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Analysis of free convection reacting flows on a porous plate in the presence of constant magnetic field.

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Abstract: We investigate free convection reacting flow under a constant magnetic field. We provide numerical solutions for both velocity and temperature fields. It was show that temperature and velocity are an increasing function of time.

Keywords: Free convection, Reacting, Porous Plate, magnetic field, Numerical Solution

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Nomenclature

English		Greek Symbols	
A	Pre-exponent factor		
C_p	specific Heat at Constant Temperature	$\beta^{'}$	Coefficient of Volume Expansion
E	Activation Energy		
G	Grashof Number	β_0	Magnetic Field
g	Gravitational Acceleration	σ	Electrical Conductivity
K	Thermal Conductivity	ρ	Density
P	Prandtl Number	υ	kinematic Viscosity
Q	Heat release Per Unit Mass	μ	Dynamic Viscosity
R	Universal gas constant	θ	Dimensionless Temperature
$T^{'}$	Temperature	$\in$	Activation energy parameter
$T_W^{'}$	Temperature of the Wall		
$T_\infty^{'}$	Temperature at Infinity		
$t^{'}$	Time		
$u^{'}$	Fluid Velocity		
$v_0^{'}$	normal Velocity of suction / Injection at Wall	$y^{'}$	Space Variable

1. Introduction

Free convective flows occur in many engineering and natural systems .Free convective flows driven by temperature or concentration differences have been studied extensively when both temperature and concentration difference occurs simultaneously, the free flow can become quite complex. (Gebhart and Pera 1971) have provided an excellent overview of this field and have indicated the important of these flows in engineering systems and nature .The process occurring in nature include photosynthetic mechanisms, clam-day evapouration and vapourisation of mist and fog while engineering applications include the chemical reaction in a reactor chamber consisting of decomposition of chemical vapour on surfaces, cooling of electronic equipment e.t.c.

Toki and Tokis (2007) examined unsteady free convection flows of a viscous and incompressible fluid near a porous infinite vertical plate under an arbitrary time-dependent heating of the plate. Exact solutions of this problem were obtained with the help of Laplace transform technique, when the plate was moving with an arbitrary time-dependent velocity and for special cases of the impulsive and the accelerated heating effects. These solutions were given in closed form for arbitrary prandtl number of the fluid and for the thermal porous wall with or without suction or injection. In much earlier work Ayeni (1978) studied the thermal runaway phenomenon while investigating the reaction of oxygen and hydrogen .He provided useful theorems on such flows. Jha and Ajibade(2010) studied free convective flow between vertical porous plates with periodic heat input under suction/injection. The temperature and

velocity fields are separated into steady and periodic parts and the resulting second order ordinary differential equations are solved to obtain the solution to the problem.

In a much recent work Omowaye(2011) extended the model investigated by Toki and Tokis (2007) to include reaction and magnetism on the flow. In this work, we analyse a free convective reacting flow under Arrhenius kinetics and constant magnetic field. We provide the velocity and temperature field of the flow.

2. Mathematical formulation of the Problem.

we consider an unsteady two-dimensional free convection flow, the coordinate origin at an arbitrary point on an infinite, porous limiting vertical plate or wall. The x'-axis is along the plate in the upward direction and the y'-axis normal towards it. The fluid is viscous and incompressible.

The flow is induced either by the motion of the plate or by heating it or by both. The plate initially at rest and with a constant temperature T_∞ is suddenly moved with the velocity $u_0 f(t')$ in its own plane along the x'-axis and its temperature is instantaneously increased (or decreased) by the quantity ($T_w' - T_\infty'$) g(t') for t >0; with u_0 being a constant velocity $T_w'(\neq T_\infty)$ a constant temperature for the plate, f (t') and g(t') two arbitrary functions of non-dimensional time t. As show in fig. 1

