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The Inverse Domination number and Independence number of some Cubic Bipartite Graphs

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Abstract

In this paper we discuss the relationship between independent set and inverse vertex dominating set of finite undirected graphs. In particular we discuss them for some cubic bipartite graphs and find that the inverse vertex domination number is $\gamma^{-1}(G) \leq \frac{p}{3}$ and independence number is $\beta(G) = \frac{p}{2}$

Keywords: Domination number,inverse dominating set,inverse domination number,independent set,independence number,bipartite graph, cubic bipartite graph.

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1

1 Introduction

The concept of inverse domination was introduced by Kulli.V.R. and Sigarkanti.S.C.[4]. Let D be a γ-set of G. A dominating set $D' \subseteq (V - D)$

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is called an inverse dominating set of G with respect to D. The inverse domination number $\gamma^{-1}(G)$ is the order of the smallest inverse domination set. The domination number of a graph G denoted as $\gamma(G)$ is the minimum cardinality of a dominating set in G. If (V-D) contains another dominating set say D', then D' is called the inverse dominating set of G with respect to D. The minimum cardinality of an inverse dominating set is called the inverse domination number of G. Every connected graph has an inverse domination set. A set of vertices in a graph G is said to be an independent set if no two vertices in the set are adjacent. A minimal dominating set from which no vertex can be removed without destroying its dominance property is called an a minimal dominating set. A maximal independent set is an independent set to which no other vertex can be added without destroying its independence property. The number of vertices in the largest independent set of a graph G is called the independence number and is denoted by $\beta(G)$. A graph is called a cubic graph (regular graph of degree 3) if the degree of each vertex is of degree three. A bipartite graph is a graph in which the vertex set can be divided into two disjoint vertex subsets X and Y such that every edge has one end in X and an another end in Y. A graph G is said to be cubic bipartite if it is bipartite as well as cubic. We use the following existing results about the independence number and inverse domination number of a graph G. Also,we discuss the relation between inverse domination number and independence number of a finite simple connected graphs.

OBSERVATION:

1) $\gamma^{-1}(K_p) = 1$ and $\gamma_i^{-1}(K_p) = 1$ where K_p is a complete graph with p vertices.
2) $\gamma^{-1}(P_p) = \lceil (P+1)/3 \rceil$ and $\gamma_i^{-1}(P_p) = \lceil (P+1)/3 \rceil$ where P_p is a path graph with p vertices.
3) $\gamma^{-1}(C_p) = \lceil (P)/3 \rceil$ and $\gamma_i^{-1}(C_p) = \lceil (P)/3 \rceil$ where C_p is a cycle with p vertices.
4) $\gamma^{-1}(W_p) = \lceil (P-1)/3 \rceil$ and $\gamma_i^{-1}(W_p) = \lceil (P-1)/3 \rceil$ where W_p is a wheel graph with p vertices.

Fundamental Results.

1)If a graph G has no isolates,then $\gamma(G) \leq \gamma^{-1}(G)$.
2)If a graph G has no isolates,then $\gamma(G) + \gamma^{-1}(G) \leq (P)$.

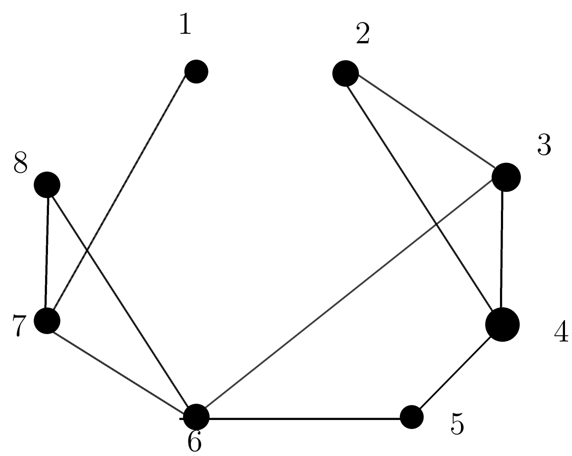

fig.1

In the above fig.1 $\gamma-$ set=$\{4,7\}$ γ^{-1}-set=$\{1,3,6\}$ independent set-$\{1,2,5\}$ maximal independent set β =$\{1,2,5,8\}$,γ =2, γ^{-1} =3 and β =4.
There are many graphs whose domination number,inverse domination number and independence number are equal.
For example: C_4, P_4

