A Fuzzy logic controller for stabilization and control of Double Inverted Pendulum (DIP) using different Membership functions (MF's)

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Abstract

Double Inverted Pendulum (DIP) on cart is a highly non-linear system widely used as a testing bed for verification of newly designed control laws and controllers. In this study control of DIP is obtained using fuzzy logic controllers having different Membership functions(MF's) i.e. triangular, trapezoidal and gbell. The effects of shape of MF's on various controlling parameters i.e. stabilization time, maximum degree of overshoot and steady state error is also illustrated. Simulation results are shown with the help of graphs and tables which proves the validity of proposed method.

Keywords- Double Inverted Pendulum, Fuzzy Logic, Membership function, Matlab-Simulink.

1.0 Introduction

The DIP is a multi-variable, unstable system which is difficult to stabilize in upright position [1]. It represents a kinematic joint for robotic knee and arm. It can also be considered as a model of human and of other animal postural control [2]. In this paper fuzzy logic reasoning is used for stabilization and control of DIP. Fuzzy logic controller is able to stabilize the non-linear systems effectively and increases their flexibility to a great extent [3]. This study shows a comparison between three different Membership functions (MF's) namely triangular, trapezoidal and gbell in terms of stabilization time, maximum degree of overshoot and steady state error. The affect of a particular MF's on performance of Inverted Pendulum system is illustrated in this study. There are several studies which have been done recently for the stabilization and control of DIP. Jianqiang Yi, Naoyoshi Yubazaki and Kaoru Hizota [4] proposed a new fuzzy controller having six inputs and one output based Single Input Rules Modules (SIRM). Ding Chengjun, Duan Ping, Zhang Ming-lu and Zhang Yan-fang [5] used genetic algorithm to optimize the weighting coefficient and MF's parameters of fuzzy controller. Sandeep Kumar Yadav, Sachin Sharma and Mr. Narinder Singh [6] performed the analysis of DIP using Linear Quadratic Regulator (LQR) controller. I.Zamani and M.H.Zarif [3] designed a fuzzy controller based on Lyapunov theorem. Atabak Nejadfard, M.J. Yazdanpanah and Iraj Hassanzadeh [7] proposed a friction compensation of DIP using neuro-fuzzy model. Ehsan kiankhah, Mohammad Teshnelab and Mahdi Aliyari Shoorehdeli [8] designed a neuro fuzzy controller using feedback-error learning for control of DIP etc.

2.0 Double Inverted Pendulum

The classical DIP system consist of a pair of rigid pendulum rods i.e. Bottom pendulum and top pendulum which are interconnected with a joint. The bottom pendulum is attached to a cart which moves in the horizontal direction [9]. Sensors are used to measure angle of each pendulum from vertical i.e. Θ_1 and Θ_2 and displacement of cart x. The final objective is to control the system such that Θ_1, Θ_2 and x should be equal to zero [1]. A view of DIP on cart is shown in figure 1.0 [6].

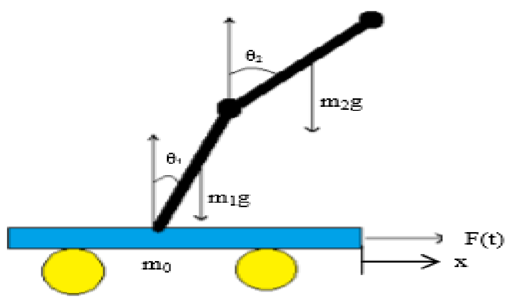

Figure 1.0 A view of DIP on cart

2.1 Mathematical model of DIP

The mathematical equations for control of DIP are derived separately for each subsystem i.e. Cart, First Pendulum and Second Pendulum. The equations are as follows:

A. Cart

$\ddot{x} = 1/M[F-N_1-b\dot{x}]$

where $\ddot{x}$ is acceleration of Cart, M is mass of Cart, F is the applied Force to Cart, N_1 is the interaction force between Cart and First Pendulum and b is friction coefficient.

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B. First Pendulum

$\ddot{\Theta}_1 = 1/I_1 \, [N_1 L_1 \cos + P_1 L_1 \sin\Theta_1 - b_1 \dot{\Theta}_1 + N_2 L_1 \cos \Theta_1 + P_2 L_1 \sin \Theta_1]$

$P_1 = m_1[-\dot{\Theta}_1^2 L_1 \cos\Theta_1 - \ddot{\Theta}_1 L_1 \sin\Theta_1 + g] + m_2[2\dot{\Theta}_1^2 L_1 \cos\Theta_1 - 2\ddot{\Theta}_1 L_1 \sin\Theta_1 - \dot{\Theta}_2^2 L_2 \cos\Theta_2 - \ddot{\Theta}_2 L_2 \sin\Theta_2 + g]$

$N_1 = m_1[\ddot{x} + \dot{\Theta}_1^2 L_1 \sin\Theta_1 - \ddot{\Theta}_1 L_1 \cos\Theta_1] + m_2[\ddot{x} + 2\dot{\Theta}_1^2 L_1 \sin\Theta_1 - 2\ddot{\Theta}_1 L_1 \cos\Theta_1 + \dot{\Theta}_2^2 L_2 \sin\Theta_2 - \ddot{\Theta}_2 L_2 \cos\Theta_2]$

