*AMO-Advanced Modeling and Optimization, Volume 16, Number 2, 2014*

# A Multi-Server Markovian Feedback Queue with Balking Reneging and Retention of Reneged Customers

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**Abstract**

The quality of service provides a competitive edge to any business firm. Lack of better service quality mechanism may lead to the dissatisfied customers (feedback customers). This causes loss of goodwill among the customers and finally, the firms may lose their potential customers. Further, due to long wait customers may get impatient, and may abandon the queueing system before getting service. This leads to the reduction in revenue of the firm. In this paper, we study a finite capacity multi-server Markovian feedback queuing model with balking, reneging and retention of reneged customers. The steady-state probability distribution of the system size is obtained and some quality of service measures like average system size, average number of customers served etc. are also derived. Finally, some special cases of this model are derived and discussed.

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* **AMO-Advanced Modeling and Optimization.ISSN:1841-4311**

**Mathematics Subject Classification:** (2000) MSC 60K25, 90B22
**Keywords:** Retention, Reneged Customers, Queueing System, Feedback, Balking, Markovian.

# 1 Introduction

Queuing models have effectively been used in the design and analysis of telecommunication systems, traffic systems, service systems and many more. A number of extensions in the basic queuing models have been made and the concepts like vacations queuing, correlated queuing, retrial queuing, queuing with impatience, and catastrophic queuing have come up. Of these, queuing with customer impatience has special significance for the business world as it has a very negative effect on the revenue generation of a firm. A customer is said to be impatient if he tends to join the queue only when a short wait is expected and tends to remain in the line if his wait has been sufficiently small. Impatience generally takes three forms. The first is balking, deciding not to join the queue at all up on arrival, the second is reneging, the reluctance to remain in the waiting line after joining and waiting, and the third is jockeying between lines when each of a number of parallel service channels has its own queue, Gross and Harris [8]. An extensive review on queuing systems with impatient customers is presented by Wang et al. [20]. They survey various queuing systems according to various dimensions like customer impatience behaviors, solution methods of queuing models with impatient customers, and associate optimization aspects.

Feedback in queueing literature represents customer dissatisfaction because of inappropriate quality of service. In case of feedback, after getting partial or incomplete service, customer retries for service. In computer communication, the transmission of protocol data unit is sometimes repeated due to occurrence of an error. This usually happens because of non-satisfactory quality of service. Rework in production operations is also an example of queues with feedback.

Recently, Kumar and Sharma [14] study a multi-server Markovian queuing systems with balking, reneging and retention of reneged customers. In this paper, the arrivals are simple Poissionian and the served customers do not require any re-service (feedback). Kumar [15] obtains the transient solution of an M/M/c/N queueing system with balking, reneging and retention of reneged customers and performs cost-profit analysis of the model. We extend the work of Kumar and Sharma [14] by incorporating the feed-back mechanism in the M/M/c/N queuing model with balking, reneging and retention of reneged customers. Further, some more quality of service measures like average number of customer served, rate of abandonment, average reneging rate, average retention rate etc. are obtained.

In this paper, we consider a single server, finite capacity Markovian feedback queuing system with reneging and retention of reneged customers in which the inter-arrival and service times follow exponential distribution. The reneging times are assumed to be exponentially distributed. After the completion of service,each customer may rejoin the system as a feedback customer for receiving another regular service with probability $p_1$ or he can leave the system with probability $q_1$ where $p_1 + q_1 = 1$. A reneged customer can be retained in many cases by employing certain convincing mechanisms to stay in the queue for completion of his service. Thus, a reneged customer can be retained in the queuing system with some probability $q_2$ and may leave the queue without receiving service with probability $p_2 = 1-q_2$. This effort to retain the reneging customer in the queue for his service has positive effect on business of any firm.

Rest of the paper is arranged as follows: in section 2, the literature review is presented; section 3 deals with the stochastic queuing model description; in section 4 the differential-difference equations of the model are derived; The steady-state solution of the queuing model is obtained in section 5; Some important quality of service measures derived in section 6; Special cases of the model are discussed in section 7, and in section 8, the paper is concluded.

