Inventory and credit strategies for day-terms credit linked demand

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Abstract

Selling on credit is a common business practice followed by the firms. Credit period through its influence on demand becomes a determinant of inventory decisions and inventory sold on credit gets converted to accounts receivable indicating the interaction between them. The close interaction between inventory and accounts receivable indicates that the efficient management of one component cannot be undertaken without the simultaneous consideration of the other component. Consequently, in this paper, using a day-terms credit linked demand function a model is developed to determine the optimal credit period and inventory decisions simultaneously in a systems perspective. The model is developed using discounted cash flow (DCF) approach and the objective is to maximize the present value of net profit per unit time. Accounts receivable carrying cost often ignored in the previous models is incorporated in order to have correct trade-off between various cost and benefits associated with the inventory-credit system. Finally, numerical example and sensitivity analysis has been done to illustrate the effectiveness of proposed model. Results shows that accounts receivable carrying cost and time value of money has significant impact on inventory-credit decisions. Results also show that credit decisions are significantly influenced by the inventory carrying cost.

Keywords: inventory, credit, accounts receivable carrying cost, credit linked demand, discounted cash flow, DCF;

AMO – Advanced Modeling and Optimization. ISSN: 1841-4311
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1. Introduction

The main objective of any inventory control system is to satisfy future demand in a most economical way. To do so many EOQ models have been developed by the researchers under various situations resembling the real world business scenario. However, the classical EOQ models assume that demand cannot be influenced by the decision maker. But in real world of business the decision maker can influence the demand of its product by giving credit period to its customers. Although many theories have been put forward for explaining why firms grant trade credit to their customers [Bougheas et al., 2009; Brennan et al., 1988; Daripa and Nilsen, 2010; Emery, 1984; Lee and Stowe, 1993; Lehar et al., 2012; Metzler, 1960; Peterson and Rajan, 1997; Schwartz, 1974; Smith, 1987; Vaidya, 2011], e.g., it can be an effective means of price discrimination, product differentiation and product quality guarantee. But the one common theme among various theories of trade credit is that it is used to stimulate the demand. Credit period can take two forms: day-terms and date-terms [Kingsman, 1983; Carlson and Rousseau, 1989]. Day-terms credit requires payment within a fixed period after the purchase and date-terms credit requires payment by a specified date, irrespective of the time of purchase.

Selling on credit without reducing selling price of the product is a common business practice followed by the firms. In credit elastic market the credit period will have a significant impact on demand which is a determinant of inventory decisions. The decision modeling in inventory system with credit linked demand has been considered by few researchers. In the beginning, [Aggarwal and Aggarwal, 1995] developed an EOQ model with date-terms credit linked demand with time discounting. [Jaggi et al. 2007] expand on this theme and developed an EOQ model with date-terms credit linked demand under two stage trade credit financing in a discounted cash flow framework. These models deal with the determination of only inventory decisions and credit period is assumed to be known. [Jaggi et al. 2008] developed an inventory model with day- terms credit linked demand under permissible delay in payments to determine optimal credit period and replenishment policy. Other relevant articles with credit linked demand are [Su et al., 2007], [Thangam and Uthayakumar, 2009], [Maiti, 2011], [Ho, 2011], [Annadurai and Uthayakumar, 2012], [Teng and Lou, 2012], [Lou and Wang, 2012], [Giri and Maiti, 2013], [Shah et al., 2014] and [Wang et al., 2014].

In all the above models with credit linked demand function, the cost of granting credit period is ignored by the researchers. However, the inventory sold on credit involves the creation of accounts receivable causing accounts receivable carrying cost to the firm. The interactive nature of inventory and accounts receivable implies that all the costs and benefits associated with the inventory-credit system must also be integrated in the model. The accounts receivable which will be converted to cash in future constitute a major category of the current assets of the firm; therefore, the cost of servicing accounts receivables forms a significant proportion of the total cost. This is a variable cost depending upon the volume of credit sales and time of carrying accounts receivable (i.e. collection period) which in turn depend upon the length of credit period given to the customers. Selling on credit will be economical for the firm if the additional revenue generated due to increased sales is sufficient to compensate the cost of granting credit period. If this is not the case, the firm would like to have cash sales program as also been shown in this paper. Hence, ignoring the cost of granting credit period is unjustified while determining the optimal values of credit period and/or inventory decisions when demand is influenced by credit period. The cost associated with carrying accounts receivable are the cost of financing accounts receivable, administrative costs in running a credit department, delinquency or collection costs and cost of default by the customers i.e. bad debt losses. Moreover, the close interaction between inventory and accounts receivable indicates that efficient management of one component cannot be undertaken without the simultaneous consideration of other component. Since inventory and credit decisions are interdependent, therefore, it seems worthwhile to coordinate inventory and credit decisions and hence make the two decisions simultaneously in a systems perspective. In addition, all the models except [Aggarwal and Aggarwal, 1995], [Jaggi et al., 2007], and [Maiti, 2011] are developed using the average cost approach. But in average cost approach the time value of money is not taken into account and there is no distinction between out-of-pocket carrying cost and opportunity cost of the fund invested in inventory and also in accounts receivable. Also, it does not capture the effect of delayed revenue realization arising due to credit period given to customers. From the financial standpoint, all cash outflows and inflows related to inventory-accounts receivable system that occurs at different point of time have different values. As a result, it is necessary to consider the effect of time value of money on inventory-credit decisions. In contrast to average cost approach, the DCF approach allows proper recognition of the financial implication of the opportunity cost and out-of-pocket costs associated with the economic system. It also permits an explicit recognition of the exact timing of each cash-flow associated with the economic system and considers the time value of money.