C. Second Pendulum

$\ddot{\Theta}_2 = 1/I_2 \, [N_2 L_2 \cos \Theta_2 + P_2 L_2 \sin\Theta_2 - b_2 \dot{\Theta}_2]$

$P_2 = m_2[-2\dot{\Theta}_1^2 L_1 \cos\Theta_1 - 2\ddot{\Theta}_1 L_1 \sin\Theta_1 - \dot{\Theta}_2^2 L_2 \cos\Theta_2 - \ddot{\Theta}_2 L_2 \sin\Theta_2 + g]$

$N_2 = m_2[\ddot{x} + 2\dot{\Theta}1^2 L_1 \sin\Theta_1 - 2\ddot{\Theta}_1 L_1 \cos\Theta_1 + \dot{\Theta}_2^2 L_2 \sin\Theta_2 - \ddot{\Theta}_2 L_2 \cos\Theta_2]$

Where Θ_1 and Θ_2 are angle of first and second pendulum angle from vertical, $\dot{\Theta}_1$ and $\dot{\Theta}_2$ are angular velocity of first and second pendulum, $\ddot{\Theta}_1$ and $\ddot{\Theta}_2$ are angular acceleration of first and second pendulum, m_1 and m_2 are the masses of first and second pendulum, I_1 and I_2 are moment of inertia of first and second pendulum, L_1 and L_2 are lengths of first and second pendulum, N_1,N_2, P_1 and P_2 are the interaction forces between two Pendulums and g is acceleration due to gravity. The values of various parameters considered are shown in Table 1.0

Symbol	Parameter	Value	Unit
M	Mass of Cart	1	Kg
m_1	Mass of 1st Pendulum	0.5	Kg
m_2	Mass of 2nd Pendulum	0.5	Kg
L_1	Length of 1st Pendulum	0.1	m
L_2	Length of 2nd Pendulum	0.1	m
I_1, I_2	Moment of inertia	0.006	kgm^2
g	Gravity	9.8	m/s^2
b	Coefficient of friction	0.1	Ns/m^2

Table 1.0 values of various parameters

3.0 Fuzzy logic controller for DIP

In this research Mamdani type Fuzzy Inference System (FIS) with triangular, trapezoidal and gbell MF's is used. Three FLC's i.e. FLC-1, FLC-2 and FLC-3 has been designed for cart, bottom pendulum and top pendulum respectively. The inputs for FLC-1 are cart position (x) and cart velocity ($\dot{x}$), for FLC-2 are bottom pendulum angle (Θ_1) and angular velocity ($\dot{\Theta}_1$) and for FLC-3 are top pendulum angle (Θ_2) and angular velocity ($\dot{\Theta}_2$). The outputs for all the controllers is Force (F).

3.1 Defining of Membership function's(MF's)

Each of the input and output variable is fuzzified with seven linguistic variables (Negative Large-NL, Negative Medium-NM, Negative Small-NS, Zero-ZE, Positive Small-PS, Positive Medium-PM and Positive Large-PL). A view of MF's of Cart controller are shown from figure 1.1 to figure 1.3. MF's for other controllers are drawn similarly.

