AMO-Advanced Modeling and Optimization, Volume 16, Number 2, 2014

An M/M/1/N Queuing System with Reverse Balking and Reverse Reneging

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Abstract

In this paper we develop and incorporate the concept of reverse balking and reverse reneging into the M/M/1/N queuing system. The concept of reverse balking and reverse reneging evolves from its application in investment business. In such a business, more number of customers associated with a firm becomes the attracting factor for the investing customers and conversely, i.e. larger the system size less is the probability of balking and similar is the case of reneging. The steady-state system size probabilities are obtained and some important measures of performance are computed. Sensitivity analysis of the model is also performed.

Mathematics Subject Classification:(2000) MSC 60K25, 90B22
Key words: Reverse Reneging, Reverse Balking, Sensitivity Analysis, Steady-State System Size, Measures of Performance.

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* **AMO-Advanced Modeling and Optimization.ISSN:1841-4311**

1 Introduction

In this era of globalization and liberalization managing business has become a challenging task. Customers have become more selective. Brand switching is more frequent. Due to higher level of expectations customers get impatient more often with a particular firm. Customer impatience has become a serious problem in the corporate world. Queuing theory offers various models that can be used in various service systems facing customer impatience. For instance, the premier work on customer impatience in queuing theory appears in [Haight, 1957, 1959], [Anker and Gafarian, 1963a, 1963b], [Barrer, 1957] etc. Since then a number of papers appear on this concept (reneging and balking). In these models, reneging and balking are the functions of system size / queue length. Larger the system size more is the reneging and similar is the case of balking.

But, when it comes to the business pertaining to investment, more number of customers with a particular firm becomes the attracting factor for investing customers. Thus, the probability of joining in such a firm is high. Modeling such a system as a queuing system indicates that the probability of balking will be low when the system size is more and vice-versa, which is balking in reverse sense (called as Reverse Balking). Further, large number of investors with an investment firm (insurance company, Mutual Fund company, banks etc.) creates trust amongst investors and lets them complete the maturity term of their policies/ bonds. That is, more is the patience when larger is the number of investing customers with a firm. Viewing such a situation as a queuing system (purchase of policy is arrival, claim processing is service) reflects that lesser will be reneging (impatience) when there are more number of customers in the system and vice-versa, which is reneging in reverse sense (called as Reverse Reneging).

Owing to the practically valid aspects of investment management as discussed in the above mentioned paragraphs, we formulate investment business with customer impatience as queuing system in this paper. For example, consider any life insurance company where the purchase of a policy refers to the arrival of a customer in the queuing system (insurance firm), the processed claim refers to as the departure from the queuing system, where the claim processing department is considered as a single server, and finite system capacity (i.e. the number of policies it can accommodate). The claims are processed in order of their arrival (i.e. the queue discipline is FCFS). We incorporate reverse balking and reverse reneging into this model. The model is based on Markovian assumptions.

Rest of the paper is structured as follows: in section 2 literature review is provided; section 3 deals with the model description; in section 4 mathematical model is formulated; section 5 deals with steady-state analysis and measures of performance; sensitivity analysis of the model is performed in section 6; conclusions and future work are provided in section 7.

2 Literature Review:

Queuing with customer impatience has special significance for the business world as it has negative impact on the performance of any business. A customer is said to be impatient if he tends to join the queue only when a short wait is expected and tends to remain in the line if his wait has been sufficiently small. Impatience generally takes three forms. The first is balking, deciding not to join the queue at all up on arrival; the second is reneging, the reluctance to remain in the waiting line after joining and waiting, and the third is jockeying between lines when each of a number of parallel service channels has its own queue [Gross and Harris, 1985].

