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A multi-objective optimization approach for determination of optimal radius of a power supply substation

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Abstract: Optimization is the discipline of adjusting a process so as to optimize some specific set of parameters, without violating some constraints. In this paper an optimization is made by means of weighted sum method, after a multi-objective programming model formed based upon the power supply radius from a substation. The main aim of the model is to enlarge the radius of power supply as much as possible with the least investment and reduction of waste. The proposed model using weighted sum method is illustrated by the numerical example.

Keywords : radius of power supply, weighted sum method, optimization.

1 Introduction

Maximum asset performance is one of the major goals for electric power distribution system operators (DSOs). To reach this goal minimal life cycle cost and maintenance optimization become crucial while meeting demands from customers and regulators. One of the fundamental objectives is therefore to relate maintenance and reliability in an efficient and effective way. This necessitates the determination of the optimal balance between preventive and corrective maintenance. The balance between preventive and corrective maintenance is approached as a multi-objective optimization problem(MOP), with the customer interruption costs on one hand and the maintenance budget of the DSO on the other hand. To solve MOP various mathematical methods have been formulated. Weighted sum method is the simplest and widely used method for solving MOPs. Due to conflicting nature of objective functions, there is no single efficient method to solve MOP. So many methods are there to solve MOPS. Geometric programming is an important optimization technique due to Duffin et al.[1] , who put a foundation stone for solving more complex type of non-linear multi-objective optimization problems. Application of GP can be found in many field such as mechanical Engineering, Civil Engineering, Chemical Engineering, Optimal control, decision making, network flow, theory of inventory, balance of machinery, analog circuitry, design theory, communication system, transportation, fiscal and monetary, management science, electrical engineering, electronic engineering, environmental Engineering, nuclear engineering and technical economical analysis. Now-a-days, application scope the GP technique continuously expanded [4]. Various dynamic and static mathematical model [2],[3],[5] have been formulated to handle long term distribution system planning pertaining to power supply in the urban as well as rural areas. So many research papers have been developed for transmission and distribution of power. Under a fuzzy environment, how power to be transmitted that has shown by Dhar [7] in his paper. J.Haddad [8] has shown in a power for long term power system planning using fuzzy environment. In the present work, we have developed a procedure to determine the radial distance of the circular power station for the supply of the electricity in a particular area. In our work, we have proposed to construct a circular power station of 110kv with minimum investment and reduction of waste for providing efficient power supply for the particular region.

The organization of this paper is as follows: Following introduction, the mathematical formulation for the optimization of power supply radius has been discussed in the Section 2. The concept of multi-objective optimization and weighted method have been discussed in sec 3 and sec 4 respectively and an illustrative example based on model is given in section 5 in order to understand the practical importance of the discussion. A brief analysis of importance of this paper has been given in sec 6 and Finally some conclusions are drawn from the discussion have been incorporated in sec 7.

2 Mathematical Formulation

The given model originally developed by Cao[6] to maximize power supply radius within a particular area. In this work, we have made an attempt based on that model, to search an alternative method other than geometric programming technique which will explain how to maximize power supply radius in minimum investment. The concept of the paper is as follows:
The power to be supplied to an area of $Q_1 km^2$ from a 110kv substation and distributed to its consumers in a city by dropping voltage method of 11kv. Based upon the above concept a model has been built using an annual-cost method.
Suppose the area transmitted by voltage district of electric distribution network is well distributed in load density, the average number is N_b and numbers n_b in unit area in the substation are shown as follows.

$$\begin{aligned} r &= \sqrt{\frac{S}{\pi\sigma K_c}}. \\ N_b &= \frac{Q_1}{\pi r^2} \\ n_b = \frac{\sigma K_c}{S} &= \frac{1}{\pi r^2} \end{aligned} \tag{2.1}$$

Here S is the capacity of in 33kV substation(kVA),σ is the mean load density(kW/km^2) and K_c is a loadable ratio. Option in 33kV substation is 2.2-2.5 this range is stipulated by National Electric Department and r is the radius of the circular substation.

Let us consider the 33kV substation as center of voltage sub-transmission from where using high-tension network voltage are lowered to 11kV net for distribution. That is substations are dropping voltage sub-transmission as well as an electricity distribution substation.
Here the investment means an annual operating cost μ arising under a certain load level without consideration of its process and the return on investment will be decided as commute years n(from 8 to 10 years), namely, the total investment is reclaimed within commute years provided the load density is well-distributed in the whole electrified net. Therefore, a static model is built by means of an annual cost as follows:

$$F = \frac{Z}{n} + \mu,$$

where Z denotes a total investment cost, and F denotes an annual total cost.