# 2 Literature Review

The notion of customer impatience appeared in queuing theory in the work of Haight [9]. He studies M/M/1 queue with balking in which there is a greatest queue length at which an arrival will not balk. Haight [10] studies queuing with reneging. Ancker and Gafarian [1] study M/M/1/N queuing system with balking and reneging and derive its steady-state solution. Ancker and Gafarian [2] obtain results for a pure balking system (no reneging) by setting the reneging parameter equal to zero. Multi-server queuing systems with customer impatience find their applications in many real life situations such as in hospitals, computer-communication, retail stores etc. Montazer-Haghighi et al. [16], Abou-El-Ata and Hariri [3], and Zohar et al. [23] study and analyze some multi-server queuing systems with balking and reneging. Chauhan and Sharma [5] perform the profit analysis of M/M/c queueing model with balking and reneging. Ward and Kumar [21] construct an admission control policy for a GI/GI/1 queue with impatient customers in heavy traffic. Al-seedy et al. [4] study M/M/c queue with balking and reneging and derive its transient solution by using the probability generating function technique and the properties of Bessel function. Xiong and Altiok [22] study multi-server queues with deterministic reneging times with reference to the timeout mechanism used in managing application servers in transaction processing environments. Choudhury and Medhi [6] study multi-server Markovian queuing system with balking and reneging, and discuss the design aspects associated with it.

Kapodistria [12] studies a single server Markovian queue with impatient customers and consider the situations where customers abandon the system simultaneously. He considers two abandonment scenarios. In the first one, all present customers become impatient and perform synchronized abandonments, while in the second scenario; the customer in service is excluded from the abandonment procedure. He extends this analysis to the M/M/c queue under the second abandonment scenario also. Kumar [13] investigates a correlated queuing problem with catastrophic and restorative effects with impatient customers which have special applications in agile broadband communication networks. Mandelbaum and Momcilovic [17] study queues with many-servers and impatient customers.

Takacs [19] studies queue with feedback to determine the stationary process for the queue size and the first two moments of the distribution function of the total time spent in the system by a customer. Davignon and Disney [7] study single server queues with state dependent feedback. Santhakumaran and Thangaraj [18] consider a single server feedback queue with impatient and feedback customers. They study M/M/1 queueing model for queue length at arrival epochs and obtain result for stationary distribution, mean and variance of queue length.

# 3 Stochastic Queuing Model

We consider multi-server single channel queueing system. The arrivals occur in accordance with Poisson process with mean arrival rate $\lambda$. The inter-arrival times are independently, identically and exponentially distributed with parameter $\lambda$. There are c servers and the service times at each server are independently, identically and exponentially distributed with parameter $\mu$. The mean service rate for $n < c$ is given by $n\mu$ and the service rate for $c \leq n \leq N$ is $c\mu$ where $n$ is the number of customers in the system and $N$ is the capacity of the system. After completion of each service, the customer can either join at the end of the queue with probability $p_1$ or he can leave the system with probability $q_1$, where $p_1 + q_1 = 1$. The customers both newly arrived and those that are fed back are served in order in which they join the tail of original queue.The queue discipline is FCFS. We do not distinguish between the regular arrival and feedback arrival. The capacity of the system is taken as finite (say, N). That is, the system can accommodate at most N customers. A queue gets developed when the number of customers exceeds the number of servers, that is, when $n > c$. Each customer upon joining the queue will wait a certain length of time for his service to begin. If it does not begin by then, he will get reneged and may leave the queue without getting service with probability $p_2$ and may remain in the queue for his service with probability $q_2(= 1 - p_2)$. The reneging times follow exponential distribution with parameter $\xi$. The arriving customer joins the system with certain balking probability. It is assumed that an arriving customer balks with probability $\frac{n}{N}$, where $n$ is the number of customers in system and therefore joins the system with probability $1 - \frac{n}{N}$ , which indicates that $N$ is the measure of customer's willingness to join the queue.