Consequently, in this paper, using DCF approach a model has been developed to jointly determine optimal credit period and inventory decisions by incorporating the accounts receivable carrying cost which has been often ignored in previously developed model. The firm purchases a single item on cash and offer equal credit period to all of its customers on the purchase of item. The demand is a function of credit period given to the customers and its effect on demand is assumed to be observed instantaneously without delay. The objective of the model is to maximize the present value of its net profit per unit time. A hypothetical numerical example, sensitivity analysis and observations are presented to illustrate the effectiveness of the proposed model. Our model establishes that accounts receivable carrying cost must be integrated in the overall cost-benefit structure associated with the inventory-credit system in order too have a proper trade-off for decision making.

2. Notations and Assumptions

The following notations and assumptions are used in developing the model

2.1 Notations

Q : ordering quantity per cycle

T : inventory cycle time

N : credit period given by the firm to the customers

O : ordering cost per order

C : unit purchase cost

P : unit selling price

k : rate of interest or discount rate per unit time

I : out- of- pocket inventory carrying charge per unit per unit time

R : out-of- pocket receivable carrying charge per unit per unit time

$I(t)$: inventory level at any time t

$R(t)$: accounts receivable level at any time t

$d(N)$: demand rate per unit time as a function of credit period

$Z(N, T)$: net profit per unit time as a function of decision variables N and T

2.2 Assumptions

1. Inventory system involves one type of item.
2. Firm purchases on cash and sell on credit.
3. The firm offer same amount of credit period to each of its credit customer. Thus, firm follows a day-terms credit policy i.e. net N, where N is the credit period.
4. The demand rate for the item increases exponentially with the customer's credit period N and is given by $d(N) = ae^{bN}$, where $a > 0$ is a demand rate per unit time firm does not offer credit period to customers, and $b \geq 0$ is a constant governing the increasing rate of demand.
5. The effect of credit policy on demand is observed instantaneously without any delay.
6. Customers settle their accounts on the last day of their credit period and there are no bad debt losses.
7. Replenishment is instantaneous.
8. Shortages are not allowed.

9. Lead time is negligible.

10. Discounted cash flow approach is used to incorporate time value of money in model.

3. Mathematical Modeling

At the start of the cycle, the inventory level is raised to Q units afterwards as time progress; the inventory decreases to fulfill demand up to 'T' and become zero at 'T'.

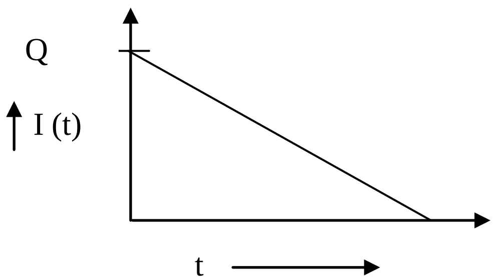

Figure-1: Inventory level vs. Time

For the firm, the present value of its net profit per unit time $Z(N,T)$, can be expressed as,

$Z(N,T) =$ Revenue from sales - Ordering cost – Purchase cost
– Inventory carrying cost – Accounts receivable carrying cost (1)

Depending upon the values of credit period (*N*) and inventory cycle length (*T*), there are two possible cases viz.

1 $N \le T$

2 $N \ge T$

We now develop mathematical formulation.

3.1. Case 1. N ≤ T

Since replenishment is instantaneous and shortages are not allowed, so the initial inventory level, $I(0)$ (i.e., the order quantity, Q) is,

$$I(0) = Q = \int_0^T ae^{bN}\,dt = ae^{bN}T \tag{2}$$

And the variation of inventory level with respect to time can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -ae^{bN} \quad 0 \le t \le T \tag{3}$$

With the boundary conditions:
$I(0) = Q = ae^{bN}T$ & $I(T) = 0$. Consequently, the solution of eq. (3) is given by

$$I(t) = ae^{bN}\left(T - t\right), \quad 0 \le t \le T \tag{4}$$

Further, taking into account the credit sales that occur during the replenishment interval, we observe that the behavior of the level of accounts receivable will be as follows: at the start of the cycle, the level of accounts receivable is zero; afterwards as time progresses it accumulates up to N due to credit sales. From N to T, two things happen simultaneously, i.e. the accounts receivable are created as a result of credit sales and also the accounts receivables are collected by the firm. Since the rate of credit sales is equal to the rate of collection of accounts receivable, therefore, the net level of accounts receivable remains constant for $N \le t \le T$. From T to $T+N$ the level of accounts receivable decreases due to their collection and becomes zero at $T+N$. The accounts receivable created from T to $T+N$ will be accounted in next cycle.