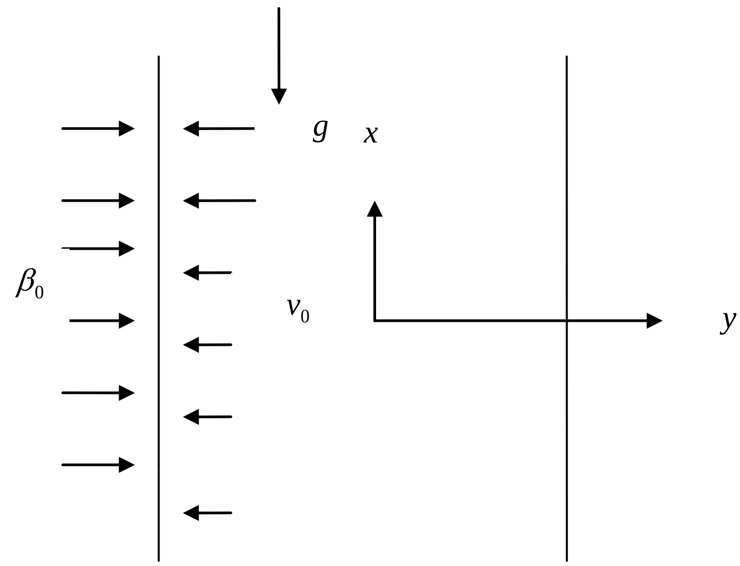

Fig.1 : Configuration of the problem

Modifying Toki and Tokis (2007), an external uniform magnetic field β_0 is applied in the positive y'-direction (Attia 2006). By assuming a very small magnetic Reynolds number the induced magnetic field is neglected (Attia 2006). Also, the fluid is reacting . On this physical grounds of the present problem all the quantities are assumed to be functions of the space coordinate y' and t' ; so that the velocity vector is given by $V = (u'(y,t),\ v'(y,t),0)$. In this case the governing equations become

The equation of continuity, on integrating becomes

$$v' = \text{constant} = v_0' \quad \text{(say)} \qquad (2.1)$$

where v'_0 is the normal velocity of suction or injection at the wall according as $v'_0 < 0$ or $v'_0 > 0$ respectively ; $v'_0 = 0$ represents the case of a non-permeable wall.

The remaining basic equations of motion and energy equation for this problem are

$$\frac{\partial u'}{\partial t'} + v_0' \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta'(T - T_\infty) - \frac{\sigma\beta_0'^2}{\rho} u' \qquad (2.2)$$

$$\frac{\partial T'}{\partial t'} + v_0' \frac{\partial T'}{\partial y'} = \frac{k}{\rho C_P} \frac{\partial^2 T'}{\partial y'^2} + \frac{\sigma\beta_0' u'^2}{\rho C_P} + \frac{QAe^{-\frac{E}{RT'}}}{\rho C_P} \qquad (2.3)$$

Assuming that no slipping occurs between the plate and the fluid the initial and boundary conditions for equations (2.1)-(2.3) are

$$u'(y',t') = 0 \quad and \quad T'(y',t') = T_\infty' \quad for \quad y' \geq 0 \ \ and \ \ t' \leq 0 \qquad (2.4)$$

$$u'(0,t') = [u_0 f(t'),0,0] \quad and \quad T'(0,t') = T_\infty' + (T_w' - T_\infty')g(t') \quad for \quad t' > 0 \qquad (2.5)$$

$$u'(\infty, t'') \to 0 \quad and \quad T'(\infty,t') \to T_\infty' \quad for \quad t' \geq 0 \quad , \qquad (2.6)$$

Where all the symbols are defined in the nomenclature. The first three terms on the right hand side of equations (2.2) are respectively viscous, Buoyancy force and Lorentz force terms while the first two terms on the right hand side of equations (2.3) represent the heat conduction and Joule dissipation terms while the last term is the Arrhenius term.