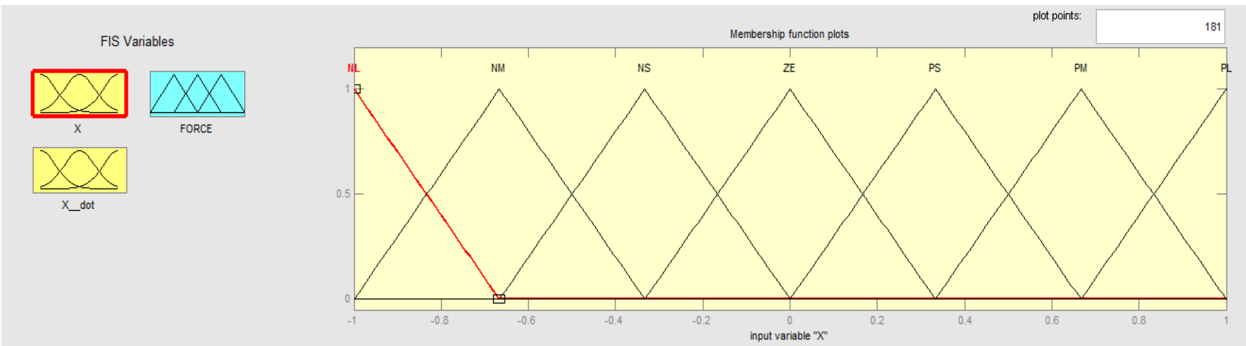

Figure 1.1 Triangular MF's for Cart Controller

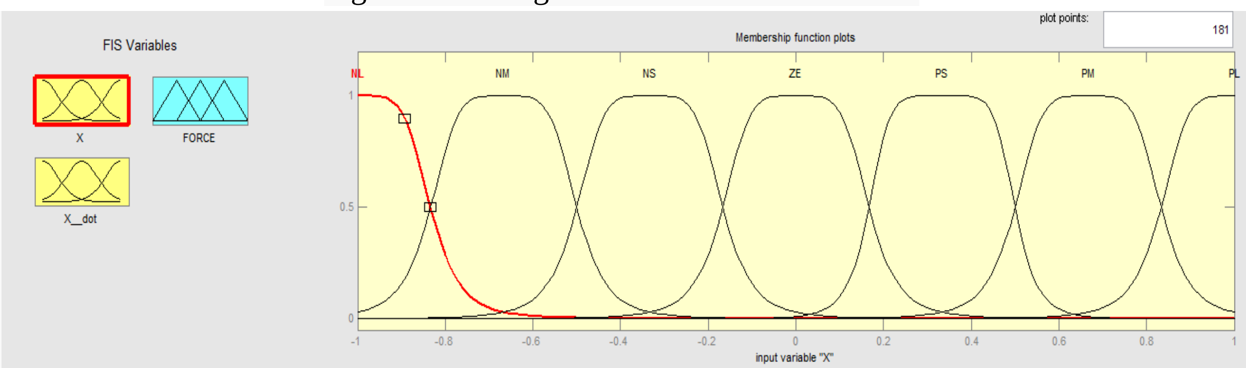

Figure 1.2 Gbell MF's for Cart Controller

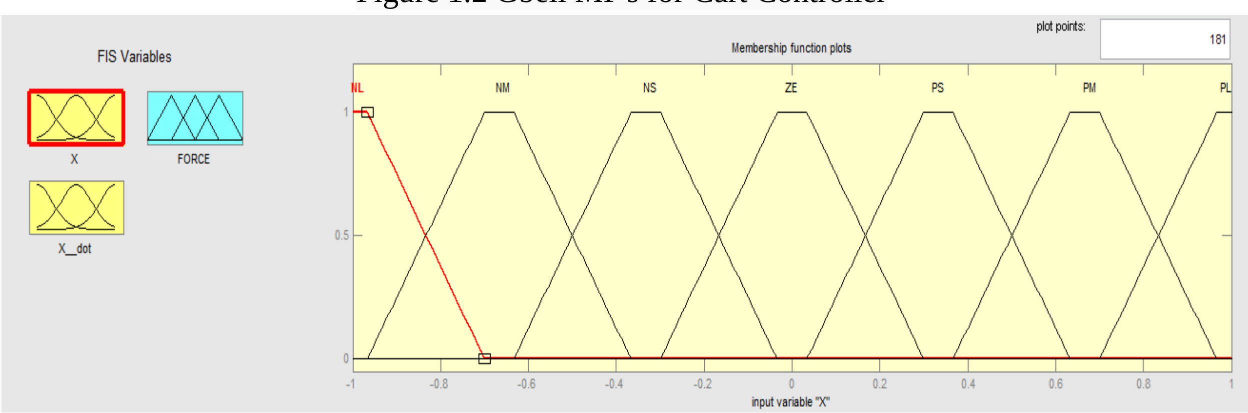

Figure 1.3 Trapezoidal MF's for Cart Controller

3.2 Defining fuzzy control rules

A total of 49 fuzzy if-then rules are used for all FLC's. A view of fuzzy control rules are shown in table 1.1

Position(X)

Del Position ($\dot{X}$)

FORCE	**NL**	**NM**	**NS**	**ZE**	**PS**	**PM**	**PL**
NL	NL	NL	NL	NL	NM	NS	ZE
NM	NL	NL	NL	NM	NS	ZE	PS
NS	NL	NL	NM	NS	ZE	PS	PM
ZE	NL	NM	NS	ZE	PS	PM	PL
PS	NM	NS	ZE	PS	PM	PL	PL
PM	NS	ZE	PS	PM	PL	PL	PL
PL	ZE	PS	PM	PL	PL	PL	PL

Table 1.1 A view of fuzzy control rules

A view of surface viewers using different MF's for cart controller is shown from figure 1.4 to figure 1.6