Let

$$R(t) = \left\{ \begin{matrix} R_1(t) = \text{ accounts receivable level at any time } t, \text{ for } 0 \le t \le N \\ R_2(t) = \text{accounts receivable level at any time } t, \text{ for } N \le t \le T \\ R_3(t) = \text{accounts receivable level at any time } t, \text{ for } N \le t \le T+N \end{matrix} \right\} \tag{5}$$

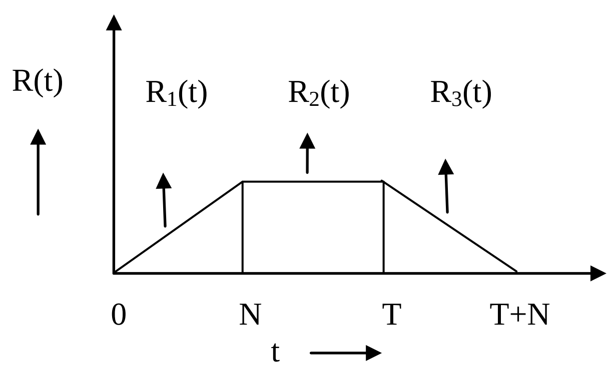

Figure-2: Accounts Receivable vs. Time

We have,

$$R_1(t) = 0 + \int_0^t ae^{bN}\,dt = ae^{bN}t, \qquad 0 \le t \le N \tag{6}$$

$$R_2(t) = \int_0^N ae^{bN}\,dt + \int_N^t \left(Sales\ rate - Collection\ rate\right)dt$$

$$= \int_0^N ae^{bN}\,dt + \int_N^t \left(ae^{bN} - ae^{bN}\right)dt, \qquad N \le t \le T \tag{7}$$

$$R_3(t) = \int_t^{T+N} ae^{bN} dt = ae^{bN}(T+N-t), \qquad T \le t \le T+N \tag{8}$$

Moreover, there is a cash outflow of ordering and purchase cost at the start of cycle and the revenue from the credit sales will be received by the firm from N to $T+N$ (Figure-3)

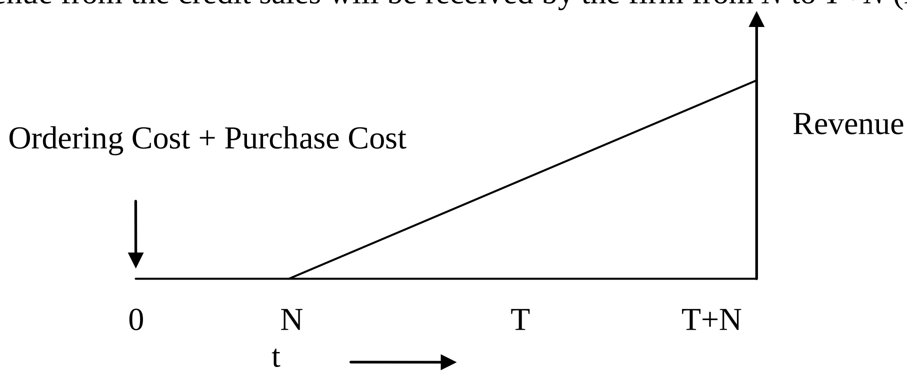

Figure-3

By using DCF approach, the various components of the profit functions are as follows:

The present value of revenue per unit time from sales

$$= \frac{P}{T}\int_N^{T+N} ae^{bN} e^{-kt} dt = \frac{Pae^{bN-k(N+T)}(e^{kT}-1)}{kT} \tag{9}$$

The present value of the ordering cost per unit time

$$= \frac{O}{T} \tag{10}$$

The present value of purchasing cost per unit time

$$= \frac{CQ}{T} = \frac{Cae^{bN}T}{T} = Cae^{bN} \tag{11}$$

The present value of inventory carrying cost per unit time

$$\begin{aligned} &= \frac{IC}{T}\int_0^T I(t)e^{-kt} dt = \frac{IC}{T}\int_0^T a e^{bN}(T-t)e^{-kt} dt \\ &= \frac{ICae^{bN}(e^{-kT}+kT-1)}{k^2 T} \end{aligned} \tag{12}$$

The present value of account receivables carrying cost per unit time

$$= \frac{RP}{T}\left(\int_0^N R_1(t)e^{-kt} dt + \int_N^T R_2(t)e^{-kt} dt + \int_T^{T+N} R_3(t)e^{-kt} dt \right)$$

$$= \frac{RP}{T}\left(\int_0^N ae^{bN} te^{-kt} dt + \int_N^T ae^{bN} Ne^{-kt} dt + \int_T^{T+N} ae^{bN}(T+N-t)e^{-kt} dt \right)$$

$$=\frac{RP}{T}\left(\frac{ae^{(b-k)N}(e^{kN}-kN-1)}{k^2}+\frac{ae^{bN}(e^{-kN}-e^{-kT})N}{k}+\frac{ae^{bN-k(N+T)}(1+e^{kN}(kN-1))}{k^2}\right) \tag{13}$$