According to Obrad[2] and Yu et al.[5], an annual cost function is a unit capacity is denoted as

$$F_0(S) = \frac{F}{S} = \frac{(Z_b + Z_1)/n + \mu_1 + \mu_2 + \mu_3}{S} \tag{2.2}$$

where

$$Z_b = a_1 + b_1 S \tag{2.3}$$

means the investment cost in the construction of a 33kV substation.here
a_1-irrelevant part coefficient to capacity of the substation in the investment.
b_1-related part coefficient to capacity of the substation in the investment
And

$$Z_1 = Er(S\ cos\phi/P_{av}a_2 + b_2 S_1) \tag{2.4}$$

denotes the money invested in 11kV, a main trunk line of medium-voltage distribution network, where
E- coefficient of terrain revision.
P_{av}-an average load per circuit(kW).
a_2- investment of each kilometer irrelevant to a wire section in the 11kV line investment.
b_2- related part coefficient with the wire section in 110kV line investment.
$S_1 = \frac{S}{\sqrt{3}jU_N}(mm^2)$ denotes the total wire section area in in the main trunk line, where 11kV is a medium-voltage distribution network in the 110kV substation, with U_N= rated voltage for the medium-voltage distribution network in 11kV; j= wire economical current density.
l=length of wire.
and

$$\mu_1 = H(Z_b + Z_1) \tag{2.5}$$

behaves as direct proportional function between total investment and the fixed parts in operating costs (heavy repair, small repair, and depreciation charge) which is used in the 11kV medium-voltage distribution network line and the 110kV substation, where H is a coefficient annually extracted from operating costs of a total investment.
And

$$\mu_2 = \triangle P\tau C_0 = 7.26\frac{E\rho j}{U_N cos^2\phi}\frac{1}{\sqrt{\rho}}S^{3/2}10^{-5}\tau C_0 \tag{2.6}$$

stands for the loss charge of the 11kV line for the whole year, where
ρ-resistance ratio in wire (Ω/km.mm^2).

τ-wastage hours in an average maximum load per year.

C_0-cost price of wastage watt-hour.

In addition, the loss charge of transformer in the 110kV substation is

$$\mu_3 = (8760C_0K_{Fe} + C_0\tau_bK_{Cu})\gamma S_{N.L}^{3/4},$$

where

K_{Fe}-an iron-loss coefficient of a transformer($\approx 0.0085kW/(kVA)^{3/4}$).

τ_b-hours equivalent to copper loss in a transformer.

K_{Cu}-a copper-loss coefficient in a transformer ($\approx 0.055kW/(kVA)^{3/4}$).

γ-number of transformers.

$S_{N.L.}$-rated load of a transformer(kVA).

S/S_N denotes an average number of transformers when a rated capacity of each transformer in 33kV substation is S_N(kVA), which serves for chosen transformers, such that μ_3 is changed into

$$\mu_3 = (8760C_0K_{Fe} + C_0\tau_bK_{Cu})\frac{S}{S_N}S_{N.L}^{3/4} \tag{2.7}$$

Therefore,the total annual operating cost of the 33kV substation and its distributing network line of medium-voltage is

$$\mu = \mu_1 + \mu_2 + \mu_3.$$

Substitute(2.3)-(2.7) for (2.2), then

$$\begin{aligned} F_0(S) &= \left(\frac{1}{n} + H\right)\left(\frac{a_1}{S} + b_1\right) \\ &+E\left[\left(\frac{1}{n} + H\right)\sqrt{\frac{1}{\pi\rho K_c}}\left(\frac{a_2 cos\phi}{P_{av}} + \frac{b_2}{\sqrt{3}jU_N}\right) + 7.26\frac{\rho j\tau C_0}{U_N cos^2\phi}\frac{10^{-5}}{\sqrt{\sigma}}\right]S^{1/2} \\ &+(8.760C_0K_{Fe} + C_0\tau_bK_{Cu})\frac{S_{N.L}^{3/4}}{S_N} \\ &= a_1\left(\frac{1}{n} + H\right)S^{-1} \\ &+E\left[\left(\frac{1}{n} + H\right)\sqrt{\frac{1}{\pi\rho K_c}}\left(\frac{a_2 cos\phi}{P_{av}} + \frac{b_2}{\sqrt{3}jU_N}\right) + 7.26\frac{\rho j\tau C_0}{U_N cos^2\phi}\frac{10^{-5}}{\sqrt{\sigma}}\right]S^{1/2} \\ &+b_1\left(\frac{1}{n} + H\right) + (8.760C_0K_{Fe} + C_0\tau_bK_{Cu})\frac{S_{N.L}^{3/4}}{S_N} \end{aligned} \tag{2.8}$$

Now the model is built as follows:

