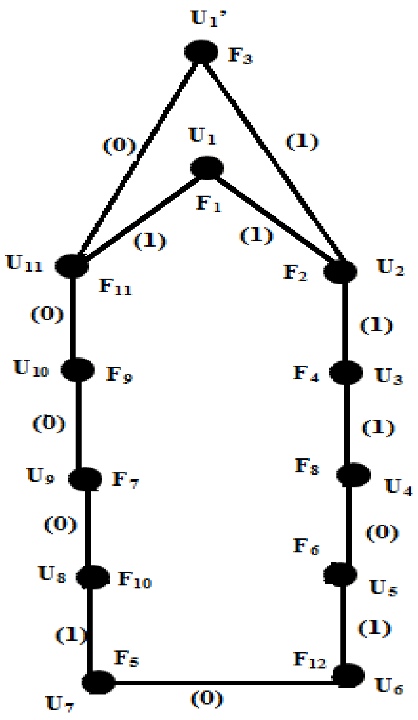
The vertices in the cycle are labeled as in (I). Here the (n+1)/2 edges in the cycle receive 1 and (n-1)/2 edges receive 0; the edge $u_1'u_2$ contributes 1 and the edge $u_1'u_n$ contributes 0.

Therefore , $e_f(1) = (n+1)/2 + 1 = (n+3)/2$

And $e_f(0) = (n-1)/2 + 1 = (n+1)/2$

So $|e_f(1) – e_f(0)| = 1$.

Illustration: Consider the graph obtained by duplication of vertex u_1 in cycle C_{11}.



Subcase III: n is a multiple of 3

We label the vertices as follows:

$f(u_1) = F_1$ and $f(u_1') = F_3$

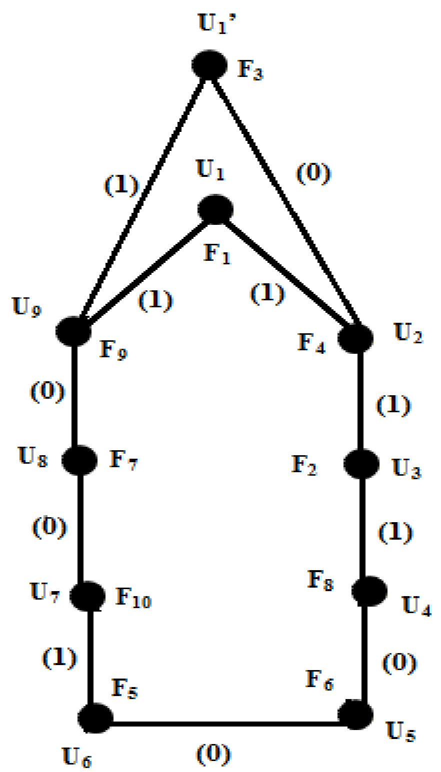
The cycle is labeled as in (I) only by interchanging the position of F_2 and F_4, i.e. , $f(u_2) = F_4$ and $f(u_3) = F_2$. Then the (n+1)/2 edges in the cycle receive 1 and (n-1)/2 edges receive 0; the edge $u_1'u_2$ contributes 1 and the edge $u_1'u_n$ contributes 0.

Therefore, $e_f(1) = (n+1)/2 + 1 = (n+3)/2$

And $e_f(1) = (n-1)/2 + 1 = (n+1)/2$

So $|e_f(1) - e_f(0)| = 1$.

Illustration: Consider the graph obtained by duplication of vertex u_1 in cycle C_9.



Thus in the above two cases ,

$|e_f(1) - e_f(0)| \leq 1$.

Hence , the graph obtained by duplication of arbitrary vertex in cycle C_n ($n \geq 5$) is a Fibonacci Divisor Cordial Graph.

Theorem 2.4: Degree Splitting graph of path P_n is Fibonacci Divisor Cordial Graph.