Using equations (9) to (13), the present value of firm's net profit per unit time, $Z_1(N,T)$ is,

$$Z_1(N,T)=\begin{bmatrix}\frac{Pae^{bN-k(N+T)}(e^{kT}-1)}{kT}-\frac{O}{T}-Cae^{bN}-\frac{ICae^{bN}(e^{-kT}+kT-1)}{k^2T}\\ -\frac{RP}{T}\left(\begin{array}{l}\frac{ae^{(b-k)N}(e^{kN}-kN-1)}{k^2}+\frac{ae^{bN}(e^{-kN}-e^{-kT})N}{k}\\ +\frac{ae^{bN-k(N+T)}(1+e^{kN}(kN-1))}{k^2}\end{array}\right)\end{bmatrix} \tag{14}$$

The necessary conditions for the maximization of $Z_1(N,T)$ are

$$\frac{\partial Z_1(N,T)}{\partial N}=0,\frac{\partial Z_1(N,T)}{\partial T}=0 \tag{15}$$

which gives,

$$\begin{bmatrix}\left(\frac{Pae^{bN-k(N+T)}\left(e^{kT}-1\right)\left(b-k\right)}{kT}\right)-\left(Cabe^{bN}\right)-\left(\frac{ICabe^{bN}\left(e^{-kT}+kT-1\right)}{k^2T}\right)\\ -\left(\frac{RPae^{(b-k)N}\left(k^2N+b\left(e^{kN}-kN-1\right)\right)}{k^2T}\right)-\left(\frac{RPae^{bN-k(N+T)}\left(-e^{kN}\left(1+bN\right)+e^{kT}\left(1+bN-kN\right)\right)}{kT}\right)\\ -\left(\frac{RPae^{bN-k(N+T)}\left(\left(e^{kN}-1\right)k+b\left(1+e^{kN}\left(kN-1\right)\right)\right)}{k^2T}\right)\end{bmatrix}=0 \tag{16}$$

and

$$\begin{bmatrix}\left(\frac{Pae^{bN-k(N+T)}\left(1-e^{kT}+kT\right)}{kT^2}\right)+\left(\frac{O}{T^2}\right)-\left(\frac{ICae^{bN-kT}\left(e^{kT}-kT-1\right)}{k^2T^2}\right)\\ -\left(\frac{RPae^{(b-k)N}\left(1-e^{kN}+kN\right)}{k^2T^2}\right)-\left(\frac{RPNae^{bN-k(N+T)}\left(e^{kN}\left(1+kT\right)-e^{kT}\right)}{kT^2}\right)\\ +\left(\frac{RPae^{bN-k(N+T)}\left(1+e^{kN}\left(kN-1\right)\right)\left(kT+1\right)}{k^2T^2}\right)\end{bmatrix}=0 \tag{17}$$

The solution of above equations gives the optimal values of *N* and *T* for the maximization of $Z_1(N,T)$ provided they satisfies the sufficiency conditions given by

$$\frac{\partial^2 Z_1}{\partial N^2} \le 0, \frac{\partial^2 Z_1}{\partial T^2} \le 0, \left(\frac{\partial^2 Z_1}{\partial N^2}\right)\left(\frac{\partial^2 Z_1}{\partial T^2}\right) - \left(\frac{\partial^2 Z_1}{\partial N \partial T}\right)^2 \ge 0 \tag{18}$$

However, it is difficult to solve the necessary conditions analytically in a closed form and also to check the validity of sufficient conditions analytically. Consequently, numerical approach is used to obtain the solution (section 3.3).

3.2. Case 2. N ≤ T

Since replenishment is instantaneous and shortages are not allowed, so the initial inventory level, $I(0)$ (i.e., the order quantity, Q) is,

$$I(0) = Q = \int_0^T ae^{bN}\, dt = ae^{bN} T \tag{19}$$

And the variation of inventory level with respect to time can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -ae^{bN} \qquad 0 \le t \le T \tag{20}$$

With the boundary conditions:
$I(0) = Q = ae^{bN} T$ & $I(T) = 0$. Consequently, the solution of eq. (20) is given by

$$I(t) = ae^{bN}\left(T - t\right), \qquad 0 \le t \le T \tag{21}$$

Further, taking into account the credit sales that occur during the replenishment interval, we observe that the behavior of the level of accounts receivable will be as follows: at the start of the cycle, the level of accounts receivable is zero; afterwards as time progresses it accumulates up to T due to credit sales and remains at this level up to *N*. From N to $T+N$ the level of accounts receivable decreases due to their cash realization and becomes zero at $T+N$. The accounts receivable created from T to $T+N$ will be accounted in next cycle.