# 4 Mathematical Formulation of the Model

In this section, we present a mathematical model consisting of differential-difference equations. These equations are derived by using the general birth-death arguments.
Define, $P_n(t), 0 \leq n \leq N$ be the probability that there are n customer in the system at time $t$. In an infinitesimally small interval $(t, t+\delta t)$,

$$P_n(t+\delta t) = Prob\{\text{there are n customers in the system at time } (t+\delta t)\}$$

When $c+1 \leq n \leq N-1$, the equation (4.3) can be derived as follows: Here,

$$\begin{aligned}
P_n(t+\delta t) &= P_n(t)\Big[\Big\{1-\Big(1-\frac{n}{N}\Big)\lambda\delta t\Big\}(1-c\mu q_1\delta t)\Big] \\
&+ P_n(t)\Big[\Big\{\Big(1-\frac{n}{N}\Big)\lambda\delta t\Big\}(c\mu q_1\delta t)\Big] \\
&+ P_{n-1}(t)\Big[\Big\{\Big(1-\frac{n-1}{N}\Big)\lambda\delta t\Big\}(1-c\mu q_1\delta t)\Big] \\
&+ P_{n+1}(t)\Big[\Big\{1-\Big(1-\frac{n+1}{N}\Big)\lambda\delta t\Big\}(c\mu q_1\delta t)\Big] \\
&+ P_n(t)\Big[\Big\{1-\Big(1-\frac{n}{N}\Big)\lambda\delta t\Big\}(1-c\mu q_1\delta t)\{(n-c)\xi q_2\delta t\}\Big] \\
&+ P_{n+1}(t)\Big[\Big\{1-\Big(1-\frac{n+1}{N}\Big)\lambda\delta t\Big\}(1-c\mu q_1\delta t)\{(n-c)\xi p_2\delta t\}\Big]
\end{aligned}$$

Finding the difference $P_n(t+\delta t) - P_n(t)$, dividing both sides by $\delta t$ and taking limit $\delta t \to 0$ leads to the differential-difference equation (4.3). $o(\delta t)$ approaches to zero as rapidly as $\delta t \to 0$. The other equations (4.1), (4.2) and (4.4) are also derived in the same way. Therefore, the differential-difference equations of the model are:

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu q_1 P_1(t) \tag{4.1}$$

$$\begin{aligned}
\frac{dP_n(t)}{dt} &= -\Big[\Big(1-\frac{n}{N}\Big)\lambda + n\mu q_1\Big]P_n(t) + (n+1)\mu q_1 P_{n+1}(t) \\
&\quad +\Big(1-\frac{n-1}{N}\Big)\lambda P_{n-1}(t), 1 \leq n \leq c
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
\frac{dP_n(t)}{dt} &= -\Big[\Big(1-\frac{n}{N}\Big)\lambda + \mu q_1 + (n-c)\xi p_2\Big]P_n(t) + [c\mu q_1 + \{(n+1)-c\}\xi p_2]P_{n+1}(t) \\
&\quad +\Big(1-\frac{n-1}{N}\Big)\lambda P_{n-1}(t), c+1 \leq n \leq N-1
\end{aligned} \tag{4.3}$$

$$\frac{dP_N(t)}{dt} = \Big(1 - \frac{N-1}{N}\Big)\lambda P_{N-1}(t) - [c\mu q_1 + (N-c)\xi p_2]P_N(t) \tag{4.4}$$

# 5 Steady-State Solution of the Model

In this section, we obtain the steady-state solution of the model iteratively. In steady-state, $\lim\limits_{t\to\infty} P_n(t) = P_n$ and therefore, the steady-state equations corresponding to equations (4.1) - (4.4) are as follows: The steady-state equations of the model are

$$0 = -\lambda P_0 + \mu q_1 P_1 \tag{5.1}$$

$$0 = -\Big[\Big(1-\frac{n}{N}\Big)\lambda + n\mu q_1\Big]P_n + (n+1)\mu q_1 P_{n+1} + \Big(1-\frac{n-1}{N}\Big)\lambda P_{n-1},\ 1 \le n < c \tag{5.2}$$

$$0 = -\Big[\Big(1-\frac{n}{N}\Big)\lambda + c\mu q_1 + (n-c)\xi p_2\Big]P_n + [c\mu q_1 + \{(n+1)-c\}\xi p_2]P_{n+1} + \Big(1-\frac{n-1}{N}\Big)\lambda P_{n-1},\ c \le n \le N-1 \tag{5.3}$$

$$0 = \Big(1-\frac{N-1}{N}\Big)\lambda P_{N-1} - [c\mu q_1 + (N-c)\xi p_2]P_N,\ n = N \tag{5.4}$$