The notion of customer impatience appears in queuing theory in the work of [Haight, 1957]. He considers an M/M/1 queue with balking in which there is a greatest queue length at which an arrival will not balk. This length is a random variable whose distribution is same for all customers. [Haight, 1959] studies a queue with reneging. [Ancker and Gafarian, 1963a] study M/M/1/N queuing system with balking and reneging and derive its steady-state solution. Ancker and Gafarian (1963b) obtain results for a pure balking system (no reneging) by setting the reneging parameter equal to zero. [Gavish and Schweitzer, 1977] consider a deterministic reneging model with the additional assumption that arrivals can be labelled by their service requirement before joining the queue and arriving customers are admitted only if their waiting time in the system does not exceed some fixed amount. [Robert, 1979] discusses in detail the reneging phenomenon of single channel queues. [Baccelli et al., 1984] consider customer impatience in which a customer gives up whenever his patience or waiting time is larger than a random threshold. [Montazer-Hagighi et al., 1986], [Abou-El-Ata and Hariri, 1992], [Falin and Artalejo, 1995], [Boots and Tijms, 1999] and [Zohar et al., 2002) study and analyze some multi-server queuing systems with balking and reneging. [Liu and Kulkarni, 2008] consider the virtual queuing time process in an M/G/s queue with impatient customers. [Altman and Yechiali, 2008] study an infinite-server queue with system's additional tasks and impatient customers. [Shin and Choo, 2009] consider an M/M/s queue with impatient customers and retrials. They describe the number of customers in orbit and service facility by a Markov chain on two-dimensional lattice space and obtain its stationary distribution. [Al-Seedy et al., 2009] study M/M/c queue with balking and reneging and derive its transient solution by using the probability generating function technique. [Armony et al., 2009] study sensitivity of the optimal capacity to customer impatience. They observe that the prevention of reneging during service can substantially reduce the total cost of lost sales and capacity. [Ibrahim and Whitt, 2009] study real time delay estimation in overloaded multi-server queues with abandonments. [Xiong and Altiok, 2009] study multi-server queues with deterministic reneging times with reference to the time-out mechanism used in managing application servers in transaction processing environments. [Choudhury and Medhi, 2010] study customer impatience in multi-server queues. They consider both balking and reneging as functions of system state by taking into consideration the situations where the customer is aware of its position in the system.

A comprehensive review on queuing systems with impatient customers is presented by [Wang et al., 2010]. They survey queuing systems according to various dimensions like customer impatience behaviours, solution methods of queuing models with impatient customers, and associated optimization aspects. [Liau, 2011] develops a queuing model for estimating business loss. Balking index and reneging rate are used in the model to represent different configurations of balking behaviour and reneging behaviour respectively for different queuing systems. [Kapodistria, 2011] studies a single server Markovian queue with impatient customers and considers the situations where customers abandoned the system simultaneously. She considers two abandonment scenarios. In the first one, all present customers become impatient and perform synchronized abandonments; while in the second scenario the customer in service is excluded from the abandonment procedure. [Kumar, 2012] investigates a correlated queuing problem with catastrophic and restorative effects with impatient customers with applications in agile broadband communication networks.

Recently, [Kumar and Sharma, 2012] introduce the concept of retention of reneged customers in queuing theory. They propose that there is a probability that a reneged customer may be retained for his service if some customer retention mechanism is employed. [Kumar, 2013] studies an M/M/c/N queuing system with balking, reneging and retention of reneged customers and performs its cost-profit analysis.

The above-mentioned papers deal with queuing models with customer impatience where reneging and balking are the functions of system size / queue length. Larger the system size more is the reneging and similar is the case of balking. But, when it comes to the business pertaining to investment, more number of customers with a particular firm becomes the attracting factor for investing customers. Thus, the probability of joining in such a firm is high. Modeling such a system as a queuing system indicates that the probability of balking will be low when the system size is more and vice-versa, which is balking in reverse sense (called as Reverse Balking). Further, large number of investors with an investment firm (insurance company, Mutual Fund company, banks etc.) creates trust amongst investors and lets them complete the maturity term of their policies/ bonds. That is, more is the patience when larger is the number of investing customers with a firm. Viewing such a situation as a queuing system (purchase of policy is arrival, claim processing is service) reflects that lesser will be reneging (impatience) when there are more number of customers in the system and vice-versa, which is reneging in reverse sense (called as Reverse Reneging).