Theorem 1.1. *An independent set of a graph G is an inverse dominating set if and only if it is maximal in $\langle V-D\rangle$.*

Proof. Let S be an independent set as well as the inverse dominating set. In other words,S is a dominating set in $(V-D)$. We have to prove that it is maximal in the induced subgraph $\langle V-D\rangle$. We prove this by the method of contradiction.
Suppose S is not maximal in $\langle V-D\rangle$. Then at least one vertex v_i can be added in S without destroying its independence property. We know that $(V-D)$ belongs to one of the following cases.

Case 1. A graph $\langle V-D\rangle$ is connected.
Since S is the inverse dominating set,all of its vertices in $(V-S)$ is adjacent to at least one vertex in S. Therefore, addition of a vertex will destroy the independence property. This is a contradiction.

Case 2. A graph $\langle V-D\rangle$ is disconnected. We consider the following subcases.

Sub-case 2.1 If it is disconnected isolated free graphs.Then we have many components such as G_1,G_2,G_3,...,G_i as graphs. Also corresponding independent sets say $S_i^{'s}$ exist.So each of the S_i is a dominating set in the graph G_i and $S_1 \cup S_2 \cup S_3 \cup, ..., \cup S_i$.Therefore it is not possible to add a vertex a vertex without destroying its independence property. This is a contradiction.

Sub-case 2.2 If $\langle V - D \rangle$ is totally disconnected. Then all the vertices form an independent set and as well as a dominating set. Therefore no vertex is left out to add into it. Thus we have the necessary part.

Conversely,let S be a maximal independent set in $\langle V - D \rangle$. We have to prove that S is an inverse dominating set or it is a dominating set in V-D. Suppose S is not an inverse dominating set,then at least one vertex $v_i \in V$ is neither in the set S or in $n(S)$. Then v_i can be added to S without destroying its independence property but contradicts the maximality of S. Thus S is the inverse dominating set.

□

Theorem 1.2. *If G is a complete graph of order $p \geq 2$,then $\gamma^{-1}G$ =$\beta(G)$.*

Proof. the vertices in a graph G are adjacent among themselves. Therefore any singleton set can be a minimum dominating set. Suppose a vertex v is a minimum dominating set D of G,and then any other vertex u is a minimum dominating set in V-D. That is the cardinality of inverse dominating setD', γ^{-1}=1. Also for any complete graph cardinality of largest independent set,β is one. Thus $\gamma^{-1}G = \beta(G)$. □

Theorem 1.3. *If every non-end vertex of a graph G is adjacent to at least one end vertex, then γ^{-1}=$\beta(G)$.*

Proof. Given all the non end vertices are adjacent to at least one end vertex. Then the graph will belongs to one of the following: (i) a graph in which each non-end vertices is adjacent to exactly one end vertex.
(ii) a graph in which some will be adjacent to exactly one end vertex and some will be adjacent to more than one end vertex (iii) a graph in which each non end vertex is adjacent to more than one end vertex to dominate these

end vertices we need to choose either end vertices or support vertices.In all these cases the number of end vertices constitutes the inverse dominating set and also the maximum independent set. Thus γ^{-1}=$\beta(G)$.

□

Theorem 1.4. *In any simple graph G, the addition of an edge decreases the number of minimal inverse dominating sets.*

Proof. We know that all the inverse dominating sets is a dominating set. We can also find a maximal independent set in (V-D).One of them will be a dominating set of (V-D) By the theorem if an edge is added ,then the number of minimal dominating set decreases by [1],addition of an edge decreases the number of minimal inverse dominating sets.

□

Independence number and Inverse dominating set of Cubic Bipartite graph.