we now introduce the following non-dimensional variables

$$y = \frac{y' u_0}{\nu}, \ t = \frac{t' u_0^{2'}}{\nu}, \ u = \frac{u'}{u_0}, \ v = \frac{v_0'}{u_0}, \ \theta = \frac{(T' - T_\infty')E}{RT_\infty'^2} \qquad (2.7)$$

Substituting equation (2.7) into (2.2)-(2.6),we have

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G \in \theta - H_a^2 u \quad (2.8)$$

$$\frac{\partial \theta}{\partial t} + v_0 \frac{\partial \theta}{\partial y} = \frac{1}{P} \frac{\partial^2 \theta}{\partial y^2} + H_a^2 \, Ec\theta u^2 + \delta \, e^{\frac{\theta}{1+\in\theta}} \quad (2.9)$$

The corresponding initial and boundary conditions are

u(y,0)=0 θ(y,0)= 0 for y≥ 0 (2.10)

u(0,t) = f(t) θ(0,t)= g(t) for t >0 (2.11)

u(∞,t) → 0 θ (∞,t) → 0 for t >0 (2.12)

where all the symbols are defined in the nomenclature.

Now, consider the asymptotic expansions of temperature θ and velocity in u λ as

Let $0 < \in < 1$

$$\lambda = e^{-\frac{1}{\in}}$$

We assume $0 < \lambda << 1$

$$H_a = O(\lambda)$$

$$\left.\begin{aligned} H_a &= \lambda H_a^* \qquad H_a^* \text{ is bounded} \\ \theta &= \theta_0 + \lambda\theta_1 + \lambda^2\theta_2 + \ldots\ldots.. \\ u &= u_0 + \lambda u_1 + \lambda^2 u_2 + \ldots\ldots. \end{aligned}\right\} \quad (2.13)$$

Substituting (2.13) into equations (2.8)-(2.12) and simplify to have

λ^0:

$$P\frac{\partial \theta_0}{\partial t} + v_0 P\frac{\partial \theta_0}{\partial y} = \frac{\partial^2 \theta_0}{\partial y^2} + \delta P\, e^{\frac{\theta_0}{1+\in\theta_0}} \qquad (2.14)$$

$$\theta_0(y,0) = 0 \qquad \theta_0(0,t) = g(t) \qquad \theta_0(\infty,t) = 0$$

λ^1:

$$P\frac{\partial \theta_1}{\partial t} + v_0 P\frac{\partial \theta_1}{\partial y} = \frac{\partial^2 \theta_1}{\partial y^2} + \delta P\, e^{\frac{\theta_0}{1+\in\theta_0}}\left(\theta_1 - \frac{\in\theta_0\theta_1}{1+\in\theta_0}\right) \qquad (2.15)$$

$$\theta_1(y,0) = 0 \qquad \theta_1(0,t) = 0 \qquad \theta_1(\infty,t) = 0$$

λ^0:

$$\frac{\partial u_0}{\partial t} + v_0\frac{\partial u_0}{\partial y} = \frac{\partial^2 u_0}{\partial y^2} + G\in\theta_0 \qquad (2.16)$$

$$u_0(y,0)=0 \qquad u_0(0,t)= f(t) \qquad u_0(\infty,t)= 0$$

λ^1:

$$\frac{\partial u_1}{\partial t} + v_0\frac{\partial u_1}{\partial y} = \frac{\partial^2 u_1}{\partial y^2} + G\in\theta_1 \qquad (2.17)$$

$$u_1(y,0)=0 \qquad u_1(0,t)= 0 \qquad u_1(\infty,t)= 0$$

We carry out one more transformation from infinite domain to finite domain using

$$x = e^{-y} \tag{2.18}$$

Using (2.18) in (2.14)-(2.17),we have

$$\frac{\partial \theta_0}{\partial t} = \frac{x^2}{p}\frac{\partial^2 \theta_0}{\partial x^2} + \frac{(v_0 P+1)}{P} x \frac{\partial \theta_0}{\partial x} + \delta\ e^{\frac{\theta_0}{1+\in\theta_0}} \tag{2.19}$$