Consider P_n with $V(P_n) = \{v_i : 1 \leq i \leq n\}$. Here $V(P_n) = V_1 \cup V_2$, where $V_1 = \{v_i : 2 \leq i \leq n-1\}$ and $V_2 = \{v_1, v_n\}$. Now in order to obtain degree splitting graph of P_n denoted by $DS(P_n)$, we add w_1, w_2 corresponding to V_1, V_2. Then $|V(DS(P_n))| = n+2$ and $E(DS(P_n)) = E(P_n) \cup \{v_1w_2, v_nw_2\} \cup \{w_1v_i : 2 \leq i \leq n-1\}$. So, $|E(DS(P_n))| = 2n-1$.

We define the vertices as follows :

$f(w_1) = F_1$

$f(w_2) = F_2$

$f(v_i) = F_{i+2}$, $1\leq i \leq n$.

Then the n edges incident to F_1 and F_2 receive 1 and the (n-1) edges of the path P_n receive 0.

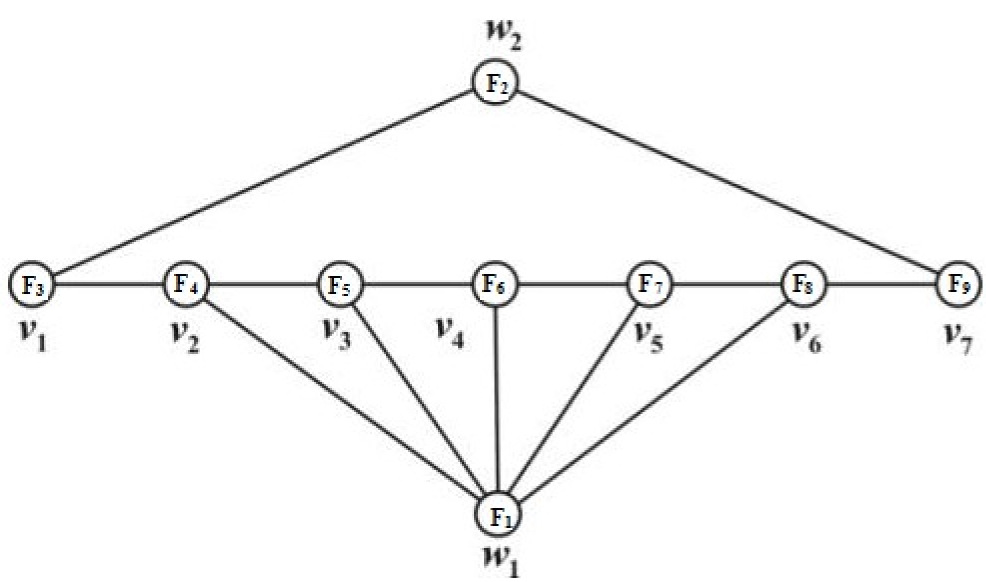
Therefore, $e_f(1) = n$

$e_f(0) = n-1$

So, $|e_f(1) – e_f(0)| \leq 1$.

Hence, the degree splitting graph of path is a Fibonacci Divisor Cordial Graph.

Illustration: Consider $DS(P_7)$



Theorem 2.5:Umbrella U(n,3) is Fibonacci Divisor Cordial Graph.

Consider U(n,3) with vertex set $V(U(n,3)) = \{u, v, w, u_i : 1\leq i \leq n\}$, so $| V(U(n,3)) | = n+3$.

$|E(U(n,3))| = 2n+1$.

Case I : n is odd

We label the vertices as follows:

$f(u) = F_1$, $f(v) = F_{n+2}$, $f(w) = F_{n+3}$, $f(u_i) = i+1$, $1\leq i \leq n$.