Let

$$R(t) = \begin{Bmatrix} R_1(t) = \text{ accounts receivable level at any time } t, \text{ for } 0 \le t \le T \\ R_2(t) = \text{accounts receivable level at any time } t, \text{ for } T \le t \le N \\ R_3(t) = \text{accounts receivable level at any time } t, \text{ for } N \le t \le T+N \end{Bmatrix} \tag{22}$$

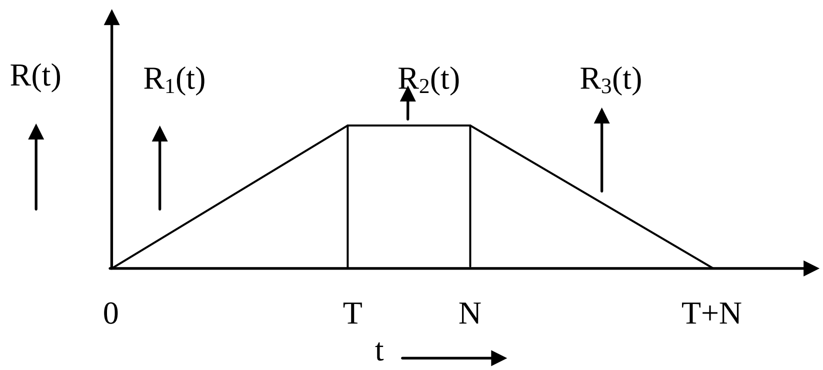

Figure-4: Accounts Receivable vs. Time

We have,

$$R_1(t) = 0 + \int_0^t ae^{bN}\,dt = ae^{bN}t, \qquad 0 \le t \le T \tag{23}$$

$$R_2(t) = \int_0^T ae^{bN}\,dt = ae^{bN}T, \qquad T \le t \le N \tag{24}$$

$$R_3(t) = \int_t^{T+N} ae^{bN}\,dt = ae^{bN}(T+N-t), \qquad N \le t \le T+N \tag{25}$$

Moreover, there is a cash outflow of ordering and purchase cost at the start of cycle and the revenue from the credit sales will be received by the firm from *N* to *T+N* (Figure-5).

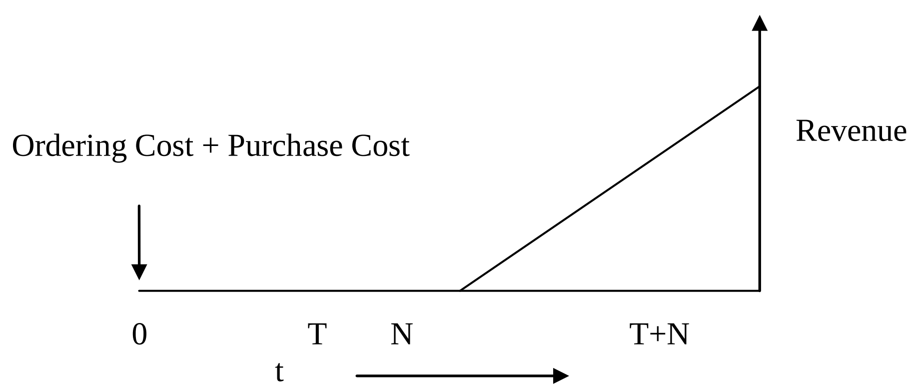

Figure-5

By using DCF approach, the various components of the profit functions are as follows:

The present value of revenue per unit time from sales

$$= \frac{P}{T}\int_N^{T+N} ae^{bN}e^{-kt}\,dt = \frac{Pae^{bN-k(N+T)}(e^{kT}-1)}{kT} \tag{26}$$

The present value of the ordering cost per unit time

$$=\frac{O}{T} \tag{27}$$

The present value of purchasing cost per unit time

$$=\frac{CQ}{T}=\frac{Cae^{bN}T}{T}=Cae^{bN} \tag{28}$$

The present value of inventory carrying cost per unit time

$$=\frac{IC}{T}\int_0^T I(t)e^{-kt}dt=\frac{IC}{T}\int_0^T a e^{bN}(T-t)e^{-kt}dt$$

$$=\frac{ICae^{bN}(e^{-kT}+kT-1)}{k^2T} \tag{29}$$

The present value of account receivables carrying cost per unit time

$$=\frac{RP}{T}\left(\int_0^T R_1(t)e^{-kt}dt+\int_T^N R_2(t)e^{-kt}dt+\int_N^{T+N} R_3(t)e^{-kt}dt\right)$$

$$=\frac{RP}{T}\left(\int_0^T ae^{bN}te^{-kt}dt+\int_T^N ae^{bN}Te^{-kt}dt+\int_N^{T+N} ae^{bN}(T+N-t)e^{-kt}dt\right)$$

$$=\frac{RP}{T}\left(\frac{ae^{bN-kT}(e^{kT}-kT-1)}{k^2}+\frac{ae^{bN}(e^{-kT}-e^{-kN})T}{k}+\frac{ae^{bN-k(N+T)}(1+e^{kT}(kT-1))}{k^2}\right) \tag{30}$$