$$\theta_0(x,0) = 0 \qquad \theta_0(0,t) = 0 \qquad \theta_0(1,t) = g(t)$$

$$\frac{\partial \theta_1}{\partial t} = \frac{x^2}{p}\frac{\partial^2 \theta_1}{\partial x^2} + \frac{(v_0 P+1)}{p} x \frac{\partial \theta_1}{\partial x} \qquad \delta\ e^{\frac{\theta_0}{1+\in\theta_0}}\left(\theta_1 - \frac{\in\theta_0\theta_1}{1+\in\theta_0}\right) \tag{2.20}$$

$$\theta_1(x,0) = 0 \qquad \theta_1(0,t) = 0 \qquad \theta_1(1,t) = 0$$

$$\frac{\partial u_0}{\partial t} = x^2\frac{\partial^2 u_0}{\partial x^2} + (v_0+1)x\frac{\partial u_0}{\partial x} + G\in\theta_0 \tag{2.21}$$

$$u_0(x,0) = 0 \qquad u_0(0,t) = 0 \qquad u_0(1,t) = f(t)$$

$$\frac{\partial u_1}{\partial t} = x^2\frac{\partial^2 u_1}{\partial x^2} + (v_0+1)x\frac{\partial u_1}{\partial x} + G\in\theta_1 \tag{2.22}$$

$$u_1(x,0) = 0 \qquad u_1(0,t) = 0 \qquad u_1(1,t) = 0$$

Numerical Solution

Assume g(t) = f(t)= 1.We shall solve equations (2.19) - (2.22) using finite difference.

We define $\frac{\partial\theta_0}{\partial t} = \frac{\theta_{0i,j+1}-\theta_{0ij}}{k}, \frac{\partial\theta_0}{\partial x} = \frac{\theta_{0i+1,j}-\theta_{0ij}}{2h}, \frac{\partial^2\theta_0}{\partial x^2} = \frac{\theta_{0i+1,j}-2\theta_{0ij}+\theta_{0i-1,j}}{h^2}$ (2.23)

$$\frac{\partial u_0}{\partial t} = \frac{u_{0i,j+1} - u_{0ij}}{k}, \frac{\partial u_0}{\partial x} = \frac{u_{0i+1,j} - u_{0ij}}{2h}, \frac{\partial^2 u_0}{\partial x^2} = \frac{u_{0i+1,j} - 2u_{0ij} + u_{0i-1,j}}{h^2}$$

Substituting (2.23) into (2.19)-(2.22)and simplify to have

$$\theta_{0i,j+1} = \theta_{ij} + \frac{(ih)^2}{ph^2}(\theta_{0i+1,j} - 2\theta_{0ij} + \theta_{0i-1,j}) + \frac{(v_0 p+1)(ih)}{2hp}(\theta_{0i+1,j} - \theta_{0ij}) + k\delta e^{\frac{\theta_{0ij}}{1+\in\theta_{ij}}} \quad \textit{(2.24)}$$

$$\theta_0(ih,0) = 0 \qquad \theta_0(0, jk) = 0 \qquad \theta_0(1, jk) = 1$$

$$u_{0i,j+1} = u_{0ij} + \frac{(ih)^2 k}{h^2}(u_{0i+1,j} - 2u_{0ij} + u_{0i-1,j}) + \frac{(v_0+1)(ih)}{2h}(u_{0i+1,j} - u_{0ij}) + kG\in\theta_{0ij} \quad (2.25)$$

$$u_0(ih,0) = 0 \qquad u_0(0, jk) = 0 \qquad u_0(1, jk) = 1$$

REMARK: Two cases of equations (2.24) and (2.25) will be considered (i)when $\in\neq 0$ (ii) when $\in\to 0$

Case I when $\in\neq 0$

We now solve equations (2.24) and (2.25) using Pascal programming Language the results are shown in figures (2) – (8) .