In this section, we discussed the inverse domination number and independence number of some cubic bipartite graphs. We know that the cubic bipartite graph starts with number of vertices $p = 6$ and cubic bipartite graph cannot have odd number of vertices.

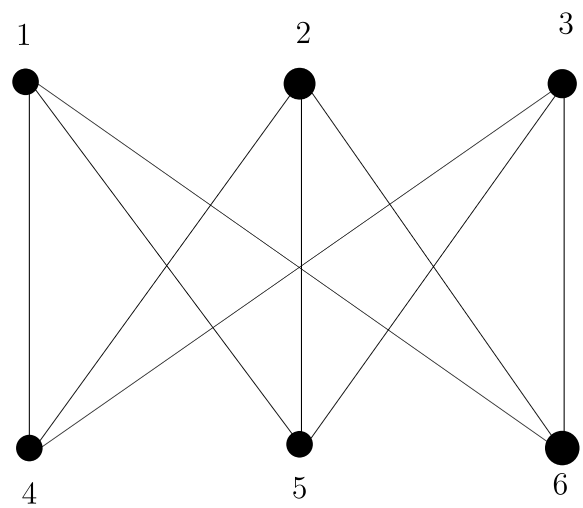

fig.2

Next,we see that the cubic bipartite graph with 8 vertices.

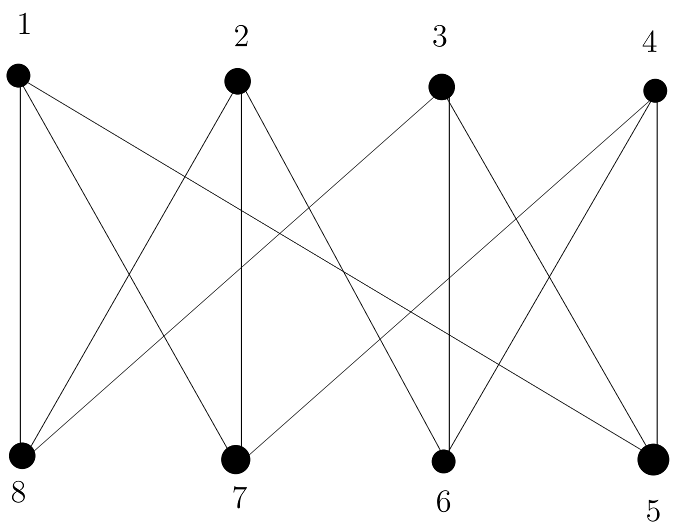

fig.3 .

Here $\{1,6\}$ can be a dominating set and $\{2,5\}$ can be an inverse dominating set.Then=$\frac{p}{3}$.
Also the largest independent set is $\{1,2,3,4\}$. Therefore $\beta^{-1}(G)=\frac{p}{2}$.

Next,we see the cubic bipartite graph with 10 vertices.