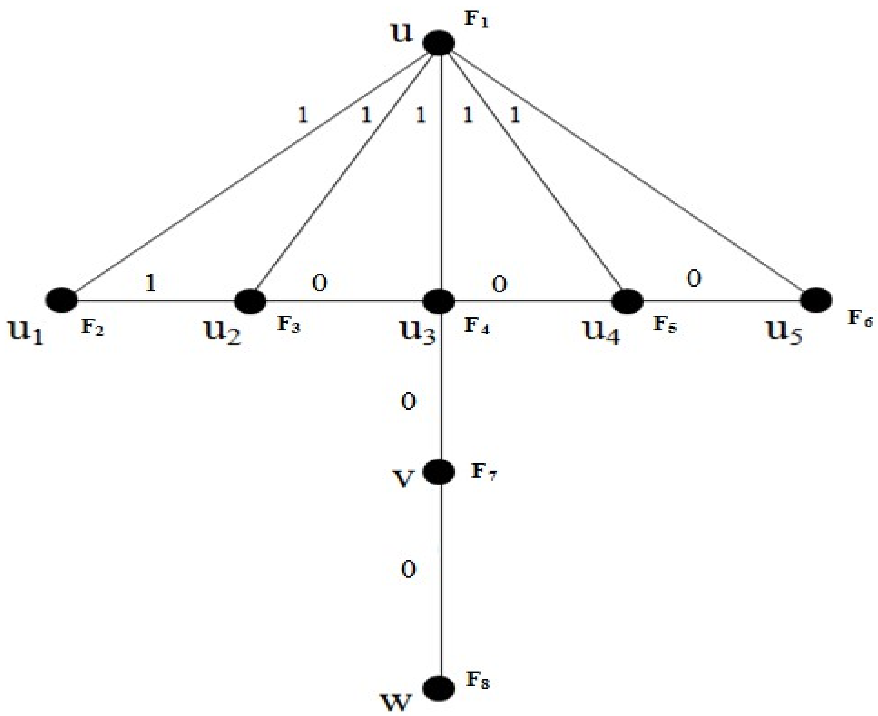
Then the n edges incident to F_1 receive 1, the edge u_1u_2 will receive 1, and the other edges of the path receive 0 and the edge $u_{(n+1)/2}v$ and vw also receives 0.

Therefore, $e_f(1) = n+1$

$e_f(0) = (n-2)+2 = n$

So, $|e_f(1) – e_f(0)| \leq 1$.

Illustration: Consider U(5,3).



Case II : n is even

Subcase I: n is a multiple of 4.

Here we consider that the path P_3 is joined at the middle odd vertex of path P_n in fan F_n. Then we label the vertices as Case I and so the result is also same as Case I.

. **Subcase II:** n is not a multiple of 4.

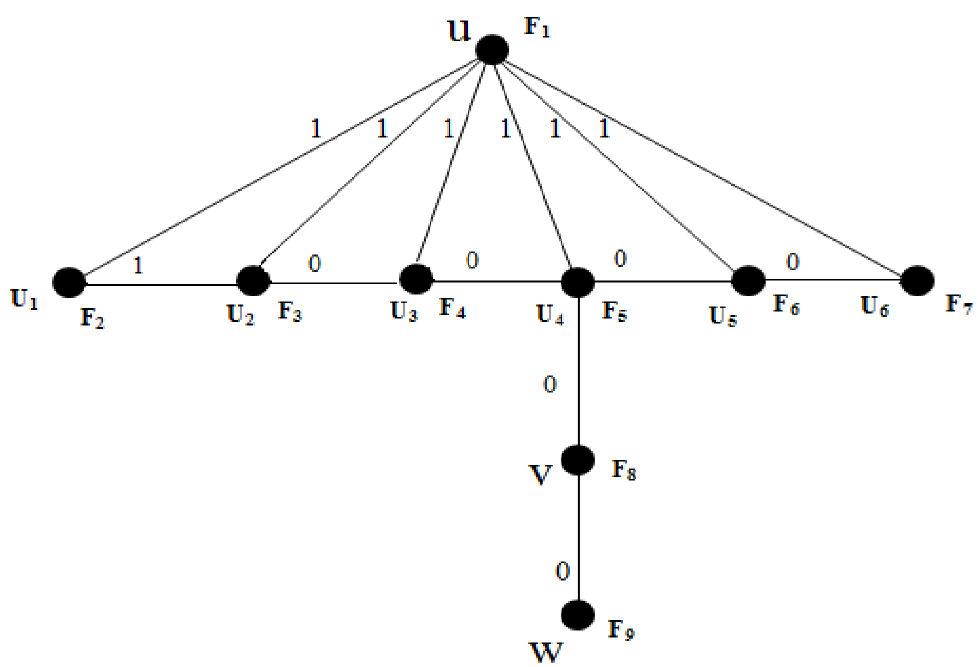
Here we consider that the path P_3 is joined at the middle even vertex of path P_n in fan F_n. Then we label the vertices as Case I and so the result is also same as Case I.