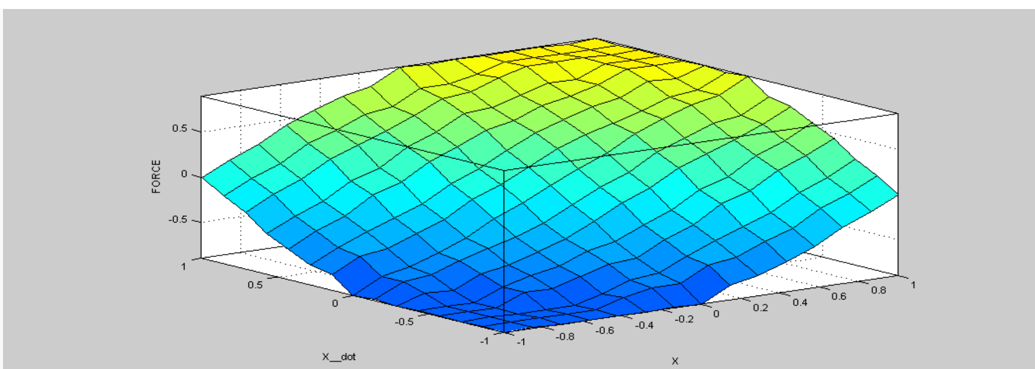

Figure 1.4 Surface viewer for triangular MF's

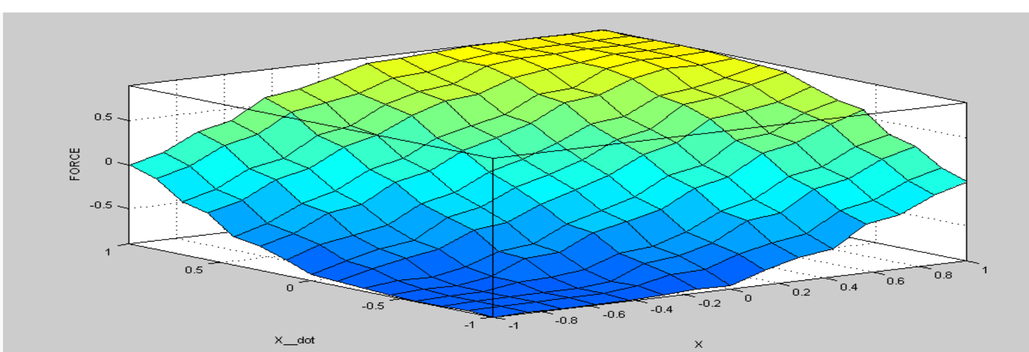

Figure 1.5 Surface viewer for Gbell MF's

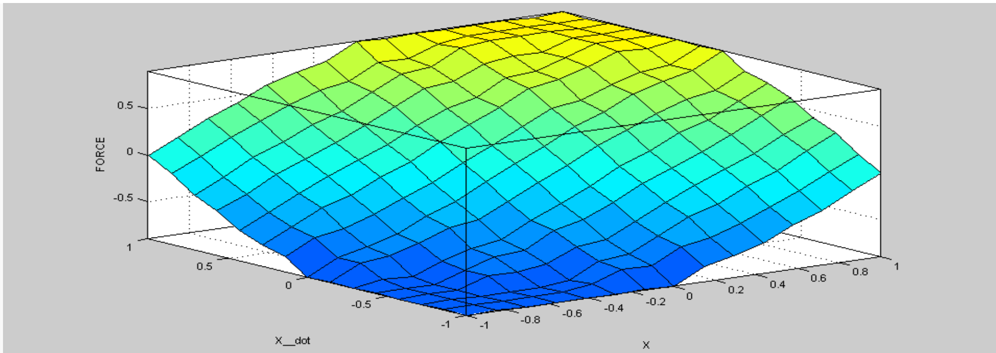

Figure 1.6 Surface viewer for Trapezoidal MF's

4.0 DIP Simulink model

The Modeling of DIP is done in Matlab-Simulink. A view of Simulink model of DIP and its subsystem is shown in figure 1.7 and figure 1.8 respectively.