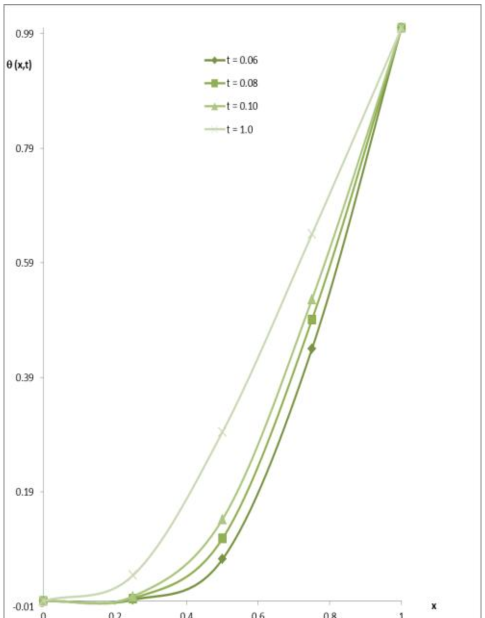

Figure 2:The graph of unsteady temperature distribution θ (x,t) against x of a reacting flow for equation 2.24 when Vo = -1.0,P = 0.71 for various t

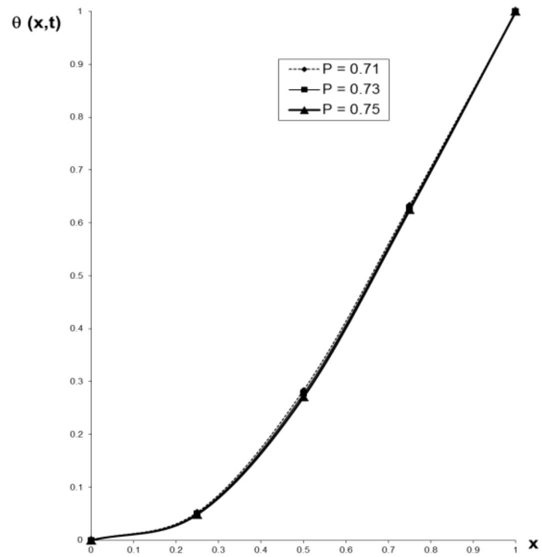

Figure 4:The graph of unsteady temperature distribution θ (x,t) against x of a reacting flow for equation 2.24 when Vo = -1.0,t = 0.2, ε = 0.1,δ = 0.1 for various P

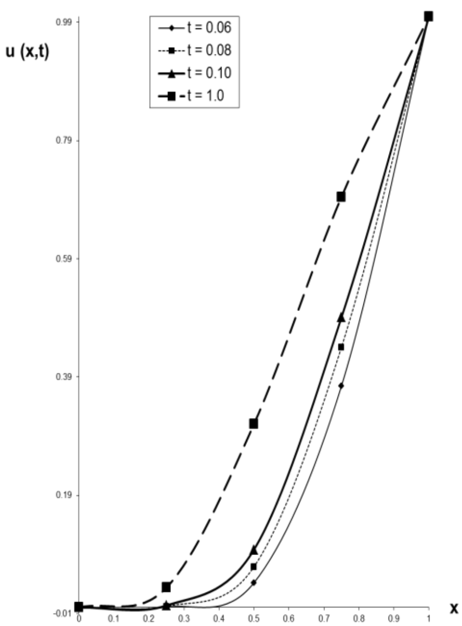

Figure 3:The graph of velocity distribution u (x,t) against x of a reacting flow for equation 2.25 when Vo = -1.0,P = 0.71,G = 2.0 for various t

