PROBABILISTIC ANALYSIS OF 3-UNIT BIOMETRIC SYSTEM

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ABSTRACT

In the present paper, reliability analysis of a biometric system is done. The system consists of three identical biometric machines which serve as hot standby units. Biometric is automatic identification of the person based on his/her physiological and behavioral characteristics like finger print etc. If one of the units fails then other two units can be used to fulfill the purpose. If all three units fail then system is declared completely failed. It is observed the system fails due to human error most of the times. Various measures of system effectiveness such as Mean Time to System Failure, availability, busy period of repairman and number of visits are estimated numerically using semi-Markov processes and regenerative point technique.

***Keywords**: Biometric System, Measures of system effectiveness, Human error, Regenerative point technique, semi-Markov processes*

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1. INTRODUCTION

Reliability of an apparatus is the ability to perform its intended function without failure. Reliability enhances cost-effectiveness of the system. It is important to assess the reliability and availability of the complex systems. A good number of researchers including Taneja, Tuteja, Goyal, Parashar etc. [1-7] have discussed various measures of system effectiveness and profit analysis for many critical and non-critical systems under different assumptions but the literature on reliability is still lacking of reliability models comprising biometric system.

The term 'biometric' means life measurement. Security is one of the most important applications associated with Biometrics. It is used to either determine the person's identity or verify the person's identity. For the present study, three identical biometric machines are considered which serve as hot standby units. If one of the three units fails, then either one of the other two units can be used to fulfill the purpose. If all three units fail then system is declared completely failed.

2. NOMENCLATURE

O	:	operative state
F_r	:	failed unit under repair
F_R	:	unit is under repair from previous state
F_w	:	failed unit is waiting for repair
λ	:	constant rate of failure of operative unit
g(t), G(t)	:	p.d.f. and c.d.f. of repair time of the unit
©, Ⓢ	:	Laplace convolution, Laplace Stieltjes convolution

3. MODEL DESCRIPTION

In present reliability model, the system under consideration is biometric system consisting of three identical units. If one of unit fails, then other two units can be used to fulfill the purpose. If all the units fail then system is declared completely failed. To study the system, information was gathered about the failures. It is observed the system fails due to human error most of the times.

The state transition diagram is shown in Fig.1 covering all the possibilities of system under consideration. The epochs of entry into states 0, 1, and 4 are regeneration points and thus 0, 1 and 4 are regenerative states. The states 0, 1, 2 and 4 are up states. States 3 is with complete failure.

The following reliability indices of system effectiveness are obtained probabilistically:

- Mean time to system failure (MTSF)
- Availability
- Busy Period of repairman
- Expected number of visits