Using equations (26) to (30), the present value of firm's net profit per unit time, $Z_2(N,T)$ is,

$$Z_2(N,T)=\left[\begin{array}{l}\dfrac{Pae^{bN-k(N+T)}(e^{kT}-1)}{kT}-\dfrac{O}{T}-Cae^{bN}-\dfrac{ICae^{bN}(e^{-kT}+kT-1)}{k^2T}\\ -\dfrac{RP}{T}\left(\begin{array}{l}\dfrac{ae^{bN-kT}(e^{kT}-kT-1)}{k^2}+\dfrac{ae^{bN}(e^{-kT}-e^{-kN})T}{k}\\ +\dfrac{ae^{bN-k(N+T)}(1+e^{kT}(kT-1))}{k^2}\end{array}\right)\end{array}\right] \tag{31}$$

The necessary conditions for the maximization of $Z_2(N,T)$ are

$$\frac{\partial Z_2(N,T)}{\partial N}=0,\frac{\partial Z_2(N,T)}{\partial T}=0 \tag{32}$$

which gives,

$$\left[\begin{array}{l}\left(\dfrac{Pae^{bN-k(N+T)}\left(e^{kT}-1\right)(b-k)}{kT}\right)-\left(Cabe^{bN}\right)-\left(\dfrac{ICabe^{bN}\left(e^{-kT}+kT-1\right)}{k^2T}\right)\\ -\left(\dfrac{RPabe^{bN-kT}\left(e^{kT}-kT-1\right)}{k^2T}\right)-\left(\dfrac{RPae^{bN-k(N+T)}\left(b\left(e^{kN}-e^{kT}\right)+e^{kT}k\right)}{k}\right)\\ -\left(\dfrac{RPae^{bN-k(N+T)}(b-k)\left(1+e^{kT}(kT-1)\right)}{k^2T}\right)\end{array}\right]=0 \qquad (33)$$

and

$$\left[\begin{array}{l}\left(\dfrac{Pae^{bN-k(N+T)}\left(1-e^{kT}+kT\right)}{kT^2}\right)+\left(\dfrac{O}{T^2}\right)-\left(\dfrac{ICae^{bN-kT}\left(e^{kT}-kT-1\right)}{k^2T^2}\right)\\ +\left(\dfrac{RPae^{bN-kT}\left(e^{kT}-kT-k^2T^2-1\right)}{k^2T^2}\right)+\left(RPae^{bN-kT}\right)-\left(\dfrac{RPae^{bN-k(N+T)}\left(e^{kT}-kT-1\right)}{k^2T^2}\right)\end{array}\right]=0 \qquad (34)$$

The solution of above equations gives the optimal values of N and T for the maximization of $Z_2(N,T)$ provided they satisfies the sufficiency conditions given by

$$\frac{\partial^2 Z_2}{\partial N^2}\le 0, \frac{\partial^2 Z_2}{\partial T^2}\le 0, \left(\frac{\partial^2 Z_2}{\partial N^2}\right)\left(\frac{\partial^2 Z_2}{\partial T^2}\right)-\left(\frac{\partial^2 Z_2}{\partial N\partial T}\right)^2\ge 0 \qquad (35)$$

However, in this case also, it is difficult to solve the necessary conditions analytically in a closed form and also to check the validity of sufficient conditions analytically. Consequently, like previous case, numerical approach is used to obtain the solution (section 3.3).

Furthermore, combining both the cases i.e. (14) and eq. (31), we get the firm's net profit per unit time, $Z(N,T)$ as:

$$Z(N,T)=\begin{pmatrix} Z_1(N,T), & N\le T \\ Z_2(N,T), & N\ge T\end{pmatrix} \qquad (36)$$

Our problem is to find the values of N and T which maximizes $Z(N,T)$.

3.3. Solution Procedure

To solve the model we solve both the cases separately and then combine the results to obtain the optimal solution. Due to highly complex and non-linear form, it is difficult to solve the model analytically in a closed form. However, the model can be solved numerically using LINGO which utilizes generalized reduced gradient algorithm as follows:

1. Maximize $Z_1(N,T)$, $Z_2(N,T)$, with respect to N and T so as to satisfy their respective conditions viz., $N \le T$ and $N \ge T$ respectively.

2. Due to complex nature of each function the optimality of the solution can only be checked graphically. Hence, to confirm optimality plot surface graphs for each case or plot a combined surface graph of all cases.