In (2.8),let

$$c_1 = a_1\left(\frac{1}{n}+H\right)$$
$$c_2 = E\left[\left(\frac{1}{n}+H\right)\sqrt{\frac{1}{\pi\rho K_c}}\left(\frac{a_2cos\phi}{P_{av}}+\frac{b_2}{\sqrt{3}jU_N}\right)+7.26\frac{\rho j\tau C_0}{U_Ncos^2\phi}\frac{10^{-5}}{\sqrt{\sigma}}\right]$$
$$c_3 = b_1\left(\frac{1}{n}+H\right)+(8.760C_0K_{Fe}+C_0\tau_bK_{Cu})\frac{S_{N.L}^{3/4}}{S_N}$$

then (2.8) is changed into

$$F_0(S) = c_1S^{-1}+c_2S^{1/2}+c_3 \tag{2.9}$$

where c_1, c_2 and c_3 are constant coefficients, and formulae (2.9) is an exponential polynomial function, which is the annual cost function concerning capacity S in the substation.

The way to determine an objective function in a static model is by making annual unit cost capacity minimal, with capacity in the substation being nonnegative, that is

$$\begin{aligned} &\min\ F_0(S) \\ &s.t.\ S > 0 \end{aligned} \tag{2.10}$$

we call (2.10) a GP model, where $F_0(S)$ is the same as (2.9). It is a posynomial where the coefficients $c_i > 0$, i.e., polynomial whose coefficients are positive numbers.

3 Multi-Objective Optimization Problem(MOOP):

The method of optimizing systematically and simultaneously a collection of objective function is called multi-objective optimization or vector optimization. A multi-objective optimization problem can be stated as:
Find $x = (x_1, x_2, ...x_n)^T$, so as to

$$\min : f_k(x), \quad k = 1, 2, ...p \tag{3.1}$$

$$Subject\ to : \quad g_i(x) \leq 0, \quad i = 1, 2, ...m \tag{3.2}$$

$$x = (x_1, x_2, ...x_n) \geq 0 \tag{3.3}$$

4 Weighted Sum Method:

Weighted sum method is the simplest method widely used to convert a set of objectives into a single objective by multiplying each objective with weights to find the non-inferior optimal solution of a multi-objective optimization problem within the convex objective space.

If $f_0^1(x), f_0^2(x), ...f_0^p(x)$ are 'p' objective functions for any vector $x = (x_1, x_2, ...x_n)^T$, then we can define weighted sum method is as follows.

$$Let \ \ W = \{w : w \in R^n, w_k > 0, \sum_{k=1}^{n} w_k = 1\} \tag{4.1}$$

be the set of non-negative weights. Using weighted sum method the multi-objective optimization problem given in sec-3 can be defined as:

$$Q(w) = \min_{x \in X} \sum_{k=1}^{p} w_k f_0^k(x) \tag{4.2}$$

$$Subject \ to: \quad g_i(x) \leq 1, i = 1, 2, ...m \tag{4.3}$$

$$x_j > 0, j = 1, 2, ...n \tag{4.4}$$

It is necessary that the objective space of original problem should convex.If non-convex,then weighting method may not capable of generating the efficient solutions on the non-convex part of efficient frontier.It must be noted that the optimal solution of a optimization problem using weighting method should not be accepted as the best compromise solution if that do not reflect in decision makers mind.

5 Numerical Example:

To illustrate the above model given by Cao following physical problem is considered. From a 33kv power substation station, power to be distributed in a city by dropping voltage of 11kv in such a way that annual unit investment construction cost in unit capacity and unit operational cost in unit capacity should not be more than 70 rupees(say) and 58 rupees(say) respectively so that total annual unit cost in unit capacity should be minimum in order to maximize power supply radius. The amount to be invested decided by National Electric Department. From the test result the following parameter values are taken, upon which a static model to be built.

$n = 10, cos\phi = 0.9, \tau = 2,400h, E = 1.3,$

$C_0 = 0.6$rupee/kWh,$H = 8\rho = 31.5\Omega$/km.mm^2, $U_N = 11kV, j = 1.15A/mm^2,$

$a_1 = 4998000$rupee, $b_1 = 490$ rupee/kVA,

$a_2 = 182672$rupee/km,$b_2 = 872$ rupee/km.mm^2

$P_{av} = 4,000kW, K_c = 2.2, Q_1 = 50km^2, \sigma = 6000kw/km^2$

$K_{Fe} \approx 0.0085kW/(kVA)^{3/4}$

$K_{Cu} \approx 0.055kW/(kVA)^{3/4}$.