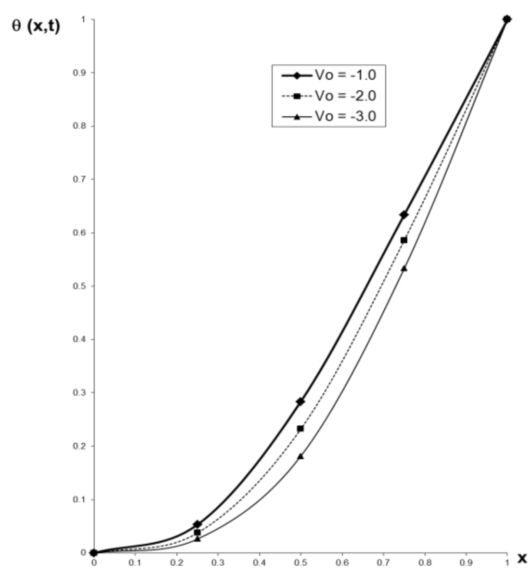

Figure 5:The graph of unsteady temperature distribution θ (x,t) against x of a reacting flow for equation 2.24 when P = 0.71,t = 0.2,δ = 0.1,ε = 0.1 for various Vo

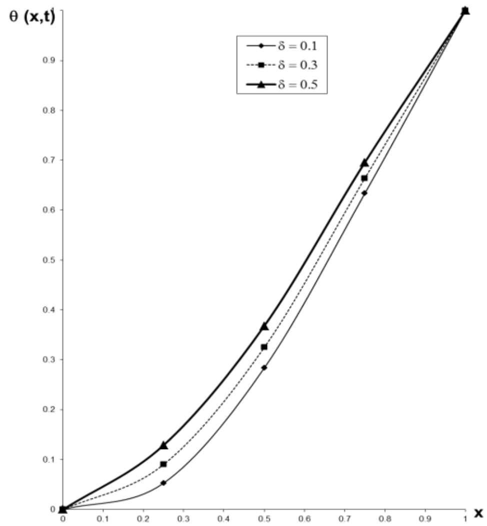

Figure 6:The graph of unsteady temperature distribution θ (x,t) against x of a reacting flow for equation 2.24 when P = 0.71,t = 0.2,ε = 0.1 for various δ

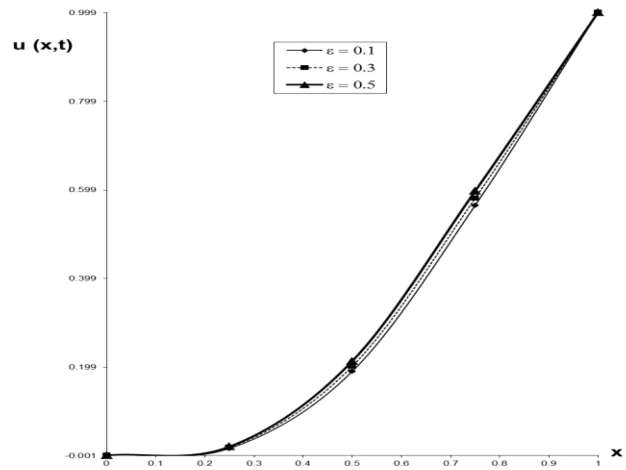

Figure 8:The graph of unsteady veocity distribution u (x,t) against x of a reacting flow for equation 2.25 when Vo = -1.0,P = 0.71,G = 2.0,t = 0.2, δ = 0.1 for various ε

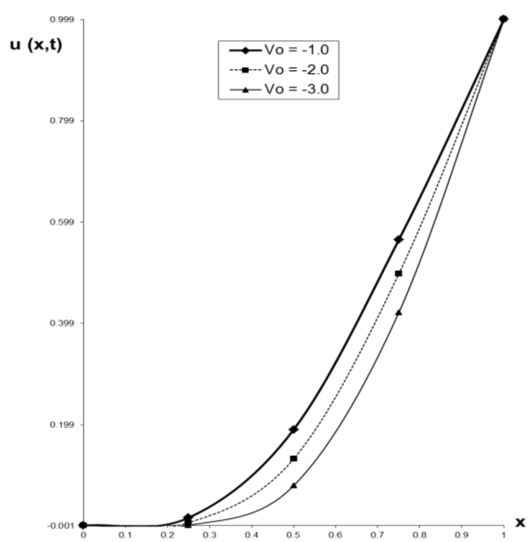

Figure 7:The graph of unsteady velocity distribution u (x,t) against x of a reacting flow for equation 2.25 when G = 2.0,t = 0.2,ε = 0.1 for various Vo