There is no study available on the concept of reverse balking and reverse reneging in the available literature. Therefore, we study a finite capacity, single server Markovian queuing model with reverse balking and reverse reneging which has its wide range applications in investment business.

3 Model Description

The queuing model is based on the following assumptions:

1. The arrival (purchase of insurance policy) to a queuing system (insurance firm) occurs one by one in accordance with a Poisson process with mean rate λ. The inter-arrival times are independently, identically and exponentially distributed with parameter λ.

2. There is a single-server (claim processing department) and the policy claims are processed one by one. The service (claim processing) times are independently, identically and exponentially distributed with parameter μ.

3. The capacity of the system (i. e. total number of policies an insurance firm can accommodate) is finite, say N.

4. The policy claims are processed in order of their arrival, i.e. the queue discipline is First-Come, First-Served.

5. (a)When the system is empty (at the start of insurance business) customers balk (do not purchase policy) with probability q' and may purchase with probability $p'(= 1 - q')$. (b) When there is at-least one customer in the system, customers balk with a probability $(1 - \frac{n}{N-1})$ and join the system with probability $\frac{n}{N-1}$. Such kind of balking is referred to as reverse balking.

6. The policy holders keeping their policies in force after some time, say T may get impatient due to certain reasons and decide to surrender their policies before maturity (i. e. customers wait up-to certain time T and may leave the system before getting service due to impatience). The reneging times (T) are independently, identically and exponentially distributed with parameter η.

4 Mathematical Model Formulation:

Define
$P_n(t)$ = The probability that there are n customers in the system.
The differential difference equations of the system are given by:

$$\frac{dP_0(t)}{dt} = -\lambda p' P_0(t) + (\mu + N\eta)P_1(t); n = 0 \tag{1}$$

$$\frac{dP_1(t)}{dt} = \lambda p' P_0(t) - \left[\left(\frac{1}{N-1}\right)\lambda + \mu + N\eta\right] P_1(t) + \left[\mu + (N-1)\eta\right] P_2(t); n = 1 \tag{2}$$

$$\frac{dP_n(t)}{dt} = \left(\frac{n-1}{N-1}\right)\lambda P_{n-1}(t) - \left[\left(\frac{n}{N-1}\right)\lambda + \mu + \{N-(n-1)\}\eta\right] P_n(t) + \{\mu + (N-n)\eta\}P_{n+1}(t); n \leq 2 \leq N-1 \tag{3}$$

$$\frac{dP_N(t)}{dt} = \lambda P_{N-1}(t) - [\mu + \eta]P_N(t); n = N \tag{4}$$

5 Steady-state Solution

In steady state $\lim_{n\to\infty} P_n(t) = P_n$ and $\lim_{n\to\infty} P'_n(t) = 0$. Therefore the equations (1) to (4) become:

$$0 = -\lambda p' P_0 + (\mu + N\eta)P_1; n = 0 \tag{5}$$

$$0 = \lambda p' P_0 - \left[\left(\frac{1}{N-1}\right)\lambda + \mu + N\eta\right] P_1 + \left[\mu + (N-1)\eta\right] P_2(t); n = 1 \tag{6}$$

$$0 = \left(\frac{n-1}{N-1}\right)\lambda P_{n-1} - \left[\left(\frac{n}{N-1}\right)\lambda + \mu + \{N-(n-1)\}\eta\right] P_n + \{\mu + (N-n)\eta\}P_{n+1}; n \leq 2 \leq N-1 \tag{7}$$

$$0 = \lambda P_{N-1} - [\mu + \eta]P_N; n = N \tag{8}$$

The equations (5) to (8) are solved iteratively to obtain:

$$P_n = \left[\frac{(n-1)!}{(N-1)^{n-1}} \prod_{i=1}^{N-1} \frac{\lambda}{\mu + [N-(r-1)]\eta}\right] p' P_0; 1 \leq n \leq N-1 \quad (9)$$

$$P_N = \left[\frac{(n-1)!}{(N-2)^{N-2}} \prod_{i=1}^{N} \frac{\lambda}{\mu + [N-(r-1)]\eta}\right] p' P_0; n = N \quad (10)$$

using condition of normality $\sum_{i=1}^{N} P_n = 1$.