3. Choose values of N and T corresponding to $Max\{Z_1, Z_2\}$.

4. The optimal value of Q can be calculated from the optimal values of N and T.

4. Numerical Example

For numerical illustration, the values of the model parameters are taken as follows:
$a = 5000$, $b = 2.2$, $O =$ 1000/order, $C =$ \$200/unit, $P =$ \$250/unit, $I = 0.3$/unit/year, $R = .2$/year, $k = 0.15$/unit/year,

Solving the model according to step 1, 2 and 3, we get

$$Z_1^*(N,T) = 222347.9\$$$
$$N_1^* = 0.06346972(years) = 23.16645(days)$$
$$T_1^* = 0.06346972(years) = 23.16645(days)$$

$$Z_2^*(N,T) = 222857.9\$$$
$$N_2^* = \left(0.09386791(years) = 34.26179(days)\right)$$
$$T_2^* = \left(0.05830318(years) = 21.28066(days)\ \right)$$

Clearly, $Max\{Z_1^*, Z_2^*\}$ is Z_2^*. Therefore,
$N^* = N_2^* = 34.26179(days)$, $T^* = T_2^* = 21.28066(days)$, & $Z(N^*,T^*) = Z_2^* = 222857.9\$$.
The optimal ordering quantity is, $Q^* = ae^{bN^*}T^* = 358.3836(Units)$

$Z(N^*,T^*)$ is optimal at (N^*,T^*) can be checked by comparing the values of the function $Z(N,T)$ at any pair of points (N,T) around (N^*,T^*) such that $N < N^*$, or $N > N^*$ and $T < T^*$ or $T > T^*$. We, therefore, have performed a grid search for each case using MATLAB and evaluated the corresponding difference "$Z(N^*,T^*) - Z(N^*,T^*)$" by taking $N = [0,1]$ years and $T = [0,1]$ years as the domain of search space and a step size approximately equivalent to one day as well as half day for verifying the results. Since inventory and credit policy decisions are short-term decisions of normally one year so it is sufficient to check the global optimality of the solution within this domain for most of the practical applications. In addition, we have generated surface graphs (Figures-6, 7 & 8) of the function using MATLAB for the parameters values taken in the numerical example. The graphs clearly shows that at (N^*,T^*) the value of $Z(N^*,T^*)$ is maximum. Thus, for the given values of parameters in the numerical example, $N^* = 34.26179(days)$ and $T^* = 21.28066(days)$ is the optimal solution.

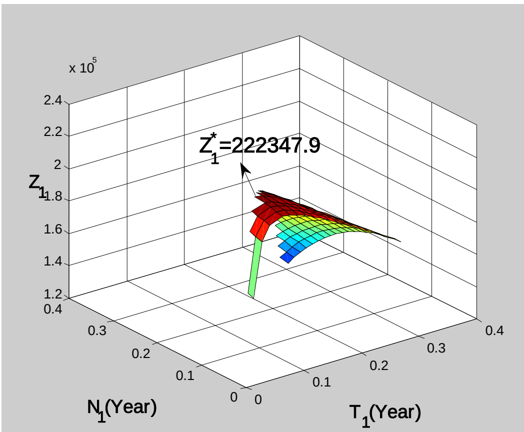

Figure-6 :Profit per unit time $(N \leq T)$

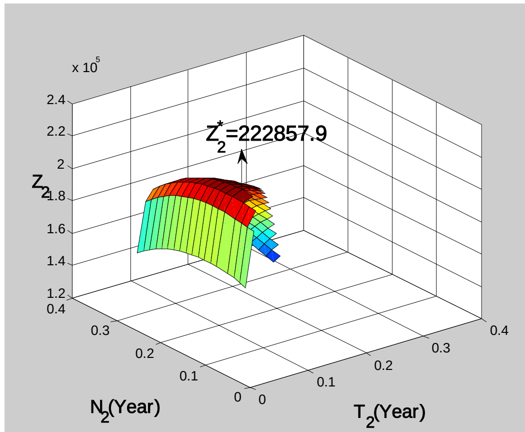

Figure-7 :Profit per unit time $(N \geq T)$

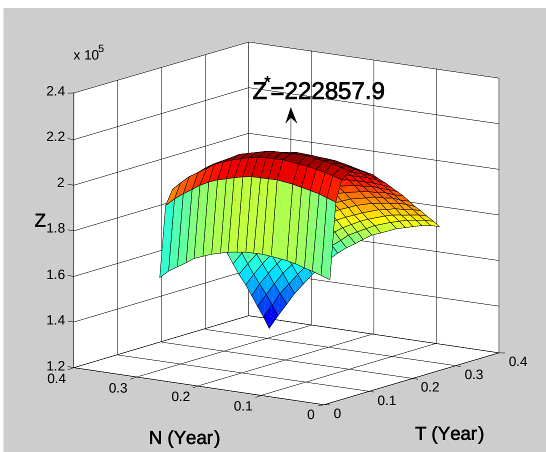

Figure-8: Profit per unit time (combined graph of both cases)

5. Sensitivity Analysis

For sensitivity analysis we consider the data as given in the numerical example. It is assumed that all other parameters are known and stationary in the time periods under consideration. The sensitivity analysis is performed by changing one parameter at a time and keeping the remaining parameters unchanged. Following tables shows the changes in optimal solution for different values of the parameters *I*, *R*, and *k*.