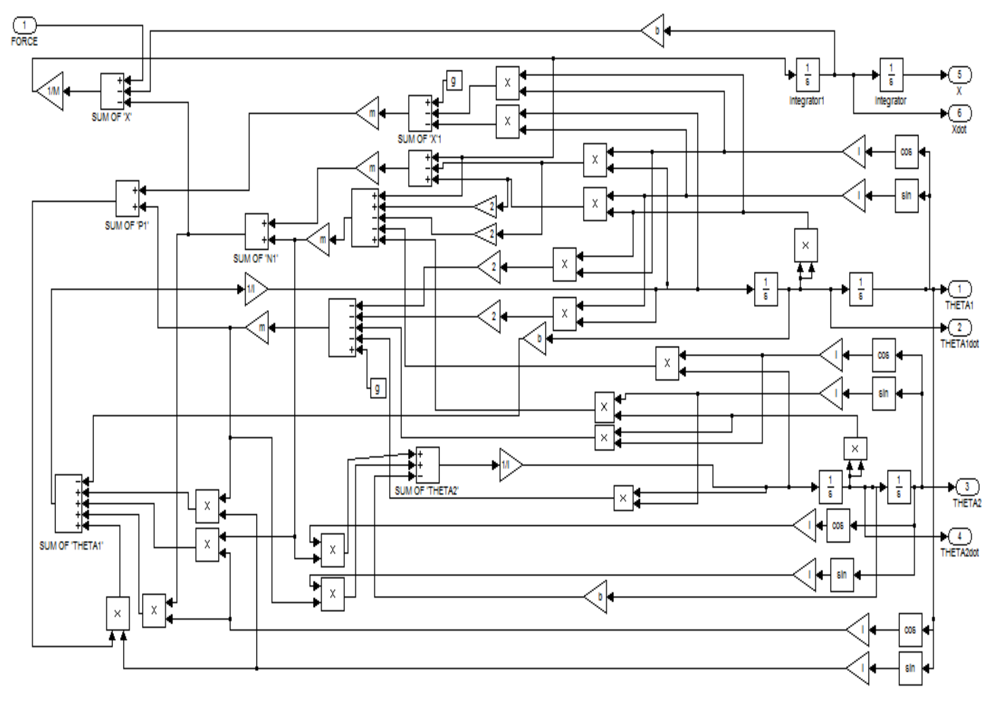

Figure 1.7 DIP Simulink model

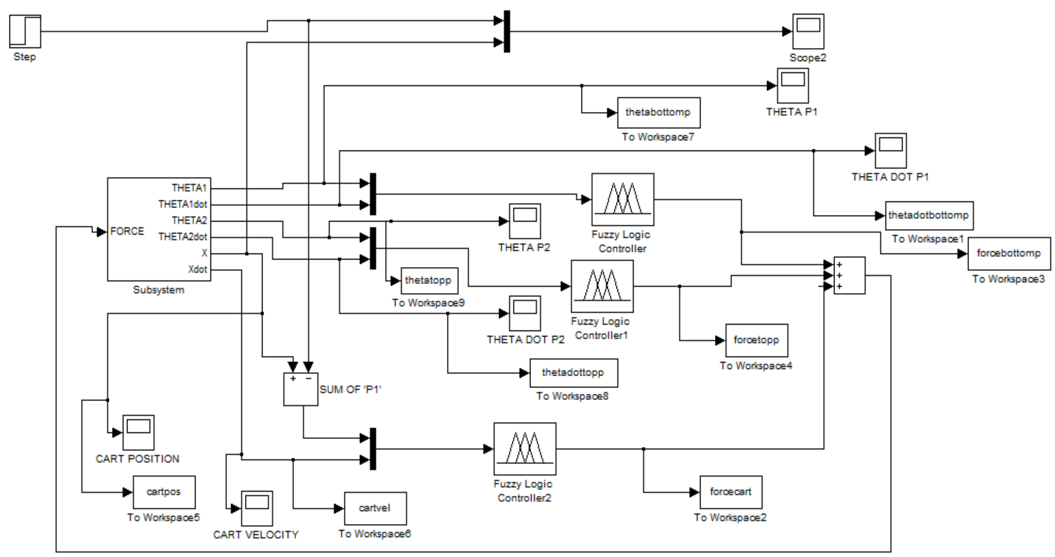

Figure 1.8 DIP Simulink subsystem model

5.0 Simulation results & Comparison

A view of simulation results and comparison between all the three controllers i.e. triangular, trapezoidal and gbell fuzzy controller are shown in graphs and tables below.

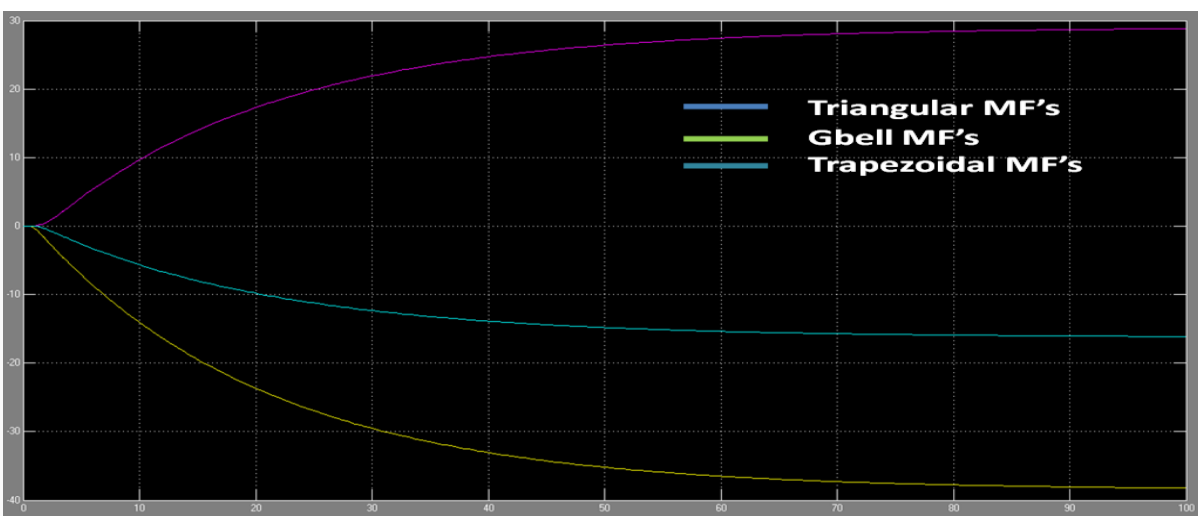

Figure 1.9 Output response for Cart Position using various controllers

Characteristics	Triangular Controller	Trapezoidal controller	Gbell controller
Settling time (s)	35	30	40
Steady state error	0	0	0
Maximum overshoot (Degree)	28	-16	-38