$$P_0 + \sum_{i=1}^{N} P_n = 1$$

$$P_0 = \frac{1}{1 + \left[\sum_{n=1}^{N-1} \left\{\frac{(n-1)!}{(N-1)^{n-1}} \prod_{i=1}^{N-1} \frac{\lambda}{\mu+[N-(r-1)]\eta}\right\} p'\right] + \left[\frac{(n-1)!}{(N-2)^{N-2}} \prod_{i=1}^{N} \frac{\lambda}{\mu+[N-(r-1)]\eta}\right] p'} \quad (11)$$

6 Measures of Performance:

In this section we derive some important measures of performance:

6.1 Expected System Size (Average Number of Policies with a Firm):

$$L_s = \sum_{n=1}^{N} nP_n$$

$$L_s = \sum_{n=1}^{N} nP_n$$

$$L_s = \sum_{n=1}^{N-1} n \left[\frac{(n-1)!}{(N-1)^{n-1}} \prod_{i=1}^{N-1} \frac{\lambda}{\mu + [N-(r-1)]\eta}\right] p' P_0 + N \left[\frac{(n-1)!}{(N-2)^{N-2}} \prod_{i=1}^{N} \frac{\lambda}{\mu + [N-(r-1)]\eta}\right] p' P_0 \quad (12)$$

6.2 Average Rate of Reverse Reneging (Mean Rate at which the Policies are Surendered)

$$R'_r = \sum_{n=1}^{N}\{N-(n-1)\}\eta P_n$$

$$R'_r = \sum_{n=1}^{N-1}\{N-(n-1)\}\eta P_n + \eta P_N$$

$$R'_r = \sum_{n=1}^{N-1}\{N-(n-1)\}\eta\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{i=1}^{N-1}\frac{\lambda}{\mu+[N-(r-1)]\eta}\right]p'P_0 + \eta\left[\frac{(n-1)!}{(N-2)^{N-2}}\prod_{i=1}^{N}\frac{\lambda}{\mu+[N-(r-1)]\eta}\right]p'P_0 \quad (13)$$

6.3 Average Rate of Reverse Balking (Mean rate at which customers do not purchase policies)

$$R'_b = q'\lambda P_0 + \sum_{n=1}^{N-1}\left(1-\frac{n}{N-1}\right)\lambda P_n$$

$$R'_b = q'\lambda P_0 + \sum_{n=1}^{N-1}\left(1-\frac{n}{N-1}\right)\lambda\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{i=1}^{N-1}\frac{\lambda}{\mu+[N-(r-1)]\eta}\right]p'P_0 (14)$$

7 Sensitivity Analysis of the Model

In this section we perform sensitivity analysis of model. We take variations in L_s w.r.t. the parameters λ, η and q'.

Table 1: Variation in L_s with respect to η

We take $\lambda = 5, \mu = 2, \eta = 0.1, N = 15$

η	L_s	R'_r	R'_b
0.10	0.2230	0.2924	4.1171
0.15	0.1882	0.3825	4.1039
0.20	0.1639	0.4522	4.0930
0.25	0.1454	0.5078	4.0841
0.30	0.1309	0.5531	4.0767
0.35	0.1190	0.5908	4.0704
0.40	0.1092	0.6226	4.0651
0.45	0.1008	0.6498	4.0605
0.50	0.0937	0.6734	4.0566
0.55	0.0875	0.6939	4.0531
0.60	0.0821	0.7121	4.0500
0.65	0.0773	0.7282	4.0472
0.70	0.0731	0.7426	4.0448
0.75	0.0692	0.7556	4.0426
0.80	0.0658	0.7673	4.0406
0.85	0.0627	0.7779	4.0387
0.90	0.0599	0.7876	4.0371
0.95	0.0573	0.7965	4.0355
1.00	0.0549	0.8047	4.0341

From table -1 it can be observed that expected system size decreases with increase in rate of reverse reneging. The average rate of reverse reneging also increases. Average rate of reverse balking decreases with increase in reverse rate of reneging; this is due to the fact that higher rate of reverse reneging leaves more space in the system and more and more customers can be accommodated in the system.