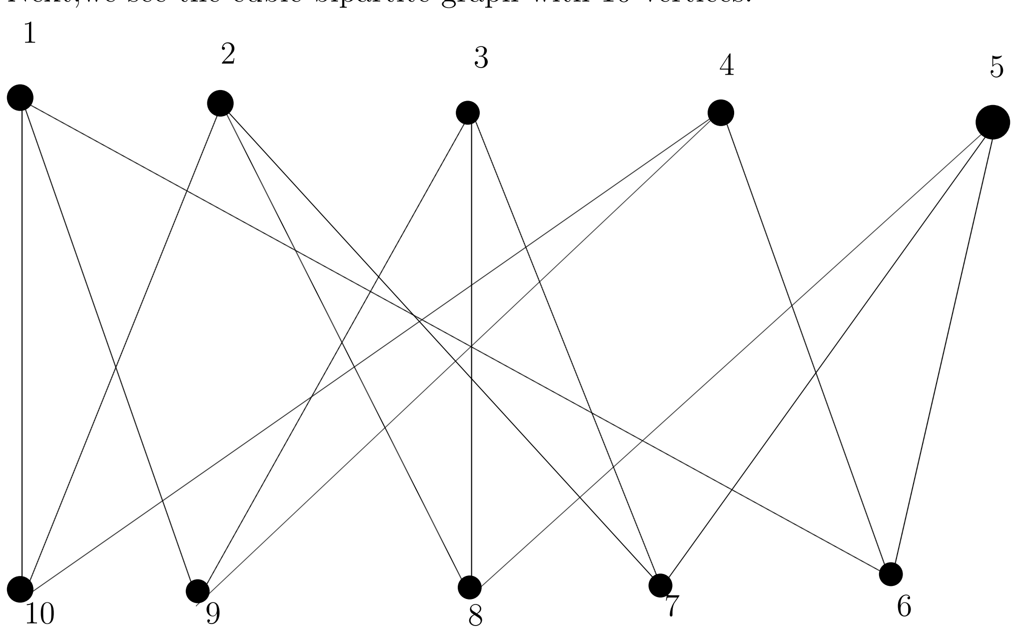

fig.4 .

Here,$\{1,7,8\}$ can be a dominating set and $\{2,9,6\}$ can be an inverse dominating set.$\gamma^{-1}\{G\}$=$\frac{p}{3}$.
Also the maximum independent set is $\{1,2,3,4,5\}$.Therefore $\beta\{G\}$ =$\frac{p}{2}$.
Next, for P=12 we have

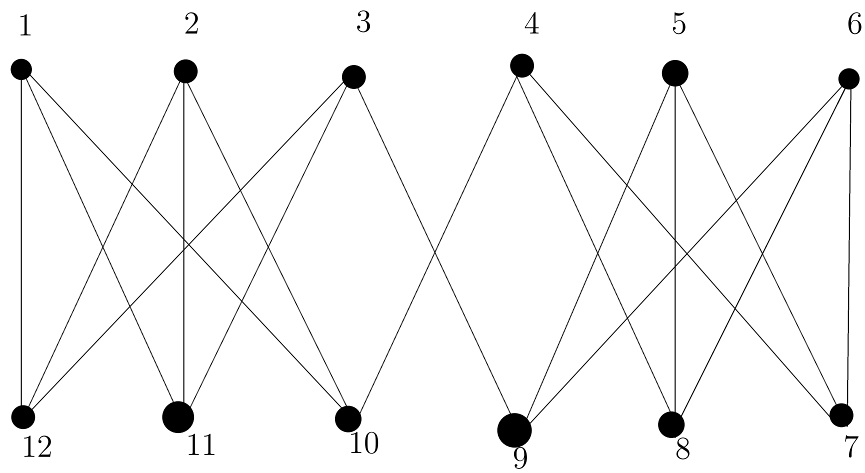

fig .5

In fig.5 $D = \{1,4,8,11\}$ can be a dominating set and $D' = \{2,5,7,12\}$ can be an inverse dominating set. Then $\gamma^{-1}(G)$=$\frac{p}{3}$.
Also the largest independent set is I= $\{1,2,3,4,5,6\}$.
Therefore $\beta(G)$=$\frac{p}{2}$.

Next, we see a cubic graph with 14 ,16,18 vertices we get the same independence number and also the same inverse domination number.

Theorem 1.5. *For any cubic bipartite graph,*$\gamma^{-1}(G) \leq \frac{p}{3}$.

Proof. Given,graph G is cubic bipartite. So vertex set can be partitioned into two subsets X and Y such that all the edges have one end in x and the other end in Y. Since the degree of each vertex is three, every vertex is adjacent to three vertices. So at most $\frac{p}{3}$ vertices sufficient in D to dominate all the other vertices. Also it is possible to choose at most $\frac{p}{3}$ vertices in $(V-D)$ for another dominating set D'.Therefore, the inverse domination number of cubic bipartite graph is at most $\frac{p}{3}$.

$\square$

Conclusion.The results of inverse domination number and largest independent set are depicted with many examples. What we have determined is that the inverse domination number of any cubic bipartite graph with 'P' vertices always have less than or equal to $\frac{p}{3}$ as its inverse domination number in G.

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