Table 1.2 Output response for Cart Position using various controllers

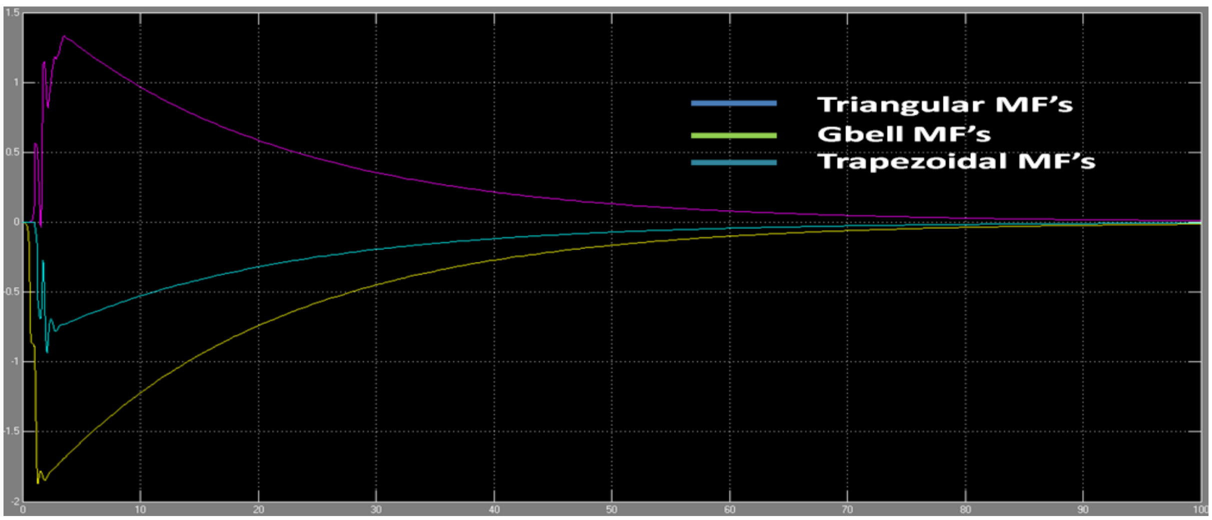

Figure 2.0 Output response for Cart velocity using various controllers

Characteristics	Triangular Controller	Trapezoidal controller	Gbell controller
Settling time (s)	40	30	45
Steady state error	0	0	0
Maximum overshoot (Degree)	1.3 to -0.1	-0.9	-1.8

Table 1.3 Output response for Cart velocity using various controllers

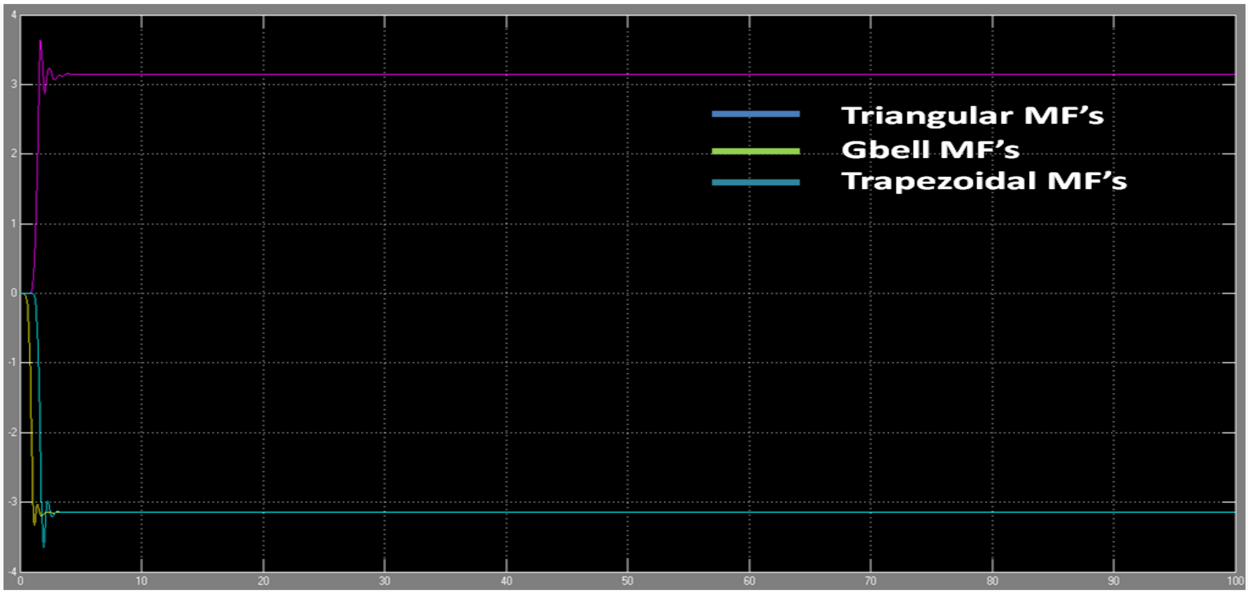

Figure 2.1 Output response for Top pendulum angle using various controllers

Characteristics	Triangular Controller	Trapezoidal controller	Gbell controller
Settling time (s)	4	3	2
Steady state error	0	0	0
Maximum overshoot (Degree)	3.6	-3.6	-3.3