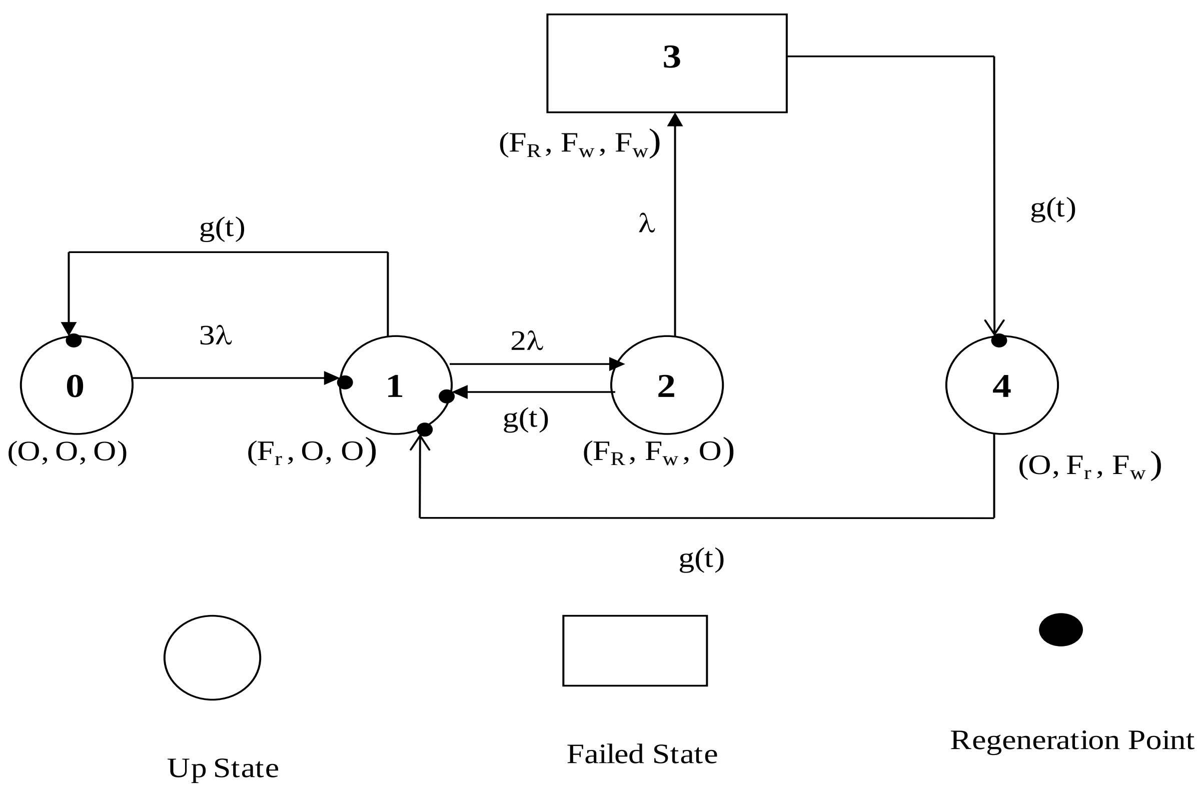

Fig. 1 State Transition Diagram

4. ASSUMPTIONS FOR THE MODEL

1. Initially all three units are operative.
2. All three units are identical.
3. The rate of failure of all three identical units is constant.
4. The repairman is readily available for repair.
5. An exponential distribution is followed by the failure times.
6. The system works perfectly after each repair.

7. Failed state is considered only if all units get failed.
8. The unit cannot fail immediately after repair.
9. All random variables are independent.

5. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The transition probabilities are given as follows:

dQ_{01} (t)= $3\lambda e^{-3\lambda t}dt$

dQ_{10} (t)= $e^{-2\lambda t}g(t)dt$

$dQ_{13}^{2}(t) = (2\lambda e^{-2\lambda t} © \lambda e^{-\lambda t})\ \overline{G}\ (t)dt$

$dQ_{11}^{2}(t) = (2\lambda e^{-\lambda t} © e^{-\lambda t})\ g(t)dt$

$dQ_{14}^{2,3}(t) = (2\lambda e^{-2\lambda t} © \lambda e^{-\lambda t} © 1)\ g(t)dt$

$dQ_{41}(t) = g(t)dt$

The non-zero elements p_{ij} can be obtained as

$$p_{ij} = \lim_{s\rightarrow 0} q_{ij}{}^{*}(s) \text{ where } \frac{dQ_{ij}(t)}{dt} = q_{ij}(t)$$

$p_{01} = 1$

$p_{10} = g^*(2\lambda)$

$p_{13}^{2} = g^*(2\lambda) - 2g^*(\lambda) + 1$

$p_{11}^{2} = 2g^*(\lambda) - 2g^*(2\lambda)$

$p_{14}^{2,3} = g^*(2\lambda) - 2g^*(\lambda) + 1$

$p_{41} = 1$

By these transition probabilities, it can be verified that

$p_{01} = 1$

$p_{10} + p_{11}^{2} + p_{14}^{2,3} = 1$

$p_{10} + p_{11}^{2} + p_{13}^{2} = 1$

$p_{41} = 1$

The mean sojourn times (μ_i) in the regenerative state i is defined as the time to stay in that state before transition to any other state. If T denotes the sojourn time in the regenerative state i, then

$$\mu_0 = \int_0^{\infty} e^{-3\lambda t} dt = \frac{1}{3\lambda}$$

$$\mu_1 = \int_0^{\infty} e^{-2\lambda t}\, \overline{G}\,(t) = \frac{1 - g^*(2\lambda)}{\lambda}$$

$$\mu_4 = \int_0^{\infty} t\, g(t)\, dt$$

The time taken by the system to transit to any regenerative state j when time is taken from the instant when the system enters the state i is given by

$$m_{ij} = \int_0^{\infty} t dQ_{ij}(t) = -q_{ij}^{*'}(0)$$

Thus,

$m_{01} = \mu_0$

$m_{10} + m_{11}^2 + m_{13}^2 = k_1$(say)