From equation (5.1), we have

$$\mu q_1 P_1 = \lambda P_0$$
$$P_1 = \frac{\lambda}{\mu q_1} P_0 \tag{5.5}$$

Substitute $n = 1$ in (5.2), we get

$$2\mu q_1 P_2 = \Big[\Big(1-\frac{1}{N}\Big)\lambda + \mu q_1\Big]P_1 - \lambda P_0$$
$$P_2 = \frac{N-1}{N}\frac{\lambda^2}{2!(\mu q_1)^2}P_0\ \ \{\text{of } (5.1)\} \tag{5.6}$$

For $n = 2$ in (5.2), we have

$$P_3 = \prod_{k=1}^{3}\frac{N-(k-1)}{N}\frac{\lambda^3}{3!(\mu q_1)^3}P_0$$

Proceeding in the same way, we get

$$P_n = \prod_{k=1}^{n}\frac{N-(k-1)}{N}\frac{\lambda^n}{n!(\mu q_1)^n}P_0, 1 \le n \le c \tag{5.7}$$

Now for $n = c$, (5.3) become

$$(c\mu q_1 + \xi p_2)P_{c+1} = -\Big[\Big(1 - \frac{c}{N}\Big)\lambda + c\mu q_1\Big]P_c - \Big(1 - \frac{c-1}{N}\Big)\lambda P_{c-1}$$

$$\{c\mu q_1 P_c = \Big(1 - \frac{c-1}{N}\Big)\lambda P_{c-1}\}$$

$$P_{c+1} = \prod_{k=c}^{c+1} \frac{N-(k-1)}{N} \frac{\lambda}{c\mu q_1 + (k-c)\xi p_2} P_c \tag{5.8}$$

Similarly, from (5.3) and (5.4), for $c + 1 \leq n \leq N$, we get

$$P_n = \prod_{k=c}^{n} \frac{\lambda}{c\mu q_1 + (k-c)\xi p_2} P_c \tag{5.9}$$

Thus, the steady-state solution of the model is

$$P_n = \begin{cases} \displaystyle\prod_{k=1}^{n} \frac{N-(k-1)}{N} \frac{\lambda^n}{n!(\mu q_1)^n} P_0, 1 \leq n < c \\ \displaystyle\prod_{k=c}^{n} \frac{N-(k-1)}{N} \frac{\lambda}{c\mu q_1 + (k-c)\xi p_2} \Big\{ \prod_{r=1}^{c-1} \frac{N-(r-1)}{N} \frac{\lambda^r}{r!(\mu q_1)^r} \Big\} P_0 \\ , c \leq n \leq N \end{cases} \tag{5.10}$$

Using the normalization condition, $\sum_{n=0}^{N} P_n = 1$, we get

$$P_0 = \frac{1}{1 + Q_1 + Q_2} \tag{5.11}$$

$$Q_1 = \sum_{n=1}^{c-1} \prod_{k=1}^{n} \frac{N-(k-1)}{N} \frac{\lambda^n}{n!(\mu q_1)^n} P_0$$

and

$$Q_2 = \sum_{n=c}^{N} \prod_{k=c}^{n} \frac{N-(k-1)}{N} \frac{\lambda}{c\mu q_1 + (k-c)\xi p_2} \Big\{ \prod_{r=1}^{c-1} \frac{N-(r-1)}{N} \frac{\lambda^r}{r!(\mu q_1)^r} \Big\}$$

Hence, the steady-state probabilities of the system size are derived explicitly.

# 6 Quality of Service Measures

(i) The Expected System Size ($L_s$)

$$\begin{aligned} L_s &= \sum_{n=1}^{c-1} n\Big\{\prod_{k=1}^{n}\frac{N-(k-1)}{N}\frac{\lambda^n}{n!(\mu q_1)^n}\Big\}P_0 \\ &+ \sum_{n=c}^{N} n\Big\{\prod_{k=c}^{n}\frac{N-(k-1)}{N}\frac{\lambda}{c\mu q_1+(k-c)\xi p_2}\Big\{\prod_{r=1}^{c-1}\frac{N-(r-1)}{N}\frac{\lambda^r}{r!(\mu q_1)^r}\Big\}\Big\}P_0 \end{aligned}$$