Table 1.4 Output response for Top pendulum angle using various controllers

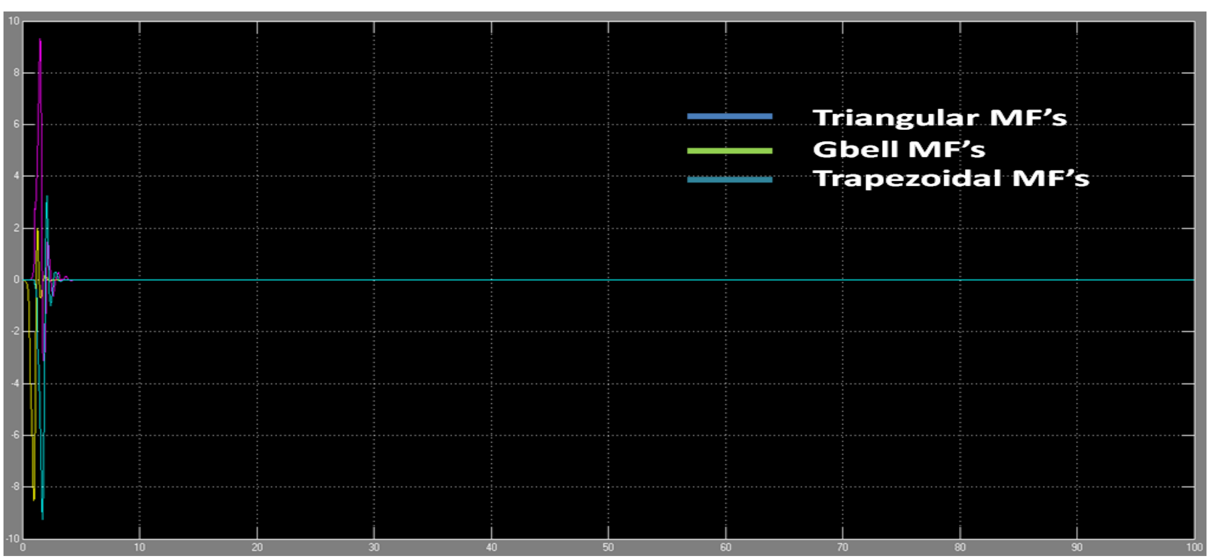

Figure 2.2 Output response for Top pendulum angular velocity using various controllers

Characteristics	Triangular Controller	Trapezoidal controller	Gbell controller
Settling time (s)	4	3	2
Steady state error	0	0	0
Maximum overshoot (Degree)	9.2 to -3.2	3.2 to -9.2	2 to -8.4

Table 1.5 Output response for Top pendulum angular velocity using various controllers

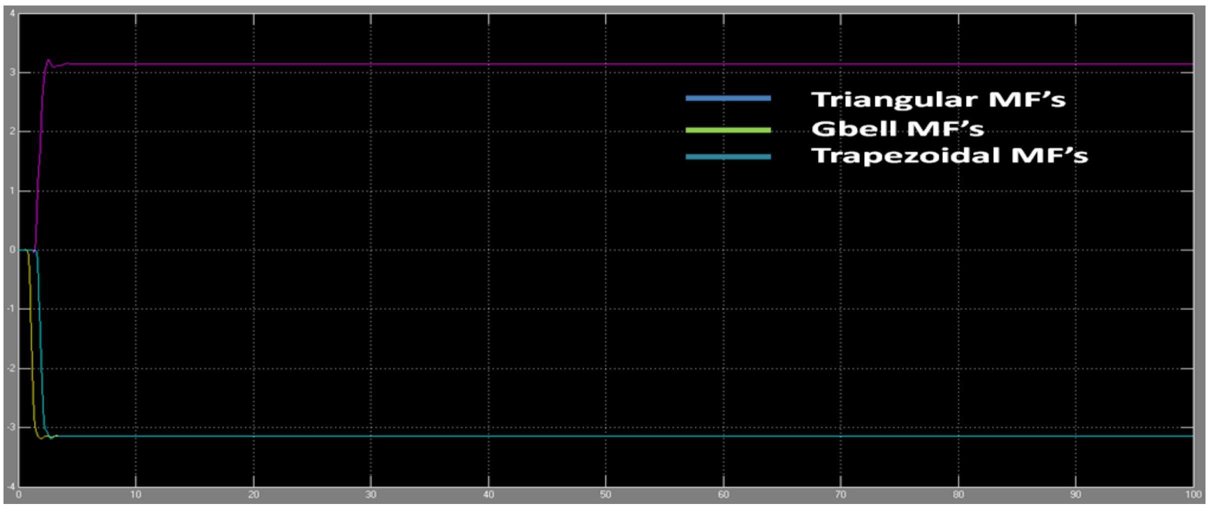

Figure 2.3 Output response for Bottom pendulum angle using various controllers

Characteristics	Triangular Controller	Trapezoidal controller	Gbell controller
Settling time (s)	3.5	3	2
Steady state error	0	0	0
Maximum overshoot (Degree)	3.2	-3.2	-3.3