$m_{10} + m_{11}^2 + m_{14}^{2,3} = k_2$(say)

$m_{41} = \mu_4$

6. MEAN TIME TO SYSTEM FAILURE (MTSF)

Regarding the failed states as absorbing states and employing the arguments used for regenerative processes, we have the following recursive relations for $\phi_i(t)$:

$$\phi_0(t) = Q_{01}(t) \circledS \phi_1(t)$$

$$\phi_1(t) = Q^2{}_{1,3}(t) + Q_{10}(t) \circledS \phi_0(t) + Q^2{}_{1,1}(t) \circledS \phi_1(t)$$

$\phi_4(t) = Q_{4,1}(t) \, Ⓢ \, \phi_1(t)$

Thus, mean time to system failure (MTSF), is as follows

$$\text{MTSF} = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{D'(0)-N'(0)}{D(0)} = \frac{N}{D}$$

$$= \frac{k_1 + \mu_0(p_{10} + p_{13}^2)}{p_{13}^2}$$

7. AVAILABILITY ANALYSIS

Let A_i (t) is the probability that the system is available at instant t, where the system entered regenerative state i at t=0, then using the probabilistic arguments of the theory of regenerative processes, we have the following recursive relations for A_i (t)

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t)$

$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + q_{11}^2(t) © A_1(t) + q_{14}^{2,3}(t) © A_4(t)$

$A_4(t) = M_4(t) + q_{41}(t) © A_1(t)$

where

$M_0(t) = e^{-3\lambda t}$

$M_1(t) = e^{-2\lambda t}\,\overline{G}(t)$

$M_4(t) = \overline{G}(t)$

For steady state, the availability of the system is given by

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1}$$

$D_1 = k_1 + \mu_4 p_{14}^{2,3} + \mu_0 p_{10}$

$N_1 = \mu_0 p_{10} + \mu_1 + p_{14}^{2,3}\mu_4$

8. BUSY PERIOD ANALYSIS

Let B_i (t) is the probability that the repairman is busy in the repair at instant t, where the system entered regenerative state i at t=0, then using the probabilistic arguments, we have the following recursive relations for B_i (t) :

$$B_0(t) = q_{01}(t) © B_1(t)$$

$$B_1(t) = W_1(t) + q^2{}_{11}(t) © B_1(t) + q_{10}(t) © B_0(t) + q_{14}^{2,3}(t) © B_4(t)$$

$$B_4(t) = W_4(t) + q_{41}(t) © B_1(t)$$

where

$$W_1(t) = e^{-2\lambda t}\overline{G}(t) + \left[2\lambda e^{-2\lambda t} © \lambda e^{-\lambda t}\right]\overline{G}(t) + [2\lambda e^{-2\lambda t} © \lambda e^{-\lambda t} © 1]\,\overline{G}(t)$$

$$W_4(t) = \overline{G}(t)$$

For steady state, the total fraction of time for which the system is under repair, is given by

$$B_0 = \lim_{s\to 0} sB_0^*(s) = \frac{N_2}{D_1}$$

$$N_2 = \left[W_1^*(0) - W_4^*(0)p_{14}^{2,3}\right]$$

and D_1 is already specified

9. EXPECTED NUMBER OF VISITS OF REPAIRMAN

Let $V_i(t)$ as the expected number of visits in (0,t], where the system entered regenerative state i at t=0, then using the probabilistic arguments, we have the following recursive relations for V_i(t):

$$V_0(t) = Q_{01}(t) \circledS (1 + V_1(t))$$

$$V_1(t) = Q_{10}(t) \circledS V_0(t) + Q^2{}_{11}(t) \circledS V_1(t) + Q_{1,4}^{2,3}(t) \circledS V_4(t)$$

$$V_4(t) = Q_{41}(t) \circledS V_1(t)$$

For steady state, the expected number of visits per unit time is given by

$$V_0 = \lim_{s\to 0} sV_0^{**}(s) = \frac{N_3}{D_1}$$

where

$N_3 = 1 - p_{11}^{2}(s)$

and D_1 is already specified

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