Table 2: Variation in L_s with respect to λ

We take $\mu = 2, \eta = 0.1, q' = 0.8, N = 15$

λ	L_s	R'_r	R'_b
2	0.0873	0.1261	1.6212
3	0.1305	0.1842	2.4460
4	0.1744	0.2395	3.2787
4.5	0.1975	0.2663	3.6974
5	0.2230	0.2924	4.1171
5.5	0.2556	0.3180	4.5359
6	0.3076	0.3429	4.9490
6.5	0.4106	0.3667	5.3425
7	0.6384	0.3879	5.6820
7.5	1.1421	0.4034	5.8903
8	2.1688	0.4065	5.8268
8.5	3.9615	0.3891	5.3231
9	6.4516	0.3479	4.3445
9.5	9.0789	0.2934	3.1384
10	11.2139	0.2423	2.0617

From table -2, it can be observed that, with increasing rate of arrival the expected system size increases. If we observe the average rate of reverse reneging, it increases initially as the customers could not build their confidence in the system but as system size increases, the customers show a higher confidence in the system and thus the reverse reneging rate decreases. Similar is the case with reverse balking. Initially, due to less number of customers in the system reverse balking rate is high. As, more and more customers join the system average rate of reverse balking decreases.

Table 3: Variation in L_s with respect to μ

We take $\lambda = 5, \eta = 0.1, q' = 0.8, N = 15$

μ	L_s	R'_r	R'_b
2	0.5080	0.3622	4.0797
2.5	0.2725	0.3244	4.1227
3	0.2230	0.2924	4.1171
3.5	0.1971	0.2662	4.1083
4	0.1782	0.2443	4.1002
4.1	0.1749	0.2404	4.0987
4.2	0.1718	0.2366	4.0973
4.3	0.1687	0.2329	4.0958
4.4	0.1658	0.2293	4.0945
4.5	0.1630	0.2258	4.0931
4.6	0.1603	0.2225	4.0918
4.7	0.1577	0.2192	4.0905
4.8	0.1552	0.2160	4.0893
4.9	0.1527	0.2130	4.0881
5	0.1503	0.2100	4.0869

It is visible from table -3 that with a high service rate the service improves and more and more customers move out of the system hence the expected system size decreases. On the other hand a high service rate also leaves a positive effect on reverse reneging and reverses balking.

Table 4: Variation in L_s with respect to q'

We take $\lambda = 5, \mu = 2, \eta = 0.1, N = 15$

q'	L_s	R'_r	R'_b
0.10	0.5944	0.7793	2.6468
0.20	0.5610	0.7356	2.7789
0.30	0.5233	0.6861	2.9284
0.40	0.4802	0.6296	3.0990
0.50	0.4305	0.5645	3.2956
0.60	0.3727	0.4887	3.5244
0.70	0.3046	0.3993	3.7942
0.80	0.2230	0.2924	4.1171
0.90	0.1237	0.1622	4.5104

It is visible from table-4 higher is the probability of balking (when there are zero customers in the system) lower is the expected system size. While reverse balking rate also increases with increase in probability of balking.

8 Conclusion and Future Work:

In this paper the concept of reverse balking and reverse reneging is incorporated into an M/M/1/N queuing system. The steady-state analysis of the model is performed and some important measures of performance are derived. Sensitivity analysis of the model is also performed. This model finds its application in investment business facing customer impatience. In future, the multiserver case of the model can be studied. Model can be studied in non-Markovian environment. This paper studies steady-state results; the time-dependent studies of the model can also be performed.

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