(ii) The Expected Number of Customers Served ($E(C.S.)$)

$$\begin{aligned} E(C.S.) &= \sum_{n=1}^{c} n\mu q_1\Big\{\prod_{k=1}^{n}\frac{N-(k-1)}{N}\frac{\lambda^n}{n!(\mu q_1)^n}\Big\}P_0 \\ &+ \sum_{n=c+1}^{N} c\mu q_1\Big\{\prod_{k=c}^{n}\frac{N-(k-1)}{N}\frac{\lambda}{c\mu q_1+(k-c)\xi p_2}\Big\{\prod_{r=1}^{c-1}\frac{N-(r-1)}{N}\frac{\lambda^r}{r!(\mu q_1)^r}\Big\}\Big\}P_0 \end{aligned}$$

(iii) Rate of Abandonment ($R_{\text{aband}}$)

$$\begin{aligned} R_{\text{aband}} &= \lambda - \sum_{n=1}^{c} n\mu q_1\Big\{\prod_{k=1}^{n}\frac{N-(k-1)}{N}\frac{\lambda^n}{n!(\mu q_1)^n}\Big\}P_0 \\ &- \sum_{n=c+1}^{N} c\mu q_1\Big\{\prod_{k=c}^{n}\frac{N-(k-1)}{N}\frac{\lambda}{c\mu q_1+(k-c)\xi p_2}\Big\{\prod_{r=1}^{c-1}\frac{N-(r-1)}{N}\frac{\lambda^r}{r!(\mu q_1)^r}\Big\}\Big\}P_0 \end{aligned}$$

(iii) Average Reneging Rate ($R_r$)

$$R_r = \sum_{n=c}^{N}(n-c)\xi p_2\Big\{\prod_{k=c}^{n}\frac{N-(k-1)}{N}\frac{\lambda}{c\mu q_1+(k-c)\xi p_2}\Big\{\prod_{r=1}^{c-1}\frac{N-(r-1)}{N}\frac{\lambda^r}{r!(\mu q_1)^r}\Big\}\Big\}P_0$$

(iv) Average Balking Rate ($R_b$)

$$\begin{aligned} R_b &= \sum_{n=1}^{c-1}\frac{n\lambda}{N}\Big\{\prod_{k=1}^{n}\frac{N-(k-1)}{N}\frac{\lambda^n}{n!(\mu q_1)^n}\Big\}P_0 \\ &+ \sum_{n=c}^{N}\frac{n\lambda}{N}\Big\{\prod_{k=c}^{n}\frac{N-(k-1)}{N}\frac{\lambda}{c\mu q_1+(k-c)\xi p_2}\Big\{\prod_{r=1}^{c-1}\frac{N-(r-1)}{N}\frac{\lambda^r}{r!(\mu q_1)^r}\Big\}\Big\}P_0 \end{aligned}$$

(v) Average Retention Rate ($R_R$)

$$R_R = \sum_{n=1}^{N}(n-c)\xi q_2\Big\{\prod_{k=c}^{n}\frac{N-(k-1)}{N}\frac{\lambda}{c\mu q_1+(k-c)\xi p_2}\Big\{\prod_{r=1}^{c-1}\frac{N-(r-1)}{N}\frac{\lambda^r}{r!(\mu q_1)^r}\Big\}\Big\}P_0$$

where $P_0$ is computed in (5.11).

**Special Cases**

1 In the absence of feedback i.e. for $q_1 = 1$, the queueing model reduces to an M/M/c/N queueing model with balking, reneging and retention of reneged customers as studied Kumar and Sharma [14].

2 In the absence of retention of reneged customer i.e. for $q_2 = 0$, our model reduces to an M/M/c/N queuing model with feedback, balking and reneging.

# 7 Conlusions

In this paper, we study a finite capacity multi-server Markovian feedback queuing model with balking, reneging and retention of reneged customers. The steady-state solution of the model is obtained and some quality of service measures are also derived. The model results may be useful in modeling various production and service processes involving feedback and impatient customers.

The model analysis is limited to finite capacity. The infinite capacity case of the model can also be studied. Further, the model can be solved in transient state to get time-dependent results.

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