Case II when $\in \to 0$

Equations (2.24) and (2.25) become

$$\theta_{0i,j+1} = \theta_{ij} + \frac{(ih)^2}{ph^2}(\theta_{0i+1,j} - 2\theta_{0ij} + \theta_{0i-1,j}) + \frac{(v_0 p + 1)(ih)}{2hp}(\theta_{0i+1,j} - \theta_{0ij}) + k\delta e^{\theta_{ij}} \tag{2.26}$$

$$\theta_0(ih,0) = 0 \qquad \theta_0(0, jk) = 0 \qquad \theta_0(1, jk) = 1$$

$$u_{0i,j+1} = u_{0ij} + \frac{(ih)^2 k}{h^2}(u_{0i+1,j} - 2u_{0ij} + u_{0i-1,j}) + \frac{(v_0 + 1)(ih)}{2h}(u_{0i+1,j} - u_{0ij}) \tag{2.27}$$

$$u_0(ih,0) = 0 \qquad u_0(0, jk) = 0 \qquad u_0(1, jk) = 1$$

equations (2.26) - (2.27) are solved using Pascal programming Language the results are presented in Figures(9-11)

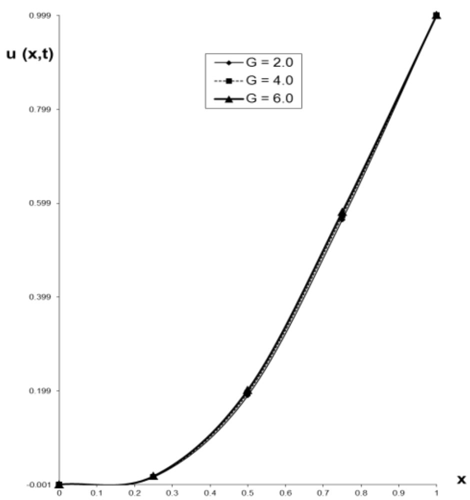

Figure 9:The graph of unsteady velocity distibution u (x,t) against x of a reacting flow for equation 2.25 when Vo = -1.0,P = 0.71,t = 0.2,ε = 0.1,δ = 0.1 for various G

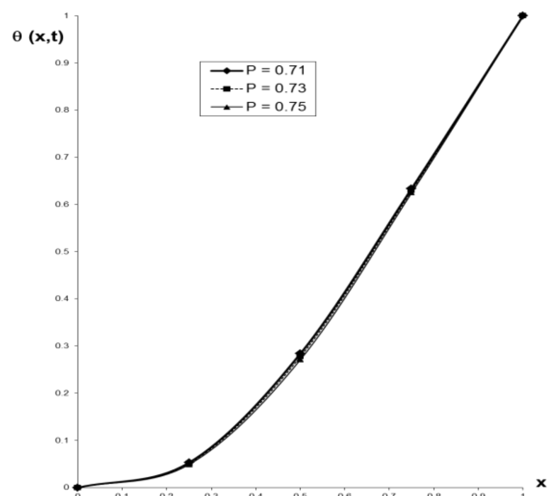

Figure 10:The graph of unsteady temperature distribution θ (x,t) against x of a reacting flow for equation 2.26 when Vo = -1.0,t = 0.2, δ = 0.1 for various P