Table 1.6 Output response for Bottom pendulum angle using various controllers

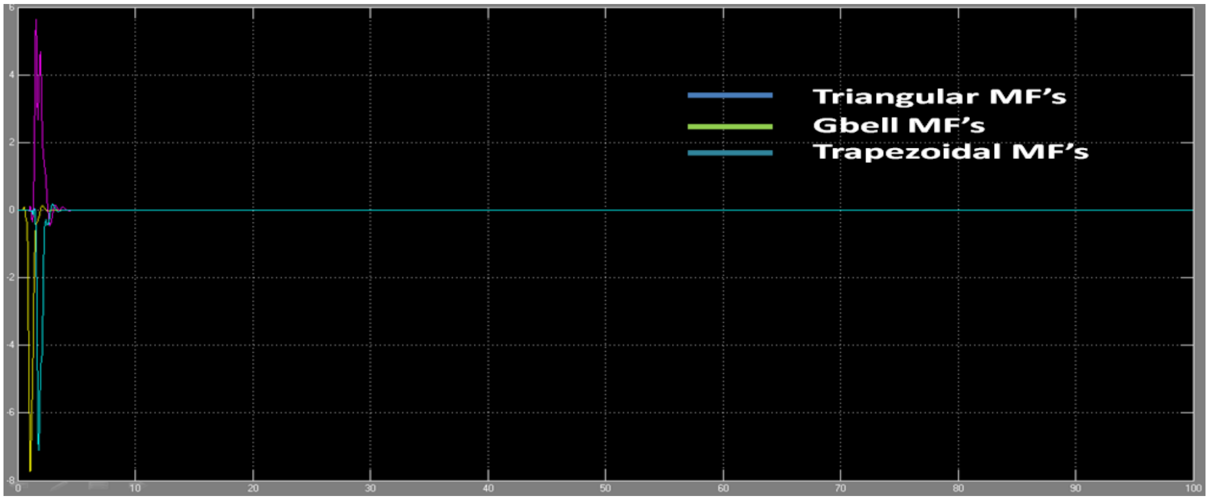

Figure 2.4 Output response for Bottom pendulum angular velocity using various controllers

Characteristics	Triangular Controller	Trapezoidal controller	Gbell controller
Settling time (s)	4	3	2.5
Steady state error	0	0	0
Maximum overshoot (Degree)	5.4 to -0.5	0.1 to -7.1	0.2 to -7.8

Table 1.7 Output response for Bottom pendulum angular velocity using various controllers

6.0 Conclusion

A fuzzy logic controller for control and stabilization of DIP has been successfully proposed. All the three controllers i.e. triangular, trapezoidal and gbell are able to stabilize nonlinear DIP system. The gbell controller gives best results and stabilizes both the pendulums within 2 seconds. Each controller shows almost a zero amount of steady state error. The triangular and trapezoidal controllers shows almost a same amount of overshoot but in different directions. The cart controller has to be optimized to reduce settling time. As a extension to future work affects of variation in type of MF's on various others process parameters for triple, rotary and quadruple inverted pendulum can be further be considered.

7.0 References

[1] Sun Qianlai, Sun Zhiyi, "Application of Multistage Fuzzy Control to a Double Inverted Pendulum", 2009 IEEE International Conference on Control and Automation Christchurch, New Zealand, December 9-11, 2009.

[2] Xinjilefu, Vincent Hayward, and Hannah Michalska, "Stabilization Of The Spatial Double Inverted Pendulum Using Stochastic Programming Seen As a Model of Standing Postural Control", Proceedings of the 9th RAS International conference on Humanoid Robots (Humanoids 09), pp. 367-372.

[3] I.Zamani and M.H.Zaril, "Nonlinear controller for fuzzy models of Double Inverted Pendulums", World Academy of Science, Engineering and Technology International Journal of Electrical, Robotics, Electronics and Communications Engineering Vol:1 No:10, 2007.

[4] Jianqiang Yia, Naoyoshi Yubazakib and Kaoru Hirotac, " A new fuzzy controller for stabilization of parallel-type double inverted pendulum system", Fuzzy Sets and Systems 126 (2002) 105–119.

[5] Ding Cheng-jun , Duan Ping , Zhang Ming-lu and Zhang Yan-fang, "Double Inverted Pendulum System Control Strategy Based on Fuzzy Genetic Algorithm", Proceedings of the IEEE International Conference on Automation and Logistics Shenyang, China August 2009 .

[6] Sandeep Kumar Yadav, Sachin Sharma and Mr. Narinder Singh, " Optimal Control of Double Inverted Pendulum Using LQR Controller", International Journal of Advanced Research in Computer Science and Software Engineering, Volume 2, Issue 2, February 2012 ISSN: 2277 128X.

[7] Atabak Nejadfard, M. J. Yazdanpanah and Iraj Hassanzadeh, "Friction compensation of double inverted pendulum on a cart using locally linear neuro-fuzzy model", Neural Computer & Application (2013) 22:337–347 DOI 10.1007/s00521-011-0686-3.

[8] Ehsan Kiankhah, Mohammad Teshnelab and Mahdi Aliyari Shoorehdeli, " Feedback-Error-Learning For Stability of Double Inverted Pendulum", Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics San Antonio, TX, USA - October 2009.

[9] S. Jadlovska and J.Sarnovsky, " Classical Double Inverted Pendulum – a Complex Overview of a System", Department of Cybernetics and Artificial Intelligence, Faculty of Electrical Engineering and Informatics, Technical University of Kosice, Kosice, Slovak Republic.