Putting the above data into (2.8) and calculating ,we got the following model as:

$$F_0(S) = \bar{F}_0(S) + 88.83 = 899640S^{-1} + 0.00832S^{1/2} + 88.83$$

$$r = 0.00491S^{1/2}$$

$$399840S^{-1} + 0.047S^{1/2} + 39.83 \leq 58$$

$$499800S^{-1} + 0.051S^{1/2} + 49 \leq 70$$

Now (2.10) is simplified into a problem: Find S so as to

$$\begin{aligned} \min f_1 :\ \bar{F}_0(S) = 899640S^{-1} + 0.00832S^{1/2} \\ \max f_2 : r = 0.00491S^{1/2} \\ 399840S^{-1} + 0.047S^{1/2} \leq 18.17 \\ 499800S^{-1} + 0.051S^{1/2} \leq 21 \\ s.t.\ S > 0 \end{aligned} \tag{5.1}$$

now above problem can be written as:

Find S so as to

$$\begin{aligned} \min f_1 :\ \bar{F}_0(S) = 899640S^{-1} + 0.00832S^{1/2} \\ \min f_2 : r_1 = 203.63S^{-1/2} \\ 399840S^{-1} + 0.047S^{1/2} \leq 18.17 \\ 499800S^{-1} + 0.051S^{1/2} \leq 21 \\ s.t.\ S > 0 \end{aligned} \tag{5.2}$$

Solution of Priamal f_1:

Find S so as to

$$
\begin{aligned}
\min f_1 : \ \bar{F}_0(S) &= 899640S^{-1} + 0.00832S^{1/2} \\
399840S^{-1} + 0.047S^{1/2} &\leq 18.17 \\
499800S^{-1} + 0.051S^{1/2} &\leq 21 \\
s.t.\ S &> 0
\end{aligned}
\tag{5.3}
$$

Solution of the primal is $f_1 = 14.651$ for $S = 72484.72$

Solution of Primal f_2:

Find S so as to

$$
\begin{aligned}
\min f_2 : r_1 &= 203.63S^{-1/2} \\
399840S^{-1} + 0.047S^{1/2} &\leq 18.17 \\
499800S^{-1} + 0.051S^{1/2} &\leq 21 \\
s.t.\ S &> 0
\end{aligned}
\tag{5.4}
$$

Solution of the primal is $f_2 = 0.756$ for $S = 72484.66$

Solution of the problem using weighted sum method:

Using Weighted mean method we can write the multi-objective programming problem as follows:

Find S so as to

$$
\begin{aligned}
\min Z = w1(899640S^{-1} + 0.00832S^{1/2}) &+ w2(203.63S^{-1/2}) \\
399840S^{-1} + 0.047S^{1/2} &\leq 18.17 \\
499800S^{-1} + 0.051S^{1/2} &\leq 21 \\
w1 + w2 &= 1 \\
s.t.\ S, w1, w2 &> 0
\end{aligned}
\tag{5.5}
$$

Solution of the above problem is given in following table-1.

Table-1

(Primal solution)

w_1	w_2	S	Z
1	0	72484.72	14.651
0.9	0.1	72484.72	13.261
0.8	0.2	72484.66	11.872
0.7	0.3	72484.68	10.482
0.6	0.4	72484.71	9.093
0.5	0.5	72484.66	7.703
0.4	0.6	72484.68	6.314
0.3	0.2	72484.75	4.924
0.2	0.1	72484.73	3.535
0.1	0.9	72484.66	2.145
0	1	72484.66	0.756

From the above table we can observe that the optimal solution of Z varies form optimal solution of f_1 to f_2 on changing the value of weights between 0 and 1.
From the above discussion, we find $\bar{F}_0(S) = 14.651$, so that annual investment construction cost in unit capacity $F(S) = 14.651 + 88.83 \approx 103.481$
Now maximum radius for power supply $r = (1/r_1) = 1.32km$ and
no.of substation required i.e $N_b \approx 9.13$

6 Result Analysis:

In this paper we have shown that the distribution of power supply providing the best as per requirement for considered the test problem and the efficiency of our method is reflected in the computed results shown in table 2 by using suitable data.

Table-2

(comparison of minimal expense and S, r, N_b)

σ	$minimalexpense$	S	r	N_b
5000	103.691	72484.66	1.44	7.67
6000	103.481	72484.72	1.32	9.13
7000	103.314	72484.66	1.23	10.52
8000	103.179	72484.66	1.14	12.24

From above table we conclude, if capacity of substation increased then there will be increase in number of substation which causes more expense in distribution of power where as there will be decrease in radial of power supply. So based on the data which obtained in the above table, we have to decide how many substation to be installed and what should be the capacity of substation in minimum investment.

7 Conclusion:

Power is the life line of modern societies. It is the base of development any economy in the world. There is a large gap in demand-supply in our country due to transmission and distribution. To reduce this gap we have to increase our installed capacity. This require huge money for implementation.The method we have considered shows that a systemic control model can be set up by means of weighted mean method, so that economical radius of power supply with small investment can be obtained under maximum economical capacity in substation.The model built in this paper contributes to asses not only to an average radius of power supply but also load to density in our economical capacity. Hope this paper will be a platform for researchers for further advance in this field.

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