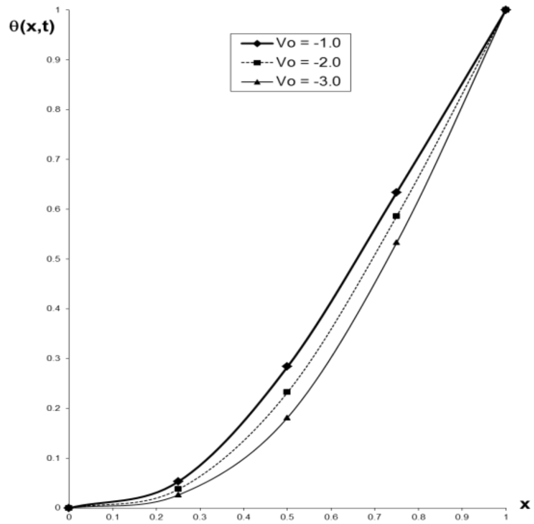

Figure 11:The graph of unsteady temperature distribution θ (x,t) against x of a reacting flow for equation 2.26 when P = 0.71,t = 0.2 ,δ = 0.1 for various Vo

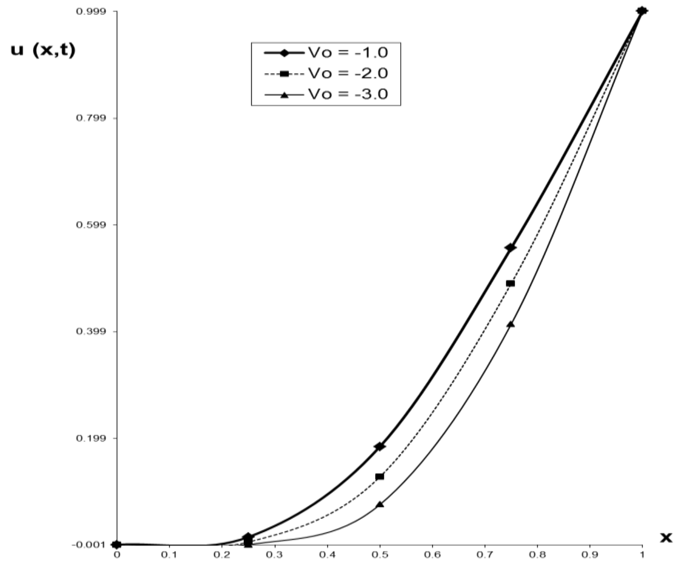

Figure 12:The graph of unsteady velocity distribution u (x,t) against x of a reacting flow for equation 2.27 when t = 0.2 for various Vo

3. Results and Discussion

In this work, the numerical solutions of the nonlinear differential equations (2.25)-(2.27) have been performed by applying finite difference method. This calculations was performed for each parameter governing the flow. The results are presented in the form of non-dimensional velocity and temperature profiles . Figures 2 and 3 show the temperature and velocity profiles for various times. The Figures show that temperature and velocity are an increasing function of time. Figures 4 – 6 show the effect of Prandtl number, suction velocity, Frank-Kamenetskii parameter on the order zero temperature profiles. As Prandtl number increases the fluid temperature decreases (Figure4)while the temperature increases as suction velocity and Frank-Kamenetskii parameter increase Figures 5, 6. Figures 7 – 9 illustrate the effect of varying suction velocity, activation energy parameter and Grashof number on order zero velocity profiles. It is obvious that, increase in suction velocity, activation energy parameter and Grashof number increase the velocity of the flow. The effect of Prandtl number and suction velocity on the order zero temperature profiles are displayed in Figures 10 and 11. It is observed that the order zero temperature increases as Prandtl number and suction velocity increase. In Figure 12 we observe that the order zero velocity increases as suction velocity increases. Finally, the solution to equations 2.20 and 2.22 is zero i.e. $\theta_1 = u_1$=0

4. Conclusion

In this paper, we have studied free convection reacting flows on a porous plate in the presence of constant magnetic field. We provided numerical solutions for both velocity and temperature fields. It was showed that increasing suction velocity ,activation parameter and grashof number increase the velocity of the flow. Finally, we show that temperature and velocity are an increasing function of time.

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