Table-1: Sensitivity analysis with respect to 'I'

I	N* (days)	T* (days)	Q* (units)	Z(N*,T*) $
0.00	39.792	34.248	596.313	236266.4
0.10	37.591	27.609	474.382	231002.5
0.20	35.807	23.814	404.8	226648.5
0.25	35.011	22.436	379.551	224695.8
0.30	34.262	21.281	358.39	222857.9
0.35	33.552	20.293	340.292	221117.3
0.40	32.877	19.436	324.597	219460.5
0.50	31.608	18.016	298.59	216357.6

Table-2: Sensitivity analysis with respect to 'R'

R	N* (days)	T* (days)	Q* (units)	Z(N*,T*) $
0.00	358.942	8.152	971.698	594047.0
0.05	210.202	12.702	617.714	340334.3
0.10	125.912	16.298	476.878	264746.7
0.15	71.829	19.092	403.23	235045.0
0.20	34.262	21.281	358.39	222857.9
0.25	6.681	23.017	328.257	218992.5
0.26	2.024	23.321	323.387	218840.7
0.28	0	23.454	321.288	218825.1

Table-3: Sensitivity analysis with respect to 'k'

K	N* (days)	T* (days)	Q* (units)	Z(N*,T*) $
0.05	119.046	19.088	535.871	266900.4
0.08	87.952	19.997	465.448	246323.0
0.10	70.338	20.467	428.403	236934.2
0.15	34.262	21.281	358.39	222857.9
0.20	6.293	21.717	308.994	217044.8
0.25	0	20.957	287.082	215083.4
0.30	0	19.977	273.658	213357.8

We obtained following observations and managerial insights from the results of numerical exercise.

- Table-1 shows that as inventory carrying cost increases the optimal value of credit period, the cycle length and ordering quantity decreases. This is quite logical because the firm would like to carry less inventory as well as order more frequently as inventory carrying cost increases and in order to do so it should reduce its demand by decreasing credit period given to the customers. This shows that when demand is influenced by credit period the credit period decision is influenced by inventory carrying cost. Therefore, decision of investment in accounts receivable should take into account the cost of carrying inventory. Results suggest that at high value of inventory carrying cost there should be less investment in accounts receivable. The optimal total profit decreases when inventory carrying cost increases, which is quite obvious and confirms to our expectations.

- From Table-2, it can be seen that as accounts receivable carrying cost increases the optimal value of credit period decreases. This is quite reasonable because at high value of accounts receivable carrying cost, the firm would like to carry lesser amount of accounts receivable and in order to do so it should give lesser amount of credit period. Consequently, the demand would decrease accompanied by a simultaneous change in inventory decisions according to the structure and parameters of the model. The results indicates that the firm should increase its cycle length to save on ordering cost in order to maximize its profit in the event of high accounts receivable carrying cost. The above analysis shows that when demand is influenced by credit period the inventory decisions are dependent on accounts receivable carrying cost and credit decisions. Therefore, accounts receivable carrying cost must be integrated in the overall cost-benefit structure while analyzing the inventory and credit decisions with credit linked demand function. The results also show that at high value of accounts receivable carrying cost the firm should invest less in accounts receivable and therefore should follow a strict credit policy. Furthermore, as accounts receivable carrying cost becomes very high, the optimal value of credit period becomes zero suggesting that the firm should go for all cash sale program at a very high value of the cost of granting credit period. The optimal total profit reduces when accounts receivable carrying cost increases, which is quite evident and confirms to economic rational.

- From Table-3, it can be seen that as the value of discount rate increases the optimal credit period and ordering quantity decreases. This is quite logical due to the fact that as the opportunity cost of fund increases, the firm would like to invest less in accounts receivable as well as in inventory. In order to do so the firm would like to give lesser amount of credit period causing the demand to decrease accompanied by a simultaneous change in inventory decisions as per the structure and parameters of the model. The results confirm that inventory- credit decisions are influenced by the time value of money. Furthermore, as discount rate becomes very high, the optimal value of credit period becomes zero suggesting that the firm should go for all cash sale program by investing less in inventory (i.e. smaller ordering quantity) for a shorter duration (i.e. smaller ordering interval) when opportunity cost of fund is very high. This is consistent with the properties of EOQ model in present value framework. Also, the optimal total profit decreases when discount rate increases, which is obvious.

6. Conclusion

Granting credit period to customers without reducing price of the product is a common business practice. Consequently, this paper examines the issue of credit period in inventory management from the view point of the provider of credit period. We have formulated an inventory-credit period decision model with day-terms credit linked demand using the DCF approach. Often ignored in previous models, the accounts receivable carrying cost resulting from credit sales is incorporated in the present model. The objective of the model is maximization of the present value of firm’s net profit per unit time by jointly optimizing the credit period and replenishment interval. Subsequently, numerical example is presented to illustrate the proposed model. Finally sensitivity analysis has been done and results are discussed which are found to be consistent with the economic rationality.

The proposed model provides a framework to coordinate and analyze inventory and credit period decisions for carrying out short term (working capital) planning activities.

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