Thus in the above two cases ,

$|e_f(1) – e_f(0)| \leq 1$.

Hence, Umbrella U(n,3) is a Fibonacci Divisor Cordial Graph.

Illustration: Consider U(6,3).



Theorem 2.6: Bistar $B_{n,n}$ is Fibonacci Divisor Cordial Graph.

Let $B_{n,n}$ be a bistar with vertex set V(G) = {u,v,u_i,v_i : 1≤ i ≤ n} where u_i, v_i pendant vertices and u, v are the apex vertices . Then |V(G)| = 2n + 2 and |E(G)|= 2n+1.

We define f as follows :

f(u) = Fibonacci number with indices largest prime number which is less than 2n + 2.

f(v) = F_1

f(v_i) = 2(i-1) + 2

f(u_1) = 2n + 2

Then u_j ; (2 ≤ j ≤ n)'s are labeled with the odd numbers one after another except that odd number which is labeled at the vertex u.

The vertex v is labeled as 1, trivially the (n+1) edges adjacent to v receive 1. Now, u has been given label by a prime number so all the adjacent edges i.e. , uv_i's receive 0.

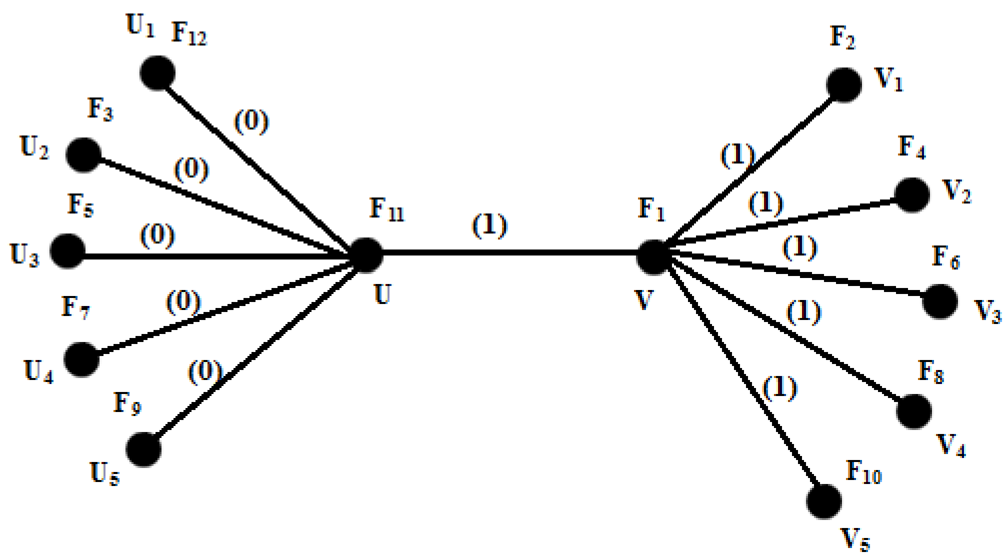
Therefore, $e_f(1)$ = n+1

$e_f(0)$ = n

So, $|e_f(1) - e_f(0)| \leq 1$.

Hence, Bistar $B_{n,n}$ is a Fibonacci Divisor Cordial Graph.

Illustration: Consider B(5,5).



References

1. Burton, D.M.,(1980) *Elementary Number Theory*, 2ed, Wm. C. Brown Company Publishers.

2. Cahit,I.,(1990) On cordial and 3-equitabel labelings of graphs, Utilitas Math,pp- 189-198.

3. Gallian , J.A.,(2009) A dynamic survey of graph labeling, Electronic Journal of Combinatorics 16, DS6.

4. Harary, F.,(1972) *Graph Theory*, Addison-Wesley, Reading Mass.

5. . Nagarjan, K., Navaneethakrishnan, S.,Varatharajan, R.,(2011)Divisor Cordial Graph , International Journal of Mathematical Comb. ,Vol. 4 , pp 15-25.

6. Navanaeethakrishnan, S., Nellaimurugan, A., Sridevi, R.(2013)Fibonacci Divisor Cordial Graphs, International Journal of Mathematics and Soft Computing, Vol. 3, No. 3.